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THE AIRLOADS ACTING ON HELICOPTER ROTOR WITH COMBINED FLAPWISE BENDING, CHORDWISE BENDING AND TORSION OF TWISTED NON UNIFORM BRADS

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Abstract

A rotor discrete free wake geometries and the airloads acting on helicopter rotor with flapwise bending has been presented in ref.[1] . Here the free vortex concept which is same to ref.[1] ,and modified to suited for the coupling elastic deformation is presented. The various connectors with the blade root, such as the device for reducing Chordwise vibration,the pitch link stiffness and the pitch device with friction, are included in the elastic motion equations. the external forces function which depend on the elastic motion parameters are unknown in the motion equations. A series of modes are used in the differential equations of

motion, then the Lagrange's ordinary equation for general coordinate is obtained. Through an iterative procedure, the general coordinate can be solved. Final, the blade airloads, the deformations in flapwise bending, chordwise bending and torsion and the pitch moment are obtained. It is demonstrated by a program for a helicopter model in forward flight.The airloads with only the flapwise bending (i.e, $v=\phi=0$) are demonstrated by comparing with the results measured from test in H-34 . And another helicopter model with a more complex on the connector in blade root and with a combined deformations is performed.

Introduction

As mentioned in the ref[1] , the blade airloads are extremely influenced by the wake geometries under rotor. Up to now many efforts on the description of wake flow have been performed by aerodynamicists, as shown in ref [2] to [8] . Early in 1960's , Scully M. P and Landgrebe A. J presented a free wake concept, and got some valuable conclusion. Especially Sadler, S. G presented a free vortex concept in 1971, which interested us. And then we were studying the distorted wake quoting the free vortex concept.

It is well known, besides the wake geometries influence on the blade airloads, the blade elastic deformation can effect on the blade airloads. In practice, the connecting conditions in the blade root, such as the damper in chordwise, the pitch link stiffness. the pitch-flap coupling parameters etc, are rather complex. Therefore, it is necessary to develop a program including the connecting conditions in blade root to determine the rotor blade airloads, elastic deformation and the pitch moment.

Symbol

a	Sound speed
A_v, A_w, A_ϕ	The mode shape quantities representing chordwise deflection flapwise deflection and torsion deformation, respectively, in a rotating blade coordinate system
b	Simichord of airfoil
C_l, C_d, $C_{m\frac{1}{4}}$	The lift, drag and pitch moment coefficient of airfoil respectively, where the $C_{m\frac{1}{4}}$ refers to a quater of chord

C	Chord of airfoil
$\overline{C}_{\ell\alpha}$	Slope of airfoil mean lift coefficient
D	Length of shed vortex element
E	Young's modulus of elasticity
e	Distance between mass and elastic axes, positive when mass axis lies ahead of elastic axis
e_0	Distance at blade root between elastic axis and shaft, Positive when elastic axis lies ahead
F_y , F_z	Airfoil airloads component in y, z-axis respectively
G	Weight of helicopter, or shear modulus of elasticity
$\dot{h}$	Plunging velocity of blade
I_1 , I_2	Bending moments of inertia about major and minor neutral axes, respectively, (both pass through centroid of cross-sectional area effective in carrying tensions)
J	Torsional stiffness constant
k_A	Polar radius of gyration of cross-sectional area effective in carrying

	tensile stresses about elastic axis
K	Number of blades
$\overline{K}$	Pitch-flap coupling parameter
jj	Last azimuthal order of full mesh for vortices .
L	Length of trailing vortex element
l_e ; l_p	Distance between midchord and elastic axis as well as pitch axis,respectively, positive when the elastic axis lies ahead of the midchord, so does the pitch axis
l_{sh}	Chordwise offset between midchord and the shaft, positive when shaft ahead
$\overline{m}$	Mass per unit length of blade
M_0 , M_ϕ	Aerodynamic moment per unit length of blade which refers to the midchord and elastic axis , respectively
N. A	Number of azimuthal step per revolution
Q_x, Q_y, Q_z	General force in x, y, z direction,respectively
Q_{HDmax}	Maximun damping in pitch hinge
$Q_{\psi\psi max}$	Maximun damping in chordwise damper
r	Radial coordinate
r_0	The distance from the shaft to the flap

	hinge
R	Radius of rotor
T	Centrifugal force of blade, or thrust of rotor
u	Resultant flow velocity normal to the leading edge of blade
U	Flow velocity in shaft rotating plane and normal to leading edge of blade
V	Flow velocity normal to shaft rotating plane
v , w	Deflection in chordwise flapwise, respectively
W_0	Mean down-wash
W	Resultant induced velocity
o-xyz	Coordinate system which rotates with blade (see fig. 5)
O-XYZ	Coordinate system fixed to shaft (see fig. 5)
α_g	Blade-section pitch angle
α	Angle of attack of blade-section
α_s	Shaft tilt angle
α_1 , α_2	Longitudinal and lateral cyclic pitch
β	Flap angle of rigid blade
Γ	Votex element circulation

γ	Circulation density of airfoil
ξ , η	Coordinate along and perpendicular to chord respectively, in which the origin is located at elastic axis of blade-section
ζ_k	General coordinate for the k-th mode
θ_1	Blade twist, positive when leading edge is upward
θ_{co}	Collective pitch
θ_0	Blade pitch
λ	Horizontal distance between pitch axis and shaft
μ	Advance ratio
ρ	Air density
χ	Blade flap angle due to elastic deflection
ϕ	Torsional deformation of blade; or velocity potential
ψ	Blade azimuth
Ω	Rotor rotating speed
ω	Natural frequency of blade

Subscript

i , r	Indicate the variable radial station

j, s	Indicate the variable azimuthal station
k	Iterative times, or order of mode
KJ	Blade ordinal number
n	Number of radial points on a blade
NW	Number of azimuthal positions for blade advancing or number of azimuthal positions in the wake

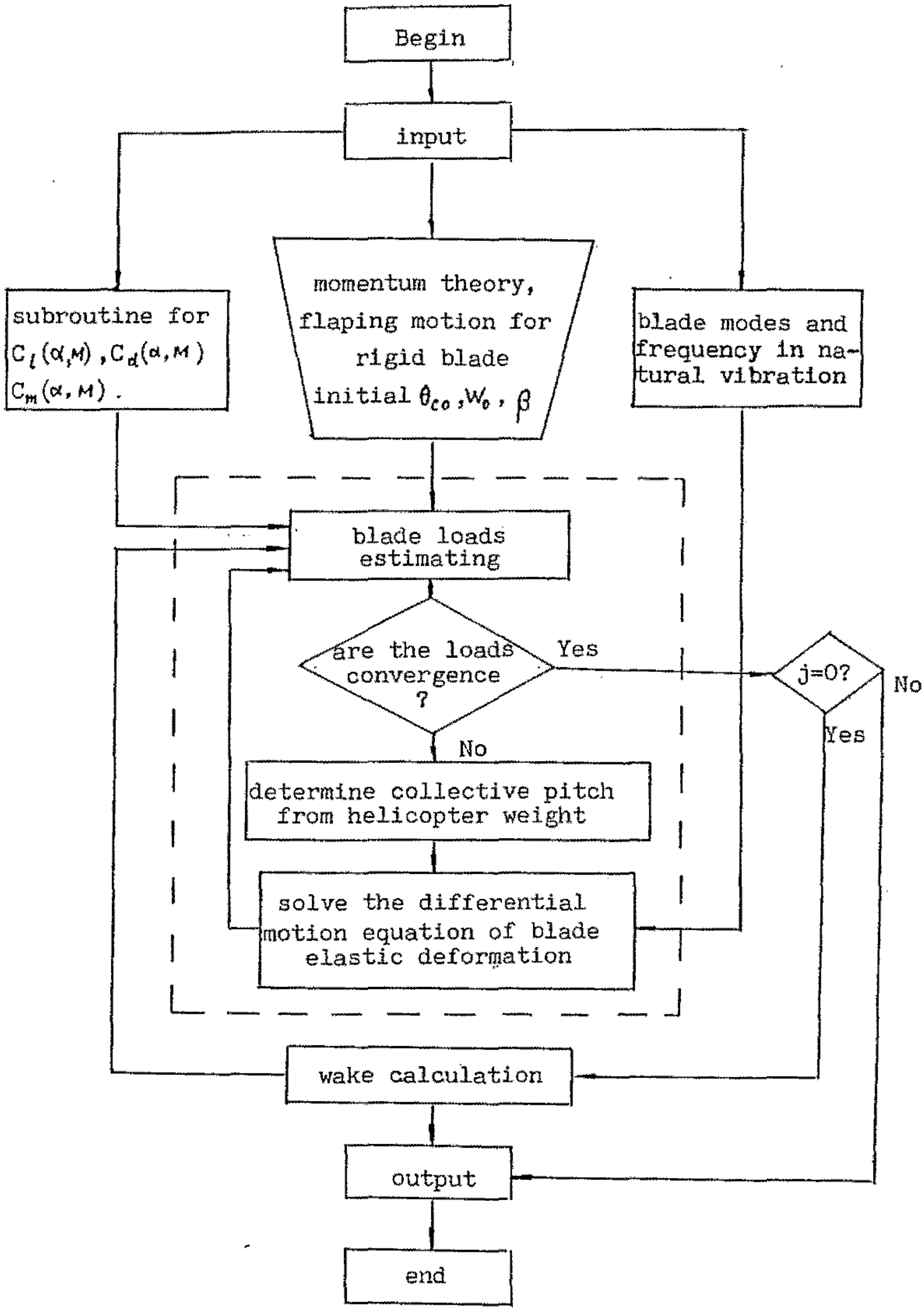


fig.1 ,the basic block diagram of computer program

Analysis

The rotor blade load is relative to the rotor wake geometry and the blade elastic motion. They interfere with each other, and from a nonlinear complex equation system. In order to obtain the blade load, a series of iterative processes are necessary. As follows, initial average induced velocity is assumed, an initial load can be obtained, so does the initial elastic deformation. Once the initial parameters are determined,an azimuthal increment is added, this same process is performed, then the rotor wake and blade circullation in this instant can be calculated. Iterations are conducted at each azimuth, until the difference between the previous iteration and the later iteration is less than a given value. Then an azimuth increment is added again and the previous process is repeated, until the wake grows to such a long distance that its influence on the blade lift turns unvaried (approximately). At this time, the process to determine wake comes to a stop. In this case the determination of blade airloads reaches a stage. Further calculation to solve the blade responses,its loads, moment and strain can be obtained.

I, Wake and Circulations.

We can consider that the wake under rotor consists of two parts. One is called full mesh beneath the rotor; the other is a concentrated vortex for each blade called blade tip vortex, which extends to the downstream.

1, The Velocity Induced by Straight Element Vortex

The induced velocity based on Laplace equation of classical three dimensional imcompressible flow is[10]:

$$\vec{q}_p = \frac{\Gamma}{4\pi}\left(\frac{\vec{\gamma}_A \times \vec{\gamma}_B}{|\vec{\gamma}_A \times \vec{\gamma}_B|^2}\right)\left[\vec{L}_A \times \left(\frac{\vec{\gamma}_A}{\gamma_A} - \frac{\vec{\gamma}_B}{\gamma_B}\right)\right] \qquad (1)$$

$$\vec{q}_p = q_{xp}\,\vec{i} + q_{yp}\,\vec{j} + q_{zp}\,\vec{k}$$

$$\begin{aligned} q_{xp} &= \nu_x G \\ q_{yp} &= \nu_y G \\ q_{zp} &= \nu_z G \end{aligned} \qquad (2)$$

where

$$L_A = \sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2 + (Z_B - Z_A)^2}$$

Similarly for γ_B and γ_A

$$\nu_x = (Y - Y_A)(Z_A - Z_B) - (Z - Z_A)(Y_A - Y_B)$$

$$\nu_y = (Z - Z_A)(X_A - X_B) - (X - X_A)(Z_A - Z_B)$$

$$\nu_z = (X - X_A)(Y_A - Y_B) - (Y - Y_A)(X_A - X_B)$$

$$G=\frac{\Gamma}{2\pi}\cdot\frac{\gamma_A+\gamma_B}{\gamma_A\,\gamma_B\,[(\gamma_A+\gamma_B)^2-\mathcal{L}_A{}^2]}$$

where the subscript P is the point interested , other symbols can be seen in fig. 2

2, The Velocity Induced by Curved Vortex Element Itself. (it was considered approximatly as the straight elements) the velocity at point P induced by the curved vortex element is [9]

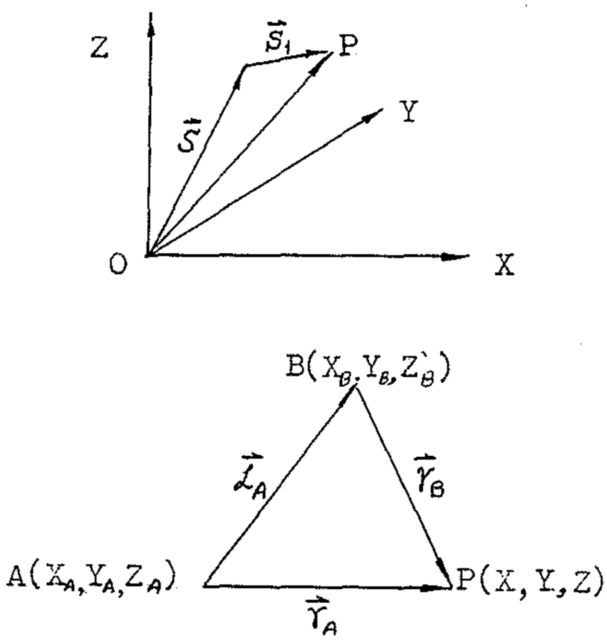


fig.2, Flow model induced by vortex.

$$q_S=\frac{1}{8\pi S}\{\Gamma_C[\ln(\frac{8S}{a_C}\,tg\,\frac{\varphi_C}{4})+\frac{1}{4}]+\Gamma_D[\ln(\frac{8S}{a_D}\,tg\,\frac{\varphi_D}{4})+\frac{1}{4}]\}\qquad(3)$$

where a_C and a_D are the vortex core radius of $\widehat{CD}$ and $\widehat{DP}$, respectively ,

$$S=\frac{\mathcal{L}_C\,\mathcal{L}_D\,\delta_C}{\sqrt{4\mathcal{L}_C^2\mathcal{L}_D^2-(\mathcal{L}_C^2+\mathcal{L}_D^2-\delta_C^2)^2}}\qquad(4)$$

$$\vec{q}_S = q_{SX}\,\vec{i} + q_{SY}\,\vec{j} + q_{SZ}\,\vec{k}$$

$$
\begin{aligned}
q_{SX} &= q_S \frac{m_x}{B} \\
q_{SY} &= q_S \frac{m_y}{B} \qquad\qquad (5) \\
q_{SZ} &= q_S \frac{m_z}{B}
\end{aligned}
$$

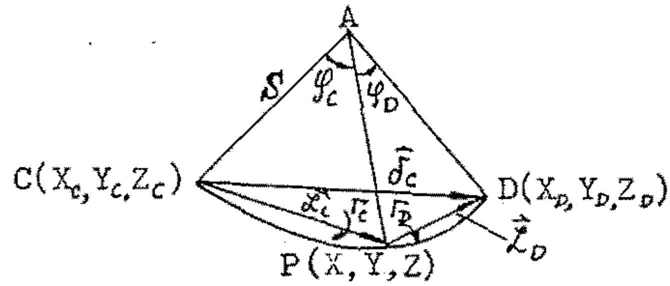


fig. 3, The Geometry of Curved Vortex

the subscript S represent the self-inducing parameter, the others can be seen in fig. 3 .

$$m_X = (Y - Y_C)(Z_D - Z) - (Y_D - Y)(Z - Z_C)$$

$$m_Y = (Z - Z_C)(X_D - X) - (Z_D - Z)(X - X_C)$$

$$m_Z = (X - X_C)(Y_D - Y) - (X_D - X)(Y - Y_C)$$

$$B = \sqrt{m_X^2 + m_Y^2 + m_Z^2}$$

$$
\operatorname{tg}\frac{\varphi_C}{4} = \begin{cases} \dfrac{2S - \sqrt{4S^2 - L_C^2}}{L_C} & \text{for } L_C^2 \leq \delta_C^2 + L_D^2 \\[2ex] \dfrac{2S + \sqrt{4S^2 - L_C^2}}{L_C} & \text{for } L_C^2 > \delta_C^2 + L_D^2 \end{cases}
$$

$$tg\frac{\varphi_D}{4}=\begin{cases}\dfrac{2S-\sqrt{4S^2-\mathcal{L}_D^2}}{\mathcal{L}_D} & \text{for } \mathcal{L}_D^2\leqslant \delta_c^2+\mathcal{L}_c^2\\ \dfrac{2S+\sqrt{4S^2-\mathcal{L}_D^2}}{\mathcal{L}_D} & \text{for } \mathcal{L}_D^2>\delta_c^2+\mathcal{L}_c^2\end{cases}$$

$$\mathcal{L}_c=\sqrt{(X-X_c)^2+(Y-Y_c)^2+(Z-Z_c)^2}$$

Similary for L_D

$$\delta_c=\sqrt{(X_D-X_c)^2+(Y_D-Y_c)^2+(Z_D-Z_c)^2}$$

3, the Resultant Velocity Induced by All Vortices Element

a, The contribution of trailing vortices $\Gamma_{t,i,j}$ to the point P

$$V_{tX}(X_{r,s},Y_{r,s},Z_{r,s})=\sum_{KJ=1}^{K}\sum_{\substack{i=1\\i\neq r}}^{N}\sum_{j=1}^{NW} q_{x,i,j}^{(KJ)}+\sum_{KJ=1}^{K}\sum_{\substack{j=1\\j\neq S-1,s}}^{NW} q_{x,i,j}^{(KJ)}+q_{S,X}(r,s) \qquad (6)$$

similary for $V_{ty}(X_{r,s},Y_{r,s},Z_{r,s})$ and $V_{tz}(X_{r,s},Y_{r,s},Z_{r,s})$, but yet the subscript variables should be changed correspondingly. In order to simplify the description, only the x-component is descripted here.

On the boundary:

$$q_{sx}(\gamma,1) = q_{sx}(\gamma,2)\Big|_{\Gamma_{t,\gamma,2}\equiv 0}$$

When $N_W=1$

$$q_{sx}(\gamma,1) = q_{sx}(\gamma,2) = 0$$

$q_{x,i,j}$ and $q_{sx}(\gamma,s)$ have been given in equations (2) and (5) respectively , but the subscript of the points A, B and P should be replaced by $A(X_{i,j}, Y_{i,j}, Z_{i,j})$, $B(X_{i,j+1}, Y_{i,j+1}, Z_{i,j+1})$ and $P(X_{\gamma,s}, Y_{\gamma,s}, Z_{\gamma,s})$ respectively, the subscript of points C and D should be replaced by $C(X_{\gamma,s-1}, Y_{\gamma,s-1}, Z_{\gamma,s-1})$ and $D(X_{\gamma,s+1}, Y_{\gamma,s+1}, Z_{\gamma,s+1})$.

Similiary for the other components.

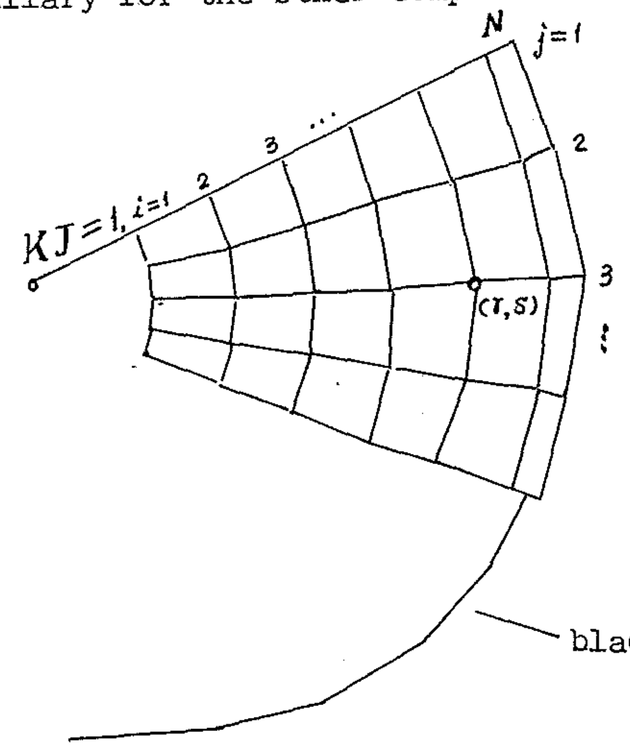


fig. 4 Illustration for the Subscript of the Wake Mesh

b, the Contribution of Shed Vortices (including bound vortices) to the Point $P_{r.s}$.

$$V_{dx}(X_{r,s}, Y_{r,s}, Z_{r,s}) = \sum_{KJ=1}^{K} \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j\neq s}}^{jj} q_{x,i,j}^{(KJ)} + \sum_{KJ=1}^{K} \sum_{\substack{i=1 \\ i\neq r-1,r}}^{n} q_{x,i.s}^{(KJ)} + q_{sx}(r, s) \tag{7}$$

where the point A and B in equation (2) are $A(X_{i,j}, Y_{i,j}, Z_{i,j})$ and $B(X_{i+1,j}, Y_{i+1,j}, Z_{i+1,j})$ respectively, and the point C and D in equation (5) are $C(X_{r-1,s}, Y_{r-1,s}, Z_{r-1,s})$ and $D(X_{r+1,s}, Y_{r+1,s}, Z_{r+1,s})$ respectively .

On the boundray:

For the inboard vortices behind the blade

$$q_{sx}(r, s) = q_{sx}(r+1, s)\Big|_{\Gamma_{d,r+1,s}\equiv 0}$$

for the outboard vortices behind the blade

$$q_{sx}(r, s) = q_{sx}(r-1, s)\Big|_{\Gamma_{d,r-2,s}\equiv 0}$$

when $S=1$

$$q_{sx}(r, 1) = 0$$

$$q_{xi,1}(X_{r,1}, Y_{r,1}, Z_{r,1}) = 0$$

The resultant induced velocity :

$$W_x(X_{r,s},\ Y_{r,s},\ Z_{r,s}) = V_{tx}(X_{r,s},\ Y_{r,s},\ Z_{r,s}) + V_{dx}(X_{r,s},\ Y_{r,s},\ Z_{r,s}) \qquad (8)$$

4. The Circulation of Vortex Element in Wake

For the bound vortex:

$$\Gamma^{(NW)}_{i,1,KJ} = \Gamma_b(\gamma_i,\ \psi_{KJ,NW})$$

where $\Gamma_b(\gamma_i,\ \psi_{KJ,NW})$ will be given in equation (17) expressed later.

For the shed vortex:

$$\Gamma^{(NW)}_{d,i,j,KJ} = \Gamma^{(NW-1)}_{i,1,KJ} - \Gamma^{(NW)}_{i,1,KJ} \qquad \text{for } j=2$$

$$\Gamma^{(NW)}_{d,i,j,KJ} = \Gamma^{(NW-1)}_{d,i,j-1,KJ} \qquad \text{for } j\geqslant 3$$

For the trailing vortex:

$$\Gamma^{(NW)}_{t,i,1,KJ} = \Gamma^{(NW)}_{i-1,1,KJ} - \Gamma^{(NW)}_{i,1,KJ}$$

$$\Gamma^{(NW)}_{t,i,1,KJ} = -\Gamma^{(NW)}_{i,1,KJ}$$ when i is at the inboard positions of the blade.

$$\Gamma^{(NW)}_{t,i,1,KJ} = \Gamma^{(NW)}_{i-1,1,KJ}$$ when i is at the outboard position of the blade

$$\Gamma^{(NW)}_{t,i,j,KJ} = \Gamma^{(NW-1)}_{t,i,j-1,KJ} \qquad \text{for } j\geqslant 2$$

Where the $\Gamma_{i,1,KJ}^{(NW)}$ expresses the circulation of KJ-th blade, which is located at azimuth $\psi_{KJ}=(KJ-1)\frac{2\pi}{K}+(NW)\cdot\Delta\psi$ and its joint in wake is at point (i, j). One can transfer $\Gamma_b(\gamma_i, \psi_{KJ,NW})$ into $\Gamma_{i,j,KJ}^{(NW)}$ according to $\Gamma_{i,j+1,KJ}^{(NW+1)}(\psi+\Delta\psi)=\Gamma_{i,j,KJ}^{(NW)}(\psi)$. Therefore, the arbitrary Γ in wake can be obtained.

For the tip vortex:

When the shed vortices are far away from the blade, its influence on blades should be alleviated, and the trailing vortices would effect each other. With the trailing vortices going down, they will roll-up and form a " vortex braid ". Its influence on blade airload is sensitive. The factors that effect the trailing vortices rolling-up are very complicated, it is concerned with viscous flow. No perfect analysis is made. In order to satisfy the need for engineering, a tip vortex is used to simulate the " vortex braid ", its circulation is equal to the maximum circulation of the bound vortex and its initial radial position can be determined as follow, like that of wing.

$$\Gamma_{t,M,jj-1}^{(NW)} = \sum_{i=i_M+1}^{i_{out}} \Gamma_{t,i,jj-1}^{(NW)}$$

where i_M means the spanwise station at which the maximum circulation of bound vortex located; i_{out} is the number of radial positions on blade tip, equal to n .

$$X_{M,jj} = \frac{\sum_{i=i_M+1}^{i_{out}} X_{i,jj} \cdot \Gamma_{t,i,jj-1}}{\Gamma_{t,M,jj-1}}$$

5, The Coordinate of Wake Joint:

The wake joints at blade:

$$\left.\begin{aligned}
X_{i,1,KJ}^{(NW)} &= r_0 \cos\psi_{KJ,NW} + \sum_{t=0}^{i-1} (r_{t+1}-r_t) \cos\chi_{KJ,NW}^{(t)} \cdot \cos\bar{\psi}_{KJ,NW}^{(t)} \\
Y_{i,1,KJ}^{(NW)} &= r_0 \sin\psi_{KJ,NW} + \sum_{t=0}^{i-1} (r_{t+1}-r_t) \cos\chi_{KJ,NW}^{(t)} \cdot \sin\bar{\psi}_{KJ,NW}^{(t)} \\
Z_{i,1,KJ}^{(NW)} &= \sum_{t=0}^{i-1} (r_{t+1}-r_t) \sin\chi_{KJ,NW}^{(t)}
\end{aligned}\right\} \quad (9)$$

$$\psi_{KJ,NW} = (KJ-1)\frac{2\pi}{K} + (NW)\Delta\psi$$

$$\bar{\psi}_{KJ,NW}^{(t)} = \upsilon'^{(t)} + \psi_{KJ,NW}$$

$$\upsilon'^{(t)} = \arcsin \frac{\upsilon^{(t+1)} - \upsilon^{(t)}}{(r_{t+1}-r_t)\cos\chi_{KJ,NW}^{(t)}}$$

The wake jonits behind blade: (for $j \geqslant 2$)

$$\left.\begin{aligned}
X_{i,j,KJ}^{(NW)} &= X_{i,j-1,KJ}^{(NW-1)} + (V_f \cos\alpha_S + W_{x,i,j-1,KJ}^{(NW-1)}) \frac{\Delta\psi}{\Omega} \\
Y_{i,j,KJ}^{(NW)} &= Y_{i,j-1,KJ}^{(NW-1)} + W_{y,i,j-1,KJ}^{(NW-1)} \frac{\Delta\psi}{\Omega} \\
Z_{i,j,KJ}^{(NW)} &= Z_{i,j-1,KJ}^{(NW-1)} + (V_f \sin\alpha_S + W_{z,i,j-1,KJ}^{(NW-1)}) \frac{\Delta\psi}{\Omega}
\end{aligned}\right\} \quad (10)$$

where the V_f is the forward flight velocity

Note : follow the example of $W_{x,i,j-1,KJ}^{(NW-1)}$, it indicates the X-Component of induced velocity at wake joint $(i, j-1)$, which is generated by KJ-th blade, and the blade located at an azimuth $\psi_{KJ,NW} = (KJ-1)\frac{2\pi}{K} + (NW)\Delta\psi$.

6, The Convergent Criteria for the Circulation $\Gamma_b(r_i, \psi_{KJ,NW})$

a, At a certain , when

$$\sum_{i=1}^{n}\left[\Gamma_{b,i,1,KJ}^{(k+1)} - \Gamma_{b,i,1,KJ}^{(k)}\right]^2 \Big/ \sum_{i=1}^{n}\left[\Gamma_{b,i,1,KJ}^{(k+1)}\right]^2 \leqslant 0.00025$$

make an increment $\Delta\psi$, then repeat the same process for other azimuth.

b, In the case of

$$\frac{\sum_{i=1}^{n}\left[\Gamma_{i,1,K}^{(NW+\frac{NA}{K})} - \Gamma_{i,1,KJ=1}^{(NW)}\right]^2}{\sum_{i=1}^{n}\left[\Gamma_{i,1,KJ=1}^{(NW)}\right]^2} \leqslant 0.00025 \qquad (11)$$

then we are sure that the wake geometry appears periodically, i.e. the wake motion is steady, and make it output the quantity $\Gamma_{i,1,KJ}^{(NW)}$, $W_{z,i,j,KJ}$, $X_{i,j,KJ}$ etc.

7, The Radius of Vortex Core.

At the continuous vortex surface, the induced velocity follows the form of $q \sim \int \frac{dx_0}{x-x_0}$. Only the two ends of the vortex surface can result in an infinite induced velocity. In the case of using discrete vortices, the induced velocity would increase infinitly near the vortex. It contradicts the practice.So a viscous vortex core should be taken into account.

Since the aerodynamic coefficients of airfoil are used, the influence of vortex core on the whole aerodynamic characteristic of the rotor is slight. Some vortex core radius are used in computation. It indicates that the differences is small. Here we assume the vortex core radius a =0.01R.

As mentioned in fig. 1 . during the stage when the blade circulation and wake geometry are determined the initial blade elastic deformation - the flapwise, choordwise and torsion--should be taken into account in that. When the wake geometry has already been determined, the final elastic deformation differential equation of blade must be solved.

II The Blade Airloads and Responses

1, The Airloads of the Blade Element.

For any blade element, the equation of airloads can be obtained from Bernoulli's equation [7]

$$\Delta p = 2\rho(u\varphi_x + \dot{\varphi}) \tag{12}$$

$$\varphi_x = \frac{1}{2}\gamma(x)$$

$$\dot{\varphi} = \frac{\partial \varphi_s}{\partial t} = \frac{\partial}{\partial t}\int_{-b}^{x} \varphi_{\xi,s}\, d\xi = \frac{\partial}{\partial t}\int_{-b}^{x} \frac{1}{2}\gamma(\xi, t)\, d\xi \tag{13}$$

where φ_S shows the velocity potential at airfoil surface.

φ_S shows $\frac{\partial \varphi_s}{\partial \xi}$

$$\gamma = 2u\left(A_0 \operatorname{ctg}\frac{\theta}{2} + \sum_{n=1}^{\infty} A_n \sin n\theta\right) \tag{14}$$

the equation for boundary condition of the flow round the airfoil:

$$\alpha_g + \frac{W_z(x)}{u} + \frac{v_i(x)}{u} = \frac{dy_m}{dx} \tag{15}$$

$$\frac{dy_m}{dx} = F'(\xi) + \frac{dy_l}{dx} \tag{16}$$

where $v_i(x)$ is the velocities perpendicular to the chord induced by distributive vortices at airfoil; $F'(\xi)$ is the derivative of the ordinate at the mean airfoil cuevature with respect to the chord without angle of attack; θ is an integrating variation. let $x=-b\cos\theta$, or $\xi=-b\cos\theta$.

In general, in order to get the circulation Γ of bound vortex, an integrated equation according to the lift line or lift-surface theory should have been used. But yet the task for the wake calculation has been considerably complicated, in addition to solve the integrated equation. the amount of calculation is too large to practise, and there is another important problem that the integrated equation is unvaluable for the angle of attack exceeding the stall angle. In order to satisfy the need for engineering, a series of synthesis expressions for lift, drag and pitch moment coefficients were used. Therefore, substitute $C_l(\alpha_{i,NW}, M_{i,NW})$ for $2\pi\alpha_{i,NW}$, through equation (12) to (16) the circulation of bound vortex can be determined, as follow:

$$\Gamma_b(r_i, \psi_{KJ,NW}) = \Gamma_{i,1,KJ}^{(NW)}$$

$$= b_i U_{i,KJ}^{(NW)} C_l(\alpha_{i,KJ}^{(NW)}, M_{i,KJ}^{(NW)}) + 2\pi b_i \Big[(l_{pi} + \tfrac{b_i}{2}) \cdot (\dot\theta_{i,KJ}^{(NW)}$$

$$- \bar K \dot\chi_{i=0,KJ}^{(NW)} + \Omega \chi_{i,KJ}^{(NW)} (l_{sh,i} + \tfrac{b_i}{2}) + (l_{e,i} + \tfrac{b_i}{2}) \dot\phi_{i,KJ}^{(NW)} \Big]$$

$$- T_{1,i,KJ}^{(NW)} \tag{17}$$

$$T_{1,i,KJ}^{(NW)} = 2U_{i,KJ}^{(NW)} \int_{-b_i}^{b_i} F'(\xi) \sqrt{\frac{b_i+\xi}{b_i-\xi}} \; d\xi$$

where Y_m is the mean airfoil curvature with angle of attack; y_1 is the ordinate of the plate wing section with angle of attack. Subscript 1 in $\Gamma_{i,1,KJ}^{(NW)}$ shows j=1. $M_{i,NW}$ is the airfoil local Mach Number at NW-th azimuth.

Note : In order to simplify writing. the subscript KJ is omitted later.

The lift of blade element

$$Y = \int_{-b}^{b} \Delta p dx = \int_{-b}^{b} \rho u \gamma(x) dx + \rho \frac{\partial}{\partial t} \int_{-b}^{b} dx \int_{-b}^{x} \gamma(\xi, t) d\xi$$

$$= \rho u \Gamma + \rho \frac{\partial}{\partial t} \int_{-b}^{b} \left[\int_{\xi}^{b} \gamma(\xi, t) dx \right] d\xi$$

$$= \rho u \Gamma + \rho \frac{\partial}{\partial t} \int_{-b}^{b} \gamma(\xi)(b-\xi) d\xi$$

$$= \rho u \Gamma + \rho \frac{\partial}{\partial t} \left[b\Gamma + \frac{M}{\rho u} \right] \tag{18}$$

$$\frac{M}{\rho u} = 2 C_m U b^2 - 2 \int_{-b}^{b} W_1 \sqrt{b^2 - \xi^2} \; d\xi$$

where W_1 is the velocity normal to the chord, which is concerned with the situation for the element blade motion.

$$M_0 = -\int_{-b}^{b} \Delta p(x,t)\, x\, dx = -\int_{-b}^{b} 2\varrho(u\varphi_x + \dot{\varphi})\, x\, dx$$

$$= -\left\{ \int_{-b}^{b} \varrho u \gamma(x,t)\, x\, dx + \varrho \frac{\partial}{\partial t} \int_{-b}^{b} \int_{-b}^{x} \gamma(\xi,t)\, d\xi\, x\, dx \right\}$$

$$= -\left\{ \int_{-b}^{b} \varrho u \gamma(\xi,t)\, \xi\, d\xi + \varrho \frac{\partial}{\partial t} \int_{-b}^{b} \gamma(\xi,t)\, d\xi \int_{\xi}^{b} x\, dx \right\}$$

$$= -\int_{-b}^{b} \varrho u \gamma(\xi,t)\, \xi\, d\xi - \frac{\varrho}{2} \frac{\partial}{\partial t} \left[\int_{-b}^{b} b^2 \gamma(\xi,t)\, d\xi - \int_{-b}^{b} \xi^2 \gamma(\xi,t)\, d\xi \right] \quad (19)$$

Substitute the expressions metioned above for the corresponding terms in equations (18) and (19), and show the radial variable i and the azimuthal variable NW, then we can obtain:

$$Y_{i,NW} = \varrho u_{i,NW} \Gamma_{i,NW} + \varrho \left\{ b_i \dot{\Gamma}_{i,NW} + 2 b_i^2 \frac{\partial}{\partial t} (C_{m i,NW} \cdot u_{i,NW}) \right.$$

$$+ \pi b_i^2 \left[l_{pi} (\ddot{\theta}_{0,NW} - \bar{K} \ddot{\chi}_{i=0,NW}) + l_{e,i} \ddot{\phi}_{i,NW} \right.$$

$$\left. \left. + l_{sh,i} \Omega \dot{\chi}_{i,NW} \right] \right\} - \dot{T}_{2,i,NW} \quad (20)$$

$$T_{2,i,NW} = 2 \varrho u_{i,NW} \int_{-b_i}^{b_i} F'(\xi) \sqrt{b_i^2 - \xi^2}\, d\xi$$

$$M_{0,i,NW} = 2C_{m,i,NW}\, \varrho\, u_{i,NW}^2 b_i^2 + \pi \varrho b_i^2 \left[l_{pi} (\dot{\theta}_{0,NW} \right.$$

$$\left. - \bar{K} \dot{\chi}_{0,NW}) + l_{e,i} \dot{\phi}_{i,NW} + l_{sh,i} \Omega \chi_{i,NW} \right] - \frac{\varrho}{4}$$

$$\left\{ b_i^2 \dot{\Gamma}_{i,NW} + \frac{\pi b_i^4}{2} (\theta_{0,NW} - \bar{K} \ddot{\chi}_{0,NW} + \ddot{\phi}_{i,NW} + \Omega \dot{\chi}_{i,NW}) \right.$$

$$- T_{3,i,NW} \Big\} - T_{2,i,NW} \cdot u_{i,NW} \tag{21}$$

$$T_{3,i,NW} = 4\,\dot{u}_{i,NW} \int_{-b_i}^{b_i} F'(\xi)\,\sqrt{b_i^2 - \xi^2}\,d\xi$$

$$C_{m,i,NW} = C_{m\frac{1}{4}}(\alpha_{i,NW}\,,\,M_{i,NW}) + C_\ell(\alpha_{i,NW}\,,\,M_{i,NW})\cdot\frac{1}{4}$$

$$X_{i,NW} = C_d\,(\alpha_{i,NW}\,,\,M_{i,NW})\,\rho\,u_{i,NW}^2\,b_i$$

where $X_{i,NW}$, $Y_{i,NW}$ and $M_{0,i,NW}$ are the drag, lift and pitch moment on the airfoil, respectively. In the C_l , C_d and C_m are got from the experiment data, then $F'(\xi) = 0$.

2, The Flow Parameters

$$\bar{\alpha}_{i,NW} = \theta_{0,NW} + \theta_{1,i} + \phi_{i,NW} - \bar{K}\chi_{0,NW} + \text{Arc tg}\,\frac{\bar{V}_{i,NW}}{U_{i,NW}}$$

$$\alpha_{i,NW} = \begin{cases} \bar{\alpha}_{i,NW} - 2\pi & \text{for } \bar{\alpha}_{i,NW} > \pi \\ \bar{\alpha}_{i,NW} + 2\pi & \text{for } \bar{\alpha}_{i,NW} < -\pi \\ \bar{\alpha}_{i,NW} & \text{for } -\pi \leqslant \bar{\alpha}_{i,NW} \leqslant \pi \end{cases}$$

$$\text{Arc tg}\,\frac{\bar{V}_{i,NW}}{U_{i,NW}} = \begin{cases} \text{arc tg}\,\dfrac{\bar{V}_{i,NW}}{U_{i,NW}} & \text{for } U_{i,NW} \geqslant 0 \\ \pi + \text{arc tg}\,\dfrac{\bar{V}_{i,NW}}{U_{i,NW}} & \text{for } \begin{matrix} U_{i,NW} < 0 \\ \bar{V}_{i,NW} > 0 \end{matrix} \\ -\pi + \text{arc tg}\,\dfrac{\bar{V}_{i,NW}}{U_{i,NW}} & \text{for } \begin{matrix} U_{i,NW} < 0 \\ \bar{V}_{i,NW} > 0 \end{matrix} \end{cases}$$

$$\overline{V}_{i,NW} = \dot{h}_{i,NW} + V_f \ \mathrm{Sin}\,\alpha_s + W_{Z,i,NW}$$

$$U_{i,NW} = r_i \Omega + V_f \ \mathrm{Cos}\,\alpha_s \cdot \mathrm{Sin}\,\psi_{NW}$$

$$\theta_{0,NW} = \theta_{co} + \alpha_1 \ \mathrm{Sin}\,\psi_{NW} + \alpha_2 \ \mathrm{Cos}\,\psi_{NW}$$

$$\mathcal{U}_{i,NW} = \left[U_{i,NW}^{2} + \overline{V}_{i,NW}^{2} \right]^{\frac{1}{2}}$$

$$\chi_{i=0,NW} = W'_{i=0,NW}$$

$$\dot{h}_{i,NW} = -\dot{W}_{i,NW} - W'_{i,NW} \ V_f \ \mathrm{Cos}\,\alpha_s \cdot \mathrm{Cos}\,\psi_{NW}$$

$$\Delta\psi = \frac{2\pi}{NA}$$

$$F_{Zi,NW} = Y_{i,NW} \frac{U_{i,NW}}{u_{i,NW}} + X_{i,NW} \frac{\overline{V}_{i,NW}}{\mathcal{U}_{i,NW}}$$

$$F_{yi,NW} = Y_{i,NW} \frac{\overline{V}_{i,NW}}{\mathcal{U}_{i,NW}} - X_{i,NW} \frac{U_{i,NW}}{u_{i,NW}}$$

$$M_{\phi,i,NW} = M_{0,i,NW} - F_{i,NW} \cdot l_{e,i}$$

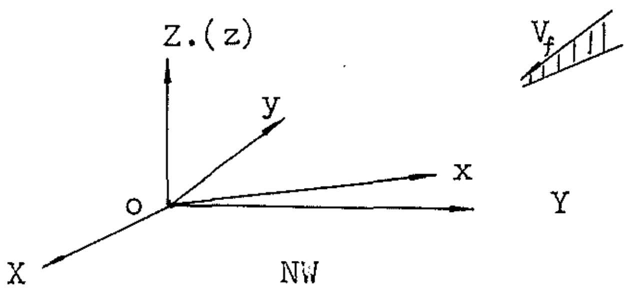


Fig.5, the Illustrate for the Relationship between the Coordinate System O-XYZ and o-xyz

3. The Responses

A motion for combined flapwise bending chordwise bending and torsion of twisted nonuniform rotor blade is to be dealt with in this paper. The external force applied to the blade is concerned with the unknown deformation of blade (it is a nonlinear function) . With the reference [11] [12], a set of natural vibration differential equations can be obtained. Then take into account the external airloads, Coriolis force and the damping moment in chordwise and in pitch device. Final a set of forced vibration differential equations can be obtained.

$$
\begin{aligned}
&-[(GJ+Tk_A^2+EB_1\theta_1'^2)\phi'+Tk_A^2\theta_1'-EB_2\theta_1'(v''\cos\theta_1+w''\sin\theta_1)]' \\
&+Te_A(v''\sin\theta_1-w''\cos\theta_1)+\Omega^2\overline{m}xe(w'\cos\theta_1-v'\sin\theta_1) \\
&+v\Omega^2\overline{m}e\sin\theta_1+\Omega^2\overline{m}[(k_{m2}^2-k_{m1}^2)\cos 2\theta_1+ee_0\cos\theta_1]\phi \\
&+(\overline{m}e\ddot{w}\cos\theta_1-\overline{m}e\ddot{v}\sin\theta_1)+\overline{m}k_m^2\ddot{\phi}=Q_X
\end{aligned}
\tag{22a}
$$

$$
\begin{aligned}
&[(EI_1\cos^2\theta_1+EI_2\sin^2\theta_1)w''+(EI_2-EI_1)\sin\theta_1\cos\theta_1\cdot v''-EB_2\theta_1'\phi'\sin\theta_1]'' \\
&-(Tw')'-(Te_A\phi\cos\theta_1)''-(\Omega^2\overline{m}xe\phi\cos\theta_1)'+\overline{m}(\ddot{w}+e\ddot{\phi}\cos\theta_1) \\
&+\{\Omega^2\overline{m}[(k_{m2}^2-k_{m1}^2)\sin\theta_1\cos\theta_1\cdot v'+(k_{m2}^2\sin^2\theta_1+k_{m1}\cos^2\theta_1)w']\} \\
&+[\overline{m}(k_{m2}^2\sin^2\theta_1+k_{m1}^2\cos^2\theta_1)\ddot{w}'+\overline{m}(k_{m2}^2-k_{m1}^2)\sin\theta_1\cos\theta_1 \\
&\cdot\ddot{v}']'=Q_Z
\end{aligned}
\tag{22b}
$$

$$[(EI_2-EI_1)\sin\theta_1\cos\theta_1\cdot w''+(EI_1\sin^2\theta_1+EI_2\cos^2\theta_1)v''+Te_A\phi\sin\theta_1$$

$$-EB_2\theta_1'\phi'\cos\theta_1]''-(Tv')'-\Omega^2\overline{m}\Omega+\Omega^2\overline{m}e\phi\sin\theta_1+(\Omega^2 mxe\phi\sin\theta_1)'$$

$$+\{\Omega^2\overline{m}[(k_{m2}^2-k_{m1}^2)\sin\theta_1\cos\theta_1\cdot w'+(k_{m2}^2\cos^2\theta_1+k_{m1}^2\sin^2\theta_1)v']$$

$$+\overline{m}(\ddot{v}-e\ddot{\phi}\sin\theta_1)+[\overline{m}(k_{m2}^2-k_{m1}^2)\sin\theta_1\cos\theta_1\cdot\ddot{w}'$$

$$+\overline{m}(k_{m2}^2\cos^2\theta_1+k_{m1}^2\sin\theta_1)\cdot\ddot{v}']'=Q_y \qquad (22c)$$

$$Q_x = M_\phi - \overline{m}k_m^2\ddot{\theta}$$

$$Q_z = F_z - (\overline{m}e\ddot{\theta}\cos\theta_1)$$

$$Q_y = F_y$$

$$B_1 = \int_{\xi_{te}}^{\xi_{le}} t\xi^2\left(\xi^2+\frac{t^2}{6}-k_A^2\right)d\xi$$

$$B = \int_{\xi_{te}}^{\xi_{le}} t\xi\left(\xi^2+\frac{t^2}{12}-k_A^2\right)d\xi$$

t is the thick of airfoil

and B_1, B_2 are small in its quantity, can be neglcted. Combine the geometry boundary conditions, if the external forces Q_x, Q_y, Q_z, equal to zero, it is a well-known Sturm-Liouville problem. It can't be solved exactly. Therefore, a mode shape superposition has been co considered here.

Let

$$\phi = \sum_{k} A_{\phi k} \zeta_k$$

$$v = \sum_{k} A_{vk} \zeta_k$$

$$w = \sum_{k} A_{wk} \zeta_k$$

We can deduce an ordinary differential equation from the set of partial differential equation (22a), (22b) and (22c) through a miscellaneous performing

$$\ddot{\zeta}_k + 2\sigma_k \omega_k \dot{\zeta}_k + \omega_k^2 \zeta_k = \frac{1}{M_k}\left[F_k(t) + \Delta F_k(t) \right] + 2\sigma_k \omega_k \dot{\zeta}_k \tag{23}$$

where $\sigma_k \omega_k$ is the supposed damping coefficient, its value does not effect the solution of (23), but does effect the rate of convergence.

$$\begin{aligned} M_k = \int_{r_0}^{R} \{ & \bar{m} k_m^2 A_{\phi k}^2 - 2\bar{m} e \left(\operatorname{Sin}\theta_1 \cdot A_{vk} \cdot A_{\phi k} - \cos\theta_1 A_{wk} A_{vk} \right) \\ & + \bar{m} \left(A_{wk}^2 + A_{vk}^2 \right) - 2\bar{m} \left(k_{m2}^2 - k_{m1}^2 \right) \operatorname{Sin}\theta_1 \operatorname{Cos}\theta_1 A'_{wk} A'_{vk} \\ & - \bar{m} \left(k_{m2}^2 \operatorname{Cos}^2\theta_1 + k_{m1}^2 \operatorname{Sin}^2\theta_1 \right) A'_{vk} - \bar{m} \left(k_{m2}^2 \operatorname{Sin}\theta_1 \right. \\ & \left. + k_{m1}^2 \operatorname{Cos}^2\theta_1 \right) A'^2_{wk} \} \, dr \end{aligned} \tag{24}$$

$$F_k(t) = \int_{r_0}^{R} \left(Q_x A_{\phi k} + Q_z A_{wk} + Q_y A_{vk} \right) dr + \Delta Q. \tag{25}$$

$$\Delta Q = \int_{r_0}^{R} 2\overline{m}\Omega w' \dot{w} \; A_{vk} \, dr \tag{26}$$

$$\Delta F_k(t) = -\left[\mathrm{Sign}(\dot{\phi}_{i=2,NW} + \dot{\theta}_{i=0,NW} - \overline{K}\dot{w}'_{i=0,NW}) \cdot Q_{HDmax} A_{\phi k} - (\mathrm{Sign}\, \dot{v}'_{i=1,NW}) \, Q_{\psi\psi max} \, A'_{vk} \, (r_1)\right] \tag{27}$$

$$\sigma_k \omega_k = -\frac{1}{2M_k} \rho \Omega b \, C_{\ell\alpha} \int_{r_0}^{R} r \, A^2_{wk} \, dr \tag{28}$$

$$T = \int_{r}^{R} \overline{m} r \Omega^2 dr$$

Since the forcing function at the right side of equation (23) is a nonlinear function, in which some unknown quantities ζ_k and $\dot{\zeta}_k$ are involved. An iterative process must be made to obtain the solution, and the ζ_k and $\dot{\zeta}_k$ at right side terms are replaced by the previous values. Where the $A_{vk}(r)$, $A_{wk}(r)$, $A_{\phi k}(r)$ and ω_k are obtained from ref. [12] .

4. The Collective Pitch Correction

Early in the wake calculating, the initial collective pitch is based on the momentum theory. It is not too accuracy. So far, a collective pitch correction is necessary.

Thrust

$$T = \frac{K}{N\,A}\sum_{NW=1}^{NA}\sum_{i=1}^{N} F_{z,i,NW}\ \frac{1}{2}(\ r_{i+1} - r_{i-1}\ )$$

where NA is the numbers of azimuthal steps per revolution. When /T-G/ greater than a prescript quantity, then

$$\Delta\theta_{co} = \frac{G - \frac{K}{N\,A}\sum_{NW=1}^{NA}\sum_{i=1}^{N} F_{z,i,NW}\ \frac{1}{2}(r_{i+1} - r_{i-1})}{\frac{K}{N\,A}\sum_{NW=1}^{NA}\sum_{i=1}^{N} \rho\, b_i\, u_{i,NW}\, U_{i,NW}\, \overline{C}_{l\alpha}\frac{1}{2}(r_{i+1}-r_{i-1})} \qquad (29)$$

$$\theta_{co}^{(k+1)} = \theta_{co}^{(k)} + \Delta\theta_{co}^{(k)}$$

Repeat this process in section II . until

$$\delta = \frac{\sum_{NW=1}^{NA}\sum_{i=1}^{N}\left[F_{z,i,NW}^{(k+1)} - F_{z,i,NW}^{(k)}\right]^2}{\sum_{NW=1}^{NA}\sum_{i=1}^{N}\left[F_{z,i,NW}^{(k)}\right]^2} \leqslant 0.001$$

5, The Synthesis Expressions for Aerodynamic Coefficients

The airfoil lift, drag and pitch moment coefficients obtained from experiment suitable for the overall range of angle of attack among $0^\circ \sim 180^\circ$ is necessary in airload compute. For the sake of saving on space. They are omitted here.

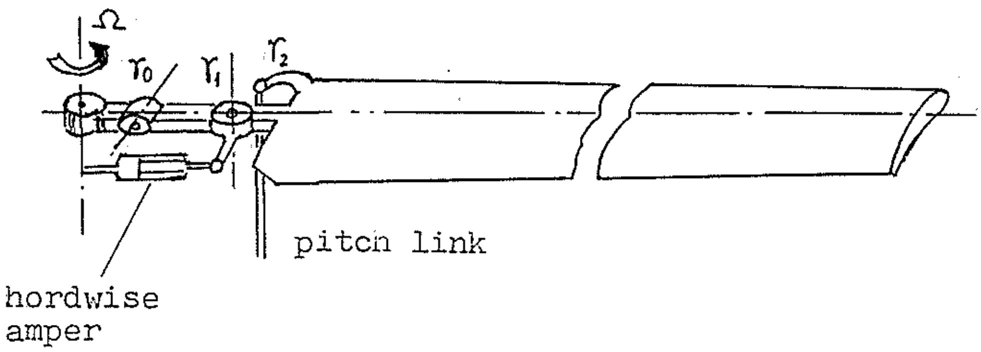
III The Resultant and Discussion

In order to check the availability of this method, two configurations for calculation are performed. the first one is H-34 [13] in μ=0.2498 and 0.288.Only the flapping deformation is considered. (i.e $v=\phi=0$. no chordwise deformation and torsion are considered) . The resultants of calculation are shown in fig. 6-8 latter, in which the resultants measured from flight test are shown in a small circle to compare with that from calculation . The second one is another helicopter model, which has a more complex connection in blade root with a combined deformations. Its results are shown in fig. 9-13 .

As mentioned above, there is a large gradient of circulation at tip , thus causing a strong tip vortex, it is concerned with the viscous flow. So far, its detail mechanism would be still unknown or known a little. It is very complicated. So does the flow pattern within the vortex core. Still,how to describe the vortex rolling-up appropriately would be interested us. further effort must be made on this subject.

IV Acknowledge

We should like to express here our appreciation to Mr. Han Qingrui for his work early in the paper. Aslo, we'd like to thank Mr Shen Xinkang for his part program in this paper.



An Illustration for the Articulations at Root

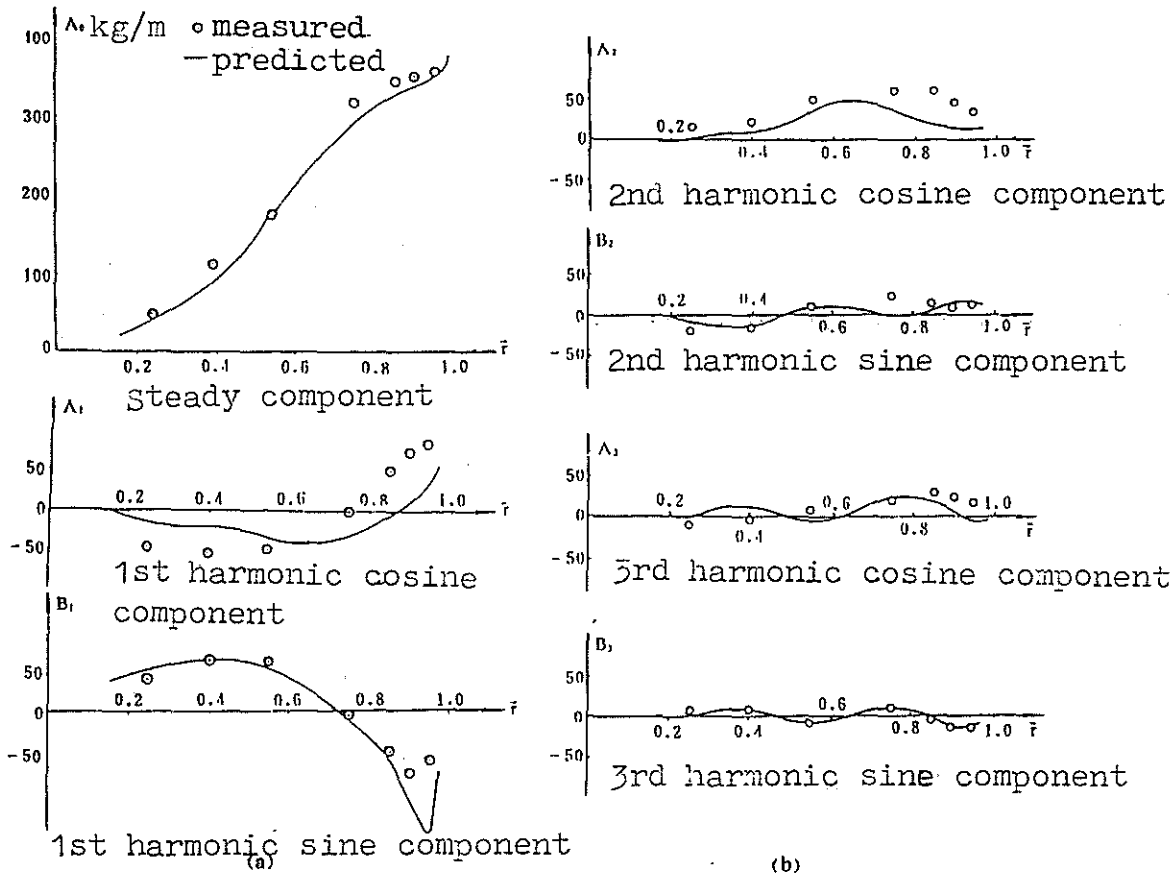


Fig. 6, H-34 helicopter rotor blade airloads various harmonic components comparision of measured and predicted (μ =0.2498)

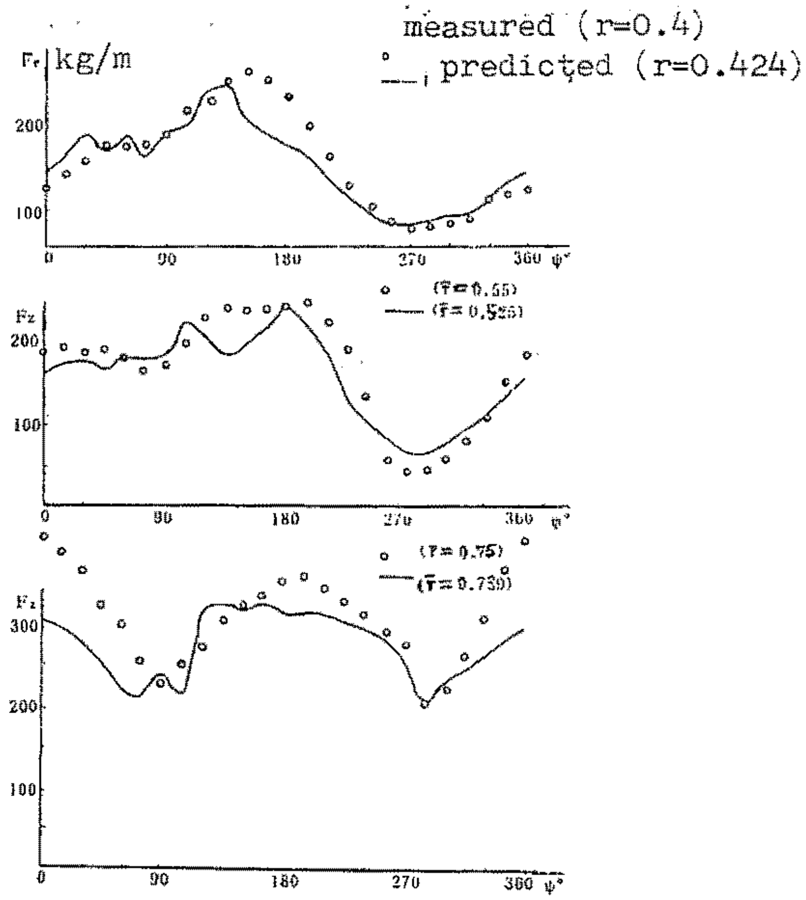


Fig. 7

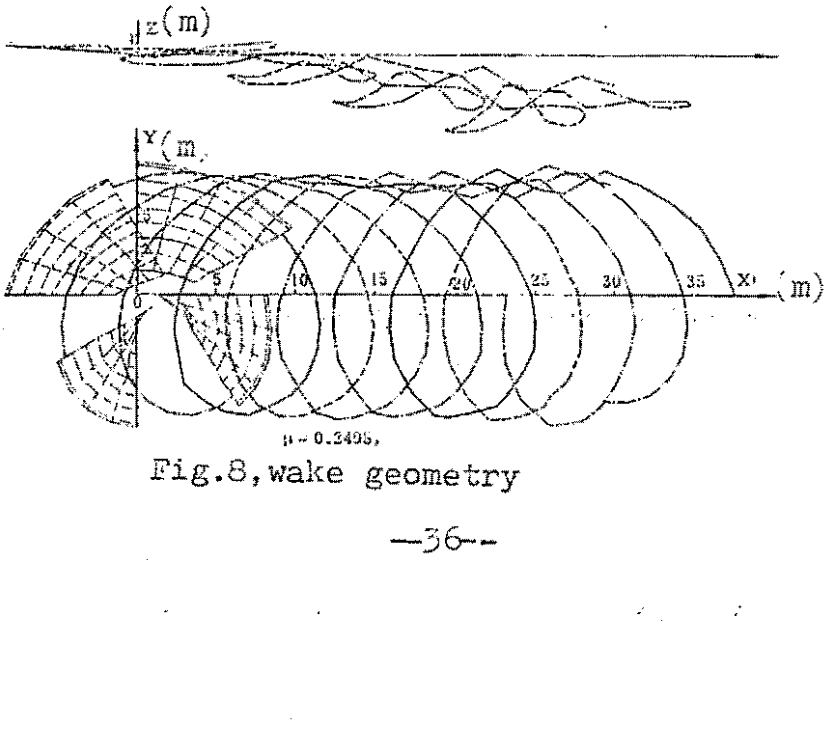


Fig.8, wake geometry

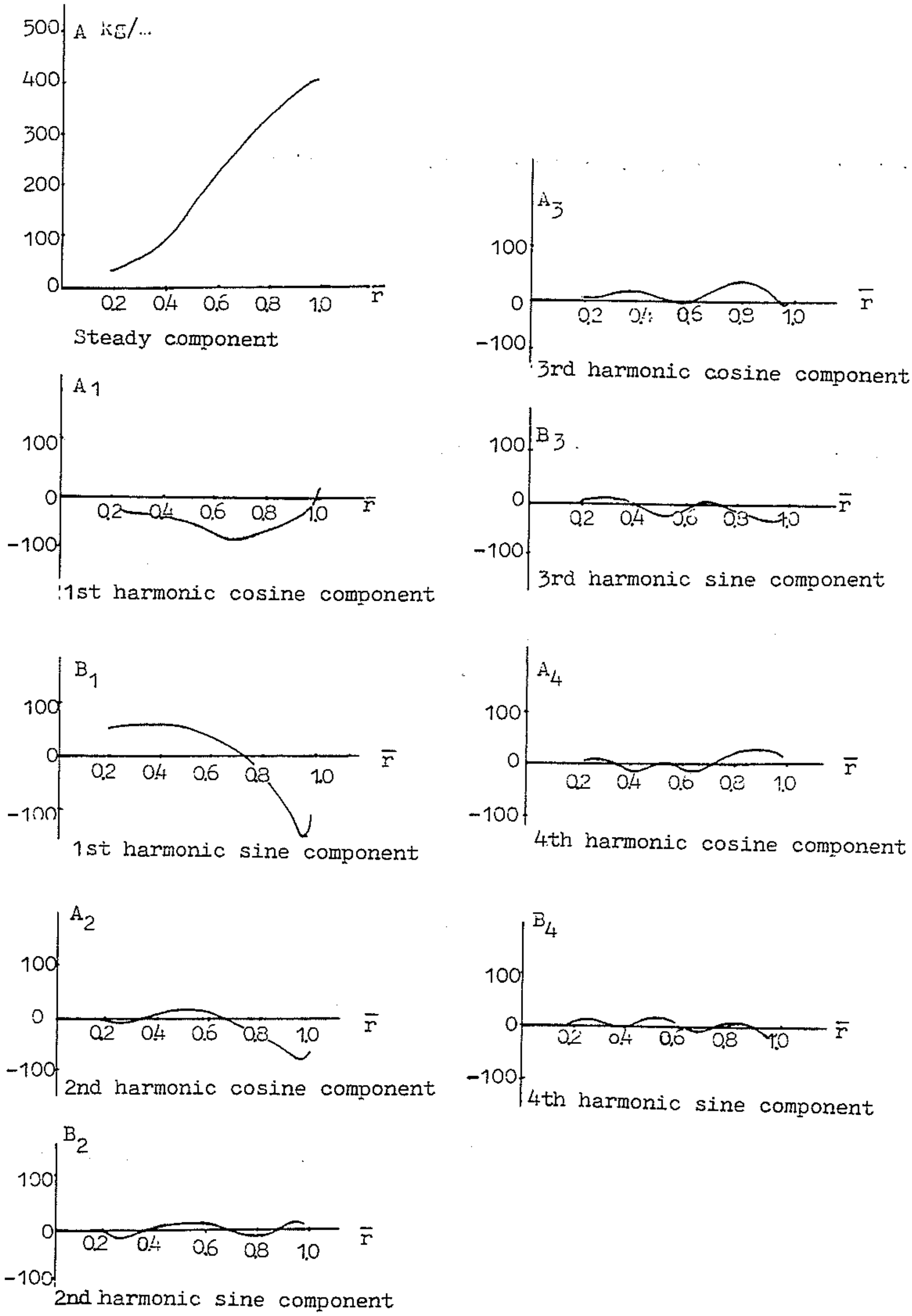


Fig.9, Some helicopter rotor blade airloads various harmonic component μ =0.2414 .

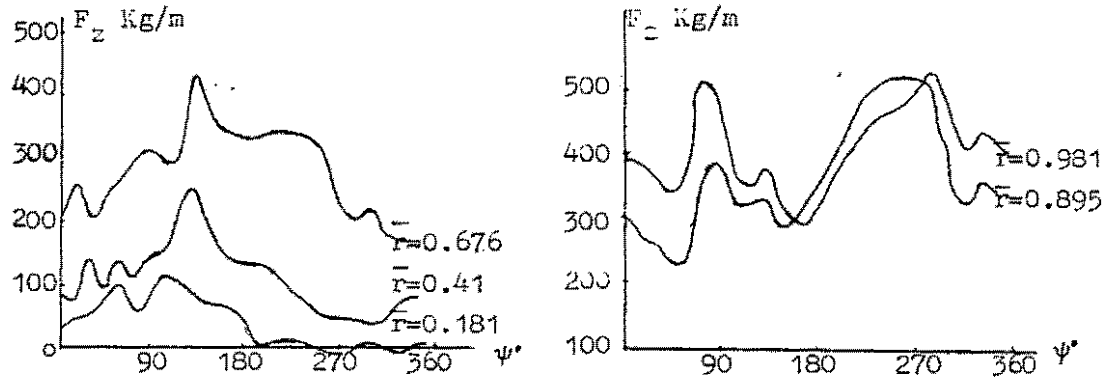


Fig.10 airloads per unit length variation with azimuth

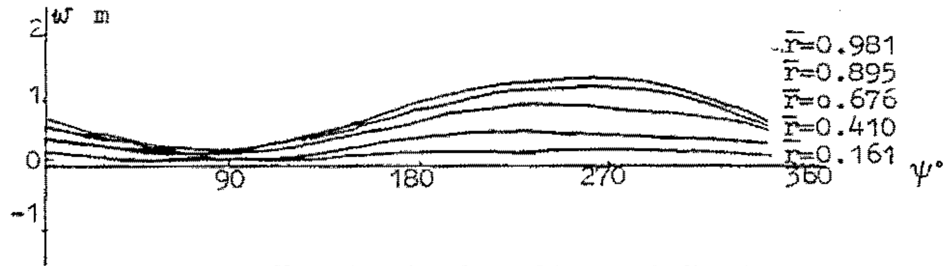


Fig.11 flapwise bending linear deflection variation with azimuth

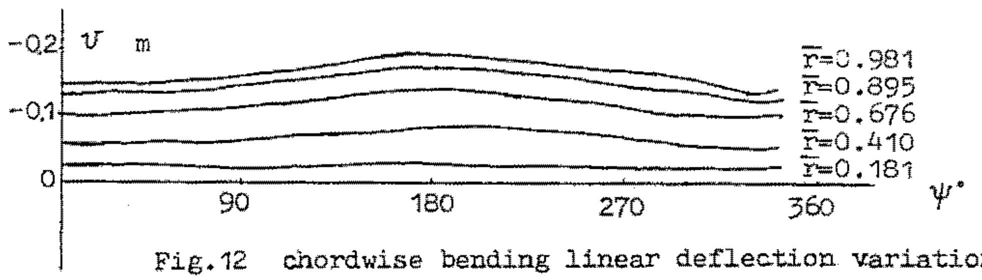


Fig.12 chordwise bending linear deflection variation with azimuth

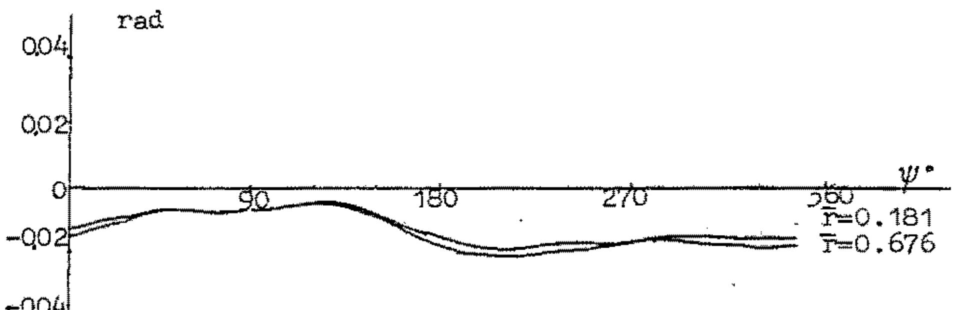


Fig. 13. torsion angular deflection variation with azimuth.

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