FXLMS vs Principal Components in On-Blade Control Applications

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Abstract

Among the various approaches to mitigate vibration, On-Blade Control (OBC) embeds actuation mechanisms on the blade in order to modify the vibratory loads at the source and achieve improved vibration reduction than conventional Higher Harmonic Control. Recent OBC studies consider state-of-the-art optimisation methods, more suitable for off-line implementations. This paper considers instead the use of two recursive methods to account for practical barriers such as computational limitations of the embedded systems and estimation errors. The considered algorithms in this study are FXLMS and Principal Components. The analysis shows that they can perform well in practice under considerable practical limitations and also discuss their benefits and disadvantages for OBC applications. A nominal stability analysis is provided and its advantages include intervals of the tuning controller parameters which guarantee nominal stability. The control algorithms and stability analysis are applied to a vibration reduction simulation example.

1 INTRODUCTION

On-Blade Control has become a significant area of research in the rotorcraft community due to its potential to offer greater benefits in terms of vibration, noise and power reduction, with vibration being usually the dominant performance aspect. The future technology requires active devices embedded in each of the main rotor blades. There exists several types of OBC devices: 1) Gurney or Micro flaps, 2) Active Trailing Edge Flaps (ATEF, see Figure 1), 3) Active Twist Rotors and 4) Active Blade Tips [4]. ATEF is perhaps the device that has received most attention, becoming the most mature OBC device due to its conceptual simplicity and low power requirements. Advantages of OBC, with respect to more traditional active vibration control methods, such as Higher Harmonic Control (HHC) and Individual Vibration Control (IBC), include lower power requirements, improved performance by originating forces and moments at the source of the considered vibration and less interference with the primary flight control system.

Existing control algorithms for OBC are similar to HHC [5], since control laws are performed in the frequency domain and target major frequency components of the vibration signals. The rotor behaviour is represented, for valid operating conditions, by an affine transformation between selected harmonic coefficients of the control actions (e.g. flap deflections for OBC with ATEF) and coefficients of dominant vibration harmonics. Typically, the algorithm is based on the unconstrained minimisation of a quadratic performance function, which encapsulates vibration levels and control energy usage. Alternative to unconstrained OBC, constrained optimisation methods, in particular Quadratic Program, have recently been used for OBC applications. The benefits of using the later approach lie on the explicit consideration of control input constraints, which can have a major effect on the achieved level of performance. For instance, if actuator limitations are not considered carefully, control algorithms can deliver very poor performance and in more undesirable scenarios, instabilities [8].

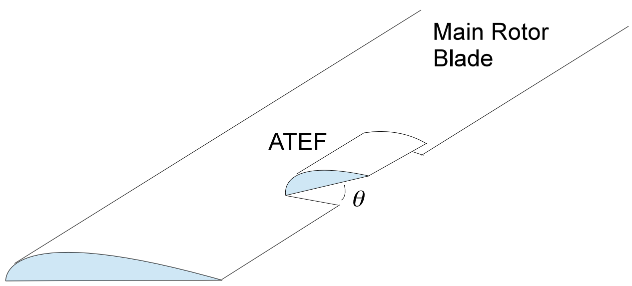

Figure 1: General schematic of an active trailing edge flap (ATEF) providing a deflection angle θ.

Although analytical or closed form solutions can be obtained for unconstrained OBC algorithms, in practical applications they are not performed because inversion of matrices is required, which increases computational efforts and times, and might be sensitive to numerical instabilities because of round-off errors, especially for ill-conditioned matrices. Practical implementations use instead *adaptive* or *recursive* forms of the unconstrained optimal control laws. These expressions usually require that new control actions are obtained by a linear combination of the previous control actions and current readings of the controlled outputs. The overall idea is that instead of obtaining the optimal solution in one major single step, the control system approaches asymptotically to the desired optimal operating point providing overall better adaptability characteristics in the presence of estimation errors and unknown disturbances and smoother transients. There are many variants of OBC algorithms. Principal Components (PC) algorithms, which are constructed on the singular value decomposition of the open-loop process, are very popular and offer an intuitive and flexible way to handle actuator limitation [1]. This is done by restricting the control to the main modes of the system, which require less control efforts. Alternatively, more recent OBC algorithms can handle actuator constraints explicitly using Quadratic Program optimisation methods [8, 9]. There are however no adaptive or recursive versions of OBC algorithms for implementations on embedded systems.

Popular steepest descent and Newton-based methods, which include Least Mean Squares (LMS) [3], can also be captured under the PC framework. Existing literature on OBC offer stability criteria for nominal analysis and in the presence of estimation errors. Such stability results are particularly useful for practical implementations of the algorithms as they can provide confidence ranges (as conservative as they might be) in the tuning of controller parameters. More recent pieces of work provide more complete results by applying advanced stability theory to obtain robustness guarantees in the presence of both modelling or estimation errors and pragmatic ways of dealing with control input limitations [10].

Two recursive implementations of unconstrained OBC algorithms, which are based on steepest descent algorithms, are Filtered-Reference Least Mean Square (FXLMS) and Principal Components (PC) [3]. The main motivation behind these implementations in more general active noise and vibration control applications is expressed in terms of cheaper computational costs. The purpose of the paper is the application of these algorithms to draw comparisons in OBC applications. We find in this paper proposal that applications of FXLMS and PC to OBC offer improved adaptation properties in comparison to closed-form or non-recursive solutions. By adaptability we refer to the capability of maintaining satisfactory levels of performance despite the presence of key limiting factors. We test the algorithms using a linear representation of the quasi-steady behaviour of the main rotor considering three performance limiting aspects: modelling errors, limited control effort and measurement noise.

The paper is structured as follows. Fristly, the modelling of the OBC process is briefly discussed in Section 2. Secondly, a brief review of both algorithms are provided: FXLMS in Section 3 and Principal Components in Section 4. In each of these sections, we will discuss the roles of major tuning parameters and key characteristics. Section 5 provides a derivation for nominal stability conditions for common choices of the considered algorithms. Section 6 provides a reference OBC problem for which both algorithms are applied. Both stability and performance assessment will be discussed in this section. The paper concludes with some final remarks in Section 7.

2 Rotor Modelling

Most OBC laws are developed from Higher Harmonic Control ideas. For vibration reduction purposes, HHC is constructed from the assumption that the relation between selected Fourier (sine and cosine) coefficients of the actuator signal and output forces and moments [5] is linear. Such representation aims to capture up to some extent the quasi-steady rotor response in cruise flight conditions. Define a complex vector $e(k)$ in phasor form as the output containing harmonic information of the vibration at the time instant indicated via the index k, with $t = k\Delta t$ and Δt representing the time gap between each implementation of the control actions. Likewise, define the input complex vector $u(k)$ containing the harmonics of a control input signal. The above assumption in the modelling of the rotor system is encapsulated in the following mathematical equation, expressed in complex form [3] as:

(1) $$e(k) = Gu(k) + d$$

d denotes the complex or phasor representation of the baseline vibration, which is equivalent to $e(k)$ when the control inputs are zero ($u(k) = 0$). Commonly, the complex matrix G is referred to as the *interaction matrix* or *sensitivity matrix* [12]. The above model is referred to by Johnson [5] as the *global model* of helicopter response and can be rewritten as

(2) $$e(k) = e_0 + G(u(k) - u_0)$$

u_0 an e_0 represent the initial control input and measured output, respectively.

Control algorithms are based on the minimisation of a performance function $J(k)$ at the time index k, which is expressed in a quadratic form for mathematical convenience, and whereby a trade-off between

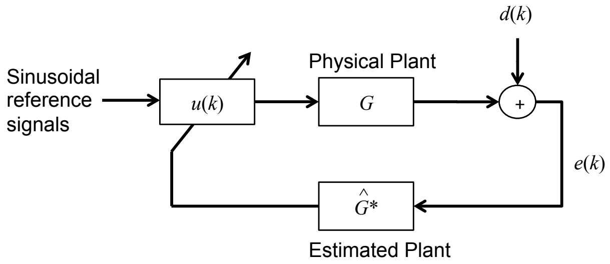

Figure 2: Block diagram of the FXLMS algorithm.

vibration reduction and control efforts is specified:

(3) $$u(k)^{\dagger} = \arg\min_{u(k)} \underbrace{e(k)^{*}He(k) + u(k)^{*}Fu(k)}_{J(k)}$$

Typically for vibration reduction, $e(k)$ contains the sine and cosine components of the N/rev hub loads and moments. The weight $H = H^T > 0$ is real and used to target specific vibration reduction among some of the vibration channels. Likewise, the weight $F = F^T > 0$ is used to specify actuator authority in the frequency domain. For instance, more weight can be associated with lower harmonics as the actuator control system is expected to perform better at such frequencies than at higher ones [7]. Often, both weights are diagonal and may be scaled differently if sensor measurements are provided in different units. A good starting point when designing the controller is to chose the same weight for all channels, which corresponds to $H = F = I$, given that all vibration measurements as well as control signals are provided in the same units and actuators have enough bandwidth.

In the case where the optimisation problem is considered without actuator constraints, an analytic solution can be found by making

(4) $$\frac{\partial J(k)}{\partial u(k)} = 0$$

Solving for $u(k)$ provides the following analytical expression for the optimal control input

(5) $$u^{\dagger}(k) = -(G^{*}HG + F)^{-1}(G^{*}H)(e_0 - Gu_0)$$

where $d = e_0 - Gu_0$. This is the classical expression of the HHC algorithm.

3 FXLMS

The philosophy of the LMS algorithms is to adapt the filter coefficients in the opposite direction of the instantaneous gradient of the mean square error with respect to the coefficients. In this case, $H = I$ and $F = 0$. The complex gradient of the performance function with respect to the control input can be written as

(6) $$\frac{\partial J_k}{\partial u_k} = g(k) = 2\left(G^{*}Gu(k) + G^{*}d\right)$$

The LMS can thus be written as

(7) $$u(k+1) = u(k) - \mu g(k)$$

with μ as the convergence factor. Assuming that the measured vibration signals $e(k)$ have time to reach their steady-state values at each iteration, the steepest-descent algorithm which minimises the sum of the squared error signals can be written as

(8) $$u(k+1) = u(k) - \alpha G^{*}e(k)$$

This is the algorithm also known as Filtered-reference LMS or the filtered-x LMS (FXLMS). The parameter $\alpha = 2\mu$ is now the convergence coefficient. In practical implementations, the true interaction matrix G would not necessarily be perfectly known, as shown in Figure 2.

4 Principal Components

PC algorithms exploit the Singular Value Decomposition (SVD) of an estimated value of the physical plant $\hat{G}$ to alleviate on-line computational burden and offer increased flexibility in the control law. The SVD of $\hat{G}$ is expressed by the following factorisation

(9) $$\hat{G} = R\Sigma Q^{*}$$

where $\Sigma \in \mathbb{R}^{m\times n}$. $R \in \mathbb{C}^{m\times m}$ and $Q \in \mathbb{C}^{n\times n}$ are orthogonal matrices ($R^{*}R = Q^{*}Q = I$). The diagonal elements of Σ are positive and known as the singular values. They are arranged in descending order. We will assume that both G and its estimate $\hat{G}$ are full rank matrices, i.e., rank(G) = rank$(\hat{G}) = \min\{m, n\}$.

Smaller singular values are usually subject to greater uncertainty and therefore attempting to control such modes leads to performance and stability degradations. For this reason the control is typically performed only on the most significant modes (hence the name PC). Mathematically, this is done by representing the SVD of G by

$$\hat{G} = \begin{bmatrix} R_r & R_{\perp} \end{bmatrix} \begin{bmatrix} \Sigma_r & \\ & \Sigma_{\perp} \end{bmatrix} \begin{bmatrix} Q_r^{*} \\ Q_{\perp}^{*} \end{bmatrix}$$

where $R_r \in \mathbb{C}^{m\times r}$ and $Q_r \in \mathbb{C}^{n\times r}$ are, respectively, the matrices containing the first r columns of R and Q. Note that the number of controlled modes are chosen by $r \leq$ rank$(\hat{G})$. The matrix Σ_r = diag$(\sigma_1, \ldots, \sigma_r) \in \mathbb{R}^{r\times r}$ where the positive value σ_i denotes the i-th singular value of $\hat{G}$.

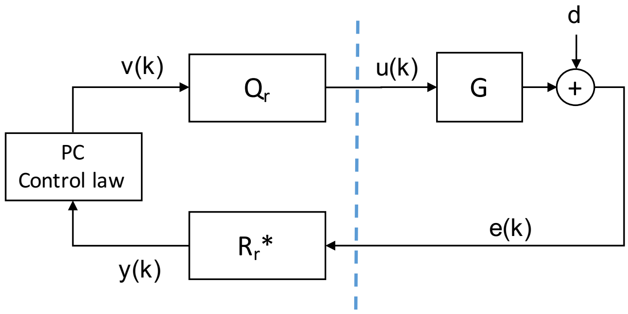

Figure 3: PC Control Architecture.

PC controllers improve the convergence speed of multichannel tonal control by transforming the input and the output signals in the so-called modal space, see [11, 2] and [13]. Such a transformation can be expressed as follows

(10) $$v(k) = Q_r^* u(k)$$

(11) $$y(k) = R_r^* e(k)$$

where $v(k) \in \mathbb{C}^r$ and $y(k) \in \mathbb{C}^r$. Figure 3 shows the block diagram for such a control architecture.

For the sake of generality in the results, the PC algorithm is described by the following control law

$$v(k+1) = W_v v(k) - W_y y(k)$$

with $W_v \in \mathbb{R}^{r \times r}$ indicating the weight associated with control efforts and $W_y \in \mathbb{R}^{r \times r}$ being the weight associated with the measured signal. We assume without loss of generality that $W_v = W_v^T > 0$ and $W_y = W_y^T > 0$.

A common choice for the weights is given below

$$\begin{aligned} W_v &= \text{diag}(1-\alpha_1\beta_1, \ldots, 1-\alpha_r\beta_r) \\ W_y &= \text{diag}(1-\alpha_1, \ldots, 1-\alpha_r) \end{aligned}$$

This algorithm was proposed by [1] and will be referred as the PC-LMS algorithm. By choosing the convergence coefficient α_i and the control effort weighting $(1-\alpha_i\beta_i)$ independently for each principal component, considerable flexibility can be introduced in the tuning of the controller, which could lead to improved performance and stability in practical applications.

5 Stability Analysis

The attention in this section is concentrated to obtain stability guarantees when there are no modelling errors in the estimation matrix $G = \hat{G}$. The results are conservative however not only because they do not account for modelling errors, but also for the signal processing blocks that estimates the required harmonic coefficients of interest, computational and communication delays and possibly scaling of the control signals. Despite such conservatism, the following stability criteria are helpful in many practical scenarios as they provide simple stability ranges for key controller parameters.

For simplicity in the following analysis, we assume only positive ad non-negative values of the tuning parameters, i.e., $0 < \alpha$ and $0 \leq \beta_i, \alpha_i$ for all modes. The dynamics of the closed-loop is characterised in the modal space by the following difference state equation

$$v(k+1) = (W_v - W_y\Sigma_r)\, v(k) - W_y R_r^* d(k)$$

The stability of the PC algorithm in this case is equivalent for the eigenvalues of the state matrix to be within the unit circle:

(12) $$|\lambda_i(W_v - W_y\Sigma_r)| < 1, \; \forall\, i = 1, \ldots, r$$

where $\lambda_i(X)$ represents the ith-eigenvalue of a valid matrix X. Simplifying the above criterion for the considered common controller choices provides:

- *FXLMS*: The analysis can be done in terms of the singular values by choosing

  $$\begin{aligned} W_v &= I \\ W_y &= \alpha\Sigma_r \end{aligned}$$

  with $r = \text{rank}(G)$. Simplifying the stability criterion leads to

  $$0 < \alpha < \frac{2}{\sigma_1^2}$$

  The above is indeed equivalent to the results presented in [3] since $\sigma_1^2 = \max\{\lambda_i(G_o^* G_o)\}$.

- *Diagonal PC*: it is common to select diagonal weights in practical implementations of PC-based algorithms. For the general diagonal case, we denote the diagonal terms for the input weight W_v and output W_y weight as w_{vi} and w_{yi}, respectively. Simplification of the general stability criterion (12) can be expressed in the following result:

  $$0 < w_{yi} < \frac{1 + w_{vi}}{\sigma_i}$$

- *PC-LMS*: Because of the positive assumptions on α_i and the positive definiteness of both W_v and W_y, the stability condition is divided into two cases for each mode:

  – If $2 \leq \sigma_i$ and $0 \leq \beta_i < \sigma_i$

  $$\frac{\sigma_i - 2}{\sigma_i - \beta_i} < \alpha_i < \min\left\{1, \frac{1}{\beta_i}, \frac{\sigma_i}{\sigma_i - \beta_i}\right\}$$

  – If $\sigma_i < 2$

  $$\begin{aligned} 0 \leq \alpha_i &< \min\left\{1, \frac{1}{\beta_i}, \frac{2-\sigma_i}{\beta_i - \sigma_i}\right\}, \sigma_i \leq \beta_i \\ 0 \leq \alpha_i &< \min\left\{1, \frac{1}{\beta_i}, \frac{\sigma_i}{\sigma_i - \beta_i}\right\}, 0 \leq \beta_i < \sigma_i \end{aligned}$$

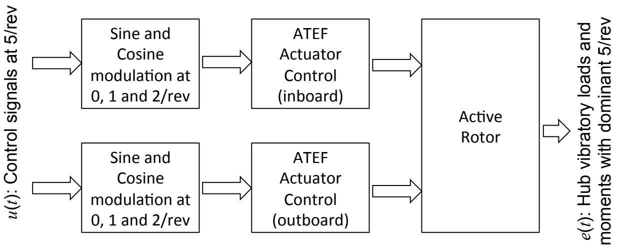

Figure 4: Schematic of the open-loop system.

Note that there does not exist a valid α_i for the case $2 \leq \sigma_i$ and $\beta_i \geq \sigma_i$.

6 Simulation Results

This section offers a test bench and compare both algorithms discussed: FXLMS and Principal Components. The simulation considers a main rotor with five blades and the target is to attenuate 5/rev components of vibrations at the rotor hub. The rotor behaviour contains natural frequencies in the range between 2 and 27.5Hz. Damping ratios are in the range between .01 and .033. We assume the blades have two sets of trailing edge flaps, placed at the inboard and outboard sections of the blades. The rotor behaviour is considered for cruise flight conditions and captured by a Linear-Time-Invariant transfer function matrix $G(s)$. The rotor operates with a constant angular velocity Ω and the steady state behaviour of the rotor is thus obtained by the complex matrix $G(jN\Omega) = G$, where $N = 5$ indicates the number of blades. The singular values of G are approximately 272, 232,115, 93, 49 and 49.

We have chosen to perform OBC with 3, 4, 5, 6 and 7/rev harmonics in order to target the 5/rev component of the vibratory hub loads [6]. In order to produce flapping signals at such frequencies, 5/rev fixed-frame control inputs are modulated with 0, 1 and 2 /rev harmonics. This control structure is shown in Figure 4 and it is the same followed in [8]. Refer to this paper for more details.

We run the simulation by first estimating the sensitivity matrix G. The steady-state behaviour of the system is estimated using a heterodyne filter, and the relative estimation error is measured using the following matrix metric

$$\|G - \hat{G}\|/\|G\| \approx 0.31$$

where the matrix norm $\|.\|$ indicates the induced 2-norm or maximum singular value matrix norm. We have also estimated the harmonics coefficients of the baseline vibration with a heterodyne filter. The error is expressed as

$$\|d - \hat{d}\|/\|d\| \approx 0.74$$

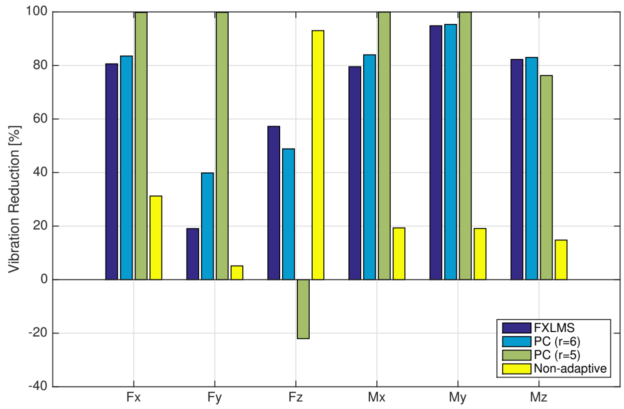

Figure 5: Steady-state Vibration Results.

where the norm in this case refers to the standard Euclidean vector norm.

Both controllers FXLMS and PC operates at a sampling frequency of $T_s = 100$ ms. In addition, we have introduced scaling for the control actions to ensure they operate within the actuator capabilities. Nominal stability conditions for the considered case are given below:

- FXLMS
$$0 < \alpha < 2.7078 \times 10^{-5}$$
- PC: We have assumed in this case $W_v = I$. The stability condition then establishes the following intervals for the diagonal elements of W_y

$$0 < w_{y1} < 73.59 \times 10^{-4}$$
$$0 < w_{y2} < 86.23 \times 10^{-4}$$
$$0 < w_{y3} < 173.38 \times 10^{-4}$$
$$0 < w_{y4} < 215.45 \times 10^{-4}$$
$$0 < w_{y5} < 404.11 \times 10^{-4}$$
$$0 < w_{y6} < 410.82 \times 10^{-4}$$

The tuning of the controller was an iterative process, ensuring that the adaptation gains were way within the ranges for the nominal stability criterion, but also where satisfactory performance was achieved. For the FXLMS, a value of $\alpha = 2 \times 10^{-6}$ was chosen. For the diagonal PC, we chose the following controller values

$$W_y = \text{diag}(6, 5, 4, 3, 2, 1) \times 10^{-4}$$

and considered the cases when $r = 6$ and $r = 5$. Choosing a lower number of principal components to control leads to very poor performance with respect to the FXLMS algorithm. The general simulation results are shown in Figure 5. The time history of the vibrations for each of the considered controller choices are shown in Figures 6- 9. The scaling factors of the controller actions are shown in Figure 10.

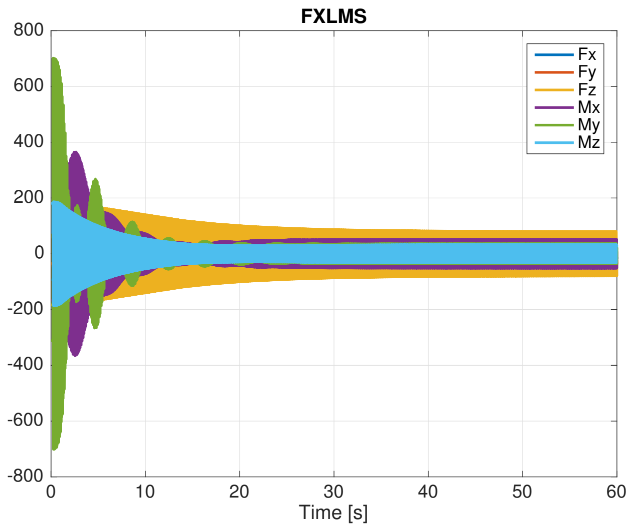

Figure 6: Vibration time history - FXLMS. Vibrations units are N and Nm.

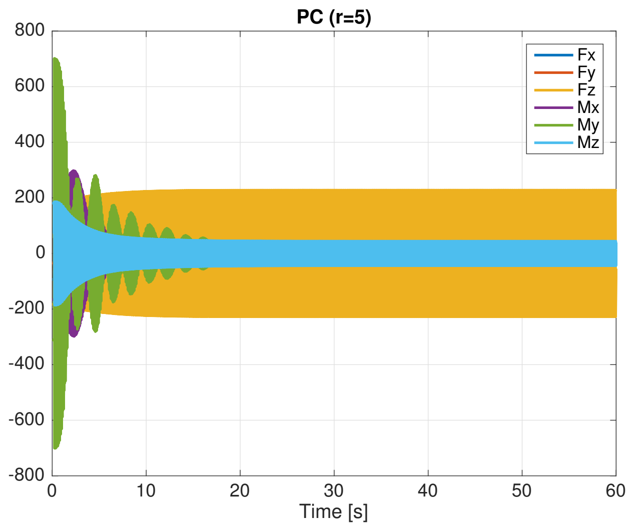

Figure 8: Vibration time history - PC with 5 modes. Vibrations units are N and Nm.

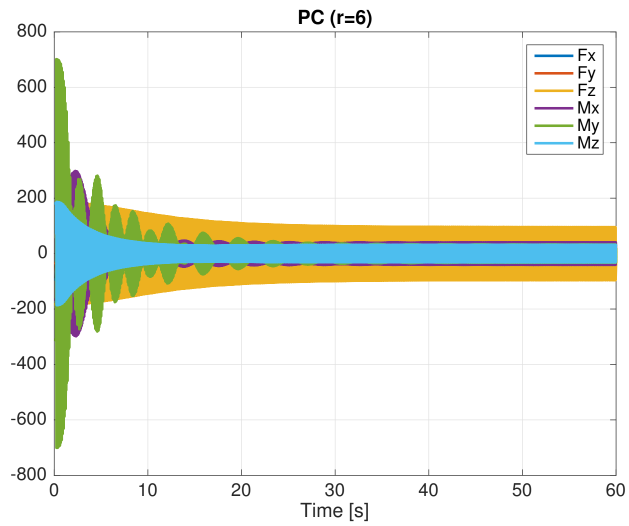

Figure 7: Vibration time history - PC with 6 modes. Vibrations units are N and Nm.

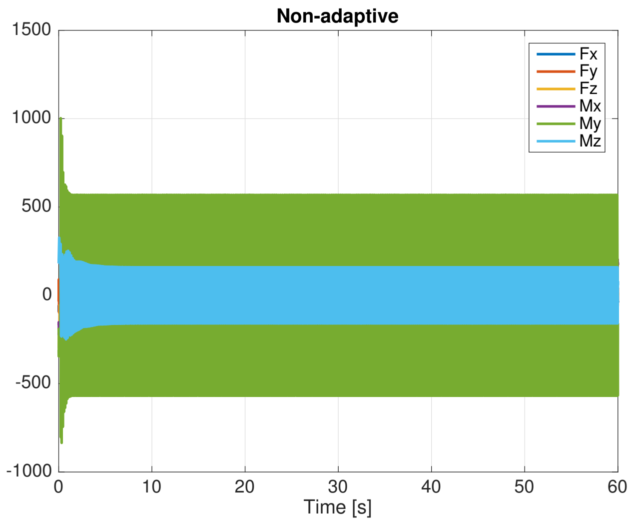

Figure 9: Vibration time history - Nonadaptive. Vibrations units are N and Nm.

It is shown that the PC can offer better performance at a lower computational costs in more realistic scenarios, when dealing with considerable estimation errors and control constraints. For the current simulation example, choosing actually a lower number $r = 5$ leads to better average vibration reduction ratio, about 76% with no saturation of the control actions occurring (scaling factor is one in this case). However, vibration in the vertical hub force component F_z is actually increased. On the other hand, FXLMS achieved a reduction ratio about 69% and saturation occurs (scaling factor becomes slightly less than 1 after 9 seconds approximately). Choosing to control all modes leads to a more balanced vibration reduction results where vibration reduction is achieved in all channels, so in practice this would become the desired controller to implement if computational capabilities allows it. Note that in this case there is also a small level of saturation. We compare also the adaptive strategies with respect to non-adaptive ones; it is clearly seen that non-recursive forms offer a poor performance at 30% due to the heavy scaling factor. Because the number of parameters to tune for the PC algorithm is larger, the design stage can become tedious, but this extra work can pay off in the end by achieving improved performance with more affordable control input energy.

7 Concluding Remarks

This paper has discussed practical considerations when applying FXLMS and Principal Components ac-

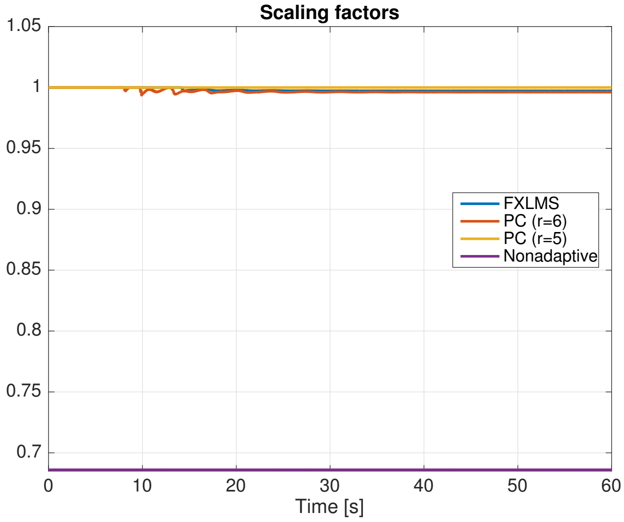

Figure 10: Scaling factor of the control actions

tive vibration algorithms for OBC applications. Because of the increased flexibility of the PC algorithms by controlling each principal coordinate separately, PC can indeed provide better performance. Tuning of PC algorithms can however be not straightforward, specially when the dimensions of the problem increases. PC provides also the possibility to exert a more refined tuning in terms of stability, which can help to automate the stability compensation process in practical implementations. The paper also discussed briefly the nominal stability criterion for each algorithm with common parameter choices. Such stability margins are very useful at the tuning stage of designing the control system. It is highlighted that adaptive forms are indeed more suitable when facing estimation errors and computational limitations as they allow faster implementation of the control algorithms and hence providing improved performance.

ACKNOWLEDGEMENTS

The author would like to thank the UK Royal Academy of Engineering for its support to this work via their Industrial Secondment Scheme Award. The author is also grateful to Leonardo Helicopters, in particular to Peter Court and Dr. Nadjib Baghdadi for their useful discussions and guidance in the simulations of this work.

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