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AN LQG-DISTURBANCE MODELLING APPROACH TO ACTIVE CONTROL OF VIBRATIONS IN HELICOPTERS

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Abstract. The application of *LQG* optimal control theory to active control of vibrations in helicopters is not straightforward due to the presence, in the rotor model, of persistent periodic disturbances which are not included in a standard *LQG* problem formulation. The purpose of the paper is to show how the theory can be tailored to achieve rejection of multiharmonic disturbances. The basic idea is to augment the system state so as to incorporate a model of the main harmonics of the vibration too (*LQG Disturbance Modelling*). The applicability and limitations of the theory are probed by means of several simulation trials on the linear dynamic model describing the influence of the swash plate collective command to the rotor hub vertical force in the helicopter Agusta A129.

1. Introduction

Helicopter vibrations reduction is of central importance not only for the improvement of the passengers comfort but also for a better behaviour of the machine.

As is well known, [1], the induced vibration can be modelled as a periodic disturbance with fundamental frequency $\Omega_o = n_b\Omega_{rot}$, where Ω_{rot} is the rotor angular speed and n_b is the number of blades.

The application of active control techniques to helicopter vibratory problems is still under study in many companies and is carried over with different control strategies and mechanical devices. In the present work, we deal with a time domain control methodology developed for the four-bladed Agusta A129 helicopter under a research contract between *Agusta Spa* and *Politecnico di Milano* (Dipartimento di Elettronica e Informazione).

Making the reasonable assumption that the main source of vibrations are the loads transmitted from the rotating frame to the fuselage and that the vertical vibration turns out to be the most disturbing one, the objective of the active control device is to cancel the vibratory components in the total vertical mast force. This can be achieved by superimposing an extra signal to the pilot's command at the swash plate. Precisely, for active control purposes we act on the collective command $u_p(\cdot)$ by adding a "small" variation $\delta u(\cdot)$ to it (Fig. 1). This is tantamount to superimposing an equivalent "small" variation $\delta\theta(\cdot)$ to each pitch angle characterizing the blades longitudinal rotation. The entity and frequency content of all this signals are constrained by the need of non-interfering with the pilot's commands (u_p).

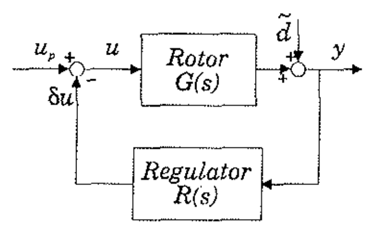

Fig. 1 - u_p=pilot's collective command (mm), δu=active control signal, u=total collective command, $\tilde{d}$=vibration (Nw), y=total vertical force at the hub (Nw)

A well known technique for vibrations reduction in helicopters is the so-called *Higher Harmonic Control* (HHC), see [2] and [3]. It is based on the estimation of a gain matrix (*T*) relating the harmonic components of the swash plate commands to those of the fuselage vibration. The control rationale is such that, at each rotor period or multiples, a small variation is superimposed to the blades commands by simply inverting the algebraic matrix computed in the previous time period. This steady-state approach has the advantage of requiring only little knowledge of the dynamics underlying the influence of the swash plate commands on the vibrations (just the *T*-matrix, gain at the frequency Ω_o). On the other side, the lack of knowledge of the dynamics governing the above relationship, makes it quite difficult to evaluate the stability margins and the response times of the overall control loop. Moreover, there is no guarantee that the control system does not interfere with the machine guidance commands.

Some novel model-based time-domain approaches have been explored. These rely on a dynamic description of the influence of the collective command on the total vertical force transmitted from the blades to the rotor mast, worked out. It turns out that, for the A129 machine, an appropriate model is a Single Input Single

Output (SISO) system with input $u(t)$ (collective swashplate command) and output $y(t)$ (total vertical mast force). Its transfer function is given by:

$$G(s) = k_G \frac{\prod_{i=1}^{9}(s-z_i)}{\prod_{j=1}^{9}(s-p_j)} .$$

The poles p_j and zeros z_i of $G(s)$ are graphically depicted in Fig. 2 and their numerical values are reported in Tab. 1. Note that the transfer function is proper (numerator and denominator with equal degree), Hurwitz (all poles in the left half plane) and non minimum phase (two zeros lie in the right half plane).

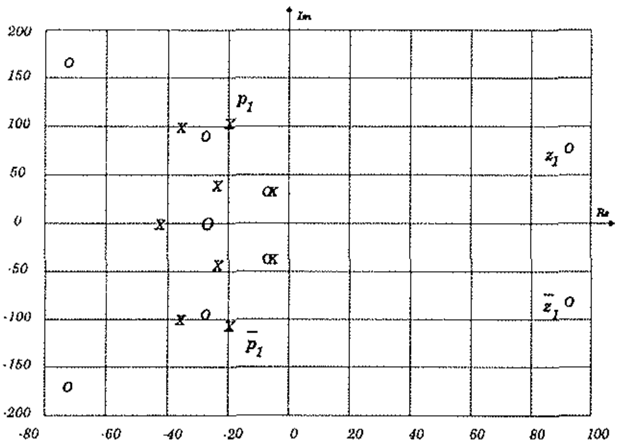

Fig. 2 - Rotor transfer function poles and zeros

Transfer constant [Nw. / mm.]	Zeros	Poles
k_G=2.47646·10^5	94.096-83.551j	-8.56-42.82j
	94.096+83.551j	-8.56+42.82j
	-9.8987-42.274j	-34.48-100.98j
	-9.8987+42.274j	-34.48+100.98j
	-29.189	-42.256
	-28.841-94.871j	-19.509-107.16j
	-28.841+94.871j	-19.509+107.16j
	-70.667-158.55j	-25.299-40.070j
	-70.667+158.55j	-25.299+40.070j

Tab. 1

For a control system to embody sinusoidal disturbance rejection capabilities, it is necessary that a dynamic block, called *Harmonic Integrator*, appears in the feedback loop, [5]. The latter has to incorporate two purely imaginary poles at the disturbance frequency so that an *N/rev* counter-vibration signal is generated within the control loop. Many disturbance rejection design techniques automatically lead to controllers incorporating harmonic integrators. This is the case of the *Observer Based Control* approach adopted in [6]. This approach consists in building a suitable model of the disturbance (vibration) of known frequency and to use an observer to produce a real time estimate of its amplitude and phase.

In this paper, we explore the possibility of using *Optimal Control* methods for helicopter vibration reduction. Such an idea has already been considered by various authors, see e.g. [7] and [8]. In [7], a solution to the narrow-band disturbance rejection problem can be found by resorting to the so-called *Frequency Shaping* control, which amounts to introducing a frequency dependent weighting of the state in the cost functional so as to impose an infinite weight in correspondence of the disturbance frequency. In [8], the idea is to augment the system with a dynamic model of the disturbance and perform estimation of the augmented state vector. The estimate of the disturbance state is then used to produce the necessary counter-vibration. Though the idea underlying this method (called *Disturbance Modelling*) is effective in principle, the mathematical formulation of the overall problem given in [8] suffers of some serious weak points, which look as major obstacles to the practical use of the approach. This is discussed in [9], where a more refined *Disturbance Modelling* approach has been proposed. Herein, the method is tested in the A129 case, by also taking into account some of the primary helicopter requirements such as the non-interaction with the pilot's commands.

The paper is organized as follows. In Sect. 2., the theoretical background of the proposed methodology is sketched for the user. Sect. 3. deals with the application of the above technique to the rejection of the Ω_o and $2\Omega_o$ harmonics of the vibration induced by the blades rotation in the helicopter fuselage. Sect. 4. contains some concluding comments on the paper.

2. The *LQG* Disturbance Modelling methodology for multiharmonic disturbance rejection

2.1. The theoretical background

As is well known, standard optimal control techniques, relying on the minimization of a quadratic performance index, give rise to stabilizing compensators whose performance can be tuned by choosing few design parameters. However, in order for the controller to ensure rejection of persistent harmonic disturbances, some further modifications are needed in the design procedure. To this purpose, several contributions are worth mentioning for the attenuation of constant and periodic disturbances, see e.g. [10] and [11]. Each of the these works explores a facet of the problem and a comprehensive analysis of the subject can be found in [12].

In this section, we will give a brief outline of the formulation and solution of the disturbance rejection problem via Linear Quadratic Gaussian Disturbance Modelling (*LQG*-DM). The general methodology will be then adapted to the particular problem of vibrations rejection in helicopters.

Consider a state-space realization of a single input-single output system subject to an output additive disturbance

$$\begin{cases} \dot{x}(t) = A\,x(t) + B\,u(t) \\ y(t) = C\,x(t) + \tilde{d}(t) \end{cases} \tag{1.1}$$

where x is the n-dimensional state vector, while u and y are the system input and output respectively. In our case this system represents the rotor dynamic behaviour, $u(t)$ being the collective swash plate command, $y(t)$ the total mast force and $\tilde{d}(t)$ its vibratory component. More in detail, we will suppose that $\tilde{d}(t)$ is a periodic function with frequency $\Omega_o = n_b\Omega_{rot}$, i.e.

$$\tilde{d}(t) = \sum_{k=1}^{N} \tilde{\alpha}_k \sin k\Omega_o t + \tilde{\beta}_k \cos k\Omega_o t \tag{1.2}$$

where only N harmonics are taken into account. Since the rotor angular velocity is known, the disturbance consists of a sum of sinusoids with known frequency and unknown amplitude and phase, i.e. Ω_o is fixed and known, while $\tilde{\alpha}_k$ and $\tilde{\beta}_k$ are unknown.

One of the main objectives of the control system is to make $y(t)$ insensitive to $\tilde{d}(t)$.

In order to simplify the subsequent analysis it is advisable to move the disturbance from the output to the input of the system, i.e.:

$$\begin{cases} \dot{x}(t) = A\,x(t) + B\,[u(t) + d(t)] \\ y(t) = C\,x(t) \end{cases} \tag{2}$$

where the $d(t)$ is an "equivalent" disturbance acting on the input of the system. Under weak assumptions on poles and zeros of the system, models (1) and (2) are input-output equivalent [12].

Notice that the $d(t)$ can be modeled as the output of an autonomous system with purely imaginary eigenvalues at frequencies $k\Omega_o$, $k=1,2, \ldots N$:

$$\begin{cases} \dot{\xi}(t) = W\,\xi(t) \\ d(t) = H\,\xi(t) \end{cases} \tag{3}$$

where

$$W = \begin{bmatrix} \begin{bmatrix} 0 & -\Omega_0^2 \\ 1 & 0 \end{bmatrix} & & 0 \\ & \ddots & \\ 0 & & \begin{bmatrix} 0 & -N\Omega_0^2 \\ 1 & 0 \end{bmatrix} \end{bmatrix}, \quad H = \begin{bmatrix} [1\ \ 0] & & 0 \\ & \ddots & \\ 0 & & [1\ \ 0] \end{bmatrix}$$

The overall system, consisting of the rotor and the disturbance (eqs. (2) and (3)), can be written in a compact state space form:

$$\begin{cases} \dot{x}_{DM}(t) = A_{DM}\,x_{DM}(t) + B_{DM}\,u(t) \\ y(t) = C_{DM}\,x_{DM}(t) \end{cases} \tag{4}$$

with $x_{DM}(t) = \begin{bmatrix} x(t) \\ \xi(t) \end{bmatrix}$ and

$$A_{DM} = \begin{bmatrix} A & BH \\ 0 & W \end{bmatrix}, \; B_{DM} = \begin{bmatrix} B \\ 0 \end{bmatrix}, C_{DM} = \begin{bmatrix} C & 0 \end{bmatrix}.$$

To cope with uncertainty in modelling and output measurement, a more refined model should also consider state and output disturbances, i.e.:

$$\begin{cases} \dot{x}(t) = A\,x(t) + B\,[u(t) + H\,\xi(t)] + v_{11}(t) \\ \dot{\xi}(t) = W\,\xi(t) + v_{12}(t) \end{cases}$$

$$y(t) = C\,x(t) + v_2(t)$$

or, in compact form,

$$\begin{cases} \dot{x}_{DM}(t) = A_{DM}\,x_{DM}(t) + B_{DM}\,u(t) + v_{DM}(t) \\ y(t) = C_{DM}x_{DM}(t) + v_2(t) \end{cases} \tag{5}$$

$v_{DM}(t) = [v_{11}(t)'\ v_{12}(t)']'$ being the state disturbance and $v_2(t)$ the measurement disturbance.

It is assumed that $v_{DM}(t)$ and $v_2(t)$ are zero-mean independent white noises with intensities

$$V_{DM} = var[v_{DM}] = \left[\begin{array}{c|c} V_{11} & 0 \\ \hline 0 & V_{12} \end{array}\right]$$

and $V_2 = var[v_2(\cdot)]$. Matrices V_{11} and V_{12} are positive semidefinite, $V_{11} \geq 0$, $V_{12} \geq 0$. Matrix V_2 is assumed to be positive definite, $V_2 > 0$. We will come back on the structure of V_{12} in Sect. 2.2.

Finally, as in any optimal control context, one has to choose the performance index. Asymptotic rejection of the disturbance $d(t)$ can be achieved only if the input signal $u(t)$ converges to $-d(t)$, so that $\{u(t) + d(t)\}$ asymptotically vanishes. Therefore a reasonable cost functional should weight the energy of the term $\{u(t) + d(t)\}$ rather than the sole energy of $u(t)$ as it is usually done in standard *LQG* problems. Hence, an appropriate quadratic criterion for the problem at hand is:

$$J_{DM} = E\left\{ \lim_{\substack{t_0 \to -\infty \\ t_f \to +\infty}} \frac{1}{t_f - t_0} \int_{t_0}^{t_f} x(t)'Qx(t) + [u(t) + H\xi(t)]'R[u(t) + H\xi(t)]\, dt \right\}. \tag{6}$$

2.2. Non-Interaction

Since the active control signal is superimposed to the pilot's guidance commands, most care has to be taken to avoid any perturbation on the pilot's action. This can be achieved by guaranteeing that the active control signal does not have frequency content at low frequencies. This goal can be achieved in the *LQG-DM* context by a suitable choice of the (fictitious) covariance matrix V_{12}. Indeed, take as V_{12} the following matrix:

$$V_{12} = \begin{bmatrix} \gamma_1^2\Omega^4 GG' & & 0 \\ & \ddots & \\ 0 & & \gamma_N^2 (N\Omega)^4 GG' \end{bmatrix} \qquad (7)$$

with $G=[g_1\ g_2]'$ and $\gamma_k \geq 0$, $k=1,2..N$. This means that the disturbance $d(t)$ is modeled as the sum of the outputs of N dynamical systems fed by uncorrelated white noises and characterized by the transfer functions

$$G_k(s) = \gamma_k (k\Omega)^2 \frac{g_1 + g_2 s}{s^2 + (k\Omega)^2}, \quad k=1,2..N.$$

Parameters g_1 and g_2 determine the position of the zero of $G_k(s)$. A good choice is $g_1=0$ and $g_2=1$. Correspondingly, the spectral density of the k-th component of the disturbance is represented in Fig. 3

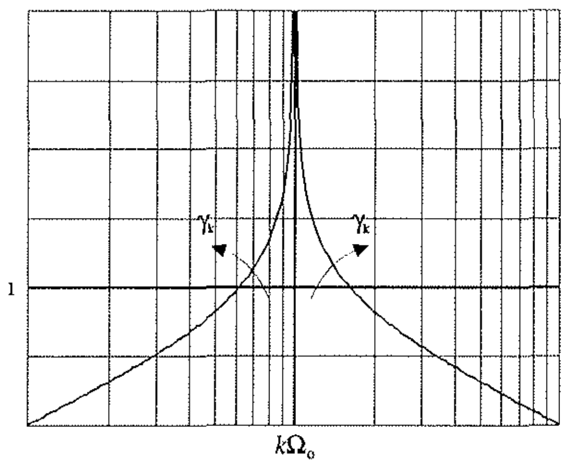

Fig. 3 - Spectral density of the k-th component of the disturbance modeled with the choice of V_{12} given by (6).

With such a disturbance modellization, the optimal control effort will concentrate mainly at those frequencies where the disturbance $d(t)$ has high energy, i.e. $\omega=k\Omega_o$, $k=1,2..N$. In contrast, thanks to the zeros in the origin in each $G_k(s)$, the controller will not produce significant changes in the low-frequency behaviour of the closed-loop system. In particular, it will be shown (Sect. 3.) that the open- and closed-loop gains are the same.

As for parameters γ_k, it could be shown that each of them approximately determines how quickly the effect of the k-th component of the disturbance is attenuated: large values of γ_k result in rapid rejection transients.

2.3. Final form of the optimal controller

The optimal *LQG* problem stated above is non-standard and calls for some care to be correctly solved, [9]. Herein, the resulting control law is described:

Kalman filter. The state of the *augmented* system (5) is estimated by means of a Kalman Filter. As a result, estimates of the rotor state ($\hat{x}$) and of the equivalent input-disturbance ($\hat{d} = H\hat{z}$) are available.

Optimal Feedback Gain. An optimal feedback gain (K) is computed for the *non-augmented* system. The final overall optimal control law is

$$u(t) = K\hat{x} - \hat{d}$$

where $K\hat{x}$ can be interpreted as a stabilizing term, while $-\hat{d}$ performs the disturbance compensation.

3. Application to the helicopter vertical vibrations reduction

3.1. Compensator design

From the above discussion, it follows that the structure of the control system for active control of vibrations is the one depicted in Fig. 4.

For the compensator specification, the designer may act on the noise intensities V_{11}, V_{12} and V_2, which affect the Kalman filter performances. As discussed in Sect. 2.2., V_{12} is chosen as in (7) with $G = [0\ 1]$. Hence, tuning V_{12} means selecting $\gamma_1, \gamma_2, ..., \gamma_N$. The designer has also to select matrices Q and R appearing in the performance index (6). By means of these matrices, the stabilizing properties of the "*LQ-regulator*" part in Fig. 4 can be modified .

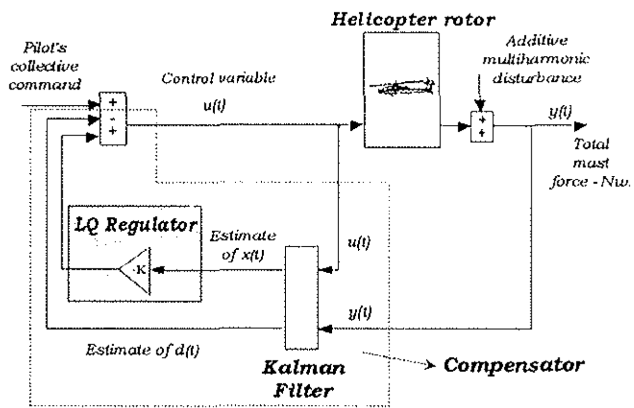

Fig. 4 - Structure of the controller

3.2. Simulation tests

In the simulations, a disturbance signal with two harmonics has been considered:

$$d(t) = a_1 \sin(\Omega_o t + \varphi_1) + a_2 \sin(2\Omega_o t + \varphi_2)$$

where Ω_o is the main vibration frequency at 4/rev (144.51 rad/sec) and $2\Omega_o$ is its first multiple (8/rev = 289.02 rad/sec). For the subsequent simulations, the numerical values of the parameters have been chosen as:

$a_1 = 4000\ Nw, \quad \varphi_1 = 70°$

$a_2 = 2500\ Nw$, $\varphi_2 = 40°$.

As a first step, we neglect the effect of the pilot's commands and we concentrate our attention just on the disturbance rejection capabilities of the closed-loop system. The free parameters have been choosen as reported in Tab. 2, where I_{9x9} denotes the identity matrix of dimension 9. The time behavior of the

$\gamma_1=\gamma_2=\gamma$	0.5
V_{11}	$3 \cdot 10^4\ I_{9x9}$
V_2	0.125
Q	I_{9x9}
R	1

Tab. 2

corresponding output hub force and of the control variable (blade's collective pitch angle) can be seen in Fig. 5.

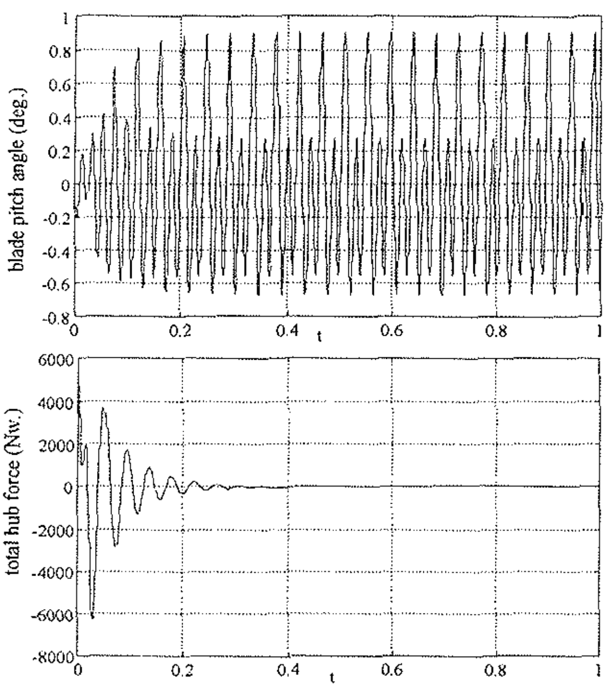

Fig. 5 - Closed-loop pitch angle (control variable) and total hub force (controlled variable)

Despite a control effort of less than 1 deg. pitch angle imposed to each blade, the output hub force turns out to be, after about 3 rotor revolutions, almost totally insensitive to the disturbance. Note that this behavior meets the constraint of achieving vibrations reduction with a limited effort (max. 3 deg.) of the control action. Thanks to *LQG* theory, it is guaranteed that the overall closed-loop system is asymptotically stable (Fig. 6).

3.3. Compensator stability

One should observe that the regulator itself turns out to be stable. This is indeed a highly desirable property of the control design. Indeed, it guarantees the boundedness of the control signal even in case of malfunctioning resulting in unexpected loop openings.

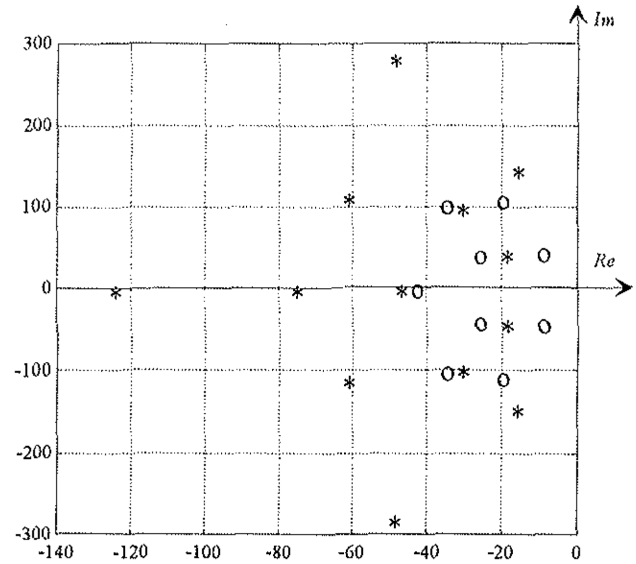

Fig. 6 - Closed-loop poles

The pole pattern of the designed regulator $R(s)$ can be seen in Fig. 7. Note that, as expected, according to the *Internal Model Principle* [13], two pairs of imaginary poles at Ω_o and $2\Omega_o$ are present so as to force two corresponding pairs of imaginary blocking zeros in the transfer function from $d(t)$ to $y(t)$.

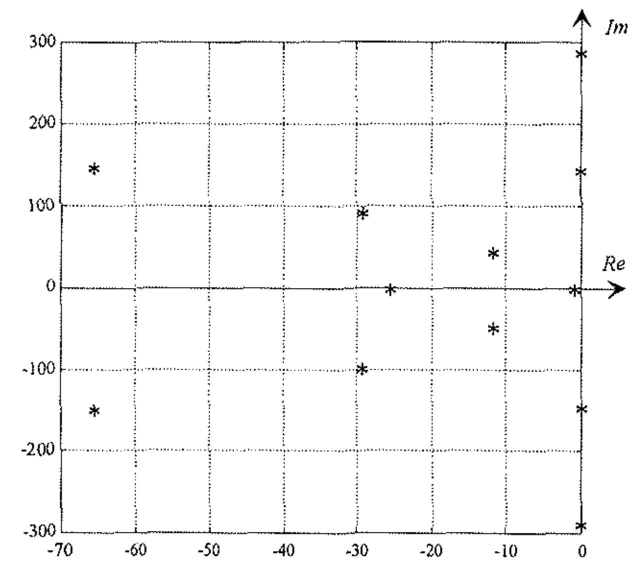

Fig. 7 - Regulator poles

3.4. Non-Interaction

Whatever values are chosen for the tuning parameters, our compensator ensures non-interaction with the helicopter guidance commands. In fact, since the feedback control signal is added directly to the pilot's input (Fig. 4), the control frequency content should not embrace the low frequency range where most probably the pilot's signal takes place. Thanks to a particular choice of the disturbance model (Sect. 2.2.), it is possible to force the action of the compensator to be almost insignificant at low frequencies.

To prove this, we have simulated the control loop when a step pitch angle of 3 deg. is imposed to each blade by the pilot. The graphical results of the simulation are depicted in Fig. 8.

By means of some trivial computations, one may verify that the regime value of the total hub force equals the product of the rotor gain and the pitch step, as desired to guarantee non-interaction.

Remark

Since now, we have supposed perfect knowledge of the model parameters (k_G, p_i, z_i). Herein, it is analyzed what happens to the closed-loop performance when the actual dominant poles are slightly different from their nominal values.

Suppose to perturbe the model dominant poles ($-9.509 \pm 107.16j$) so as to weaken their damping (increase their imaginary part of 15%).

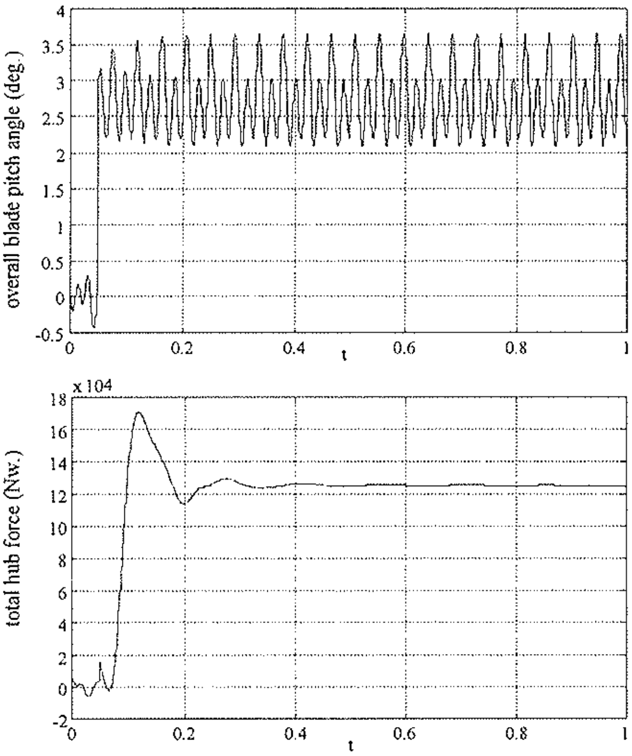

Fig. 8. Closed-loop pitch angle (control variable) and total hub force (controlled variable)

The simulation performed under these conditions (Fig. 9) shows that the compensator has mantained its rejection capabilities but the transients are longer than before. However, the fact that both the closed-loop system and the regulator keep on being asymptotically stable, suggests that a fairly reasonable stability margin has been obtained.

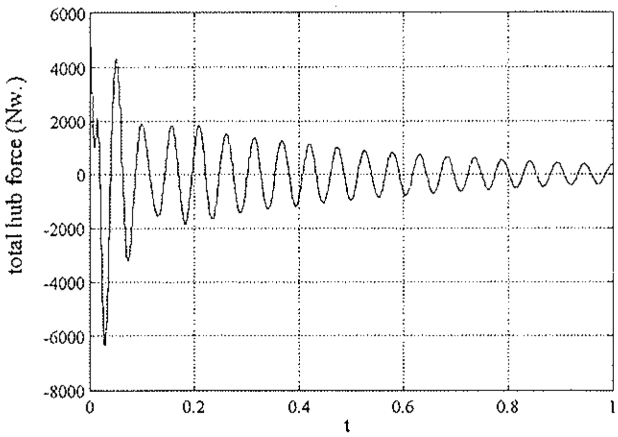

Fig. 9. Closed-loop response with poles perturbation

4. Concluding remarks

In this paper, an optimal control technique for helicopter vibrations reduction has been outlined. Differently from widely known steady-state algorithms, such as *HHC*, the proposed scheme is based on a suitable description of the main dynamics governing the rotor behavior. The vibration is described as a multiharmonic signal and then modelled as output of a suitable dynamic system.

In the paper we have shown how a vibration rejection compensator can be designed by means of an *LQG* rationale. One has to solve a standard control problem to ensure stability of the feedback system plus a filtering problem, for the whole augmented system, to estimate both the unmeasurable rotor states and the disturbance. The control action is the sum of two different terms: the first one accounts for closed-loop stabilization and the second for the generation of a countervibration at the disturbance frequencies. Some tuning parameters are available to find the "optimal" trade-off between closed-loop stability degree, disturbance rejection time and control effort. Interestingly enough, the designed controller is itself stable, which is a guarantee against possible faults in the closed-loop functioning.

The effectiveness of this control technique has been analyzed by means of some simulation trials. Particular attention has been devoted to the low frequency non-interaction between the compensator and the pilot action.

5. Acknowledgements

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