Paper Number: 060

Fuzzy Approach for Uncertainty Analysis of Thin-Walled Composite Beams

Prashant M. Pawar[1] , Sung Nam Jung[2] and Babruvahan P. Ronge[1]

[1]*Department of Mechanical Engineering*
SVERIs College of Engineering, Pandharpur 413304, Dist. Solapur, India

[2]*Department of Aerospace Information Engineering*
Konkuk University, Seoul 143-701, Korea

Abstract
In this study, an analytical approach is developed to evaluate the influence of material uncertainties on cross-sectional stiffness properties of thin-walled composite beams. Fuzzy arithmetic operators are used to modify the thin-walled beam formulation, which was based on a mixed force and displacement method, and to obtain the uncertainty properties of the beam. The resulting model includes material uncertainties along with the effects of elastic couplings, shell wall thickness, torsion warping and constrained warping. The membership functions of material properties are introduced to model the uncertainties of material properties of composites and are determined based on the stochastic behaviors obtained from experimental studies. It is observed from the numerical studies that the fuzzy membership function approach results in reliable representation of uncertainty quantification of thin-walled composite beams. The propagation of uncertainties are also demonstrated in the estimation of structural responses of composite beams.

Authors keywords: A. Uncertainty; B. Composite material blades; C. Vibratory hub load

INTRODUCTION

Thin walled composite beams have been extensively used in engineering structures, such as helicopter blades, wings, trusses in space structures, antenna legs, submarine hulls, cooling tower shafts, medical tubing, connecting shafts, transmission poles, tail boom of helicopter, tube like structures in missiles and launch vehicles. Composite materials display superior fatigue characteristics, greater damage tolerance, and higher stiffness to weight ratio over the conventional metal materials. However, these materials show a larger variation in material properties and mechanical behavior, due to the presence of large design variables, variable manufacturing tolerances, and lack of experience and precise test data. These variations essentially increase the level of uncertainties encountered in the design and construction stages of composite materials which needs thorough investigation to understand reliability of structures with sufficient confidence. Some studies show that even relatively minor levels of variability existed in system parameters, loads, and boundary constraints can induce significant changes in the system stability [13,14,16].

Generally, stochastic approaches [2,15,21,22] are used to quantify the scatter in composite materials. These approaches needs exact probability distribution of uncertain parameters which might need considerable amount of data which is either impossible or unrealistic. In some cases, it is difficult to fit well-defined probability distribution to the basic constituent properties such as fiber modulus, fiber longitudinal tensile strength, ply thickness and unidirectional laminate strength. This type of problems can be handled using a fuzzy set with specific preference values for different values in the observed range of that variable to model behaviors accurately. Fuzzy approach can also be used when the uncertain parameters are described in a qualitative or linguistic form. Fuzzy approach can be considered, in some sense as the most general type of uncertainty analysis. Computational efforts required for stochastic analysis are quite high as compared to the fuzzy analysis [10].

Fuzzy approach for uncertainty analysis faces various challenges due to limited understanding of the meaning of fuzzy numbers and definition of their applications [5]. Another challenge is to understand the relation of fuzzy numbers to conventional uncertainty modeling approaches. It is well accepted that the fuzzy set theories have a sound mathematical foundation as compared to probabilistic theories except the problem of less clarity and understanding of semantics [5]. Rao and Sawyer [17] developed a fuzzy finite element approach for imprecisely defined systems which was demonstrated for stress analysis problems involving vaguely defined geometry, material properties, external loads, and boundary conditions. Further, Chen and Rao [3] demonstrated use of fuzzy approach for improving computational effectiveness for the analysis of problems involving dynamics. The use of fuzzy approach for problems involving uncertain boundary conditions was demonstrated by Cherki et al. [4]. Akpan et al. [1] developed a practical approach for analyzing the response of structures with fuzzy parameters by integrating finite element model with response surface analysis and fuzzy analysis. Savoia [20] proposed a procedure to perform reliability analysis using extended fuzzy operation and demonstrated for buckling problem with very few data defining the imperfection. Massa et al. [11] developed an efficient methodology to calculate fuzzy eigenvalues and eigenvectors of finite element structures defined by imprecise parameters. Similarly, Moens [12] introduced a numerical algorithm to calculate frequency-response functions of damped finite element models with fuzzy uncertain parameters.

Although several problems have been solved using fuzzy approach in structural mechanics, this approach was not much explored for the composite structures. Rao and Liu [19] proposed fuzzy approach to the mechanics of fiber-reinforced composite materials. They used basic fuzzy operations to modify the composite mechanics starting from the fuzzy properties of fiber and matrix of lamina to obtain membership functions of stresses in transverse and longitudinal loading conditions. They have also derived the laws of mechanics using fuzzy approach for obtaining fuzziness in the coefficient of linear thermal expansion and stress-strain relationships of thin orthotropic lamina. Further, Liu and Rao [10] developed a fuzzy finite element approach for the analysis of laminated beams, involving fuzziness, possibly in the boundary conditions as well which can undergo axial, bending, and transverse shear deformations. They developed a fuzzy beam element using the basic concepts of the deterministic finite element theory, fuzzy computations and fuzzy matrix operations. The use of fuzzy beam element was numerically demonstrated for the static and eigenvalue analysis of beams involving imprecise data or information.

Several thin walled beam theories have been developed in the literature based on stiffness, flexibility and mixed beam approaches [6,7,8,9]. Uncertainty modeling in these beams is less explored area which was initiated by Murugan et al. [13] for finding the effects of uncertainty in composite material properties on the cross-sectional stiffness properties, natural frequencies and aeroelastic response of a composite helicopter rotor blades. The elastic modulii and Poisson's ratio of the composite material were considered as random variables with coefficient of variation around 5 percent. A finite element method based on variational asymptotic procedure is used for evaluating the blade cross

-sectional properties. In another study by You et al. [23], an assessment was made to quantify the influence of random material properties and fabrication / manufacturing uncertainties on the aeroelastic response and hub vibratory loads of composite rotor blades. The random variables include lamina stiffness properties, ply thicknesses and fiber orientation angles of the laminate structures, and the elastic-axis offset from the aerodynamic center in the section of the blade. Both these studies were based on Monte-Carlo approach which is computationally cumbersome and give less understanding about propagation uncertainty. Fuzzy approach which helps to overcome these drawbacks of Monte-Carlo simulation can be promising methods for modeling uncertainty in the thin walled beam structures.

In this study, fuzzy approach is used for uncertainty analysis of thin walled composite beams whereby the basic fuzzy arithmetic operations are used to derive a closed-form solution for thin walled composite beam structures with uncertainties in material properties. The closed-form solution for thin walled composite beam is obtained using a mixed force and displacement method. The formulation includes the effects of elastic coupling couplings, shell wall thickness, transverse shear deformation, warping, and constrained warping. The force-displacement relation of the beam is obtained using Reissners semi-complementary energy functional. The approach is demonstrated using the membership functions of material properties to obtain the membership functions of cross-sectional stiffness properties and performances of the beam.

FUZZY ARITHMETIC OPERATIONS

Fuzzy arithmetic operations for α-cut are explained briefly in this section. The α-cut A_α of A which is fuzzy set of crisp values X is original set of membership values greater than some threshold $\alpha \in [0, 1]$.

The typical fuzzy arithmetic operations involved in the derivation of mathematical expression include fuzzy addition, subtraction, multiplication, and division. While deriving these expressions these operations are denoted as $(**)$, where $**$ represents deterministic arithmetic operations such as $+, -, *, /$. For example, if $+$ denotes the deterministic addition, then the $(+)$ represents the fuzzy addition. The fuzzy arithmetic operation of two fuzzy numbers $A_\alpha = [a_1^\alpha \quad a_2^\alpha]$ and $B_\alpha = [b_1^\alpha \quad b_2^\alpha]$ is defined as [19]

Addition $(+)$

$$A_\alpha(+)B_\alpha = [a_1^\alpha + b_1^\alpha \quad a_2^\alpha + b_2^\alpha] \tag{1}$$

Subtraction $(-)$

$$A_\alpha(-)B_\alpha = [a_1^\alpha - b_2^\alpha \quad a_2^\alpha - b_1^\alpha] \tag{2}$$

Multiplication $(\cdot)$

$$\begin{aligned} A_\alpha(\cdot)B_\alpha = [&min(a_1^\alpha b_1^\alpha, a_1^\alpha b_2^\alpha, a_2^\alpha b_1^\alpha, a_2^\alpha b_2^\alpha) \\ &max(a_1^\alpha b_1^\alpha, a_1^\alpha b_2^\alpha, a_2^\alpha b_1^\alpha, a_2^\alpha b_2^\alpha)] \end{aligned} \tag{3}$$

Division $(/)$

$$\begin{aligned} A_\alpha(\cdot)B_\alpha = [&min(\frac{a_1^\alpha}{b_1^\alpha}, \frac{a_1^\alpha}{b_2^\alpha}, \frac{a_2^\alpha}{b_1^\alpha}, \frac{a_2^\alpha}{b_2^\alpha}) \\ &max(\frac{a_1^\alpha}{b_1^\alpha}, \frac{a_1^\alpha}{b_2^\alpha}, \frac{a_2^\alpha}{b_1^\alpha}, \frac{a_2^\alpha}{b_2^\alpha})] \end{aligned} \tag{4}$$

The powers of fuzzy numbers are computed by repetitive multiplication operations whereas summation is computed by repetitive additions. Fuzzy trigonometry operations for $X_\alpha = [x_1^\alpha, x_2^\alpha]$ can be computed using.

$$\sin(X_\alpha) = [min(\sin(x_1^\alpha, \sin(x_2^\alpha))] \tag{5}$$

$$\cos(X_\alpha) = [min(\cos(x_1^\alpha, \cos(x_2^\alpha))] \tag{6}$$

The multiplication of deterministic number $k \in \mathbf{R}^+$ with fuzzy number $X_\alpha = [x_1^\alpha, x_2^\alpha]$ can be

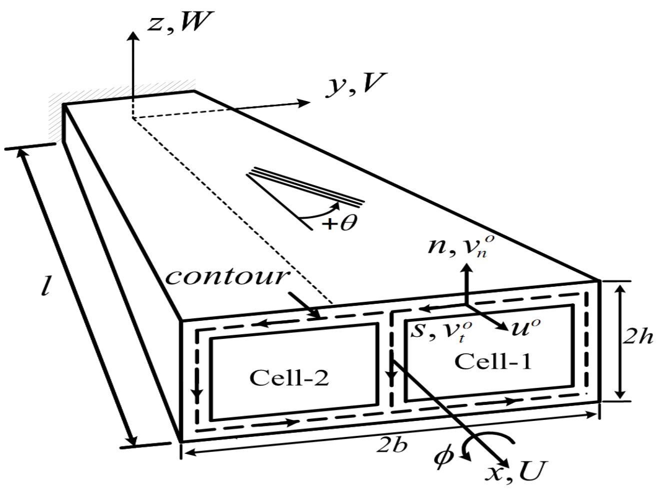

Figure 1. Geometry and coordinate systems of a composite box beam.

given as

$$k(\cdot)(X_\alpha) = [k, k](\cdot)[x_1^\alpha, x_2^\alpha] = [k \cdot x_1^\alpha, k \cdot x_2^\alpha] \quad (7)$$

FUZZY THIN WALLED BEAM THEORY

Fuzzy arithmetic is used to derive the closed-form force-displacement relation of thin walled composite structure shown in the Figure 1. The beam coordinate system represented using Cartesian system (x,y,z) whereas the curvilinear system (x,s,n) is used for the shell section of the beam as shown in Figure 2. The global fuzzy deformations of the beam are represented as $(U_\alpha, V_\alpha, W_\alpha)$ along the x, y and z axes and the fuzzy elastic twist as ϕ_α. The midplane fuzzy shell deformations are $(u^0_\alpha, v^0_{t\alpha}, v^0_{n\alpha})$ along the x, s, and n directions, respectively which can be represented using beam displacements and rotations:

$$\begin{aligned} v^0_{t\alpha} &= V_\alpha(\cdot)y_{,s}(+)W_\alpha(\cdot)z_{,s}(+)r\phi_\alpha \\ v^0_{n\alpha} &= V_\alpha(\cdot)z_{,s}(-)W_\alpha(\cdot)y_{,s}(-)q\phi_\alpha \end{aligned} \quad (8)$$

The strain-displacement and curvature-displacement relations for the shallow shell segment are given by

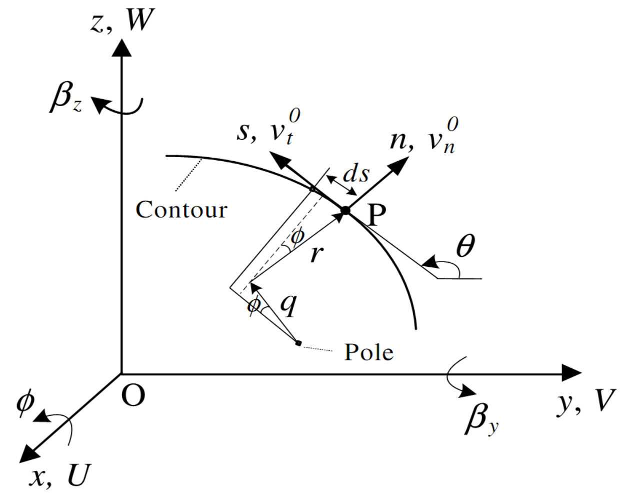

Figure 2. The definition of beam and section variables

$$\begin{Bmatrix} \epsilon_{xx\alpha} \\ \epsilon_{ss\alpha} \\ \gamma_{xs\alpha} \end{Bmatrix} = \begin{Bmatrix} u^0_{,x\alpha} \\ v_{t,s\alpha} \\ u^0_{,s\alpha}(+)v^0_{t,s\alpha} \end{Bmatrix}$$

$$\gamma_{xn\alpha} = \gamma_{xy\alpha}z_{,s}(-)\gamma_{xz\alpha}y_{,s} \quad (9)$$

and

$$\begin{Bmatrix} k_{xx\alpha} \\ k_{ss\alpha} \\ k_{xs\alpha} \end{Bmatrix} = \begin{Bmatrix} \Psi^0_{x,x\alpha} \\ \Psi_{s,s\alpha} \\ \Psi_{x,s\alpha}(+)\Psi_{s,x\alpha} \end{Bmatrix} = \begin{Bmatrix} -v^0_{n,xx\alpha} \\ -v_{n,ss\alpha} \\ -2v^0_{n,xs\alpha}(+)\gamma_{xn,s\alpha} \end{Bmatrix} \quad (10)$$

Where $\Psi_{x\alpha}$ and $\Psi_{s\alpha}$ are the fuzzy rotations of a general shell segment about s and x coordinates, respectively. The fuzzy cross-section rotations $\beta_{y\alpha}$ and $\beta_{z\alpha}$ about the y and z axes can be obtained using the fuzzy shear strains of the beams $\gamma_{xy\alpha}$ and $\gamma_{xz\alpha}$, respectively.

$$\begin{aligned} \beta_{y\alpha} &= \gamma_{xz\alpha}(-)W_{x\alpha} \\ \beta_{z\alpha} &= \gamma_{xy\alpha}(-)V_{x\alpha} \end{aligned} \quad (11)$$

Using the beam-shell displacement, strain-displacement and shell rotation relations, the fuzzy shell strain-beam displacement relation can be given as

$$\begin{aligned} \epsilon_{xx\alpha} &= U_{,x\alpha}(+)z\beta_{y,x\alpha}(+)y\beta_{z,x\alpha} - \bar{\omega}\phi_{,xx\alpha} \\ \gamma_{xs\alpha} &= u^0_{,s\alpha}(+)V_{,x\alpha}y_{,s}(+)W_{,x\alpha}z_{,s}(+)r\phi_{,x\alpha} \end{aligned}$$

$$
\begin{aligned}
k_{xx\alpha} &= \beta_{z,x\alpha} z_{,s} (-) \beta_{y,x\alpha} y_{,s} (+) q\phi_{,xx\alpha} \\
k_{xs\alpha} &= 2\phi_{,x\alpha} (+) (\beta_{z\alpha} y_{,s} (+) \beta_{y\alpha} z_{,s} (-) r\phi_{,x\alpha})/a \\
\epsilon_{ss} &= k_{ss} = 0
\end{aligned} \quad (12)
$$

Where $\bar{\omega}$ is the sectorial area of the section. The strain-displacement relations have used following geometrical relations

$$
\begin{aligned}
y_{,s} &= \cos\theta, \quad z_{,s} = \sin\theta \\
y_{,ss} &= -z_{,s}/a, \quad z_{,ss} = y_{,s}/a \\
r &= y\sin\theta - z\cos\theta, \quad q = y\cos\theta + z\sin\theta
\end{aligned} \quad (13)
$$

The general constitutive relation for the shell wall of the section with fuzzy α-cut is given by

$$
\begin{Bmatrix} N_{xx\alpha} \\ N_{ss\alpha} \\ N_{xs\alpha} \\ M_{xx\alpha} \\ M_{ss\alpha} \\ M_{xs\alpha} \end{Bmatrix} = \begin{bmatrix} A_{11\alpha} & A_{12\alpha} & A_{16\alpha} & B_{11\alpha} & B_{12\alpha} & B_{16\alpha} \\ A_{12\alpha} & A_{22\alpha} & A_{26\alpha} & B_{12\alpha} & B_{22\alpha} & B_{26\alpha} \\ A_{16\alpha} & A_{26\alpha} & A_{66\alpha} & B_{16\alpha} & B_{26\alpha} & B_{66\alpha} \\ B_{11\alpha} & B_{12\alpha} & B_{16\alpha} & D_{11\alpha} & D_{12\alpha} & D_{16\alpha} \\ B_{12\alpha} & B_{22\alpha} & B_{26\alpha} & D_{12\alpha} & D_{22\alpha} & D_{26\alpha} \\ B_{16\alpha} & B_{26\alpha} & B_{66\alpha} & D_{16\alpha} & D_{26\alpha} & D_{66\alpha} \end{bmatrix} (\cdot) \begin{Bmatrix} \epsilon_{xx\alpha} \\ \epsilon_{ss\alpha} \\ \gamma_{xs\alpha} \\ k_{xx\alpha} \\ k_{ss\alpha} \\ k_{xs\alpha} \end{Bmatrix} \quad (14)
$$

with

$$
\begin{Bmatrix} N_{sn\alpha} \\ N_{xn\alpha} \end{Bmatrix} = \begin{bmatrix} A_{44\alpha} & A_{45\alpha} \\ A_{45\alpha} & A_{55\alpha} \end{bmatrix} (\cdot) \begin{Bmatrix} \gamma_{sn\alpha} \\ \gamma_{xn\alpha} \end{Bmatrix} \quad (15)
$$

Where $A_{ij\alpha}, B_{ij\alpha}, D_{ij\alpha} (i, j = 1, 2, 6)$ and $A_{ij\alpha}$ are the fuzzy inplane, bending-inplane coupling, bending or twisting, and thickness-shear stiffness, respectively.

$$
\begin{aligned}
(A_{ij\alpha}, B_{ij\alpha}, D_{ij\alpha}) &= \sum_m \int_{z_{\alpha m}}^{z_{\alpha m+1}} Q_{ij\alpha}^{(m)} (\cdot) (1, z_\alpha, z_\alpha^2) dz, \\
A_{ij\alpha} &= \sum_m \int_{z_{\alpha m}}^{z_{\alpha m+1}} k_i k_j Q_{ij\alpha}^{(m)} dz, (i, j = 1, 2, 6)
\end{aligned} \quad (16)
$$

The material and ply orientation uncertainty or fuzziness can be introduced to the constitute relation through $Q_{ij\alpha}^{(m)}$, stiffness coefficients of mth layer in the shell wall sections [19]. The ply thickness uncertainty can be introduced through $z_{\alpha m}$, the distance from the midplane to the lower bottom surface of the mth layer [19].

With the assumption that the hoop stress flow $N_{ss\alpha}$ and the shear flow N_{sn} are negligible, the constitutive relation reduces to

$$
\begin{Bmatrix} N_{xx\alpha} \\ N_{xs\alpha} \\ M_{xx\alpha} \\ M_{ss\alpha} \\ M_{xs\alpha} \end{Bmatrix} = \begin{bmatrix} A'_{11\alpha} & A'_{16\alpha} & B'_{11\alpha} & B'_{12\alpha} & B'_{16\alpha} \\ A'_{16\alpha} & A'_{66\alpha} & B'_{16\alpha} & B'_{26\alpha} & B'_{66\alpha} \\ B'_{11\alpha} & B'_{16\alpha} & D'_{11\alpha} & D'_{12\alpha} & D'_{16\alpha} \\ B'_{12\alpha} & B'_{26\alpha} & D'_{12\alpha} & D'_{22\alpha} & D'_{26\alpha} \\ B'_{16\alpha} & B'_{66\alpha} & D'_{16\alpha} & D'_{26\alpha} & D'_{66\alpha} \end{bmatrix} (\cdot) \begin{Bmatrix} \epsilon_{xx\alpha} \\ \gamma_{xs\alpha} \\ k_{xx\alpha} \\ k_{ss\alpha} \\ k_{xs\alpha} \end{Bmatrix} \quad (17)
$$

and

$$
N_{xn\alpha} = A'_{55\alpha} (\cdot) \gamma_{xn\alpha} \quad (18)
$$

The terms with the primes are obtained after condensation of matrix with the assumptions that the $N_{ss\alpha} = 0$ and $N_{sn\alpha} = 0$. As the formulation is based on 1) displacement based parameters $\epsilon_{xx\alpha}, k_{xx\alpha}, k_{xs\alpha}$ and $\gamma_{xn\alpha}$ along with 2) force based parameters $N_{xs\alpha}$ and $M_{ss\alpha}$ derived from equilibrium equations of shell wall, the constitutive equation takes following semi-inverted form

$$
\begin{Bmatrix} N_{xx\alpha} \\ N_{xs\alpha} \\ M_{xx\alpha} \\ M_{ss\alpha} \\ M_{xs\alpha} \end{Bmatrix} = \begin{bmatrix} C_{n\epsilon\alpha} & C_{nk\alpha} & C_{n\phi\alpha} & C_{n\gamma\alpha} & C_{n\tau\alpha} \\ C_{nk\alpha} & C_{mk\alpha} & C_{m\phi\alpha} & C_{m\gamma\alpha} & C_{m\tau\alpha} \\ C_{n\phi\alpha} & C_{m\phi\alpha} & C_{\phi\phi\alpha} & C_{\phi\gamma\alpha} & C_{\phi\tau\alpha} \\ -C_{n\gamma\alpha} & -C_{m\gamma\alpha} & -C_{\phi\gamma\alpha} & C_{\gamma\gamma\alpha} & C_{\gamma\tau\alpha} \\ -C_{n\tau\alpha} & -C_{m\tau\alpha} & -C_{\phi\tau\alpha} & C_{\gamma\tau\alpha} & C_{\tau\tau\alpha} \end{bmatrix} (\cdot) \begin{Bmatrix} \epsilon_{xx\alpha} \\ \gamma_{xs\alpha} \\ k_{xx\alpha} \\ k_{ss\alpha} \\ k_{xs\alpha} \end{Bmatrix} \quad (19)
$$

The beam formulation is developed based on the semi-inverted matrix using Reissner functional $\Phi_R \alpha$

$$
\begin{aligned}
\Phi_{R\alpha} = \frac{1}{2} [& C_{n\epsilon\alpha} (\cdot) \epsilon_{xx\alpha}^2 (+) 2C_{nk\alpha} (\cdot) k_{xx\alpha} (\cdot) \epsilon_{xx\alpha} (+) \\
& C_{n\phi\alpha} (\cdot) k_{xs\alpha} (\cdot) \epsilon_{xx\alpha} (+) 2C_{n\gamma\alpha} (\cdot) N_{xs\alpha} (\cdot) \epsilon_{xx\alpha} (+) \\
& C_{n\tau\alpha} (\cdot) M_{ss\alpha} (\cdot) \epsilon_{xx\alpha} (+) C_{mk\alpha} (\cdot) k_{xx\alpha}^2 (+) \\
& 2C_{m\phi\alpha} (\cdot) k_{xx\alpha} (\cdot) k_{xs\alpha} (+) 2C_{m\gamma\alpha} (\cdot) k_{xx} (\cdot) N_{xs} (+) \\
& C_{m\tau\alpha} (\cdot) k_{xx\alpha} (\cdot) M_{ss\alpha} (+) C_{\phi\phi\alpha} (\cdot) k_{xs}^2 (+) \\
& 2C_{\phi\gamma\alpha} (\cdot) k_{xs} (\cdot) N_{xs} (+) 2C_{\phi\tau\alpha} (\cdot) k_{xs} (\cdot) M_{ss} (+) \\
& C_{xn\alpha} (\cdot) \gamma_{xn\alpha}^2 (-) C_{\gamma\gamma\alpha} (\cdot) N_{xs\alpha}^2 (-) \\
& 2C_{\gamma\tau\alpha} (\cdot) N_{xs\alpha} (\cdot) M_{ss\alpha} (-) C_{\tau\tau\alpha} (\cdot) M_{ss\alpha}^2]
\end{aligned} \quad (20)
$$

The Reissner functional is used to obtain the stiffness matrix relating beam forces to beam displacements

$$\delta \int_0^l \oint \{\Phi_{R\alpha}(+)\gamma_{xs\alpha}(\cdot)N_{xs\alpha}(+)k_{ss}(\cdot)M_{ss}\}dsdx = 0 \quad (21)$$

Integrating above equation by part with x and substituting using shell strain- beam displacement relation results in equilibrium equations of an element of the shell wall. Using these equilibrium equations along with continuity conditions results in shear flows and hoop moments vectors $\{n_\alpha\}$ as [8]:

$$\{n_\alpha\} = [Q_\alpha]^{-1}(\cdot)[P_\alpha](\cdot)\{q_\alpha\} = [e_\alpha](\cdot)\{q_\alpha\} \quad (22)$$

Where $\{q_\alpha\}$ is a generalized beam displacement vector given as

$$\{q_\alpha\} = [U_{,x\alpha} \quad \beta_{y,x\alpha} \quad \beta_{z,x\alpha} \quad \phi_{,x\alpha} \quad \phi_{,xx\alpha}]^T \quad (23)$$

Substituting the hoop stress and hoop moment relations in the Equation 21, the cross-sectional stress resultants can be obtained as

$$\begin{aligned} N_\alpha &= \oint N_{xx\alpha}ds \\ M_{y\alpha} &= \oint (N_{xx\alpha}z(-)M_{xx\alpha}y_{,s})ds \\ M_{z\alpha} &= \oint (N_{xx\alpha}y(+)M_{xx\alpha}z_{,s})ds \\ M_{\omega\alpha} &= \oint (-N_{xx\alpha}\bar{\omega}(+)M_{xx\alpha}q)ds \\ T_{s\alpha} &= \oint 2M_{xs\alpha}ds \end{aligned} \quad (24)$$

Where N_α is the axial force, $M_{y\alpha}$ and $M_{z\alpha}$ are the bending moments about y and z axes, respectively, $T_{s\alpha}$ is the St. Venant twisting moment and $M_{\omega\alpha}$ is the Vlasov bi-moment. By Substituting strain-displacement relation and semi-inverted constitutive relation in Equation 24, the resultant beam force-displacement relation is obtained as

$$\{F_\alpha\} = [K_\alpha](\cdot)\{q_\alpha\} \quad (25)$$

Where $\{F_\alpha\}$ is the generalized beam force vector given as

$$\{F_\alpha\} = [N_\alpha \quad M_{y\alpha} \quad M_{z\alpha} \quad T_{s\alpha} \quad M_{\omega\alpha}]^T \quad (26)$$

The stiffness matrix $[K]_\alpha$ is inclusive of the uncertainties and fuzziness at α-cut. It can be noted that with the introduction of fuzzy arithmetic for the mixed beam thin walled composite approach [8] derived for deterministic structure can be converted for estimating the uncertainties in the thin walled structures without loss of generality. It can also be noted the flow of uncertainty introduced in the constitutive relation of the shell wall propagates in the final beam force-displacement relation of thin walled beam structure.

NUMERICAL RESULTS

The fuzzy membership functions of material properties are developed using the stochastic behavior of composite materials available in the literature. Based on these membership functions, the membership functions of the cross-sectional stiffness properties which are out-of-plane bending rigidity $EI_{y\alpha}$, inplane bending rigidity $EI_{z\alpha}$ and torsional rigidity GJ_α of a thin walled beam, are obtained. The mean and coefficient of variation (COV) of experimental values [2,21,22] of E_1, E_2, G_{12} and ν_{12} for graphite/epoxy material are given in Table 1. Uncertainty propagation through fuzzy membership function is demonstrated using a thin walled box beam modeled as a single-cell box beam with outer width = 203.2 mm and outer depth = 38.1 mm, having 28 plies with ply thickness =0.127 mm and a balanced layup as $[0_4/(15/-15)_3/(30/-30)_2]_s$ in all the walls.

The thin walled beam theory modified using fuzzy approach for uncertainty analysis is used for obtaining membership functions of cross-sectional

Table 1
Stochastic material properties of graphite/epoxy

Material properties	Mean	COV
E_1	141.9 GPa	3.39
E_2	9.78 GPa	4.27
G_{12}	6.13 GPa	4.27
ν_{12}	0.42	3.65

Table 2
Statistics of cross-sectional stiffness values

C/S Stiffness	Mean (Nm^2)	COV
EI_y	47.8e3	6.1
EI_z	761e3	6.1
GJ	22.8e3	5.7

stiffness properties for given material properties in terms of their membership functions. The membership functions of material properties are used for transmitting the uncertainty to **A**,**B**, **D** matrices as given in Equation 16.

As shown in Equation 14 and 15, the membership functions of **A**,**B**, **D** transmit the uncertainties to the cross-sectional stiffness properties of thin walled composite beams. The results obtained through fuzzy thin walled beam analysis developed in this study as a membership functions of cross-sectional stiffness properties of composite box beam are shown Figure 3. From the figure, it is noted that the distributions of membership functions of cross-sectional stiffness properties remain almost same as that of distributions of material properties. However, COV values are changed as shown in Table 2.

Further, to demonstrate the propagation of uncertainty in the performance of the thin walled beams, the fuzzy membership functions of cross-sectional stiffness properties are used to find out the uncertainty propagation in the slopes of the beam under out-of-plane, in-plane and torsion loading. Typically, the tailored composite beams will have bending-torsion couplings for which the force-displacement matrix given in Equation 25 reduces to

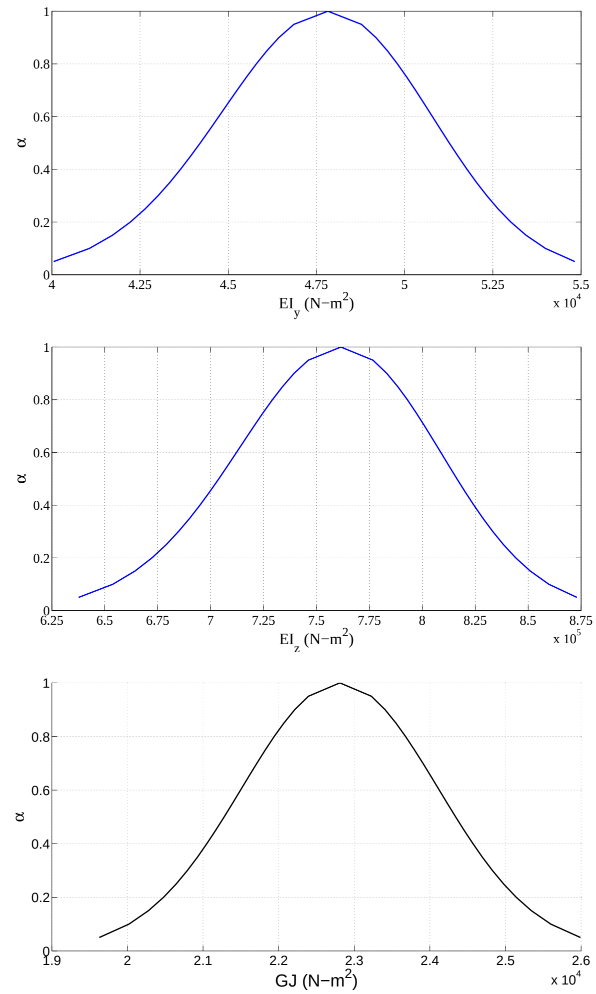

Figure 3. Membership functions of cross-sectional stiffness properties

$$\begin{Bmatrix} M_{y\alpha} \\ M_{z\alpha} \\ T_{s\alpha} \end{Bmatrix} \begin{bmatrix} EI_{y\alpha} & 0 & K_{24\alpha} \\ & EI_{z\gamma\alpha} & K_{34\alpha} \\ SYM & & GJ_{\alpha} \end{bmatrix} (\cdot) \begin{Bmatrix} \phi_{y\alpha} \\ \phi_{z\alpha} \\ \phi_{s\alpha} \end{Bmatrix} \quad (27)$$

The force-displacement relation given in Equation 27 gives the formulas for out-of-plane bending, in-plane bending and torsion under unit load as shown in Equation 28. From these formulas, it can be noted that if the couplings term becomes negligible, the performance of the beam is only a function of the rigidity in that direction. However, in presence of coupling term, the membership function of the performance becomes a function of membership functions of bending and torsion rigidities along with membership func-

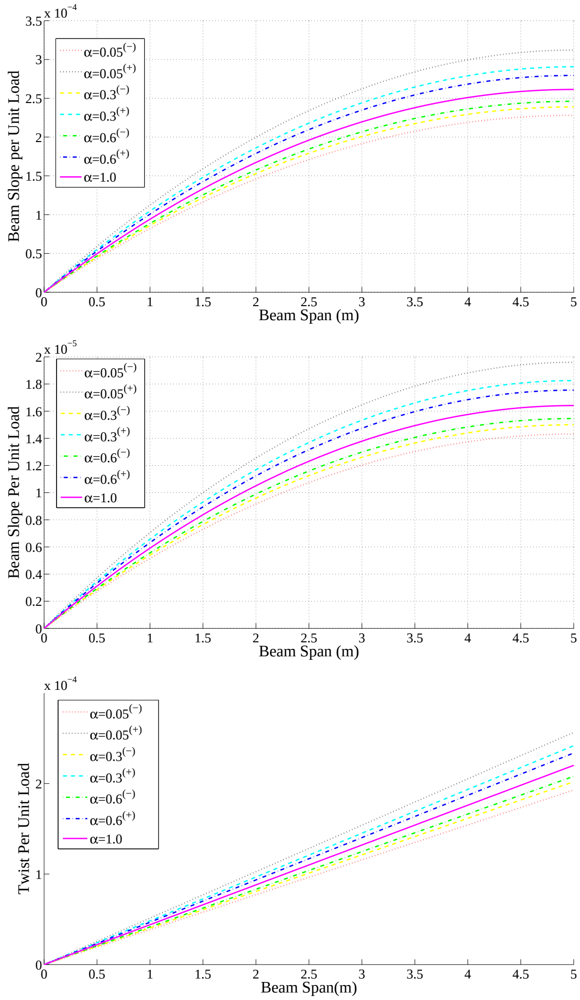

Figure 4. Beam slopes and along the span for various α-cut values

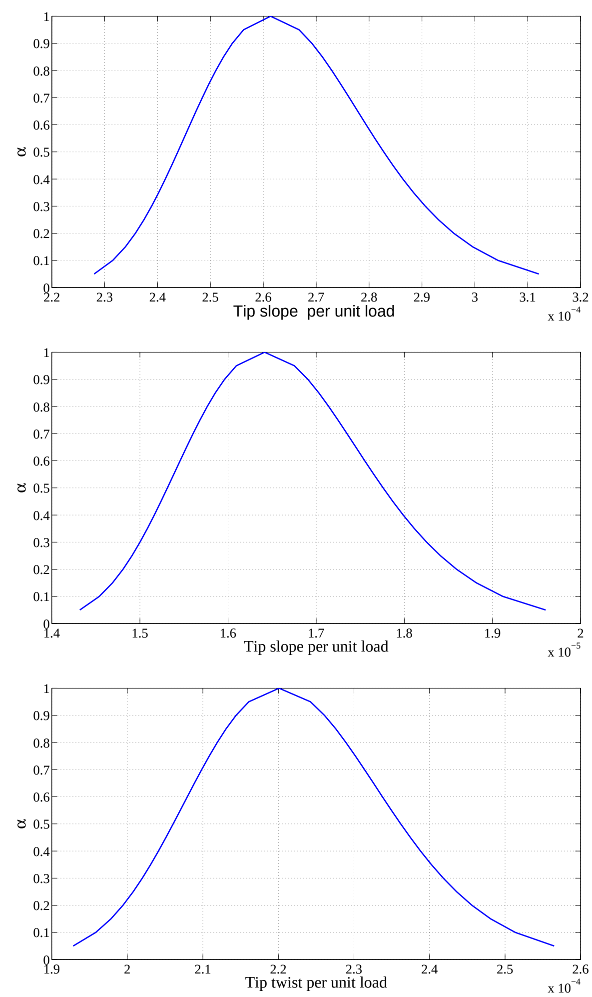

Figure 5. Membership functions of tip slopes for out of plane bending, inplane bending and twist per unit load

tion of coupling term. Figure 4 shows bending slopes and twists of a cantilever beam having a span of 5m under unit loading for four α-cut values of 1, 0.6, 0.3, 0.05. The membership functions of tip slope and twist are shown in Figure 5.

$$\begin{aligned}
\phi_{y\alpha} &= L^2/2(EI_{y\alpha}(-)K_{24\alpha}(\cdot)K_{24\alpha}(/)GJ_\alpha) \\
\phi_{z\alpha} &= L^2/2(EI_{z\alpha}(-)K_{34\alpha}(\cdot)K_{34\alpha}(/)GJ_\alpha) \\
\phi_{s\alpha} &= L/(GJ_\alpha(-)K_{24\alpha}(\cdot)K_{24\alpha}(/)EI_{y\alpha}(-) \\
&\quad K_{34\alpha}(\cdot)K_{34\alpha}(/)EI_{z\alpha})
\end{aligned} \quad (28)$$

To demonstrate the interaction of coupling terms with the beam performance, a thin walled beam is modified to generate inplane bending-torsion coupling $K_{34\alpha}$ without affecting the values of bending and torsion rigidities. The beam with coupling will have top and bottom wall layup as $[0_4/(15/-15)_3/(30/-30)_2]_s$, right wall layup as $[0_4/(-15)_6/(30/-30)_2]_s$ and left wall layup as $[0_4/(15)_6/(30/-30)_2]_s$. The membership function of the resulting coupling term is shown in Figure 6. It is observed that the membership function of coupling term shows normal distribution with mean value of 8177 N-m^2 and COV of about 4.9%. The membership functions of tip slope and tip twist for this case are shown in Figure 7.

CONCLUSIONS

This study demonstrated use of fuzzy approach

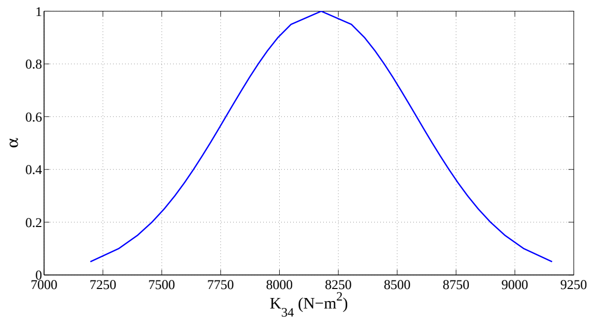

Figure 6. Membership function of inplane bending–torsion coupling term

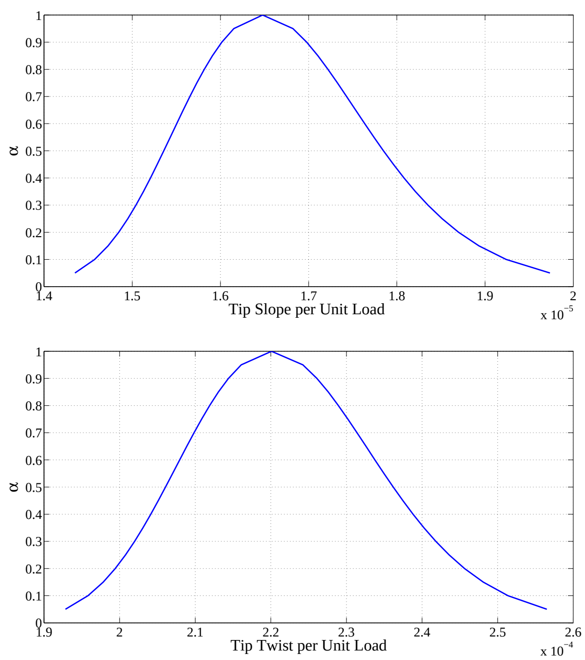

Figure 7. Membership functions of tip slope for inplane bending and tip twist per unit load for beam with coupling

for modifying the thin-walled beam analysis to get the uncertainty propagation in cross-sectional stiffness properties. The basic fuzzy arithmetic operations were used to derive a closed-form solution for thin-walled composite beam with uncertain material properties. The closed-form solution for thin-walled composite beam was obtained using a mixed force and displacement method. Through numerical results it was demonstrated that using the fuzzy membership functions of material properties, membership functions of cross-sectional stiffness properties viz. bending and torsional rigidities could be obtained. It was observed that the resulting membership functions show similar distribution as that of input membership functions whereas the COV values were increased. Further, it was also demonstrated that the bending and torsional rigidities could be used for estimation of membership functions of beam responses under out-of-plane bending, inplane bending and torsion loading. Finally, it was shown that in case of elastic coupling, the response was influenced by membership functions of bending and torsional rigidities along with membership function of coupling term. This study gives an example of rederiving existing analytical expressions [8] using fuzzy approach to incorporate uncertainties in the responses analytically. As compared to Monte-Carlo method which required about 6000 runs of beam stiffness evaluations [23], current fuzzy approach needs just 20-50 runs of thin walled beam code. Hence, it can be concluded that the fuzzy approach helps in improving computational efficiency.

ACKNOWLEDGEMENTS

This work was supported by Aeronautics Research and Development Board (Structures Panel), Government of India grant.

References

[1] Akpan, U. O., Koko, T. S., Orisamolu, L. R., and Gallant, B. K., Practical Fuzzy Finite Element Analysis of Structures, Finite Elements in Analysis and Design, Vol. 38, No. 2, 2001, pp. 93111.

[2] Antonio, C. C., and Hoffbauer, L. N., From Local to Global Importance Measures of Uncertainty Propagation in Composite Structures, Composite Structures, Vol. 85, 2008, pp. 213225.

[3] Chen L. and Rao S. S. Fuzzy finite-element approach for the vibration analysis of imprecisely-defined systems Finite Elements in Analysis and Design Volume 27, Issue 1, 30 September 1997, Pages 69-83

[4] Cherki, A., Plessis, G., Lallemand, B., Tison, T., Level, P., Fuzzy behavior of mechanical systems with uncertain boundary conditions , Computer Methods in Applied Mechanics and Engineering, Vol. 189,No. 3, 2000,pp. 863-873

[5] Dubois, D. and H. Prade (1997). The three semantics of fuzzy sets. Fuzzy Sets and Systems 90 (2), 141150.

[6] Jung, S. N. and Park, I. J., "A Simple Mixed-Based Approach for Thin-Walled Composite Blades with Two-Cell Sections", Journal of Mechanical Science and Technology (KSME Int. J.), Vol. 19, No. 11, 2005, pp. 2016-2024.

[7] Jung, S. N., Nagaraj, V. T. and Chopra, I. "Refined Structural Dynamics Model for Composite Rotor Blades", AIAA Journal, Vol. 39 (2), 2001, pp. 339-348.

[8] Jung S. N., Park I. J., Shin E. S. Theory of thin-walled composite beams with single and double-cell sections Composites: Part B 38 (2007) 182192

[9] Jung S. N., Nagaraj V. T. and Chopra I. Assessment of Composite Rotor Blade Modeling Techniques Journal American Helicopter Society 44, 188- 916 (1999)

[10] Liu, Q., and Rao S S., Fuzzy Finite Element Approach for Analysis of Fiber-Reinforced Laminated Composite Beams, AIAA Journal, Vol. 43, No. 3, 2005, pp. 651-661.

[11] Massa, F., Lallemand, B., Tison, T., Level, P., Fuzzy eigensolutions of mechanical structures, Engineering Computations, 2004,Vol. 21,No. 1, pp.66 - 77

[12] Moens, D., Vandepitte, D., Fuzzy finite element method for frequency response function analysis of uncertain structures, 2002 AIAA Journal, Vol.40, No.1, 2002, pp.126-136.

[13] Murugan, S., Ganguli, R., and Harursampath, D., Aeroelastic Response of Composite Helicopter Rotor with Random Material Properties Journal of Aircraft, Vol. 45, No. 1, 2008, pp. 306322.

[14] Murugan, S., Harursampath, D., and Ganguli, R., Material Uncertainty Propagation in Helicopter Nonlinear Aeroelastic Response and Vibration Analysis AIAA Journal, Vol. 46, No. 9, 2008, pp. 23322344.

[15] Onkar, A. K., and Yadav, D., Forced Nonlinear Vibration of Laminated Composite Plates with Random Material Properties, Composite Structures, Vol. 70, No. 3, 2005, pp. 334342.

[16] Petit, C. L., "Uncertainty Quantification in Aeroelasticity: Recent Results and Research Challenges", Journal of Aircraft, Vol.43, No. 5, 2004, pp. 1217-1229.

[17] Rao, S. S. , Sawyer, J. P., Fuzzy Finite Element Approach For The Analysis Of Imprecisely Defined Systems AIAA Journal ,Vol.33,No.12,1995,pp. 2364-2370

[18] Rao S. S. , Chen, Li., Numerical Solution Of Fuzzy Linear Equations In Engineering Analysis International Journal For Numerical Methods In Engineering Vol. 43, No. 3,pp.391408

[19] Rao, S. S., Liu, Q., Fuzzy approach to the mechanics of fiber-reinforced composite materials, AIAA JOURNAL Vol. 42, No. 1, pp. 159-167 2004

[20] Savoia, M., Structural reliability analysis through fuzzy number approach, with application to stability Computers and Structures, 2002,Vol. 80,No. 12,pp. 1087-1102

[21] Shaker, A., Abdelrahman, W. G., Taw, A., and Sadek, E., Stochastic Finite Element Analysis of the Free Vibration of Laminated Composite Plates Computational Mechanics, Vol. 41, No. 4, 2008, pp. 493501.

[22] Vinckenroy, G. V., and de Wilde, W. P., The Use of Monte Carlo Techniques in Statistical Finite Element Methods for the Determination of the Structural Behavior of Composite Materials Structural Components, Composite Structures, Vol. 32,Nos. 14, 1995, pp. 247253.

[23] You Y. H. ,Jung S. N., Pawar P. M. and Shin E. S. Effect of Uncertainty on Hub Vibration Response of Composite Helicopter Rotor Blades, Journal of Aircraft, Vol. 47, No. 1, JanuaryFebruary 2010