FAILURE ANALYSIS METHOD FOR THE PRESIZING OF MULTI-ROTORS eVTOL

Pierre-Marie BASSET, Raphaël PERRET

ONERA – The French Aerospace Lab, FRANCE

ABSTRACT

The new kind of aircraft electric Vertical Take-Off and Landing (eVTOL) concepts are well suited for the Advanced Air Mobility including Urban Air Mobility. The new paradigm of design for these Advanced Aircraft no longer relies on autorotation capability, but on redundancy. Redundancy is one required condition, but it may not be enough to ensure a safe controlled flight in case of failure of one or more thrust generator systems, i.e. {energy system – controller - motor – propeller}. This paper focuses on the crucial issue of setting a methodology to assess if an eVTOL configuration is able to cope with one or more thrust generator failure(s). This is a crucial step for their predesign because the thrust generators must be sized to withstand the most demanding conditions, which are imposed by the worst critical failure cases. Therefore, the analysis of the different failure cases is needed before the sizing of the rotors and more generally of the thrust generator system. A numerical method is presented as well as results giving a comprehensive overview for the axi-symmetrical circular multi-rotors configurations with or without co-axial rotors and more generally for eVTOL with all rotors thrusting always in the same unique direction.

INTRODUCTION

eVTOL stands for electric Vertical Take-Off and Landing aircraft. This VTOL capability is of course needed to avoid the dependency to runways or other infra-structures like catapult and recovery systems. Rotary wings aircraft have not only this advantage, they also have a very good maneuverability at low speeds and they are less noisy than other VTOL concepts like turbine jet aircraft. That is why most of the eVTOL concepts use rotary wings (rotors or propellers) which are distributed around the aircraft center of Gravity (cG) to create forces and moments to control its six degrees of freedom by tilting or not the aircraft body (e.g. Volocopter concept with its 18 lifting propellers).

In the case of missions requiring significant cruise phases, some concepts combine rotary wings and fixed wings which remain the most efficient mean to create lift in forward flight. These aerodynamic components (rotary and/or fixed wings) can be tilted on some concepts (e.g. the Vahana concept with four tilting wings each carrying two propellers tilting with the wings).

Helicopters, Tandems, Tilt-Rotor (e.g. XV15, V22) and Compound Helicopters (e.g. X2 and X3) have used these basic ideas for a long time in order to combine the hover and low speed flying capability with higher speeds (see an overview on the rotorcraft concepts explored by the helicopters pioneers in [1]).

These conventional rotorcraft rely on one or two thermal engines (piston only for light helicopters or turboshaft engines for the others) which rotate at a constant revolution speed few rotors through heavy and complex mechanical systems. They are equipped with one or two large lifting rotors controlled in collective and cyclic pitch angles through a complex swashplate system. Thanks to these blade pitch angle controls and enough inertia, these large rotors are able of autorotation.

The new kind of aircraft studied here are based on the principle of Distributed Electric Lift and Propulsion (DELP) with electric motors placed at the nearest of the propellers (most of the time with a direct coupling or more rarely with a gearbox) and the energy transmitted by electric cables from an energy source (batteries etc.). By this way DELP avoids mechanical loss and weights inherent to mechanical transmission. The lifting and propulsive propellers are controlled in rotation speed (rpm) with usually a fixed blade pitch, and are not able of autorotation. Therefore, the new paradigm of design with DELP relies on redundancy.

Redundancy is one required condition, but it is not a sufficient condition by itself to ensure a safe controlled flight in case of failure of one or more thrust generator systems, i.e. {energy system – Electronic Speed Controller - motor – propeller}. This paper focuses on the crucial issue of setting a methodology contributing to the presizing of eVTOL configurations able to cope with one or more thrust generator failure(s). This step is crucial for the predesign of such new rotorcraft concepts. Indeed the thrust generators must be sized on the most demanding conditions, which are imposed by the worst critical failure cases. Therefore, the analysis of the different failure cases is needed before the sizing of the rotors and more generally of the thrust

Presented at the 48th European Rotorcraft Forum, Switzerland, September 6-9, 2022.

generator systems: the maximum thrust and torque impacts mainly the motor presizing while the maximum power and energy demand impact mainly the power-energy system.

Considering Urban Air Mobility (UAM) applications, the flights over dense populated areas require that the aircraft system must be safe with a probability of catastrophic failure below 10^{-9} (by fly hour). Taking the realistic assumption that an electrical thrust generator system is safe with a probability of failure about 10^{-4} (by fly hour), that leads to the condition that these new advanced aircraft for UAM must be resilient to any two thrust generators failures.

A methodology for thrust generators failure analysis has been developed based on what we initiated in [2]. The first part of this paper describes it carefully as well as the models and algorithms. The second part demonstrates its interest mainly on the practical case of axi-symmetrical circular multi-rotors eVTOL configurations. These examples of application provide a comprehensive understanding on the optimal redistribution of the thrusts and the effect on the torques and powers as well.

1 METHODOLOGY FOR THRUST FAILURE ANALYSIS

The issue is: when one or two thrust generators are in failure, what is the optimal redistribution of the rotor thrusts in order to still maintain a steady flight condition. This crucial question raises from the early stages of the presizing, while trying to dimension the propellers, motors and energy system in terms of maximum thrust, maximum torque, maximum power and energy.

A schematic view of a first presizing loop (for a predesign candidate converged on the Design Gross Weight DGW) is shown on Figure 1.

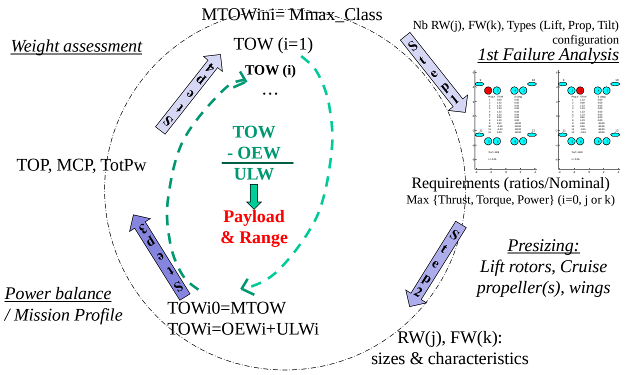

Figure 1: First Presizing Loop, convergence on DGW

The step 1 is a first failure analysis in order to get a first assessment of the ratios of maximum thrust, torque and power relative to the nominal case:

$$Tmax/T_0,\ Qmax/Q_0,\ Pmax/P_0$$

with T_0 a very first estimate of the nominal thrust of the n identical lifting rotors : $T_0 = DGW/n = M.g / n$

and Q_0, P_0 the corresponding nominal torque and required power on one of the n lifting rotors for the “All Engine Operating” reference case. Thus, even if at this early stage the lifting rotors are still not well presized, the critical effect of the failure on the maximum thrust, torque and power can be taken into account in relative value. For the failure condition contributing to the most demanding flight cases (together with other conditions like Maximum Take-Off Weight, high and hot atmospheric condition, winds etc.), these ratios ($Tmax/T_0$, $Qmax/Q_0$, $Pmax/P_0$) are key inputs for the presizing in the step 2 of Figure 1.

The Hover Out of Ground Effect (HOGE) flight case is one of the most demanding one because of the high values of the required power by the lifting rotors. That is why this flight condition (HOGE) is first considered here without wind in order to set up the methodology on a simple yet relevant flight case for the presizing needs. The hovering or forward flights with wind could be dealt with by adapting the methodology presented below.

In this very first thrust failure assessment of a configuration (before its accurate presizing), its capability to maintain a trimmed flight in HOGE is evaluated as a first example of needs regarding its controllability. For this purpose, a flight dynamics code *DynaPyVTOL* (ONERA in-house code) is used. It allows simulating the flight dynamics of any aircraft by using models adapted to the presizing studies. Multi-fidelity models are available from the simplest ones (e.g. analytical rotor model based on momentum theory) to numerical models based on lifting line theory. It is able to calculate the trim of the 6 degrees of freedom of the aircraft.

For example for the case of axi-symmetrical circular multi-rotors configurations with n identical lifting rotors each generating a vertical thrust force (T_k) and a torque (Q_k), the 6 rigid body flight dynamics trim equations are :

$$\text{Lift} \quad \sum_{k=1}^{n} T_k = W + Drag_z$$

$$\text{Longitudinal} \quad \sum_{k=1}^{n} T_k \sin\theta = Drag_x$$

$$\text{Lateral} \quad \sum_{k=1}^{n} T_k \sin\phi = Drag_y \qquad \text{(eq. 1)}$$

$$\text{Roll} \quad \sum_{k=1}^{n} T_k y_k = 0$$

$$\text{Pitch} \quad \sum_{k=1}^{n} T_k x_k = 0$$

$$\text{Yaw} \quad \sum_{k=1}^{n} Q_k = 0$$

Where W is the weight force, ($Drag_x$, $Drag_y$, $Drag_z$) are the aerodynamic drag forces. In HOGE without wind, there is no horizontal nor vertical airspeed, therefore no drag and thus no pitch and bank angles ($\theta=\phi=0$). Hence 4 equations remain for n unknowns which are the revolution speeds (RPM controls) of the n lifting rotors.

As soon as the number of lifting rotors is above 4, the system is underdetermined in the nominal ("All Engines Operating: AEO" case). In the case of one thrust generator failure ("One Engine Inoperative: OEI"), the system is underdetermined for (n>5). For two thrust generator failures ("Two Engine Inoperative: TEI"), the system is underdetermined for (n>6). Furthermore, the eVTOL multi-rotors configurations with only lifting rotors, (thrust only in the rotor axial direction without tilt system), have an even number n of contra-rotating lifting rotors to ensure the yaw stability.

Therefore, there is in principle an infinite number of solutions in terms of rotor controls as soon as the number of operating rotors is above 4: $n-n_i > 4$ with n_i the number of inoperative rotors. In order to find an optimal solution, different metrics have been considered. For example, hereafter (in part 2), results are compared with two different metrics:

- The L_2 norm (the Euclidian norm) of the vector of the thrust values which aims at minimizing the root of the sum of the squares of thrusts,
- The L_∞ norm of the vector of the thrusts, which aims at minimizing the maximum of the thrusts.

The L_2 norm can be seen as a "global energy metric" as it will give the preference to solutions minimizing the total power, hence minimizing the required energy to maintain the hover.

The L_∞ norm can be seen as a "power metric" as it will minimize the maximum power required in order coping with a failure.

A result of the study is that the L_∞ norm is the preferred metric for the presizing because by minimizing the maximum thrust (reducing the sizing demand on the rotors), it minimizes also the maximum torque (reducing the sizing constraint on the motors) and the maximum power (reducing the installed power). Even if the L_2 norm minimizes in principle the total power required, it could lead to heavier sizing wrt. the L_∞ solution because a higher maximum thrust will required heavier rotors, a higher maximum torque, heavier motors as well as a higher maximum power would need heavier power batteries.

For solving this optimization problem of finding the optimal redistribution of thrusts in case of failure, different algorithms have also been tested. In [2] we used a Newton algorithm, but the gradient methods are of course sensitive to initial values.

At first an iterative method based on the Jacobian pseudo inverse was used, to solve this almost linear problem. Indeed, depending on the rotor model, the relation between the torque *(Qk)* and the thrust *(Tk)* may not be linear. However, it soon appeared that the more complex configurations required better algorithm, and we thus turn to the use of optimization algorithms, with first *SLSQP* and finally *COBYLA*, which are algorithms available in the python module *scipy*.

2 EXAMPLES OF APPLICATION

The methodology proposed above has been successfully put into practice on typical configurations corresponding to the nowadays most popular eVTOL concepts:

- the case of axi-symmetrical circular multi-rotors configurations,
- the case of "Lift + Cruise" configurations with dedicated propellers (lifting / propulsive) and combining rotary wings and fixed wings.

For the sake of brevity, and in order to keep the paper in the imposed limits, mainly examples of the first kind, i.e. on axi-symmetrical circular rotors configurations, will be presented here.

2.1 AXI-SYMMETRICAL CIRCULAR MULTI-ROTORS

The quad and hexa rotors configurations are not able to cope with all double failures. That is why the study starts with the practical case of Octo-rotors configurations. The figure hereafter shows an example of Octo configurations ("Octo+"): 8 lifting rotors axi-symmetrically distributed around the centre of gravity with alternate directions of rotation starting from *rotor 1* in the front position with a clockwise direction (in red and noted "P" hereafter), then rotor 2 with anti-clockwise direction (in blue, noted "N" in the following).

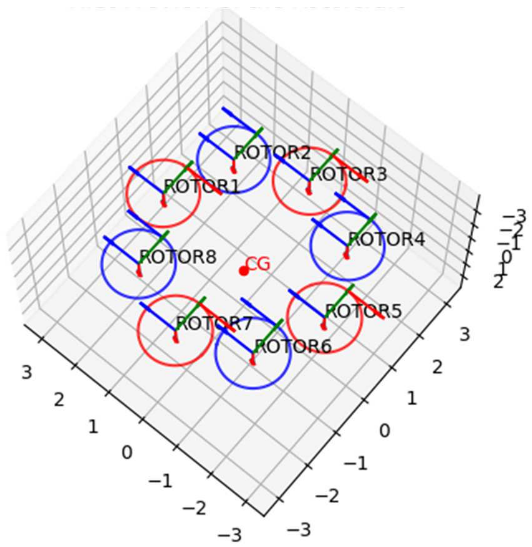

Figure 2: Octo+ configuration ("PNPN")

2.1.1 Effect of the metric (L_2 or L_∞)

The effect of choosing the L_2 or L_∞ metric is shown on Figure 3 for the case of one thrust failure (rotor 6 is stopped). Beyond the fact that the algorithm (here a continuation algorithm with multi-starts) converges toward two different thrusts redistributions with of course a lower maximum (about 7% lower) with L_∞ wrt. L_2, the logics of optimal redistribution is slightly simpler with L_∞.

Whatever the rotor which is in failure among the eight (here for example rotor 6), the logics of thrust redistribution in the OEI case have been identified as follows.

The L_∞-optimal logic is to:

- increase of the thrust of the four nearest rotors to the one in failure (rotors 4, 5 & 7, 8 by ~+40%),
- increase of the thrust of the diametrically opposite rotor (rotor 2 by ~+10%) because it contributes to the yaw trim as it turns in the same direction as the one in failure,
- decrease the thrusts of the two rotors (1 & 3 by about -35%) on each side of the opposite rotor (2) wrt. the one in failure (6).

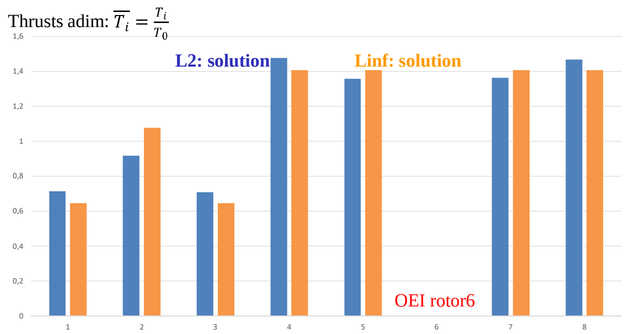

Figure 3: Example of two optimal L_2 or L_∞ thrusts redistributions for a failure on rotor 6 in the "Octo+" case.

The L_2-optimal logic brings a difference mainly on the discrepancy of the thrust increases on the 4 rotors on the same side as the one in failure: the two closest on each side of the failed one are less increased (~+36%) in order to let the two next (rotors 4 & 8 here) make growth more (~+48%) because they turn in the same direction as the one stopped. On the opposite side, the thrust of the three rotors are decreased, but again less the opposite rotor (2) as it turns in the same direction as the one stopped.

Yet for some failure cases, the two metrics can lead to the same optimal solution. This is for example the case for the worst case of two thrust generators failures ("TEI").

2.1.2 Worst TEI case on Octo configurations

The TEI cases are numerous and no longer symmetrical, even for axi-symmetric configurations. We will therefore seek and study the worst TEI case, which is the most dimensioning case.

The worst TEI case on any circular axi-symmetrical multi-rotors configuration is logically the one where the two failed rotors are the closest on the same side and turning in same direction because it destabilizes both the roll/pitch axes and the yaw trim.

On the Octo-rotors with alternate rotation directions, as the one shown on Figure 2, it happens for any couple of rotors separated by one rotor. For example for the case of the rotors 1 & 3 in failure, the optimal thrusts redistribution according to the L_∞ metric is given on Figure 4.

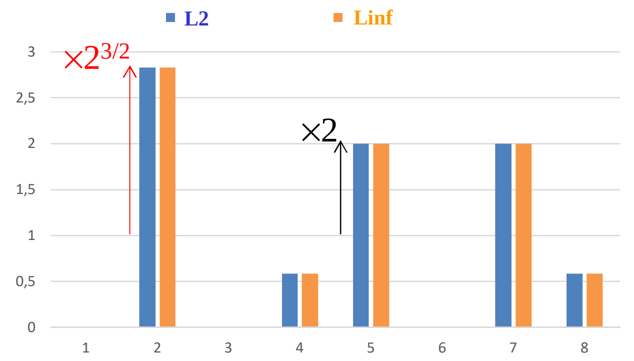

Figure 4: Normalized thrusts redistribution for rotors 1 & 3 in failure – Worst case for the Octo+ of **Figure 2**

The maximum thrust increase is of course on the rotor between the two failed ones (here rotor 2). The ratios of increase wrt. the nominal case are high with nearly a factor 3 on the thrust:

$$\frac{T_{max}}{T_0} = 2^{\frac{3}{2}} \sim 2.83 \sim 3$$

and about a factor five on the power (normalized by the required power P_0 on the nominal AEO case):

$$\frac{P_{max}}{P_0} = \left(2^{\frac{3}{2}}\right)^{\frac{3}{2}} \sim 4.76 \sim 5$$

Indeed in hover, the power is mainly the induced power which depends on the thrust with exponent (3/2) as can be easily seen from the momentum theory:

$$P_{i0} = T \times V_{i0} = T \times \sqrt{\frac{T}{2.\rho.S}} \cong a \times T^{3/2}$$

A ratio of 3 on the thrust and 5 on the power will lead to a heavy aircraft with such a configuration (Octo+ “PNPN”) in order to be resilient to all double thrust failure.

A geometrical explanation on why the required maximum thrust increased is $(2 \times \sqrt{2})$ is provided hereafter.

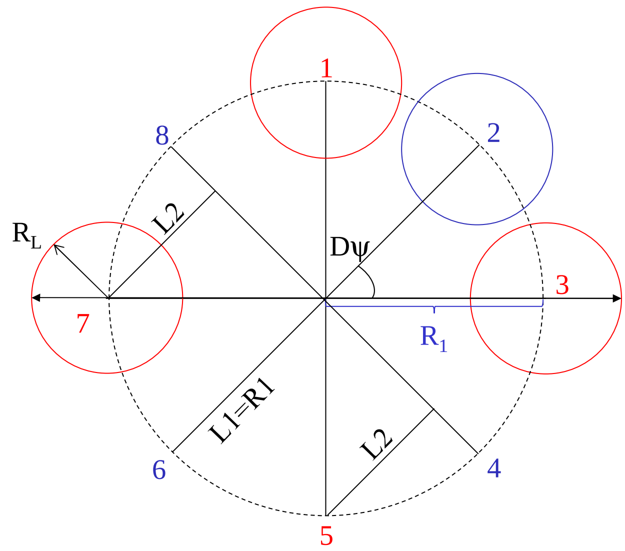

Figure 5: Geometrical description of the Octo case.

For the present example with rotors 1 & 3 in failure, only the rotor 2 remains on this side wrt. the axis given by rotors 4-8. The strong thrust increase requested on rotor 2 for compensating the lack of thrust on this side has for corollary the increase of torque. Therefore on the opposite side, rotor 6 which turns in same direction as rotor 2 is stopped for ensuring both the trim wrt. “axis 4-8” and the yaw trim.

The torques given by the rotors turning in clockwise direction (“P” in red) being only provided by the two remaining rotors 5 & 7, increasing their thrust by a same factor 2 allows compensating the lack of torque in this direction due to the failed rotors 1 & 3 while still contributing to the lift equilibrium and trim around “axis 2-6”.

Therefore by noting: $\overline{T_i} = \frac{T_i}{T_0}$

$\overline{T_1} = \overline{T_3} = 0$ and yaw trim and trim “axis 4-8” give:

$$\overline{T_6} = 0 \text{ and } \overline{T_5} = \overline{T_7} = 2$$

Hence for the trim about “axis 4-8”, on the side of the failed rotors, there is only rotor 2 with a lever-arm L1 (equal to the radius of the circle where are located the rotors centers) facing on the other side the rotors 5 and 7 which have a smaller lever arm L2:

$$L_2 = L_1 \times sin(D_\psi)$$

With the angular pace: $D_\psi = \frac{360°}{n} = \frac{360°}{8} = 45°$

Thus: $L_2 = L_1 \times \frac{\sqrt{2}}{2}$

The trim equation around axis 4-8 is:

$$\overline{T_5} \times L_2 + \overline{T_7} \times L_2 = \overline{T_2} \times L_1$$

Hence:

$$\overline{T_2} = 4 \times \frac{L_2}{L_1} = 2 \times \sqrt{2}$$

Previous results and the trim about "axis 2-6" lead to:

$$\overline{T_4} = \overline{T_8}$$

With the vertical trim (or lift) equation:

$$2 \times \overline{T_4} + 2 \times \overline{T_5} + \overline{T_2} = n = 8 \Longrightarrow \overline{T_4} = 2 - \sqrt{2}$$

The ratios of maximum thrust in case of a double failure on any of the eight rotors of the Octo "PNPN" (Figure 2) configuration are presented on Figure 6 for example for the case of rotor 1 in failure and any one of the seven others.

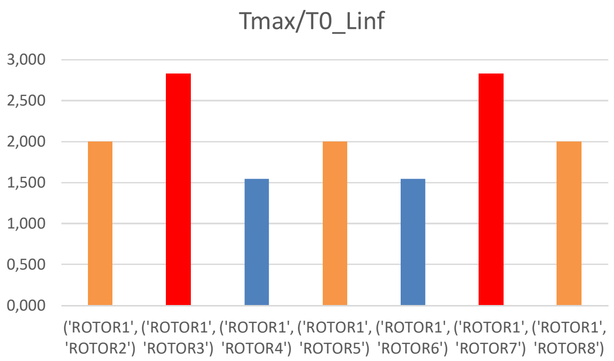

Figure 6: Maximum thrust ratios depending on which couple of rotors is in failure for the Octo "PNPN"

For each of the eight rotors, the worst case of double failure appears of course two times, i.e. on each side with one of the two closest rotors turning in same direction (here for rotor 1 with rotor 3 on one side and rotor 7 on the other side). For both of these two worst cases as demonstrated geometrically before:

$$\overline{T_{max}} = \frac{T_{max}}{T_0} = 2 \times \sqrt{2} \approx 2.83$$

Then there are three cases with : $\overline{T_{max}} = 2$ and two cases for nearly opposite rotors turning in opposite direction for which: $\overline{T_{max}} \approx 1.55$

Another case of Octo configuration uses side by side rotors turning in same direction as shown hereafter ("PPNN" instead of "PNPN").

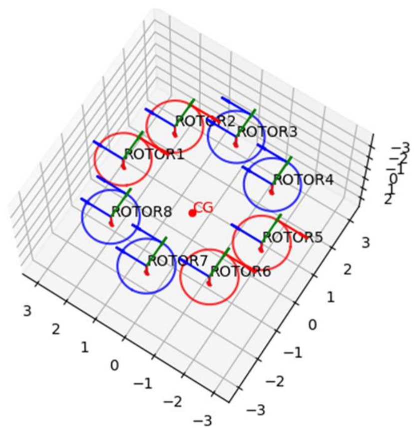

Figure 7: Octo+ configuration ("PPNN")

The results obtained with the proposed method (optimal redistribution of the thrusts using the L_∞ metric) are shown on Figure 8. A critical point is that this kind of Octo configurations "PPNN" is not resilient to all double failure cases in the sense that they are not able to keep hovering when two side by side rotors turning in same direction are in failure. Indeed there is no trim solution for all these four worst cases: rotors 1&2, 3&4, 5&6, 7&8.

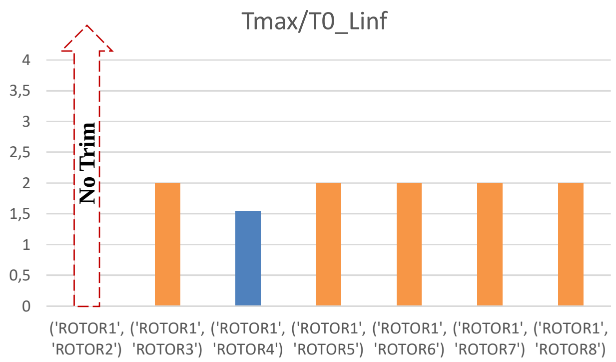

Figure 8: Maximum thrust ratios depending on which couple of rotors is in failure for the Octo "PPNN"

Besides these four untrimmable worst cases, all other double failures require: $\overline{T_{max}} = 2$ except when the pair of failed rotors is composed of nearly opposite rotors turning in opposite direction as the rotors 1&4 in the example given on Figure 8.

In the Octo "PNPN" configuration, there are 8 worst cases (2 for each of the 8 rotors hence 8 combinations rotors i & i+2), whereas on the Octo "PPNN" there are only four worst cases. That is why in table 2 of [3] the percentage of fault tolerant cases, i.e. roll-pitch-yaw controllable for two random rotor failures, is higher with the "PPNN" configuration than with the "PNPN" one.

However the "PPNN" configuration is in principle clearly less resilient to double rotor failures because there are four cases for which, whatever the performance of the thrust generators is, the configuration cannot be trimmed in HOGE, i.e. is roll-pitch-yaw uncontrollable. The percentages obtained in [3] depend on the thrust generators performance of the peculiar Octo-UAS on which the experiments have been done. The "PNPN" configuration can in principle cope with all double failure. But in practice as experimented in [3], if the rotors are not able to generate at least $\overline{T_{max}} \approx 2.83$, which means a factor nearly 5 on the power wrt. the nominal, then this particular Octo UAS will not succeed to maintain the trim (i.e. will not be roll-pitch-yaw controllable).

Once the method has been set and explained on the Octo case, in the following part, results with higher numbers of axi-symmetrical rotors are provided.

2.1.3 Results for 8 to 20 axi-symmetrical rotors

First are considered the cases of circular axi-symmetrical rotors configurations without coaxial rotors. They are geometrically defined as on Figure 9.

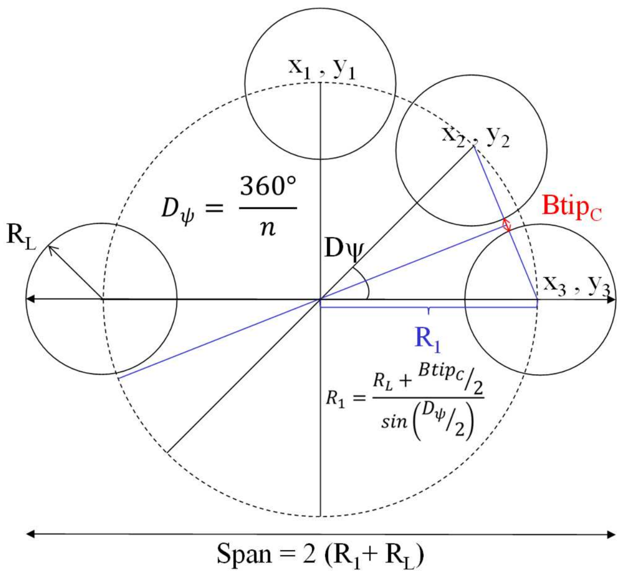

Figure 9: circular axi-symmetrical configurations for any number of n lifting rotors

With R_L the radius of each lifting rotor, D_ψ the pace angle between the n lifting rotors, $Btip_C$ is the blade tip clearance, R_1 the radius on which the n identical lifting rotors are regularly distributed. The choice has been done to use always the same R_L whatever the number n of lifting rotors. This is because the Step 1 "First failure analysis" (Figure 1) is focused on the configuration, i.e. the geometrical arrangement of the rotors, not on the rotor sizing. The purpose is not to compare their performance but their resilience to rotor failures. That is why the outputs of Step 1 are ratios in relative values (not absolute values) and therefore the size of the rotor (R_L) does not have an impact on the following results.

The Table 1 shows the results in terms of maximum ratios for thrust, power on one rotor and total power, for $n \in \{8, 10, 12, 14, 16, 18, 20\}$ given by the proposed methodologyusing the L_∞-optimal thrusts redistribution.

TEI cases \ Metrics		Linf			
Nb of rotors	N° rotors in failure for T&Pmax	Tmax/T0	Pmax/P0	PtotEI/Ptot0	N° rotors in failure for PtotMax
8 (PPNN)	**[1, 2]**	No Trim			
8 (PNPN)	**[1, 3]**	2^(3/2)	(2^(3/2))^(3/2)	2^(1/2)	**[1, 3]**
10	**[1, 2]&[1, 3]**	10/6	(10/6)^(3/2)	(10/6)^(1/2)	**[1, 2]&[1, 3]**
12	**[1, 3]**	1,566	1,961	(12/8)^(1/2)	**[1, 2]**
14	**[1, 2]&[1, 3] [1, 5]&[1, 7]**	14/10	(14/10)^(3/2)	(14/10)^(1/2)	**[1, 2]&[1, 3]**
16	**[1, 3]**	1,353	1,573	(16/12)^(1/2)	**[1, 2]**
18	**[1, 2]&[1, 3] [1, 5]&[1, 7] [1, 9]**	18/14	(18/14)^(3/2)	(18/14)^(1/2)	**[1, 2]&[1, 3]**
20	**[1, 3]**	1,258	1,412	(20/16)^(1/2)	**[1, 2]**

Table 1: worst TEI cases for circular axi-symmetrical rotors configurations (the number in square brackets are the rotor number starting from 0)

The graphical representation on Figure 10 shows the trend of these maximum ratios with the number of lifting rotors, for the worst case. We denote this worst case value for the adimensioned L_∞ norm as $\overline{T_\infty}$. Therefore $\overline{T_\infty}$ represents the maximum thrust ratio for all rotors, and for all TEI cases, for the solutions minimizing the L_∞ norm.

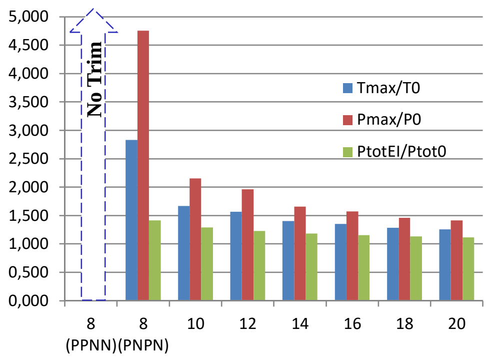

Figure 10: maximum ratios of thrust, power and total power for the worst double rotor failure for different numbers of rotors

The results in the T_{max}/T_0 column of Table 1, gives the values of $\overline{T_\infty}$ for various axisymmetric configurations. The values in blackcorrespond to the cases of the configurations with opposite rotors turning in opposite direction (cases with 10, 14, 18 rotors with alternate rotation direction “PNPN”). In these cases $\overline{T_\infty}$ reaches its lowest value for this kind of configuration as explained hereafter.

Indeed, if the geometrically opposite rotors turn in opposite direction, the optimal redistribution simply consists in stopping the opposite rotors. It stabilizes the roll, pitch and yaw degrees of freedom. Then all the *(n-4)* remaining rotors just have to produce enough lift for the vertical trim, thus the thrust increase is the same for all with:

$$\overline{T_\infty} = \frac{n}{n-4}$$

For rotors arrangements with opposite rotors turning in same direction, i.e. for configurations with a number of rotors multiple of 4: $n \in \{8, 12, 16, 20 \dots\}$ and alternate direction of rotation (“PNPN…”), this simple strategy cannot be applied and thus:

$$\overline{T_\infty} > \frac{n}{n-4}$$

Of course, this increase in maximum thrust value decreases with the number of rotors. It is a factor $2^{(1/2)}$ in the Octo case (i.e. nearly 41% of increase with respect to the lowest theoretical value 8/(8-4)= 2) and only 0.8% in the case with n=20.

An example showing the more complex logics of redistribution of thrusts, for these cases where the opposite rotors do not turn in opposite direction wrt. the two rotors in failures, is shown on Figure 11 and Figure 12 for the case with 16 rotors. The TEI worst cases are still the ones where two rotors on the same side and turning in same direction are in failure. That is why the case of rotors 1 & 3 in failure is given as example. The logics is as follows:

- the « triangular opposite » to the two in failure (n°1&3) is stopped (n°10, see Figure 12) ;
- the two diametrically opposite are reduced (9&11),
- but less than the 2 following turning in opposite direction wrt. the 2 in failure (8&12)

Indeed when the opposite rotors turn in same direction, they cannot be stopped otherwise it would increase the yaw instability.

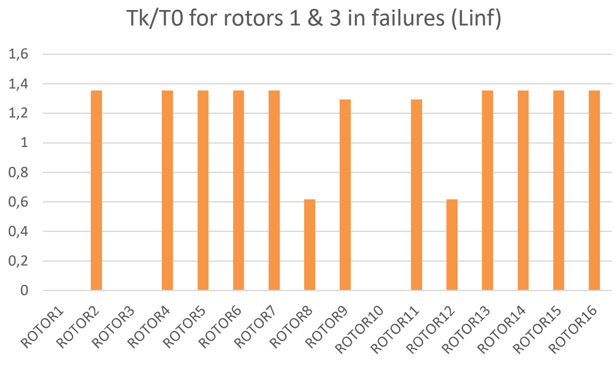

Figure 11: Thrusts redistribution for the case with 16 rotors and rotors 1 & 3 in failure

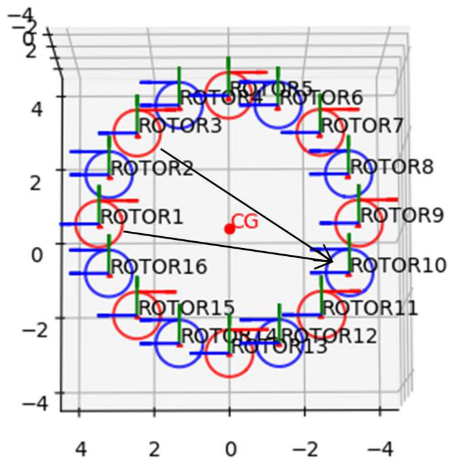

Figure 12: Scheme of the 16 rotors with rotors 1 & 3 in failures

An analytical demonstration about why $\frac{T_{max}}{T_0} = \frac{n}{n-4}$ is the lowest attainable increase of thrust ratio for coping with the worst double failure in the case of these configurations with only lifting rotors (generating only axial thrust in same direction, with no tilt or canted angles) is proposed hereafter.

2.1.3.1 Analytical demonstration

Considering the equations of interest described in section 1 *(eq. 1)*, one can manipulate the equations as follows, considering only the weight and torque equations and that the torque generated by a rotor is proportional to the thrust, i.e. $Q_k = (-1)^{i_k} \alpha T_k$, with i_k defining the direction of rotation of the rotors ($i_k = 1$ for anti-clockwise, and $i_k = -1$ for clockwise), the equations can be written has:

$$\sum_{k=1}^{n} \overline{T_k} = n$$

$$\sum_{k=1}^{n} (-1)^{i_k} \overline{T_k} = 0$$

Where $\bar{T}_k = T_k / T_0$. Combining those two equations, we have:

$$\sum_{\substack{k=1 \\ i_k=1}}^{n} \overline{T_k} = \frac{n}{2} \qquad \textit{(eq. 2a)}$$

$$\sum_{\substack{k=1 \\ i_k=-1}}^{n} \overline{T_k} = \frac{n}{2} \qquad \textit{(eq. 2b)}$$

This general relationship shows that given any configuration, under the given assumptions, the total thrust generated by the clockwise rotors is fixed as well as the total thrust generated by the counter clockwise rotors.

In the case where the number of clockwise rotors equals the number of counter clockwise rotors (i.e. n/2), and where two rotors with the same direction of rotation, say clockwise, are inoperative, the corresponding equation (for clockwise rotors *(eq. 2b)*) is constrained with two rotors thrusts being null. In this case, the best solution for the L_∞ norm, considering only this equation is: $\overline{T_k} = \frac{n}{n-4}, \forall k$. Indeed, minimising the L_∞ norm for this equation is equivalent to having all rotors at the same value, which, considering they are $\frac{n}{2} - 2$, gives the expected result. The fact that the L_∞ norm is minimized by having all rotors at the same value can be justified by considering that if all rotors thrusts are such that $\forall k, \overline{T_k} < \frac{n}{n-4}$. This would give:

$$\sum_{\substack{k=1 \\ i_k=-1}}^{n} \overline{T_k} < \sum_{\substack{k=1 \\ i_k=-1}}^{n} \frac{n}{n-4} = \left(\frac{n}{2} - 2\right)\left(\frac{n}{n-4}\right) = \frac{n}{2}$$

Which is not possible if we wish to respect the corresponding equation (e.g. *(eq. 2b)*). Furthermore, one can derive the same reasoning with a different number of clockwise and counter-clockwise rotors, by replacing the n in the denominator by:

$$n_{min} = 2 \times \min(n_{clockwise}, n_{counter-clockwise}).$$

Then the L_∞ optimum to cope with the most demanding double failure is such that:

$$\overline{T_\infty} \geq \frac{n}{n_{min} - 4}$$

This gives a general lower bound on $\overline{T_\infty}$, which does not mean that this value is attainable for any configuration, but for this case of failure, one can not expect any better result than this for the configurations having all rotors thrusting in same direction in HOGE. The roll and picth equations (see eq. 1) will just influence the thrusts redistribution depending on the positions of the rotors, but they will not change this $\overline{T_\infty}$ lower bound.

The same reasoning gives that the lower bound for a single rotor failure is $\overline{T_\infty} \geq \frac{n}{n-2}$, and can be generalized to any number of failures with $\overline{T_\infty} \geq \frac{n}{n-2n_i}$, where n_i is the number of inoperative rotors. So at the step 1 of the presizing loop (Figure 1), the maximum thrust ratio for that kind of configurations (with all the thrusts fixed in the same direction) is at best, in the worst case of failure and if the configuration is well chosen:

$$\overline{T_\infty} = \frac{n}{n - 2n_i}$$

Those results are highly dependent on the assumption that the torque depends linearly on the thrust generated. This assumption can be verified analytically by considering that the rotors have no pitch control, and thus have a fixed C_{zm} value (meaning that the thrust varies with the RPM without changing the mean lift coefficient of the rotor C_{zm}). Indeed as already explained in [2], if the rotor model is based on momentum theory, the definitions of C_{zm} and of the rotor Figure of Merit *FM* provide a linear relationship between the torque *Q* and the thrust *T*:

$$Q = \frac{1}{2}\sqrt{\frac{\sigma C_{zm}}{3}} \frac{R}{FM} \times T \approx C^{ste} \times T$$

With *R* the rotor radius, σ the rotor solidity (ratio of blades surface on rotor disk surface). It turns out to be verified experimentally by observing the results of the UIUC database [4] in hover conditions.

Finally, those results beg the question: can one find a configuration reaching this optimum for a given number of rotors ? A partial solution is explored in the following section.

2.1.3.2 *Configurations with coaxial rotors*

As seen previously, the configurations having opposing rotors turning in the same direction do not reach the lower bound of $\overline{T_\infty}$. However, instead of single rotors, using pairs of coaxial contra-rotating rotors is a simple way to reach this lower bound for the configurations of circular axi-symmetrical rotors with *n* a multiple of 4 (i.e. with opposite rotors turning in same direction). For these cases shown in red in *Table* 1 ($n \in \{8, 12, 16, 20 \ldots\}$), running the algorithms on the coaxial rotors gives that those configurations reach $\overline{T_\infty} = \frac{n}{n-4}$, as shown in *Table 2*, because there are always opposite rotors turning in opposite direction wrt. the ones in failure.

TEI cases \ Metrics	Linf		
Nb of rotors	Tmax/T0	Pmax/P0	PtotEl/Ptot0
8 (PPNN)	**No Trim**		
8 (PNPN)	2^(3/2)	(2^(3/2))^(3/2)	2^(1/2)
8 Coax (PNPN)	8/4	2^(3/2)	2^(1/2)
10 Coax	**1,809**	2,434	1,251
10	10/6	(10/6)^(3/2)	(10/6)^(1/2)
12	1,566	1,961	(12/8)^(1/2)
12 Coax	12/8	(12/8)^(3/2)	(12/8)^(1/2)
14 Coax	**1,431**	1,713	1,154
14	14/10	(14/10)^(3/2)	(14/10)^(1/2)
16	1,353	1,573	(16/12)^(1/2)
16 Coax (PNPN)	16/12	(16/12)^(3/2)	(16/12)^(1/2)
16 Coax (PNNP)	16/12	(16/12)^(3/2)	(16/12)^(1/2)
18	18/14	(18/14)^(3/2)	(18/14)^(1/2)
20	1,258	1,412	(20/16)^(1/2)

Table 2: worst TEI cases for circular axi-symmetrical rotors configurations including coaxial.

For 8 coaxial rotors: $\overline{T_\infty} = \frac{8}{4} = 2$,

For 12 coaxial rotors get: $\overline{T_\infty} = \frac{12}{8} = 1.5$,

For 16 coaxial rotors get: $\overline{T_\infty} = \frac{16}{12} = 1.33 \ldots$

However, grouping the rotors by pairs of coaxial contra-rotating rotors can induce a loss of geometrical symmetry for the configurations where *n* is not a multiple of 4. Indeed, for those configurations, there is no longer a symmetric rotor to respond to a given failure. That is why the results are worst for these cases compared to the configurations using only single rotors and for which the opposite rotors turn in opposite direction, and where the lower bound of $\overline{T_\infty}$ is attainable without using coaxial arrangement (see *Table 2* results in red bold).

For example, for the case of 10 coaxial rotors, there are 5 pairs and thus only 5 positions for the rotors thrusts (see Figure 13).

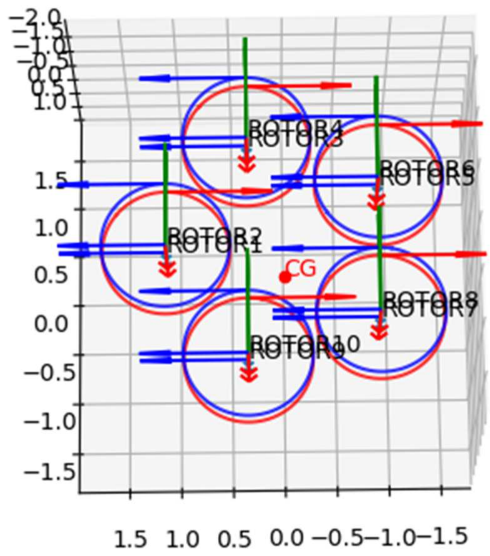

Figure 13: configuration with 10 coaxial rotors.

When a pair of coaxial rotors is in failure (e.g. rotors 1 & 2), the two closest pairs (3&4 and 9&10) are pushed to the maximum: $\frac{T_{max}}{T_0} \sim 1.81$, which is clearly above the optimum 10/(10-4)~1.66, whereas the thrusts on the two pairs on the opposite side (5&6 and 7&8) are reduced as shown on Figure 14.

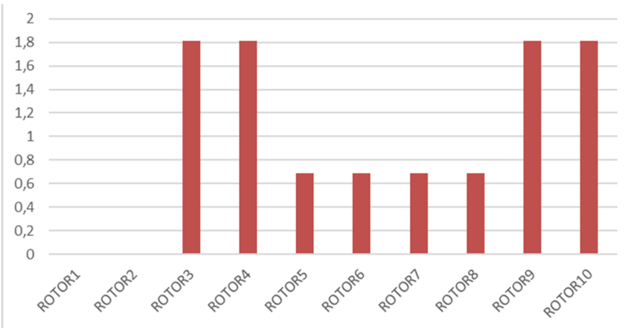

Figure 14: thrusts optimal redistribution on the 10 coaxial rotors configuration for coping with rotors 1&2 failures

For the case with 8 coaxial rotors (4 pairs), it must be underlined that the roll-pitch equilibrium is unstable when a pair of coaxial rotors is in failure. Indeed, the most efficient solution being to stop the opposite rotors, then only the rotors on the other axis (roll or pitch) remain active. They are not able to counter a perturbation on the axis of the double failure.

For these reasons, the 12 coaxial configuration is identified as an interesting resilient and controllable coaxial arrangement as shown hereafter (Figure 15). The interest wrt. single rotors configurations is a more compact geometry (6 rotors positions) with a smaller span. The drawback is that the probability to lose two coaxial rotors in the event of a bird strike, for example, could be higher than in the case of single rotors.

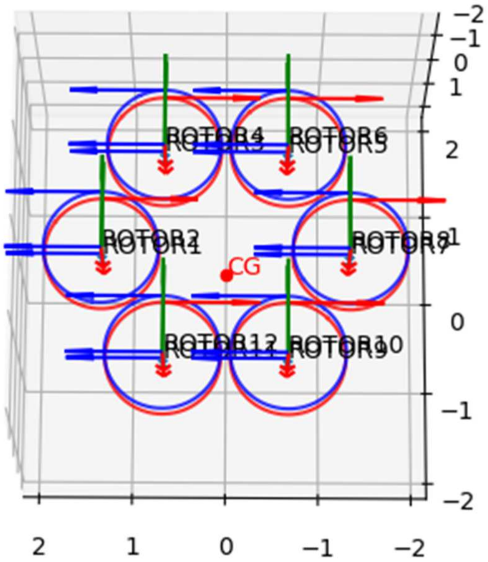

Figure 15: configuration with 12 coaxial rotors.

The ratios of $\frac{T_{max}}{T_0}$ for all TEI cases with rotor 1 in failure are shown on Figure 16.

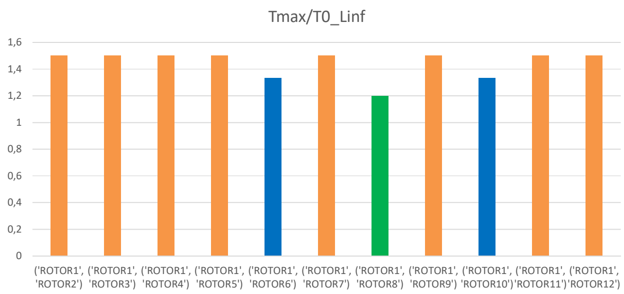

Figure 16: $\frac{T_{max}}{T_0}$ for the 11 cases of double failures with the 12 coaxial rotors configuration.

The results in orange on Figure 16 correspond to the cases where the optimal-L_∞ solution consists in stopping the opposite rotors therefore: $\frac{T_{max}}{T_0} = \frac{12}{12-4} = 1.5$. The other results are for the TEI cases where the failures occur on nearly symmetrical rotors for the blue results (1&6 and 1&10) and on fully symmetrical rotors for the result in green (1&8).

For the results in blue on Figure 16, the optimal-L_∞ redistribution consists in reducing the thrust on the pair of rotors which is the "triangular opposite" (e.g. 9&10) of the two in failure (e.g. 1&6). It is enough to maintain the roll-pitch trim. The yaw trim being kept by the fact that the two rotors in failure turn in opposite direction, all other thrusts are increased by the same value for the vertical trim as appears on Figure 17.

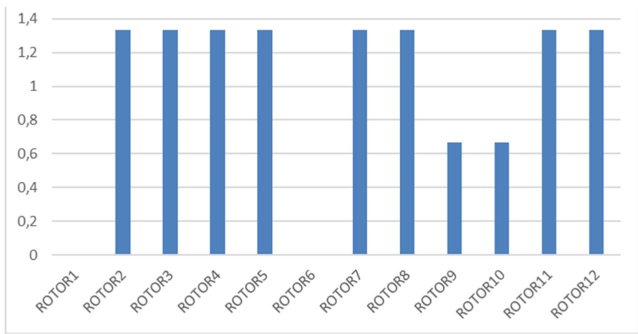

Figure 17: Optimal-$\boldsymbol{L_\infty}$ thrust redistribution for the nearly symmetrical cases of double failure (e.g. 1&6).

The result in green on Figure 16 corresponds to the lucky case where the two failures occur on symmetrical rotors turning in opposite direction (e.g. 1&8). In these cases the perturbation is only on the vertical trim, thus the full trim is kept just by readjusting the thrusts to compensate the lack of two lifting forces with $\frac{T_{max}}{T_0} = \frac{12}{12-2} = 1.2$. It is also the kind of optimal-L_∞ redistribution for a OEI case on such configuration (just stop the opposite rotor turning in opposite direction).

We can therefore see that this configuration does indeed reach its lower bound with respect to its number of rotors, i.e. $\overline{T_\infty} = \frac{12}{12-4} = 1.5$.

2.2 MORE GENERAL CONFIGURATION WITH NON-CIRCULAR DISTRIBUTION

Our approach can be applied to any configuration with all rotors pointing in the same direction. To illustrate its interest and generality, it is applied to the case of a "Lift + Cruise" configuration similar to the Cora (formerly known as Kittyhawk Cora and now as Wisk Cora) with 12 lifting rotors and a pushing propeller. Here, it is a simplified academic case with all the 12 lifting rotors producing vertical thrusts (without the canted angles used in the original Cora design). In HOGE the pusher propeller is stopped and aerodynamic interferences are neglected in a first approximation.

The geometry of this simplified Cora configuration is shown on Figure 18 with the original choice of rotors direction of rotation.

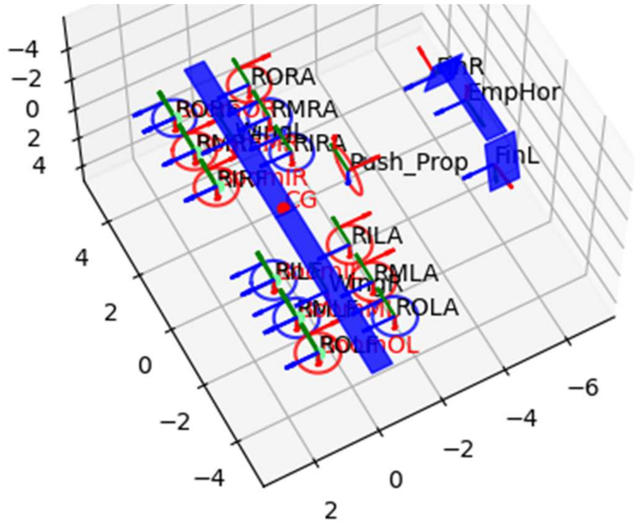

Figure 18: Simplified Cora configuration with original directions of rotation

Considering double failure cases, there are in principle 66 combinations of two rotors between the 12 lifting rotors. All these cases have been dealt with, but for the sake of clarity hereafter are shown the 33 cases considering the left – right symmetry of the aircraft.

The more axes are destabilized by the failure, the higher the $\overline{T_{max}}$. Indeed the most demanding cases are the ones with two side by side rotors turning in same direction (there are four cases but only two are shown in red on Figure 19 because of the right/left symmetry):

for these four worst cases: $\frac{T_{max}}{T_0} \sim 1.65$

The most numerous cases are the ones with two destabilized axes, they are shown in orange on Figure 19, they reach the lower bound described above, with: $\frac{T_{max}}{T_0} = \frac{12}{12-4} = 1.5$

In blue are the cases where only one axis is destabilized (roll axis), with a value of $\overline{T_{max}}$ depending on the distance of the failed rotors wrt. the cG: $1.3 < \frac{T_{max}}{T_0} < 1.4$

In green, the least demanding cases are presented with nearly opposite rotors turning in opposite direction:

$$1.23 < \frac{T_{max}}{T_0} < 1.28$$

Hence, for this configuration $\overline{T_\infty} \sim 1.65$ which is higher than the theoretical lower bound identified above.

Now if the same aircraft is considered, but with opposite rotors turning in opposite direction as illustrated on Figure 20, $\overline{T_\infty}$ reaches the lower bound, as can be seen on the obtained $\overline{T_{max}}$ values for all double failure cases presented on Figure 21.

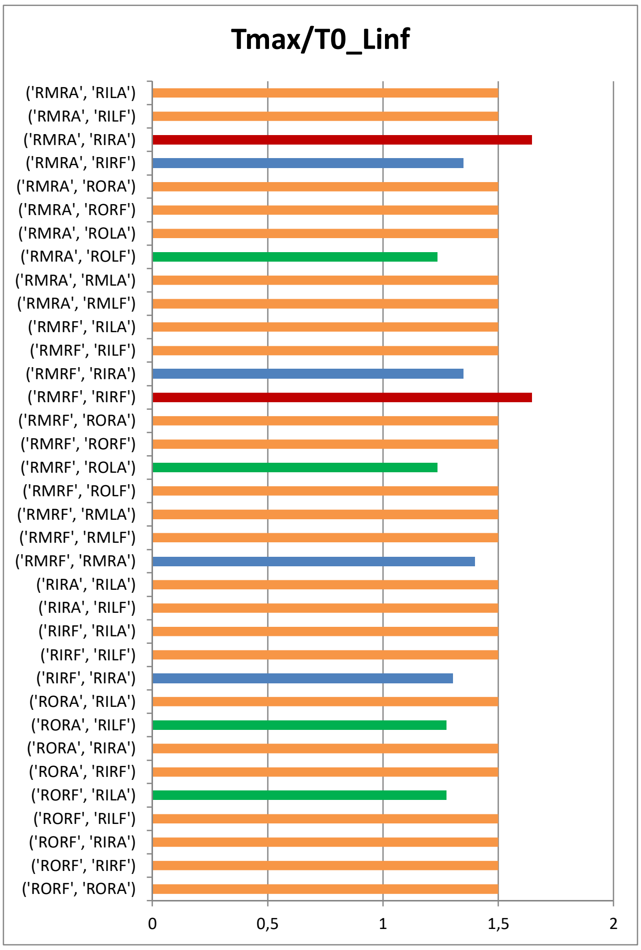

Figure 19: ratios of maximum thrust augmentation for coping with a double rotor failure (half cases presented taking into account the right/left symmetry, case of configuration on Figure 18)

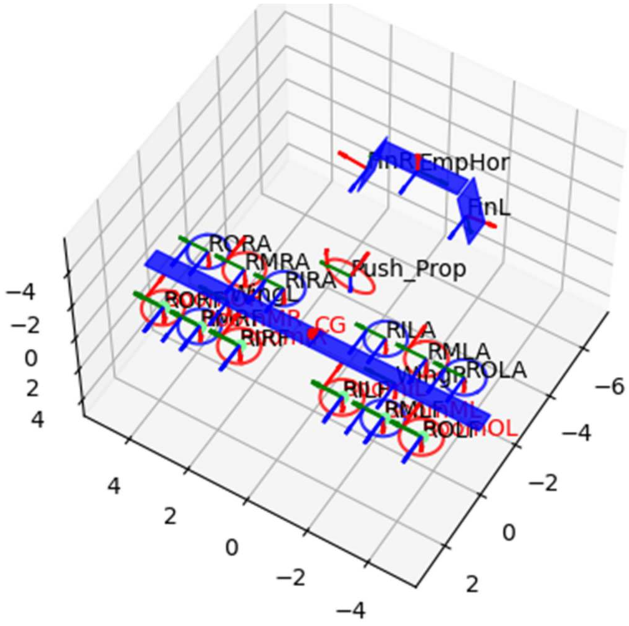

Figure 20: fully symmetrical configuration with diametrically opposite rotors turning in opposite direction

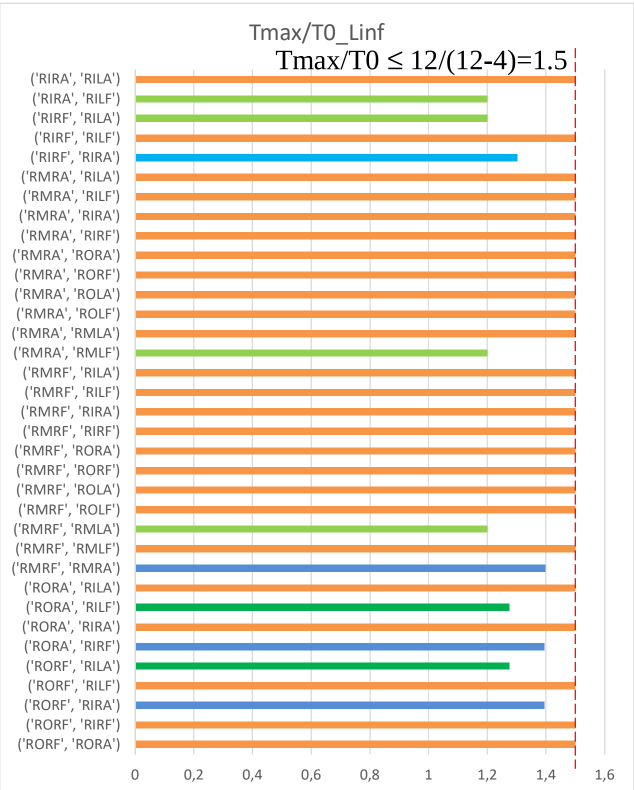

Figure 21: ratios of maximum thrust augmentation for coping with a double rotor failure (half cases presented taking into account the right/left symmetry, case of configuration on Figure 20)

With this fully symmetrical configuration, all the thrust augmentations remain below the lower bound limit:

$$\frac{T_{max}}{T_0} \leq \frac{12}{12-4} = 1.5 = \overline{T_\infty}$$

The least demanding cases, i.e. the ones with by luck two opposite rotors in failure, reach the lowest bound of thrust increase for coping with one rotor failure (i.e. just have to stop the diametrically opposite rotor for stabilizing all axes):

$$\overline{T_{max}} = \frac{n}{n-2} = \frac{12}{10} = 1.2$$

A configuration with opposite rotors turning in opposite direction may thus seem to be more optimal regarding this first failure analysis criterion. Indeed with such arrangement, the maximum increase on thrust, torque and power being lower, the sizing demand on the rotors, motors and power generation system will be in principle lower which potentially may lead to a lighter sizing.

However, a complementary criterion wrt. the optimality of the control may be required. Indeed, with opposite rotors turning in same direction, in order to control the roll or pitch degree of freedom on a configuration with circular rotors distribution, one just has to increase the lift on one rotor and to decrease it on the opposite rotor. On a configuration with opposite rotors turning in opposite direction, such symmetrical variation of thrust will induce a yaw acceleration which will have to be compensated by changing the thrust and torque of other rotors. This will require a more complex control law, but also likely more power.

3 ATTAINABLE CONTROL SET: A COMPLEMENTARY MATHEMATICAL TOOL FOR FAILURE ANALYSIS

From the previous sections, we are able to evaluate the increase in thrust, torque and power required by a TEI failure case. Furthermore, we can judge if a configuration is optimal in a certain sense, by evaluating $\overline{T_\infty}$ with respect to its lower bound. However, and as we have seen in the case of the different octo configurations, this does not indicate the controllability of the failed configuration. Furthermore, modifying a configuration to reach the lower bound of $\overline{T_\infty}$ could be detrimental to the control of the aircraft.

In order to remedy to this problem, and to measure its impact, we propose the computation of the Attainable Control Set (ACS), which is a way to assess the useful reachable domain in terms of forces and moments or accelerations. The definition of the ACS is the subspace in terms of thrust and moments (or accelerations) that are attainable while being in the boundary of the possible generated forces by the rotors. Examples of application to multi-rotor configurations can be found in the UAS literature for example in [3]. Beyond the analysis of the capability of a configuration to maintain a steady trim position after one or two rotor failure(s), its remaining performance for maneuvers in order to insure a safe landing or for facing wind gusts has to be evaluated.

Therefore the ACS is a complementary tool to the proposed first failure analysis method giving a more comprehensive assessment of the maneuvering capability of a configuration, i.e. its controllability. For example, it can be applied to compare the ACS with thrust generator failure (OEI or TEI etc.) wrt. the ACS in nominal use (AEO).

Here again, at this early stage of the presizing, that kind of assessment should be done in relative value because neither the weight nor the aerodynamic characteristics are still well known. Thus the ACS is determined in relative value wrt. the ACS in nominal use.

Computing the ACS for the three different configurations of Octo in their worst case of failure shows the uncontrollability of the coaxial case, and the controllability of the PNPN case, although its $\overline{T_\infty}$ value is above the lower bound.

It is to be noted that, in the method presented above, the inverse problem is solved: for a given acceleration (trim means null accelerations and here we only considered hover), the optimal thrust distribution is calculated to have the lowest maximum in terms of thrust (optimal-L_∞ thrust distribution) which minimizes also the maximum needed torque and power on each rotor. We need this first study to identify the worst failure cases, and then plot a meaningful ACS.

The next step will be to generalize the method in order to evaluate not only the optimality of a configuration wrt. to its resilience to thrust generator(s) failure(s), but also to assess more widely its controllability, ideally merging all information in a single criterion specific to a family of configurations.

CONCLUSIONS

An efficient and robust general method is proposed for calculating the optimal redistribution of the thrusts in case of rotor failure for any eVTOL configuration. As a very first step for the presizing, by using as criterion the L_∞ norm, it provides the maximum thrust ratio increase wrt. the nominal case as well as for the torque and power. These maximum ratios of increase are crucial inputs for the presizing of the rotors, motors and power-energy system.

In the cases of eVTOL configurations with all *n* rotors thrusting in the same direction, the lowest maximum ratio in the optimal-L_∞ thrusts redistribution to cope with the worst failure of any n_i rotors, $\overline{T_\infty}$, can be shown to be such that:

$$\overline{T_\infty} \geq \frac{n}{n-2n_i}$$

Configurations can reach the lower bound for this optimal-L_∞ thrusts redistribution if the diametrically opposite rotors turn in opposite direction. For circular axi-symmetrical configurations with alternate direction of rotation, when *n* is a multiple of 4, the opposite rotors turn in same direction. A way to reach the optimal lowest bound limit is then to use coaxial rotors. The first truly interesting such coaxial reconfiguration is the one with 12 coaxial rotors (the case with 8 coaxial rotors being not roll-pitch-yaw controllable for all double failures). The generality of the approach is also illustrated on non-circular configuration.

As shown in [2], it is possible to define configurations for which it is not necessary to stop rotors which are able to thrust and thus get below the presented lower bound for $\overline{T_\infty}$. This is done to the price of more complexity with configurations using rotors not thrusting in the same direction (pushing propellers, tilting rotors etc.). The method will be applied also to that kind of configurations in order to get a more comprehensive assessment.

Further work is required to generalize the method, in order to evaluate not only the “trimmability” of a configuration, but also the optimality of its controllability. Indeed, after any rotor(s) failure, these Advanced Aircraft will have not only to be able to maintain hover, but also to perform a safe landing or to go on a trajectory within an imposed corridor even in presence of wind gusts. Therefore their ability to generate forces and moments (i.e. accelerations) will have to be evaluated more widely in order to find the optimal configuration for a set of missions.

REFERENCES

[1] R.L. Robb: *"Hybrid Helicopters: Compounding the Quest for Speed", Vertiflite,* Vol. 52, No. 2, Summer 2006.

[2] P.-M. Basset, B. Dang Vu, P. Beaumier, G. Reboul, B. Ortun: "Models and Methods at ONERA for the Presizing of eVTOL Hybrid Aircraft Including Analysis of Failure Scenarios", AHS International 74th Annual Forum & Technology Display, Phoenix, Arizona, USA, May 14-17, 2018.

[3] T. Schneider, G. Ducard, K. Rudin, P. Strupler "Fault-tolerant Control Allocation for Multirotor Helicopters Using Parametric Programming", Conference Paper, July 2012.

[4] J.B. Brandt, R.W. Deters, G.K. Ananda, O.D. Dantsker, and M.S. Selig, *UIUC Propeller Database, Vols 1-4*, University of Illinois at Urbana-Champaign, retrieved from https://m-selig.ae.illinois.edu/props/propDB.html