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AN ANALYSIS OF HELICOPTER ROTOR RESPONSE DUE TO GUSTS AND TURBULENCE

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ABSTRACT

Analyses are presented for the rotor and vehicle response to gusts and turbulence. The nature of helicopter alleviation factors is discussed and the rotor response to sinusoidal gusts is used to build up the aircraft spectral behaviour. The formulation of a more detailed study of step and ramp gust response is described briefly and preliminary results presented.

Notation

C_T	Thrust coefficient, $C_T = T/\rho(\omega R)^2\pi R^2$
T	Rotor thrust
ρ	Atmospheric density
ω	Rotor angular velocity
R	Rotor radius
a_o	Blade section lift curve slope
σ	Solidity, $\sigma = bcR/\pi R^2$
b	Number of blades
c	Blade chord
θ	Collective pitch
λ	Inflow ratio, positive for flow up through the rotor
μ	Tip speed (advance) ratio, $\mu = U/\omega R$
U	Forward speed
r_H, r_F	Relative gust sensitivity for the helicopter and the fixed wing aircraft respectively
$\Delta n_H, \Delta n_F$	Incremental load factors for the helicopter and fixed wing aircraft respectively
w_G	Gust velocity, positive upwards
K_H, K_F	Alleviation factors for the helicopter and aircraft respectively
$(W/S)_{HB}$	Blade loading
$(W/S)_F$	Aircraft wing loading
T_R	Rotor thrust lag time constant
K_R	Rotor gain constant
m	Helicopter mass
W	Fuselage vertical velocity, positive downwards
D	Operator, $D \equiv \frac{d}{dt}$
r	Radial co-ordinate
x	Non-dimensional radial co-ordinate, $x = r/R$
γ	Lock Number, $\gamma = \rho a_o c R^4/I_\beta$
ψ	Blade aximuthal angle, zero at the rear of the disc
I_β	Blade moment of inertia about the flap hinge
β	Flap angle
K_1	Wave number of a gust varying sinusoidally in space
$J_o(k_x)$, $J_1(k_x)$	Bessel functions of the first kind
K	Non-dimensional wave number, $k = k_1 R$
p	Non-dimensional rotational frequency
$\varepsilon_o, \varepsilon_1$	Phase angles

λ_G	Gust velocity amplitude non-dimensionalised with respect to ωR
ϕ_N	Power spectral density of the normal load factor increment
ϕ_G	Power spectral density of the turbulence
$Z_1(p), Z_2(p)$	Frequency response functions
L	Turbulence scale
L_G	Gust ramp length
a_0f_1, a_1f, b_1f	Fourier coefficients defining blade flapping
S'	Laplace transform **variable**

1. Introduction

The presence of gusts in the plane of and normal to the rotor disc of a helicopter can be readily accommodated by considering the changes of incidence and dynamic head at each blade section. Provided the gust description is accurate, the detailed blade response stresses and transmitted forces can be estimated and critical design cases and fatigue loadings determined. For fuselage critical loading, a sharp edged gust which instantaneously immerses the rotor uniformly over the disc is used. Gross rotor response and gust grading can be incorporated, as in fixed wing aircraft, into an alleviation factor. However, the basis for this factor is not well established and in U.K. Civil Airworthiness Requirements a unity factor is required.[1] It has been suggested[2] that current U.S. factors are conservative and based on the wrong parameter. In the present paper, an attempt is made to assess some of the major contributions to rotorcraft alleviation factors.

Sharp edged gust response involves transient system response. Classically, fixed wing aircraft flight in continuous turbulence employs random but stationary inputs[3] resulting in a steady state forced response with spectral content built up from the frequency response to gusts distributed sinusoidally in space. A more realistic presentation of turbulence[4] involves discrete regions of high intensity and both transient and spectral components must be considered. In the case of rotorcraft there is little reported study of turbulence response, even using the spectral approach. Extra complications exist. For example, the flapping motion is a non-stationary process itself because the governing differential equation, although linear, has time varying coefficients[5]. Moreover, the flapping moment produced by a sinusoidal gust is a complex function of forward speed and gust wavelength as a ratio of rotor radius. This second complication is considered in the present paper on the basis of a simplified rotor and fuselage response model. Finally, the philosophy for a more exact numerical study is described. This program should be capable ultimately of dealing with a full range of step, ramp and sinusoidal gust inputs.

2. Gust Sensitivities and Comparison with Fixed Wing Aircraft

The manoeuvre and gust envelopes of conventional fixed wing aircraft are well-established and proven. By contrast it is difficult to define generally critical gust cases for helicopters and envelopes do not exist. The blade loads resulting from gusts must be estimated for design purposes in a different way to the fuselage loads arising from the same gust. This distinction is recognised in the British Civil Airworthiness Requirements[1] by the use of a 35ft/sec step gust with unity alleviation factor for fuselage stressing and a ramp gust of 50ft/sec in 100ft for individual blade stressing. It has been suggested[2] that current U.S. military specifications result in conservative alleviation factors which are based, incorrectly, on disc loading. It is therefore instructive to consider some basic features of gust sensitivity and compare with fixed wing results.

The relative sensitivity to vertical and fore/aft gusts can be assessed for the articulated rotor from the form of the thrust coefficient (see the list of Notation for the definition of symbols):-

$$C_T = \tfrac{1}{2}a_o\sigma\left(\frac{\theta}{3} + \mu^2\frac{\theta}{2} + \frac{\lambda}{2}\right)$$

A measure of the relative sensitivity is:-

$$r_H = \frac{\partial C_T}{\partial \mu} \Big/ \frac{\partial C_T}{\partial \lambda}$$

$$= \frac{12}{a_o}\,\frac{\mu}{(2 + 3\mu^2)}\,\frac{2C_T}{\sigma} \qquad \text{for small } \lambda$$

The mean blade lift coefficient $2C_T/\sigma$ is seen to be an important parameter here and maximum values are shown in Fig.1, for a conventional articulated rotor and for an advanced rotor such as the ABC concept in which lift capability at high μ is exploited. The effect on relative sensitivity is shown in Fig.2. For the advanced rotor, in-plane gusts produce a significant change in the thrust and shaft transmitted load at high forward speed but for the articulated rotor, response is dominated by the vertical gust. In-plane gusts will result largely in disc inclinations for the articulated rotor, with associated body moments. For the fixed wing aircraft, relative gust sensitivity is given approximately by:-

$$r_F = 2C_L/a_1$$

where C_L is the flight lift coefficient and a_1 is the lift curve slope for the complete aircraft. In cruise, r_F is typically 0.1 to 0.2 and vertical gusts dominate response but in landing or take-off configuration the value may approach unity and augmented lift STOL aircraft will be highly sensitive to fore and aft gusts.

The absolute incremental load factors due to a vertical gust can be expressed as:-

$$\Delta n_H = \tfrac{1}{2}\rho a_o \frac{\omega R}{(W/S)_{HB}}\, w_G\, K_H \qquad \text{for the helicopter}$$

$$\Delta n_F = \tfrac{1}{2}\rho a_1 \frac{U}{(W/S)_F}\, w_G\, K_F \qquad \text{for the fixed wing aircraft}$$

For the aircraft, gust sensitivity is reduced by a high wing loading $(W/S)_F$. The helicopter gust response depends on blade loading rather than disc loading and is to a first order independent of forward speed. Direct comparison can be made between the two types of vehicle using this approach[c] but the form and magnitude of the helicopter gust alleviation factor K_H remains uncertain. The factor K_H is considered further in the next section.

3. A Simplified Analysis of Body Response to Sharp Edged Gusts

The major contributions to the alleviation factor arise largely from the rotor characteristics such as blade flap, bending, lagging and torsion; section lift, wake adjustment and dynamic stall. The relative importance of these can be assessed using the simplified model presented in Fig.3. The rotor is represented as a simple transfer function with a general time constant T_R characterising the lag between the step gust input w_G and the rotor thrust increment T. The rotor gain constant is given by:-

$$K_R = \frac{1}{4} \rho\omega R\pi R^2 \sigma a_o$$

The aerodynamic force on the fuselage due to its vertical velocity w is neglected and it is assumed that the motion is controlled by the fuselage mass m. The governing equation for w can be readily obtained and is second order in the operator D. The solution for a step input can be found subject to the initial conditions that the velocity w and the incremental thrust are both initially zero. The latter condition implies that the initial acceleration is also zero. The incremental load factor can be obtained from the velocity solution as:-

$$\Delta n(t) = -\frac{Dw}{g}$$

This reaches a maximum value which can be normalised with respect to its value when $T_R = 0$, to give the alleviation factor. The factor estimated in this way is presented in Fig.4 and is a unique function of the non-dimensional variable $4K_RT_R/m$. The time constant T_R is more easily appreciated if expressed in terms of an angular lag in azimuthal position ψ_R where $\psi_R = \omega T_R$. The upper scale in Fig.4 is obtained if values appropriate to the Sea-King are used. A close approximation to the form of the alleviation factor is:-

$$K_H = 1 - 0.532\sqrt{K_R T_R/m}$$

The magnitude of the rotor lift time constant T_R will depend on the blade lift build-up and any resultant blade response. The major effects are summarised as follows:-

Blade lift development

1) Section lift. The two-dimensional wake adjustment will result in a lag in lift formation. For 90% lift development at the tip, the Kussner function for step gusts gives $\psi_R \simeq 3c/R$ radians and is therefore typically about 10^o and small.

2) Dynamic stall. The dynamic delay in stall means that the lift increase due to an upgust may not be limited as it would be by static maximum lift coefficient.[2]

3) Trailing vortex wake. There is a delay in the wake strength adjustment to new blade lift states and a corresponding delay in induced velocity change at the rotor. The rotor lift is therefore closer to its quasi-steady value and there is an effective small negative contribution to the lag time constant.

4) Gust penetration. The time taken for complete disc immersion in a sharp edged gust results in an effective lag $\psi_R = 2/\mu$ radians. With multi-

bladed rotors the build-up will be more rapid than this but values in excess of 360° would be normal.

5) Gust grading. If the gust is represented as a linear ramp function of length L_G, then the immersion process results in a delay

$$\psi_R = \frac{2}{\mu} + \frac{L_G}{\mu R}$$

Blade flap and bending response

If the blade lift were capable of development in a step manner, then this step would be transmitted as a step rotor lift with no alleviation due to flap or flexural response. However, the blade lift must develop in a finite time from the above considerations and the flap response will occur thereby alleviating the rotor lift. The flap time constant is given approximately by $\psi_R \simeq \gamma/8$ radians.

From the above considerations, it can be seen that the most significant contributions towards gust alleviation are the penetration time and gust ramp length. To this must be added the flap response when the initial blade lift build-up is not too rapid. Fig.4 shows that a significant amount of alleviation will occur.

4. Response to Sinusoidal Gusts and Turbulence

To formulate the problem in a way which is analytically tractable yet capable of realistic interpretation, the following assumptions are made:-

1) The rotor is articulated, untwisted, two-bladed with zero offset and zero hinge stiffness. The blades are rigid.

2) The fuselage motion is limited to vertical translation and no consideration is given to the other longitudinal modes of motion. For most practical situations the fuselage velocity will be sufficiently small that it has no effect on blade flapping motion.

3) The blade flap response can be determined by an expansion in powers of the tip speed ratio. The zero order forced equation then becomes:-

$$\beta'' + \frac{\gamma}{8}\beta' + \beta = \frac{\gamma}{2\omega R}\int_0^1 x^2\, w_G(x,\psi)\,dx \qquad (1)$$

where a prime denotes differentiation with respect to ψ, β is that part of the flap motion resulting from the gust, γ is the Lock Number and $w_G(x,\psi)$ is the up-gust velocity at a non-dimensional radial station x when the blade is at azimuthal angle ψ. In a full analysis, terms of order μ could be included to enable the determination of disc tilt response. At very high forward speed it would be necessary to use the flapping equation with periodic coefficients[5].

The turbulence is represented in the classical way as a spatially frozen pattern of sinusoidal gust components through which the helicopter is flying. The turbulence is assumed to be one dimensional and only wavelengths down to the rotor radius are considered. This is adequate in evaluating the effect of turbulence on fuselage stressing, passenger and crew comfort and interference with controllability. A wider range of wavelengths is required for blade stressing[6,7].

A rotor moving with tip speed ratio μ through a sinusoidal gust of wave number k_1 will have a gust velocity distribution at the blade which varies with radial position and has an infinite number of frequency components. It can be shown that for wavelengths greater than the rotor radius the components with the two lowest frequencies contribute to w_G as follows:-

$$w_G(x,\psi) = \omega R\ \lambda_G\{J_o(kx)\sin(p\psi + \varepsilon_o) + J_1(kx)\sin\left[(1 - p)\psi + \varepsilon_1\right]\} \qquad (2)$$

where λ_G is the non-dimensional gust amplitude, $J_o(kx)$ and $J_1(kx)$ are Bessel functions of the first kind, $k = k_1R$ and $p = \mu k$. Here p is less than unity and all other components have frequencies above the rotational speed of the rotor ($p = 1$). The frequency p is produced as in fixed wing aircraft by the flight at speed U through the gust of wave number k_1. In the case of the rotorcraft, there is an extra term with frequency given by the difference term $1 - p$. The steady state flap response can be readily found from equations (1) and (2). The rotor lift input can then be determined and hence the fuselage acceleration response. The fuselage is modelled as a simple mass and aerodynamic forces due to vertical velocity are neglected. The acceleration will have the same two frequency components but, because they will not be the same in general, there will be no contribution to the mean statistical power from the cross product. The power spectral density ϕ_N of the normal load factor increment can therefore be obtained from the turbulence power spectral density ϕ_G from the form:-

$$\phi_N = \{|Z_1(p)|^2 + |Z_2(p)|^2\}\phi$$

where $Z_1(p)$ and $Z_2(p)$ are the frequency response functions for the p and $(1 - p)$ frequency terms respectively. The form of ϕ_N is shown in Fig.5 for a helicopter with the characteristics:-

$$\mu = 0.2 \qquad C_T = 0.005 \qquad a_o = 5.8 \qquad \frac{c}{R} = 0.05 \qquad \omega R = 700 \text{ ft/sec.}$$

The von Karman[3] model was adopted for the turbulence, a range of turbulence scales (L) used and the rms turbulence level set at 5 ft/sec. The effect of the $(1 - p)$ component is evident in the second hump at frequencies just below $p = 1$ and this makes a significant contribution to the overall rms load factor increment shown in Fig.6 as a function of turbulence scale. Since the present analysis is intended only to lay the foundations for a more detailed investigation, extensive comparison with fixed wing sensitivity has not been undertaken but the indications are that the helicopter is more sensitive to turbulence at the larger scales. The analysis can be readily extended to include the full fuselage and rotor motions so that full comparison with flight data and fixed wing results can be made.

5. An Analytic Method for Helicopter Gust Response

The analysis to be described is concerned with a more detailed examination of the stick-fixed response of a helicopter when entering a gust. Initially the fuselage was assumed fixed but fuselage motion in one direction was considered later to estimate gust alleviation factors. The initial fixed fuselage model is appropriate to a rotor rig test.

In analysing the steady state flapping motion of a rotor blade, a flapping angle of the following form

$$\beta = a_o f - a_1 f.\cos\psi - b_1 f.\sin\psi \qquad (3)$$

$$(a_o f,\ a_1 f,\ b_1 f \text{ constant})$$

is substituted into the flapping equation of motion. Coefficients of 1, $\cos\psi$, and $\sin\psi$ are equated on both sides of the equation. This gives three simultaneous algebraic equations, the solving of which determines $a_o f, a_1 f, b_1 f$. The transient flapping motion of a rotor as it changes from one steady state to another is determined in a comparable manner. The flapping angle expression (3) is assumed, however $a_o f, a_1 f, b_1 f$ are allowed to be functions of rotor rotation ϕ from the instant of gust entry. The gust is assumed to envelop the rotor uniformly.

Similar substitution and manipulation leads to three simultaneous differential equations for $a_o f(\phi)$, $a_1 f(\phi)$ and $b_1 f(\phi)$. These can be solved analytically for a step gust by a Laplace transform method which reduces the differential equations to algebraic equations. The rotor behaviour to a general gust can be evaluated from the step gust response by using the Duhammel superposition integral. Once the flapping behaviour has been determined, the individual blade and rotor thrusts can be ascertained.

Computer programs have been written to perform the above analysis on step and ramp gusts and consist of the following steps.

From the initial flight data, the pre-gust motion steady state is calculated. The gust is superimposed and the final steady state evaluated (this provides an independent check on the validity of the calculated transients).

The rotor induced downwash is assumed uniform over the rotor disc and remains constant throughout the transient motion. (This implies the wake adjustment is slower than the rotor adjustment).

The result of the application of Laplace transforms to the three simultaneous differential equations requires inversion of the following S function

$$L(S') = \frac{P(S')}{S.Q(S')}$$

where $P(S')$ and $Q(S')$ are polynomials.

Since $L(S')$ is menomorphic the inversion applied residue theory to the

Bromwich Integral thus:

$$\int_{Z-i\infty}^{Z+i\infty} L(S')e^{S'\phi}\, dS'$$

this gives

$$\mathcal{L}^{-1}\{L(S')\} = \sum_j \left. \frac{P(S')}{\frac{d}{dS'}\left[(S'Q(S'))\right]} e^{S'\phi} \right)_{S'=\alpha_j}$$

where α_j are the roots of $S'Q(S') = 0$.

The trivial root $S' = 0$ gives a constant term in the inversion which represents the difference between the initial and final steady state values. The six roots of the polynomial $Q(S')$ give the transient motion linking these two steady states.

Once the flapping motion is obtained, the individual blade and rotor thrust history can be determined.

The time periods to attain final steady state flapping are small enough to justify a fixed fuselage undergoing a step gust. However for a ramp gust of large extent the time period is substantially longer, and the fuselage motion, which is such to alleviate the gust, will have become an important factor. Hence the results produced by the ramp gust programme with a fixed fuselage must be treated with caution.

In order to gain more realistic results for the ramp gust and to obtain estimates for gust alleviation factors, rectilinear fuselage motion along the shaft axis was introduced into the equations, and the computer programs extended to cater for the more involved analysis.

Figure 7 shows the thrust histories of the Sea King rotor calculated for the four gust cases (vis. fixed/free fuselage and step/100 foot ramp gust). Figure 8 shows the variation in rotor thrust lag (fixed fuselage) from the instantaneous steady state for a range of Lock Numbers, flapping frequencies and ramp lengths. High stiffness and a low Lock number produce the lowest thrust lag, but this lag is of longer duration than with a longer maximum thrust lag value. Lower thrust lag values are produced with a longer ramp length.

Figure 9 shows the calculated gust alleviation factors for the same rotor characteristics and ramp lengths as Figure 8. The greatest alleviation is produced by high Lock number, low stiffness and a large ramp length.

6. Conclusions

Current articulated and semi-rigid rotors are insensitive to in-plane gusts. Projected high lift rotors may be more sensitive. Normal gust load factors for the helicopter are roughly independent of forward speed and inversely proportional to blade loading (rather than disc loading). Rotor penetration time, ramp length and flap response are the major factors in gust alleviation and significant amounts of alleviation will occur. Response to a sinusoidal gust has two major components (of differing frequency) in the flap motion and rotor lift. This results in a double peaked power spectral density for the turbulence response. The method is suitable for extension to a full analysis of helicopter response to turbulence for comparison with flight data and fixed wing results.

The trends shown by the preliminary results of the more detailed approach to step and ramp gusts have already been outlined. It is apparent with the free fuselage model that a significant time period is required to reach the final steady state (1-2 seconds). This is due to the extra root of the denomination being real and of small magnitude. The fuselage motion equation can be reasonably decoupled giving a solution containing the slow decay root. This will then affect the flapping motion via the other equations.

The fixed fuselage model when examining the effect of Lock number produced a change in the roots at a value of γ in the region of 20. For lower γ values, six complex roots were obtained, higher γ values produced four complex and two real roots, one being of slow decay.

7. References

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MEAN BLADE LIFT COEFFICIENT
MAXIMUM CAPABILITY

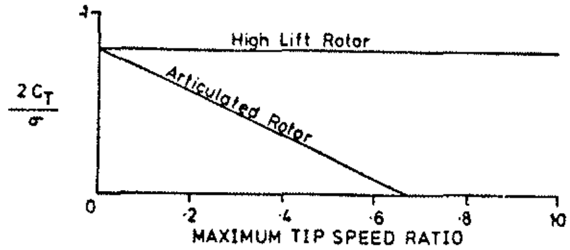


Fig 1

RELATIVE GUST SENSITIVITY

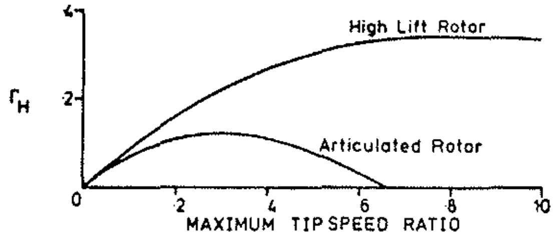
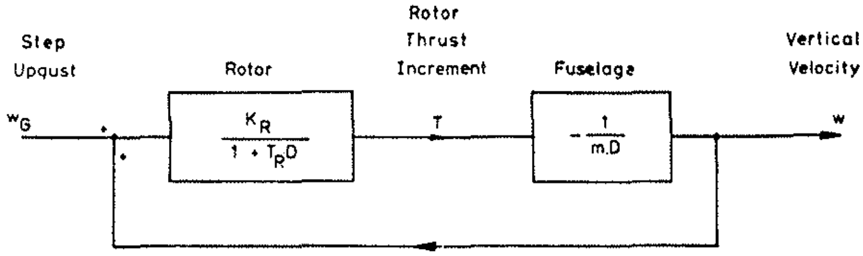


Fig 2



SIMPLIFIED GUST RESPONSE MODEL

Fig 3

ALLEVIATION FACTOR

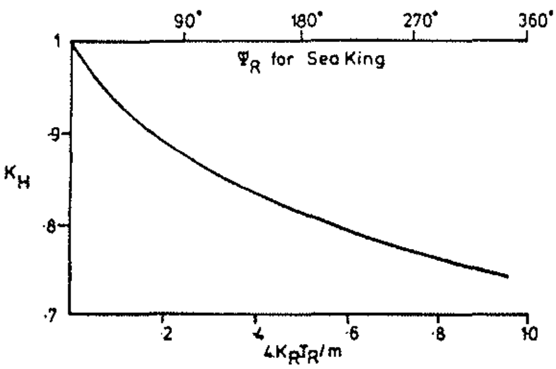


Fig 4

POWER SPECTRAL DENSITY
OF LOAD FACTOR

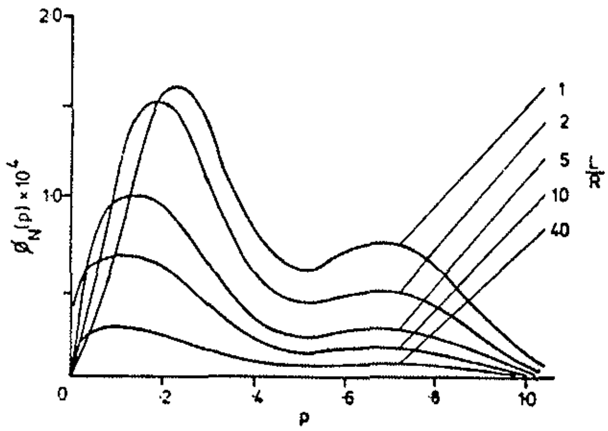


Fig 5

R.M.S LOAD FACTOR INCREMENT

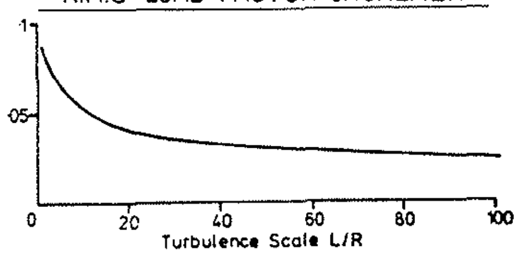


Fig 6

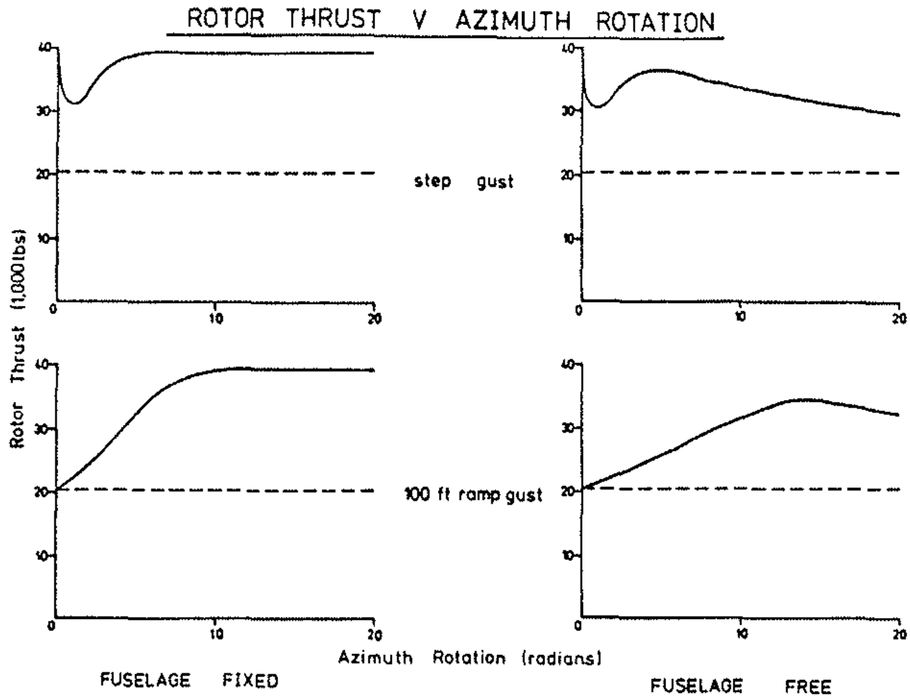

ROTOR THRUST V AZIMUTH ROTATION
Rotor Thrust (1,000 lbs)
step gust
100 ft ramp gust
Azimuth Rotation (radians)
FUSELAGE FIXED
FUSELAGE FREE

Fig 7

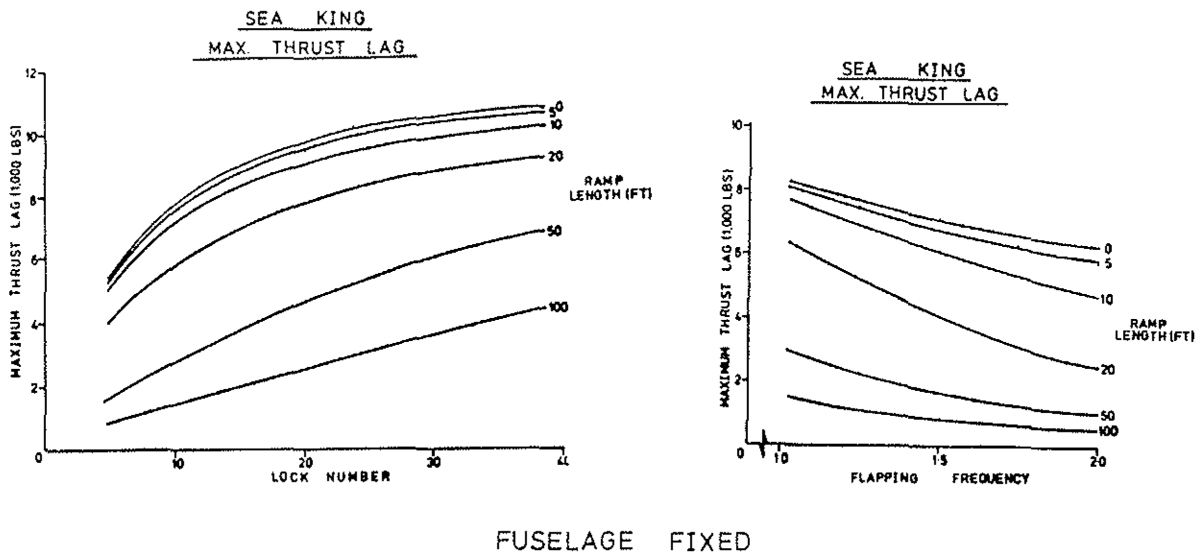
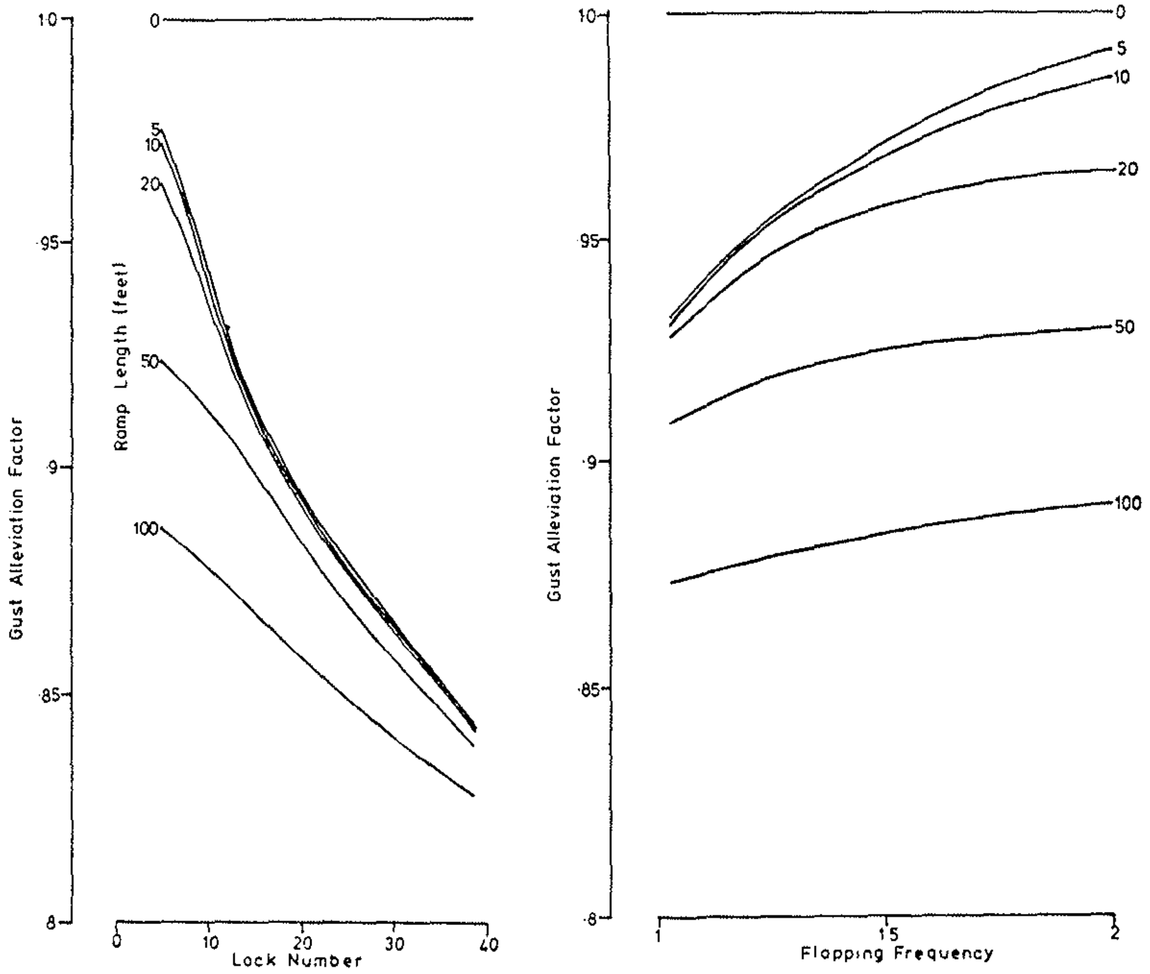

SEA KING
MAX. THRUST LAG
MAXIMUM THRUST LAG (1,000 LBS)
LOCK NUMBER
RAMP LENGTH (FT)
SEA KING
MAX. THRUST LAG
MAXIMUM THRUST LAG (1,000 LBS)
FLAPPING FREQUENCY
RAMP LENGTH (FT)
FUSELAGE FIXED

Fig 8

GUST ALLEVIATION FACTORS



FUSELAGE FREE

Fig 9