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NUMERICAL CALCULATION OF HELICOPTER EQUATIONS AND COMPARISON WITH EXPERIMENT

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(D Petot - J Bessone / ONERA)

1 - Introduction

The configuration of helicopter rotors, of wind tunnel rotor models or of wind turbines can have very different aspects. In the past, the study of these configurations [ref1] often required some adaptation of the model in order to meet the kinematic requirements of the program.

It therefore became useful to extend to more general structures the technique that gave access to the numerical equations of a rotor. This has been done by describing the structure through the assembly of elementary units which often consist only of simple rotations or translations.

The significance of this development is that it leads directly to the equations of the model, equations that can then be used for any needed purpose. Starting from this central core, different applications have been written which give access to:

- The stability in hover, by calculating the equations' eigen values and eigen vectors,
- The forced periodic response in forward flight through an iterative procedure between the displacement field and the aerodynamic forces,
- The rotor transient response through time integration by a predictor-corrector scheme.

2 - Description of the studied structure

The equations of a structure are written by describing the way all the *dm* elements move. The displacements of every point of the model are defined relative to any other by the transformation that relates the two points. When all the transformations of the model are defined, it becomes possible to write the displacement of any point relative to the general reference frame by jumping from one transformation to another. The displacement of a *dm* element is finally defined through a "chain of transformations".

Describing a structure through the product of a set of transformations is usefully general for a helicopter model since we can have:

- Transformations with degrees of freedom such as blade flapping,
- Transformations with several degrees of freedom such as the one that binds a blade element *dm* to the blade root, and depends on the n basic blade deflections,
- Time dependant transformations, such as the rotor rotation or the cyclic pitch angle in forward flight,
- Time dependant transformations can also have a degree of freedom, such as the rotor rotation in some applications,
- Some of the transformations can have pseudo degrees of freedom and are defined here just to simulate the action of experimental strain gauges. They lead to local stresses.

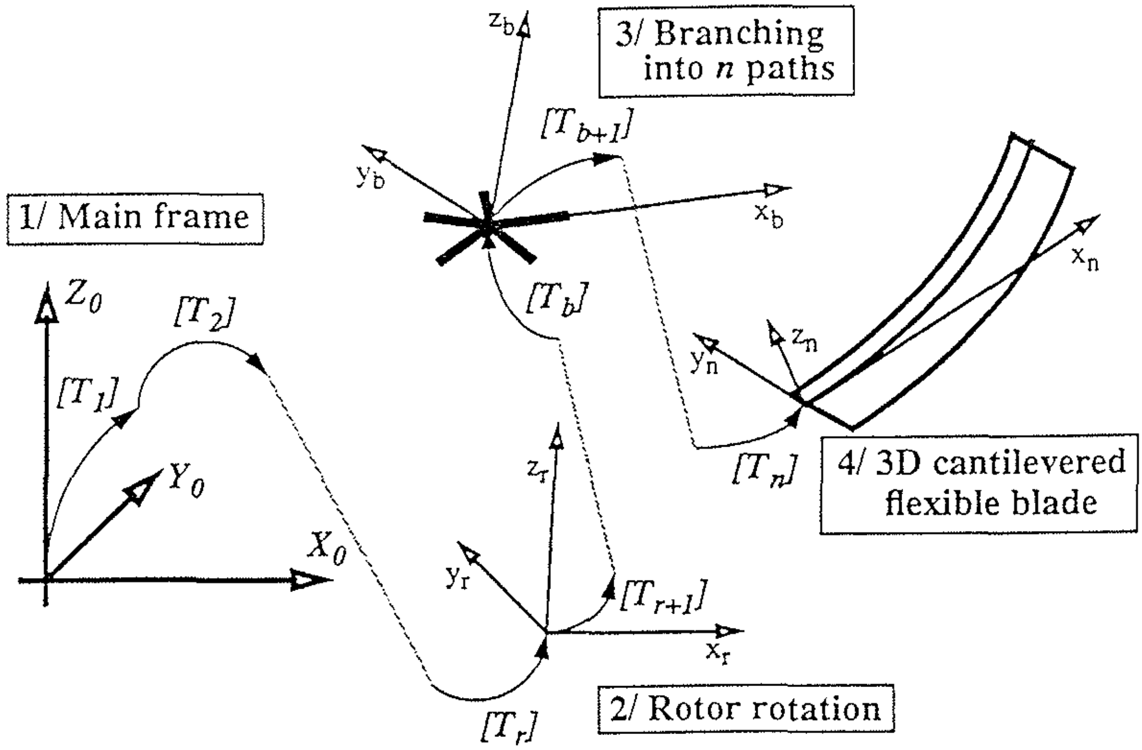


Figure 1 - Description of the studied structure

3 - Notations

- T is a transformation that defines how a reference frame behaves relative to another
- $[T]^T$ is the translation associated with T,
- $[T]^R$ is the rotation matrix (3,3) associated with T,
- $[T]^{(2)}$ is the second column of the rotation matrix associated with T.
- X is the list of degrees of freedom of one chain of transformations (relative to one blade)
- Y is the list of degrees of freedom for the whole model; $X = A(t) \cdot Y$

4 - Equations for structural dynamics

The model is divided into elements dm which are not necessarily small, but assumed rigid. Whenever dm is a blade element, it is small along the span but is rigid and may be large chordwise.

For each degree of freedom, a linearization of the position of the element dm is conducted up to the second order in order to write the Lagrange equation of the system. The problem of the large amplitude of the flapping angle in forward flight is solved by linearizing around its large deflection, which thus needs to be known.

Several quantities relative to the transformations are to be defined: the set of transformations T_i themselves, their derivatives versus the various degrees of freedom q_i: $\partial T_i / \partial q_j$, $\partial^2 T_i / \partial q_j \partial q_k$ and their derivatives versus time: dT_i / dt, $d^2 T_i / dt^2$ and $d/dt . \partial T_i / \partial q_j$ whenever necessary.

A certain product $\langle U, V \rangle$ between two transformations U and V has to be defined to take into account the mass properties of the elements dm upon which they act (mass, inertias and static moments).

Finally, A is the matrix for transforming the blade coordinates to multiblade coordinates and T stands for the general transformation that leads to the blade element dm: $T = T_1 \bullet T_2 \bullet \dots \bullet T_n$.

This being given, summing the equations derived for a dm element leads to the global equations which are written under the classical form:

$$M_{mech} \cdot Y'' + (B_{Mech} + B_{Aero}) \cdot Y' + (K_{Mech} + K_{Aero}) \cdot Y = F_{Mech} + F_{Aero}$$

With:

$$M_{Mech} = \bar{A} \cdot \langle \frac{\partial T}{\partial q}, \frac{\partial T}{\partial q} \rangle \cdot A$$

$$B_{Mech} = 2 \cdot \bar{A} \cdot \{ \langle \frac{\partial T}{\partial q}, \frac{d}{dt}\frac{\partial T}{\partial q} \rangle \cdot A + \langle \frac{\partial T}{\partial q}, \frac{\partial T}{\partial q} \rangle \cdot A' \} + \bar{A} \cdot D \cdot A$$

$$K_{Mech} = \bar{A} \cdot \left(\langle \frac{d^2 T}{dt^2}, \frac{\partial^2 T}{\partial q \partial q} \rangle \cdot A + \langle \frac{\partial T}{\partial q}, \frac{d^2}{dt^2}\frac{\partial T}{\partial q} \rangle \cdot A + 2 \cdot \langle \frac{\partial T}{\partial q}, \frac{d}{dt}\frac{\partial T}{\partial q} \rangle \cdot A' + \langle \frac{\partial T}{\partial q}, \frac{\partial T}{\partial q} \rangle \cdot A'' \right) + \bar{A} \cdot U \cdot A + \bar{A} \cdot D \cdot A'$$

$$F_{Mech} = \bar{A} \cdot \langle \frac{d^2 T}{dt^2}, \frac{\partial T}{\partial q} \rangle$$

D is a diagonal matrix that takes into account viscous damping of the degrees of freedom considered. U is the stiffness expressed for these degrees of freedom.

5 - Equations for aerodynamics

Aerodynamics are linearized around the local flow incidence. The aerodynamic forces are assumed to remain perpendicular to the wind velocity during the vibration; they depend on the wind conditions at the fore quarter-chord. The work of the aerodynamic forces and moments is calculated for a virtual displacement of a blade element at the aerodynamic centre. In order to write the aerodynamic matrices obtained, some notations are defined in section 3.

Then:

$$(1) \qquad F_{\text{Aero}} = \int_{\text{Blades}} \bar{A} \cdot \left(\overline{\left[\frac{\partial T}{\partial q}\right]_{\text{Ac}}^{T}} \cdot [T]_{\text{Ac}}^{R} \cdot F_0 + \overline{\left[\frac{\partial T}{\partial q}\right]_{\text{Ac}}^{(2)}} \cdot [T]_{\text{Ac}}^{R} \cdot M_0^{t} \right) \cdot ds$$

$$K_{\text{Aero}} = \int_{\text{Blades}} \bar{A} \cdot \left(\overline{\left[\frac{\partial T}{\partial q}\right]_{\text{Ac}}^{T}} \cdot [T]_{\text{Ac}}^{R} \cdot F_1 + \overline{\left[\frac{\partial T}{\partial q}\right]_{\text{Ac}}^{(2)}} \cdot [T]_{\text{Ac}}^{R} \cdot M_1^{t} \right) \cdot (V_X \cdot A + V_{X'} \cdot A') \cdot ds$$

$$+ \int_{\text{Blades}} \bar{A} \cdot \left(\left[\frac{\partial^2 T}{\partial q \partial q}\right]_{\text{Ac}}^{T} \cdot [T]_{\text{Ac}}^{R} \cdot F_1 + \left[\frac{\partial^2 T}{\partial q \partial q}\right]_{\text{Ac}}^{(2)} \cdot [T]_{\text{Ac}}^{R} \cdot M_1^{t} \right) \cdot {}^{m}A \cdot ds$$

$$B_{\text{Aero}} = \int_{\text{Blades}} \bar{A} \cdot \left(\left[\frac{\partial T}{\partial q}\right]_{\text{Ac}}^{T} \cdot [T]_{\text{Ac}}^{R} \cdot F_0 + \left[\frac{\partial T}{\partial q}\right]_{\text{Ac}}^{(2)} \cdot [T]_{\text{Ac}}^{R} \cdot M_0^{t} \right) \cdot V_{X'} \cdot A \cdot ds$$

In these formulas, the subscript "Ac" denotes a value taken at the aerodynamic centre, and lift and moment are assumed to be given by:

$$F = F_o + F_1 \cdot (V_X \cdot X + V_{\dot{X}} \cdot \dot{X}) \qquad \text{and} \qquad M^t = M_o^t + M_1^t \cdot (V_X \cdot X + V_{\dot{X}} \cdot \dot{X})$$

More complex aerodynamics can be treated. Neither 'damping' nor 'stiffness' terms are then used (which excludes direct stability calculations) and the aerodynamic efforts are directly introduced into the right hand side of the equations as in eq[1]. Iterations are then performed in order to balance aerodynamics and displacement fields.

ONERA has developed a simple dynamic stall model [ref2] which can also be used. This stall model requires a more general input than the classical angle of attack. If w stands for the additional downward velocity imposed by the presence of the airfoil (a simple pitch implies $w = -V\dot{\theta}$), we need to know: w, dw/dt, $\partial w/\partial y$, and $d/dt . \partial w/\partial y$. Expressions for these quantities have been derived as general functions of the basic transformations, but are not reported here.

6 - Structure/aerodynamics coupling

Only linear equations are directly obtained by the method. However, non-linearities are important in helicopter applications due to the inescapably large angles of attack found in certain regions of the rotor disk. The coupling with the non-linear terms is performed using an iterative technique named 'CI', which was developed by DAT and TRAN [ref3].

The iterations are straightforward: one starts with a certain state of the model, which determines the value of the external non-linear forces. Applying these external forces determines a new response of the model and thus new external forces... etc... Of course, convergence does not occur if the external forces consist of the total aerodynamics.

However, convergence is readily achieved if the linear aerodynamics (or better: a certain amount of them) are included in the model. The non-linear part of the aerodynamics only remains in the external field, which is then far less disturbing. However, it should be mentioned that convergence difficulties may still appear if the moment is very non-linear at the blade tip and if the blade is flexible...

7 - Using the equations

Several applications have been or will to be developed:

• **Stability** calculations are performed in hover by looking at the eigen values of the equations which have constant coefficient once the multiblade coordinates are used.

• **Stability in forward flight** will be analysed through a classical Floquet method.

• **Advancing flight periodic response** is obtained through the inversion of a matrix once the linearized equations are found. For a non linear calculation, iterations through the CI method yield the solution. Helicopter calculations often require to trim the model. At present, this trimming is performed in a loop outside the program, which much increase computation time. It may be possible to incorporate trimming right into the CI iterations.

• **Direct time integration** of the equations is also performed, which gives access to the transient response of the rotor to a gust or manoeuvres for example.

• The linearized equations can be used as **rotor transfer functions** to be exported to an accurate fuselage model for coupled fuselage/rotor calculations, something we intend to do. A simplified fuselage model (fuselage defined by its modes and static deflections) can already be taken into account in the code by projecting the fuselage basis on the first transformations used, consisting for example of three translations and three rotations at the hub centre.

8 - Testing the code

The first tests of the program have been to reproduce the results of an earlier imposed geometry stability code (see for example [ref6]).

Among many other tests, we can mention that the theoretical rotating frequencies of the LAURENSON model [ref4] are perfectly reproduced. This model consists of a rotating flexible blade mounted with pre-lag or pre-cone angles varying between 0 and 180 degrees. The frequencies were calculated by considering pre-lag or pre-cone either as a particular transformation or as part of a 3D blade. Some non-linear calculations, taking into account the geometry of blades bent by centrifugal forces, have reproduced the non-linear rotating finite element results of LAULUSA [ref5].

Numerous tests in stability have been successfully performed so far. Fig[2] shows for example the stability of one configuration of an experimental model [ref7]. The instability in hover was due to the out of the plane mass of the blade caused by an inertia ring added at the blade root for lowering its torsional frequency. The calculated frequencies and dampings predict reasonably well the measurements performed in the wind tunnel.

Computation time

For a typical application with quasi-steady aerodynamics, SUN4 workstations deliver all the helicopter modes in hover in 1 or 2 minutes and a rotor periodic response in 5 to 10 minutes.

Applications with the dynamic stall model require one hour and the trimming of the helicopter multiplies computation time by a factor of 10 as at present the procedure is external (it may be included into the code and will then save much time).

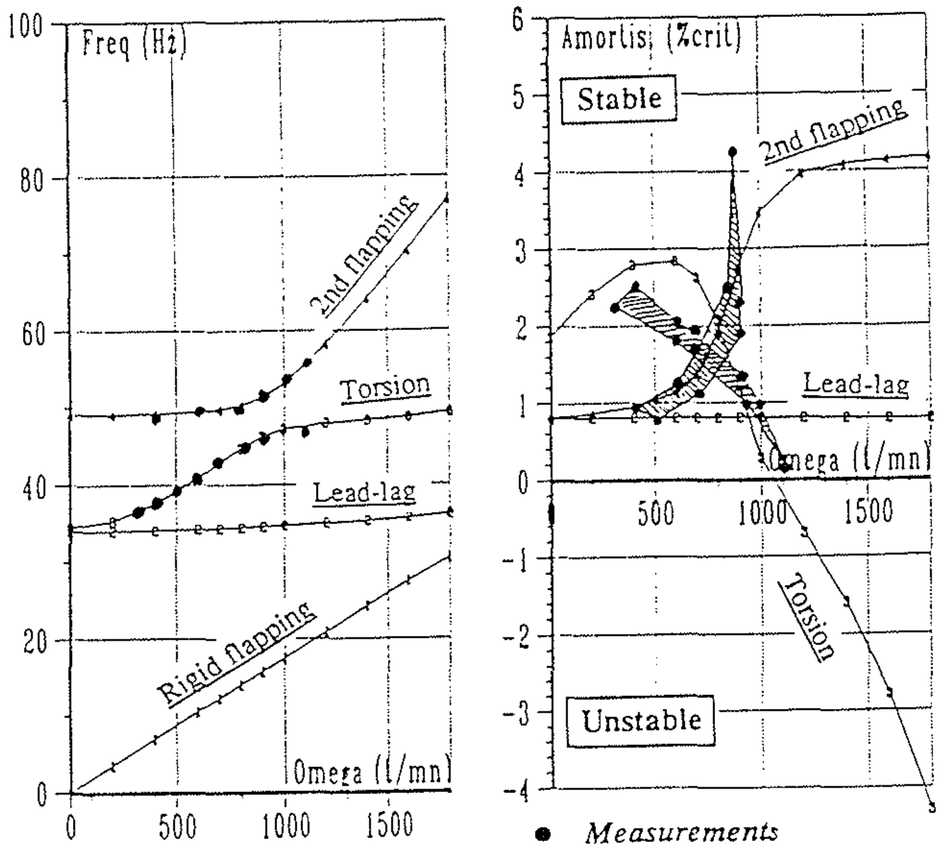


Figure 2 - EDY rotor frequencies and dampings

9 - Influence of unsteady aerodynamics

The ONERA dynamic stall model modelizes aerodynamics with two differential equations:

- A first order equation that calculates aerodynamic efforts as if the airfoil were never stalled (the expression for moment is explicit). This part of the model acts as THEODORSEN equations but can be applied to non-sinusoidal movements and to unsteadiness of the wind velocity,
- A second order equation that calculates the additive effort due to stall.

Both equations give unsteady effects that are interesting to analyse.

Linear unsteady aerodynamics

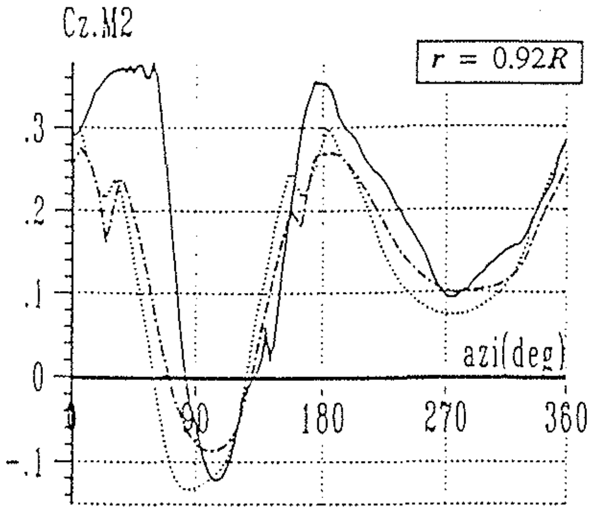
For stability calculations, the introduction of the linear unsteady aerodynamic moment stabilizes the torsional mode which the analytical codes often find unsteady. The effect is just sufficient for slender blades, but has been found to be very important on the very soft in torsion ROSOH rotor (torsion at 2.6Ω) [ref13], a fact that may explain the stability of this rotor in the wind tunnel.

The effect of the unsteadiness of wind velocity has been checked on the 2m diameter rotor of the Modane tests at the very high advance ratio $\mu = 0.50$ where it is maximum.

Fig3 shows that the lift deficiency on the advancing blade occurs 15 degrees later and is much less pronounced. The experiment shows a much larger delay for this lift deficiency. This may also have another cause. The section at $0.92R$ is the only one available experimentally at $\mu = 0.50$.

Figure 3 - Influence of linear unsteady aerodyanmics ($\mu = 0.50$ and $C_T/\sigma = 0.075$)

Modane wind tunnel rotor test [ref10]



——— *Experiment*
········ *Quasi-steady Aero.*
—·—·— *Dynamic stall model*

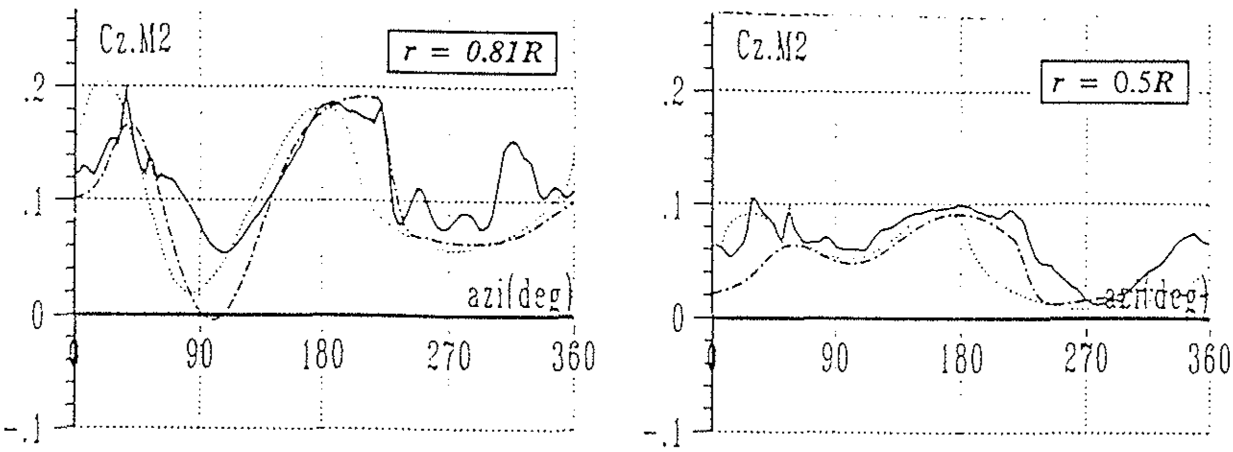


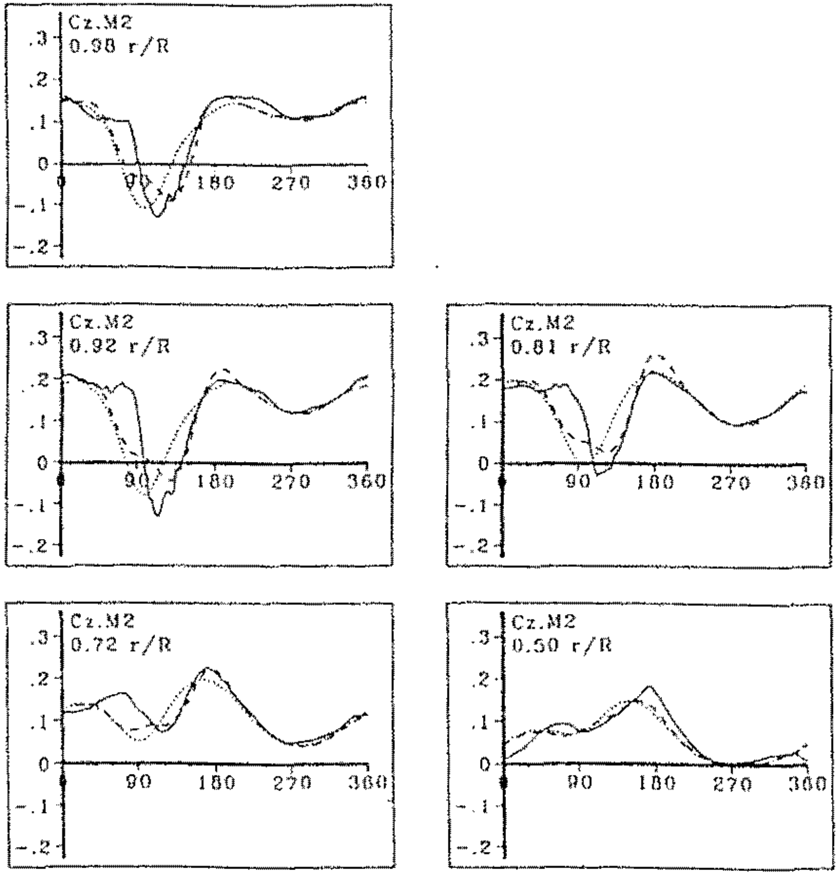
Figure 4 - Influence of dynamic stall ($C_T/\sigma = 0.145$ and $\mu = 0.30$)

Dynamic stall

The effect of dynamic stall is analyzed here on a highly loaded rotor ($C_T/\sigma = 0.145$) of the wind tunnel test of ref10. Calculations with and without the ONERA dynamic stall model are compared to experiment in fig4.

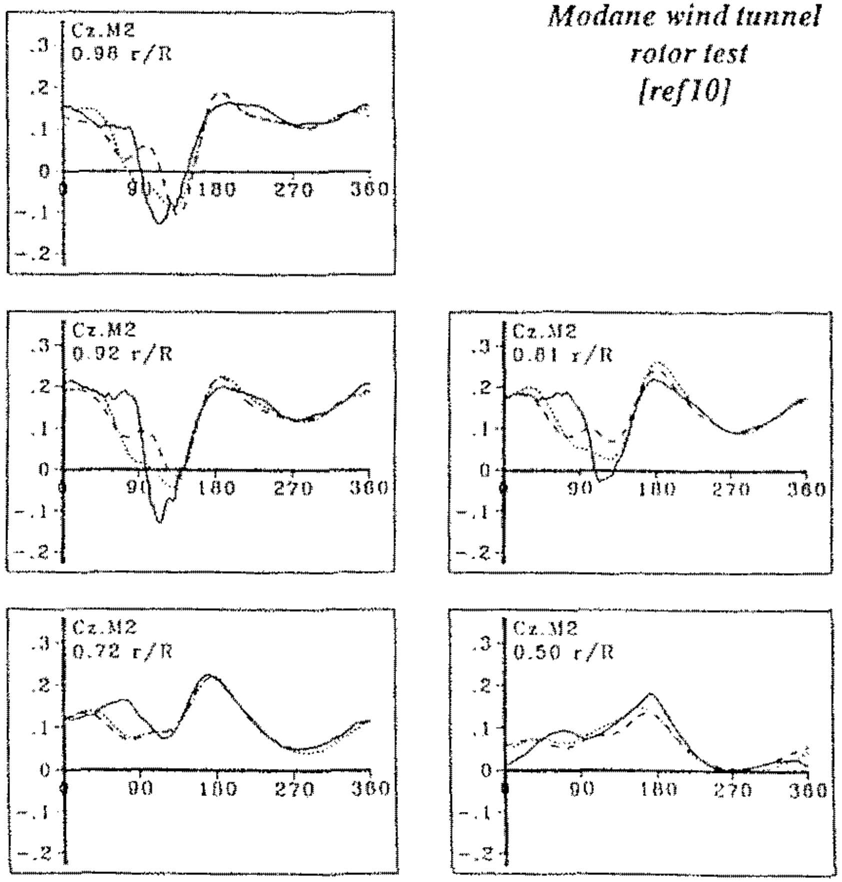
Three main effects are to be found:

- A lift deficiency at azimuth 30 degrees, due to the elastic response of the blade, itself due to badly predicted aerodynamic moments,
- A stall delay at azimuth 180 - 200 degrees, which correlates well with experiment,
- A phase shift at azimuth 90 degrees, which follows the experimental trends.



—— *Experiment*
.... *Unsteady Aero.*
– – -·- *Same + measured blade torsion*

Modane wind tunnel rotor test [ref10]



—— *Experiment*
.... *Meijer-Drees induced velocity*
–·–·- *Metar induced velocity*
(Calculations with unsteady Aero. + measured torsion)

Figure 5 - Aerodynamic rotor loads at several span locations (μ = 0.40 and C_T/σ = 0.075)

The conclusion of the Modane rotor tests [ref10] is that dynamic stall effects become sensitive only at the highest loads. Experiments usually show that rotors can have much higher thrusts than predicted by quasi-steady calculations. This does not seem to result only from a larger stall delay due to rotation (which does not seem to be the case in fig4), but also from the fact that stalled region tends to give more lift than expected...

10 - S1 Modane tests [ref10]

The 11th Modane wind tunnel test of June 91 is a large scale test of 4m diameter, 4-bladed helicopter rotors in true flying conditions which allows us to examine in detail the possibilities of the code in its present state. We have concentrated our efforts on the rotor with rectangular blades tips. The data available concerns the aerodynamic local loads and the blade deformations [ref11]. The experimental hub vibratory loads are not yet available.

Blade loads

Predicted blade loads always exhibit the same behaviour relative to experiment. This is illustrated by fig5 which corresponds to a cruising flight at $\mu = 0.40$ with a load factor $C_T/\sigma = 0.075$. The helicopter is trimmed and the control law is such that $\beta_c = \theta_s$ and $\beta_s = 0$.

It can be seen that the calculated and measured blade loads agree well all along the blade, except for a delay in the lift deficiency of the advancing blade. Analysis finally shows that:

- This delay would have been worse if quasi-steady aerodynamics had been used,
- When the measured blade torsion [ref11] (which cannot be predicted by 2D analysis) is introduced in the calculation, correlation greatly improves at azimuth 150 degrees. A lift bump still remains around azimuth 70 degrees...
- This bump seems typical of vortex problems, especially at this azimuth where the opposite blade vortex happens to come almost parallel to the blade... This is why an induced velocity field issued from a sophisticated induced velocity field (METAR from Eurocopter-France, ref12) had been used... It reveals the existence of a vortex interaction at, but the outer quarter of the blade only and 20 degrees later than expected.
- At last, the non negligible measured lateral force may be related to this lift deficiency delay which may come from the fact that the expected control law was not exactly respected in the experiment but was assumed in the calculations.

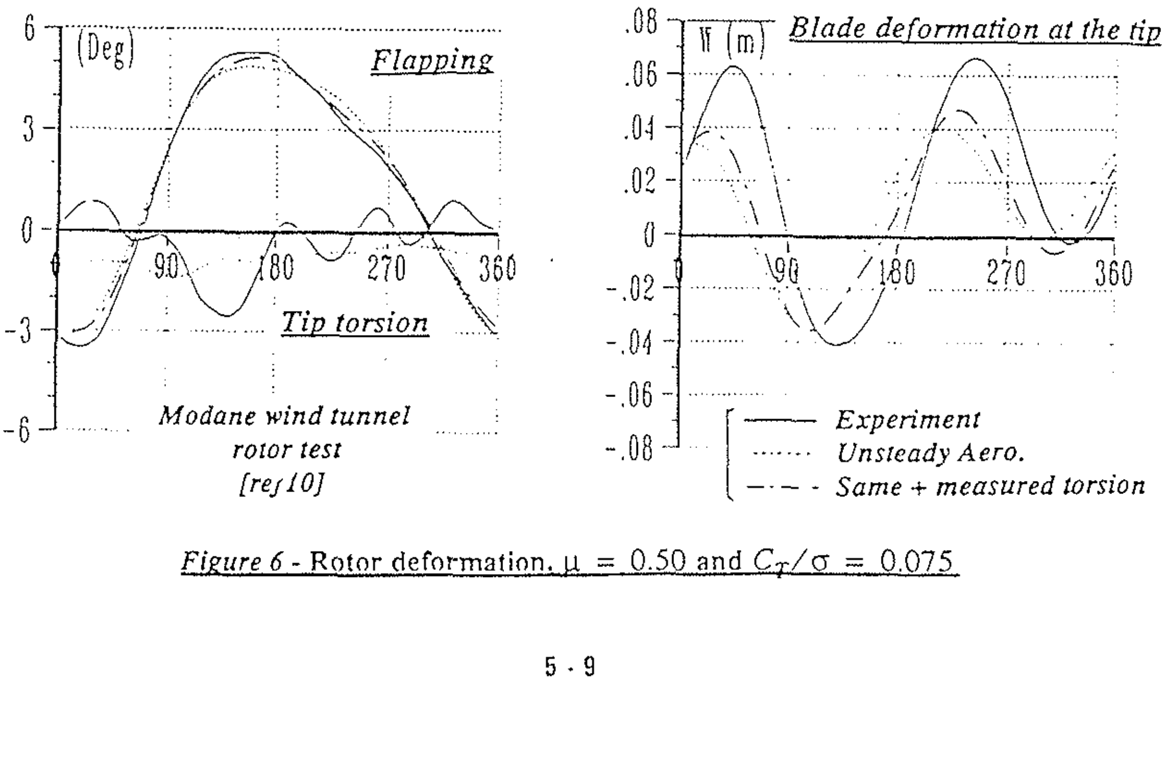


Figure 6 - Rotor deformation, $\mu = 0.50$ and $C_T/\sigma = 0.075$

Rotor deformation

Fig6 shows the rotor deformation via the flapping angle, the torsion and the deformation W at the blade tip, all quantities that were measured [ref11]. As has already been mentioned, the blade torsion is not at all predicted, certainly because its excitation by the aerodynamic moment is not a 2D phenomenon. The flapping angle is at 1Ω and the blade deformation at 2Ω both in the experiment and in the calculations.

The blade deformation, and thus the blade stresses for further analysis, is very sensitive to the calculation hypothesis. As for the $Cz \cdot M^2$ curves, the calculated value are found to be shifted from the experiment (in addition to some underestimation). Analysis gives the same conclusions:

- The shift would have been worse if quasi-steady aerodynamics had been used,
- The use of the measured torsion in the calculation improves all the rotor deformations. Flapping becomes excellent.

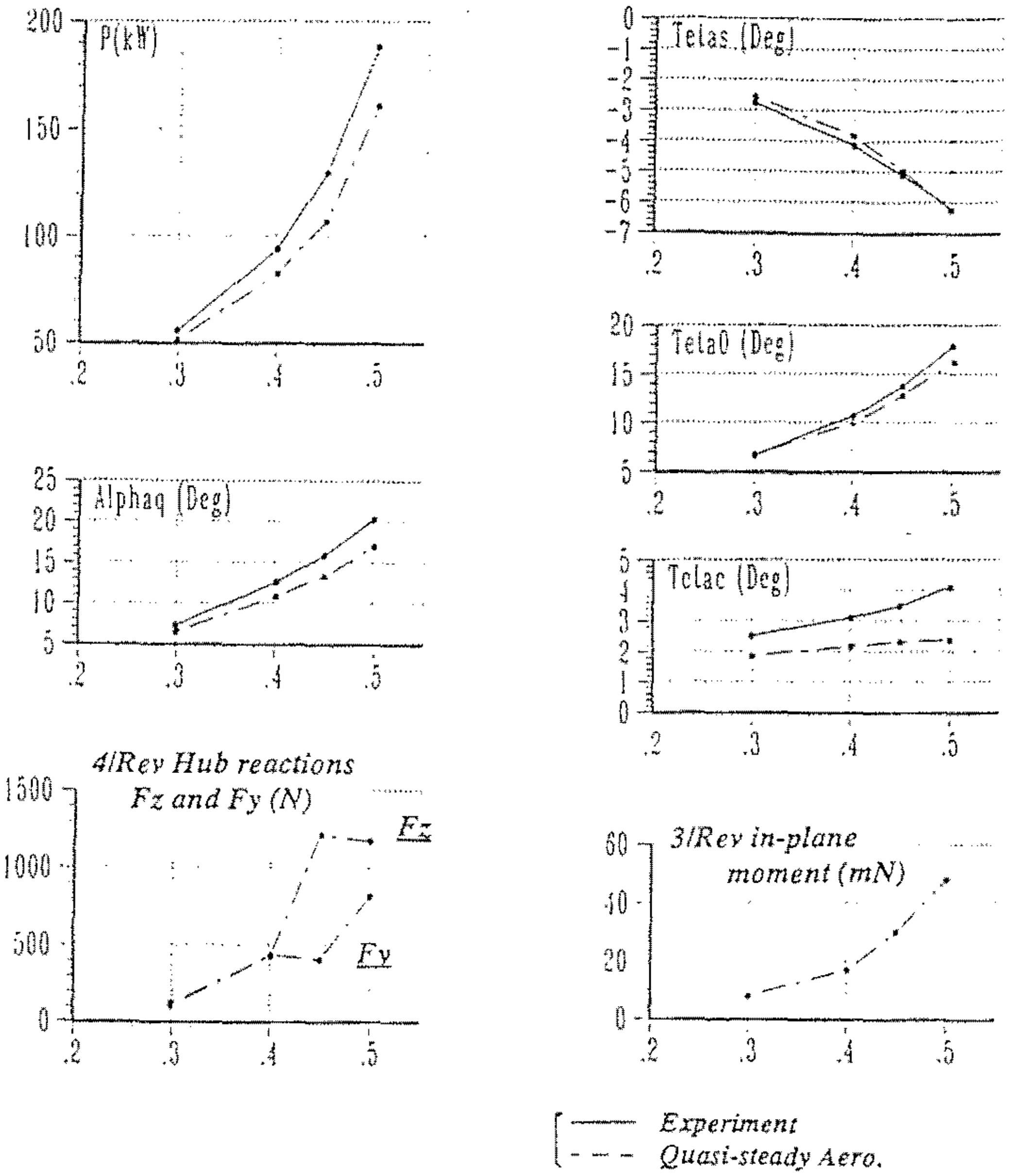


Figure 7 - Evolution versus advance ratio at $C_T/\sigma = 0.075$

Hub vibratory loads

Introducing into the program pseudo degrees of freedom to simulate the action of strain gauges gives access to local strain. Fig7 shows the vertical and lateral forces obtained at 4Ω and the in-plane moment at 3Ω. Experimental values are not yet available. Fig6 corresponds to an evolution in cruising speed at a constant $C_T/\sigma = 0.075$.

Rotor trim and power

Rotor trim and power are reported here through the evolution in advance ratio μ at constant $C_T/\sigma = 0.075$. Calculated and measured values of power, α_q, and blade pitch are shown in fig7. Although differences can be seen, all the trends are very good.It appears that for an unknown reason the experiment is obliged to put a lateral force through θ_c whereas the application of the control law in the calculation prevents such a lateral force... The 10% precision on power was first explained by the sensitivity to blade tip aerodynamics, and it has later been found that the use of a METAR [ref12] induced velocity brought a large increase in power... METAR is not yet fully implemented in the code.

We can mention here that the introduction of the dynamic stall model does not change the rotor trim.

11 - Future developments

At the moment, our code is not ready to work with transformations that are not simple translations or rotations up to the blade root. Using more general intermediate transformations should permit the introduction of complex pitch link geometry, or take into account the behaviour of a flexible blade root arm such as at the root of a BMR, provided that the different transformation derivatives [section 3] can be written.

A precise prescribed wake induced velocity is to be used too (METAR algorithm from Eurocopter France). Due to the possibilities of the code for stability, it is planned to search for the feasibility of taking into account a dynamic wake.

A study of the rotor/fuselage vibration coupling is to be done where an accurate finite element description of the fuselage will be coupled with the rotor equations. This study will aim at determining the minimum characteristics that have to be kept for both the rotor and the fuselage in order for the coupling to be meaningful.

Some calculations with complex CFD aerodynamics will be undertaken. A coupling with the 3D dynamic stall model of COSTES [ref8, 9] is to be completed by the end of the year.

12 - Conclusions

ONERA has now a set of research codes which is well adapted to helicopter applications and very open to extensions. It is numerically well validated. The helicopter equations obtained are ready to be used for further applications.

Correlation with the are very satisfactory both on the experimental and calculation point of view. The difficulties that have been encountered have been explained. Some hypothesis need to be improved for a better fit. More complex induced flow fields have to be used and a good aerodynamic moment needs be introduced... This last point will require a heavy coupling with CFD calculations...

13 - References

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