TWENTY-FIFTH EUROPEAN ROTORCRAFT FORUM

Paper No. M 12

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SEPTEMBER 14-16, 1999
ROME
ITALY

ASSOCIAZIONE INDUSTRIE PER L'AEROSPAZIO, I SISTEMI E LA DIFESA
ASSOCIAZIONE ITALIANA DI AERONAUTICA ED ASTRONAUTICA

An Advanced Analytical Tool for the design of Aerospace Double Helical Gears

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Abstract

Double helical gears represent an alternative approach to single helical gears in last stages of aerospace transmission where the weight penalties due to excessive axial thrust have to be avoided.
This paper presents an analytical tool to predict the kinematical behaviour of meshing double helical gears considering all the effects (large stresses, misalignments, surface corrections, ..) of modern aerospace applications.

Introduction

Next generation aerospace transmission systems will require increased power density, reduced noise, improved reliability and reduced maintenance within specific cost targets.
In order to fulfil these requirements, aerospace transmission technology is evolving toward new advanced concepts: Split Torque design, bearingless arrangements, reduced number of stages.
Some advanced helicopter transmission studies are carried out aiming at the overall reduction of weight and manufacturing complexity; these designs are based on the reduction of the number of stages by means of the application of modern gear concepts like high ratio bevel gears or face gears.
Reducing the number of stages implies the increase of the transmission ratio in each stage: as a consequence, large torque and loads have to be reacted in the last stage of the transmission.
Many turboprops and modern helicopter transmissions are designed with a final collector gear - usually an helical gear - transmitting the torque to the main rotor or the propeller.
Nevertheless, due to the axial loads, this solution calls for very stiff gear webs and adequate axial bearings: thus introducing undesired penalties on the system.
The alternative application of double helical gears can overcome these penalties.
Double helical gears are then proposed for applications where the axial thrust component of the load on single helical gears is excessive. Axial thrust is avoided by making the net axial component of the loads on the respective halves to be equal and opposite. This is accomplished as the pinion is able to float axially in its bearings while the position of the gear is fixed (Figure (1)).
Up to now, no literature could be found on the analysis of the kinematical behaviour of double helical gears.
This paper presents the development of an analytical method for the complete evaluation of the kinematical behaviour of meshing double helical gears.
A detailed Tooth Contact Analysis, including tooth contact ellipses, hertzian stresses, sliding velocities and transmission errors will be presented; three numerical examples will be discussed.

Formulation of the analytical approach

In order to define gear position and the effective tooth topology, some reference frames and variables will be defined. On the basis of these definitions, the behaviour of the double helical meshing gears will be simulated by means of a Tooth Contact Analysis. Contact forces, contact ellipses, maximum hertzian stresses and transmission errors will be derived.

Reference frames and variables

Four main reference frames will be defined:
S_1: rigidly connected to the pinion, z-axis is the rotation axis of the pinion.
S_2: rigidly connected to the gear, z-axis is the rotation axis of the gear.
S_f: the fixed frame, z-axis is the common generatrix to the pitch cylinders without misalignments.
S_q: an auxiliary fixed frame, to take into account gear misalignments.
Matrices M define the relations among the co-ordinates of these frames.
The main variables are listed in Table (1).

θ	[rad]	Roll angle
u	[mm]	Axial co-ordinate (distance from the symmetry plane of gear or pinion)
s	[mm]	Axial displacement of the pinion
ϕ	[rad]	Gear or pinion rotation

Table (1): list of main variables

Tooth surfaces and their modifications

Perfectly aligned and rigid gears can mesh with theoretical surfaces. In common aerospace application some misalignments will take place due to the flexibility of the supporting structures and of the gear itself; besides some assembly position errors could occur.

Gear tooth flexibility under load will also introduce mismatch error in adjacent couples of teeth. Tip interference could occur.

To avoid these undesired effects, without increasing the weight of the components, adequate tooth surface modifications will be introduced.

In common practice, tooth surface modifications usually apply only to pinion (fewer teeth); however it could also apply to gear, if needed. Theoretical pinion tooth surface is defined with an involute helicoid surface and its theoretical unit normal. In a reference frame (S_1) rigidly connected to the pinion, the theoretical position vector of the pinion can be expressed[1] as:

$$ {}^{(k)}\mathbf{r}_1^{(1t)}\left(u_1^{(k)},\vartheta_1^{(k)}\right) \tag{1} $$

and the theoretical unit normal vector:

$$ {}^{(k)}\mathbf{n}_1^{(1t)}\left(\mathrm{u}_1^{(k)},\vartheta_1^{(k)}\right)=\left[\frac{\partial\,{}^{(k)}\mathbf{r}_1^{(1t)}}{\partial\,\vartheta_1^{(k)}}\right]\times\left[\frac{\partial\,{}^{(k)}\mathbf{r}_1^{(1t)}}{\partial\,\mathrm{u}_1^{(k)}}\right] \tag{2} $$

Modified tooth surfaces are obtained by adding to the theoretical ones the product of a scalar function $f^{(1,k)}\left(u_1^{(k)},\vartheta_1^{(k)}\right)$, called modification function, by the theoretical normal unit vector. The modification function is the sum of two polynomial expressions, one function of the roll angle only and one of the axial co-ordinate along the face width:

$$ f^{(1,k)}\left(u_1^{(k)},\vartheta_1^{(k)}\right)=f_u^{(1,k)}\left(u_1^{(k)}\right)+f_\vartheta^{(1,k)}\left(\vartheta_1^{(k)}\right) \tag{3} $$

Analytically, this approach leads to the following expression for the modified tooth surface:

$$ {}^{(k)}\mathbf{r}_1^{(1)}\left(u_1^{(k)},\vartheta_1^{(k)}\right)={}^{(k)}\mathbf{r}_1^{(1t)}\left(u_1^{(k)},\vartheta_1^{(k)}\right)+f^{(1,k)}\left(u_1^{(k)},\vartheta_1^{(k)}\right)\cdot{}^{(k)}\mathbf{n}_1^{(1)}\left(u_1^{(k)},\vartheta_1^{(k)}\right) \tag{4} $$

The "actual" unit normal vector can be computed as:

$$ {}^{(k)}\mathbf{n}_1^{(1)}\left(\mathrm{u}_1^{(k)},\vartheta_1^{(k)}\right)=\left[\frac{\partial\,{}^{(k)}\mathbf{r}_1^{(1)}}{\partial\,\vartheta_1^{(k)}}\right]\times\left[\frac{\partial\,{}^{(k)}\mathbf{r}_1^{(1)}}{\partial\,\mathrm{u}_1^{(k)}}\right] \tag{5} $$

Figure (2) shows a plot of the modification function.

Tooth Contact Analysis (T.C.A.)

The developed T.C.A. algorithm is aimed to the determination of the set of instantaneous contact ellipses and the transmission error, under the condition that gear and pinion tooth surfaces are in point contact at any instant.

Pinion and gear teeth are assumed as rigid bodies in point contact at any instant, without friction and lubrication. As a consequence, only two couple of teeth (one for each face-width) will mesh at any instant. The procedure considers only one couple of opposite teeth for the pinion and one couple for the gear.

Pinion rotates about a fixed axis (z_1) located in S_q and can also translate along this axis. Misalignment is referred to the pinion; the location and orientation of S_q with respect to S_f simulate the misalignment of the gear drive. Thus, the relationship between frames S_1 and S_f is expressed by the following matrix equation:

$$ \mathbf{M}_{f1}\left(\phi_1,s_1\right)=\mathbf{M}_{fq}\cdot\mathbf{M}_{q1}\left(\phi_1,s_1\right) \tag{6} $$

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[1] The symbol: ${}^{(k)}\mathbf{r}_i^{(j)}$ designate a vector **r** in the reference frame S_i for the gear wheel j and its k-th face-width; the scalar variable: $u_j^{(k)}$ is referred to the k-th face-width of the j-th gear wheel.

In reference frame S_f, a "double" family of pinion tooth surfaces is generated. The family of these surfaces may be determined by the matrix equation[2]:

$$\mathbf{r}_f^{(1)}(u_1,\vartheta_1,\phi_1,s_1)=\mathbf{M}_{f1}(\phi_1,s_1)\cdot\mathbf{r}_1^{(1)}(u_1,\vartheta_1) \tag{7}$$

and the unit normal to pinion tooth surface is represented in S_f by the matrix equation:

$$\mathbf{n}_f^{(1)}(u_1,\vartheta_1,\phi_1,s_1)=\mathbf{L}_{f1}(\phi_1,s_1)\cdot\mathbf{n}_1^{(1)}(u_1,\vartheta_1) \tag{8}$$

The gear rotates about another fixed axis (z_2) in S_f, and its tooth surface generates a family of surfaces in S_f, that may be represented by the matrix equation:

$$\mathbf{r}_f^{(2)}(u_2,\vartheta_2,\phi_2)=\mathbf{M}_{f2}(\phi_2)\cdot\mathbf{r}_2^{(2)}(u_2,\vartheta_2) \tag{9}$$

and the unit normal:

$$\mathbf{n}_f^{(2)}(u_2,\vartheta_2,\phi_2)=\mathbf{L}_{f2}(\phi_2)\cdot\mathbf{n}_2^{(2)}(u_2,\vartheta_2) \tag{10}$$

Matrices L are derived from matrices M. The position of the instantaneous contact point and the rotations of the wheels can be derived from the equations of tangency of the contacting surfaces, considered as rigid bodies. The solution at any instant is obtained as their position and normal vectors coincide at any instant. These equations must be written for both the meshing face-widths. The following system of vector equations has to be solved:

$$\begin{cases} {}^{(1)}\mathbf{r}_f^{(1)}\left(u_1^{(1)},\vartheta_1^{(1)},\phi_1,s_1\right)={}^{(1)}\mathbf{r}_f^{(2)}\left(u_2^{(1)},\vartheta_2^{(1)},\phi_2\right) \\ {}^{(1)}\mathbf{n}_f^{(1)}\left(u_1^{(1)},\vartheta_1^{(1)},\phi_1,s_1\right)={}^{(1)}\mathbf{n}_f^{(2)}\left(u_2^{(1)},\vartheta_2^{(1)},\phi_2\right) \\ {}^{(2)}\mathbf{r}_f^{(1)}\left(u_1^{(2)},\vartheta_1^{(2)},\phi_1,s_1\right)={}^{(2)}\mathbf{r}_f^{(2)}\left(u_2^{(2)},\vartheta_2^{(2)},\phi_2\right) \\ {}^{(2)}\mathbf{n}_f^{(1)}\left(u_1^{(2)},\vartheta_1^{(2)},\phi_1,s_1\right)={}^{(2)}\mathbf{n}_f^{(2)}\left(u_2^{(2)},\vartheta_2^{(2)},\phi_2\right) \end{cases} \tag{11}$$

The main difference with respect to T.C.A. equations for other kind of gears is represented by the degree of freedom corresponding to the axial displacement of the pinion, which allows the system to reach a "kinematic equilibrium position" at any instant. After solving the system (11) for a set of different values of the rotational degree of freedom, the solution functions can be pointed out:

$$\left\{u_1^{(1)}(\phi_1),\vartheta_1^{(1)}(\phi_1),u_1^{(2)}(\phi_1),\vartheta_1^{(2)}(\phi_1),u_2^{(1)}(\phi_1),\vartheta_2^{(1)}(\phi_1),u_2^{(2)}(\phi_1),\vartheta_2^{(2)}(\phi_1),s_1(\phi_1),\phi_2(\phi_1)\right\}\in C^1 \tag{12}$$

The transmission error (T.E.) can now be calculated as the difference between the theoretical gear rotation ($z_1\ \phi_1/z_2$) and the calculated one ($\phi_2\ (\phi_1)$).
Figure (2) shows the T.E. curves for three adjacent couples of teeth called (-1), (0) and (1).
As shown in [1], provided gear teeth are perfectly spaced and identical, T.E. and the functions listed in Equation (12) for the two couples (1) and (-1) can be calculated by only adding or subtracting one angular pitch to pinion rotation angle (ϕ_1). The crossing points of the T.E. curves are defined as "transfer points" because contact shifts from one couple to the following. As a consequence, only the part of meshing between transfer points A and B will be considered. T.E. for a whole rotation of the pinion can be derived simply shifting the part A-B.
An axial jump in transfer points A and B is also obtained, as a consequence of the theoretical approach. Nevertheless, the actual behaviour of double-helical meshing gears will not show any axial discontinuities because of the tooth elastic compliance.

Determination of forces and bearing contact

The instantaneous point of contact and the contact ellipses on pinion and gear tooth can be calculated by means of Hertz Theory and the approach based on Rodrigues' Formula to compute principal curvatures. To define the contact ellipses, torque has been assumed equally shared between the two face-widths of the meshing gears .

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[2] Henceforth, when equations can be written for both face-widths, the index that designates the face-width is omitted.

The meshing force on pinion and gear is calculated neglecting the load sharing effects due to teeth deflections. Under the hypothesis of frictionless and non-lubricated localized contact between two rigid bodies, the contact force on teeth is directed as the common normal in the contact point. Assuming that the torque acting on each face-width of the pinion is constant, and is half of the whole torque (T_1), the tangential component of the normal contact force is expressed by the equation:

$$F_{1,t}^{(k)}(\phi_1)=\frac{T_1/2}{\rho_1^{(k)}(\phi_1)} \tag{13}$$

Here, $\rho_1^{(k)}(\phi_1)$ is the radius of the instantaneous contact point. The other components can be easily derived.
Starting from equation (13), the instantaneous contact ellipses can be determined.
Following the calculation of instantaneous contact forces, the loads on bearing supports will then be carried out.
In case of misalignment or when modification functions are not symmetrical, a small net axial thrust is obtained. It comes out from the assumption that the torque is equally shared between the face-widths.
This residual net axial thrust is negligible (usually less than 1% of the axial load on each face-width) (Figure (12)); the assumption of equally shared torque can be accepted.
Maximum hertzian stress is localized on high curvature areas; modification functions with low curvatures are then suggested in the design phase.
Instantaneous sliding velocity has been calculated.
Maximum hertzian stress and sliding velocity, can be used to evaluate scoring / pitting index, even if neglecting load sharing and dynamical effects.

Numerical examples and discussions

All the following numerical examples are relevant to the development of a double helical gear set that will be manufactured for aerospace applications.
The main dimensions of a double helical gear set are listed in Table (2).

Pinion

Number of teeth		49
Helix angle	[deg]	20.86
Normal pressure angle	[deg]	22.50
Normal module	[mm]	2.54
Limit diameter	[mm]	128.05
Effective outside diameter	[mm]	138.60

Gear

Number of teeth		196
Helix angle	[deg]	20.86
Normal pressure angle	[deg]	22.50
Normal module	[mm]	2.54
Limit diameter	[mm]	512.26
Effective outside diameter	[mm]	554.51

General data

Torque	[Nm]	1228
Pinion speed	[rpm]	1450
Center distance	[mm]	332.98
Face-width	[mm]	20.00
Distance between face-widths	[mm]	22.00

Table (2): Sample gear dimensions

The simulation starts with the example of perfectly aligned meshing gears. The results of this analysis will be assumed as reference.
Results are reported in Figures (5), (6) and (7). The selected tooth surface modifications are represented in Figure (2) and apply to both pinion loaded tooth flanks. As expected, in the opposite face-widths of each meshing gear, the bearing contact is symmetrical, while stresses, forces and sliding velocities are identical. Figure (5) shows the path of contact, the extension of the contact ellipses and the T.E. curves. Point (1) and point (2) are transfer points. Figure (6) shows that the maximum hertzian stress is located on high curvature areas. This figure also shows the magnitude of the sliding velocity (zero at pitch point). Components of the contact force are reported in Figure (7). The axial components on the two face-widths have opposite values, so that the net axial thrust is null. The pinion axial displacement is also null at any instant. It must be remembered that while double-helical gear eliminate the net axial load on the shafts, the two face-widths internally react the full thrust load. The design of the gear blank must be carefully evaluated to provide enough rigidity with respect to the calculated axial thrust of each face-width.
When introducing some misalignment, the kinematical behaviour will modify. Optimized surface modifications have to minimize the kinematical effects.

In the second example, an angular misalignment $\gamma_p = 5'$ (5 minutes of arc) has been applied. The path of contact is now located in the lower part of the face-width and has a different shape. The "flat top" of the T.E. curves is reduced and the T.E. value at the transfer point (the measured peak to peak value) is higher (Figure (8)). The hertzian stress and the sliding velocity have about the same maximum values with respect to the previous example, although in different positions of the meshing due to the different position of the path of contact. Figure (9) shows the axial displacement of the pinion, which displaces about a non-zero mean value.
The components of the contact force (Figure (11)) have about the same values of the perfectly aligned situation. Figure (12) shows that there is a small non-zero net axial thrust (about 0.6 % of the axial load on a face width) that oscillates about zero.
In the third example, an angular misalignment $\gamma_p = 10'$ has been applied. This misalignment can be considered as an off-design value for common helicopter gearbox structures. The path of contact is rather below the previous ones. Figure (13) shows that the contact ellipses exceed teeth dimensions: edge-contact will take place.
Also in this case, Figure (12), the net axial thrust is negligible with respect to the axial load on each face-width.
A considerable overall kinematical behaviour has been achieved by means of the assigned tooth surface modification; no relevant differences between bearing contact, hertzian stresses, sliding velocities, contact forces and T.E. curves can be found when increasing misalignment.

Conclusions

Double helical gears can represent an alternative solution to single helical gears in last stages of aerospace transmissions where weight and configuration penalties due to excessive axial thrust have to be avoided.
Up to now, most of the technical literature on double helical gear applications is quite general.
This paper is focused on the analysis of the kinematical behaviour of meshing double helical gears, taking into account all the effects (misalignments, surface corrections, large contact paths, ...) of modern aerospace applications. An analytical formulation of the modified tooth surface has been defined. The behaviour of the double helical meshing gears has been simulated by means of an innovative T.C.A. algorithm. Contact forces, contact ellipses, maximum hertzian stresses and transmission errors have been derived. The application of this analytical tool has lead to a double helical gear design with improved performances.

References

[1] Chen, J.-S., Litvin, F. L., Shabana, A. A., *Computerized simulation of meshing and contact of loaded gear drives*, 1994 International Gearing Conference, 1994, Newcastle upon Tyne
[2] Litvin, F. L., *Gear geometry and applied theory*, Prentice Hall, 1994
[3] Thomas, J., Houser, D., *A procedure for predicting the load distribution and transmission error characteristic of double helical gears*

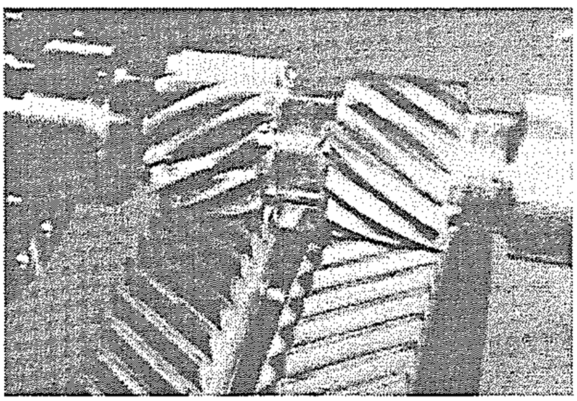


Figure 1: Double helical gear in aerospace application

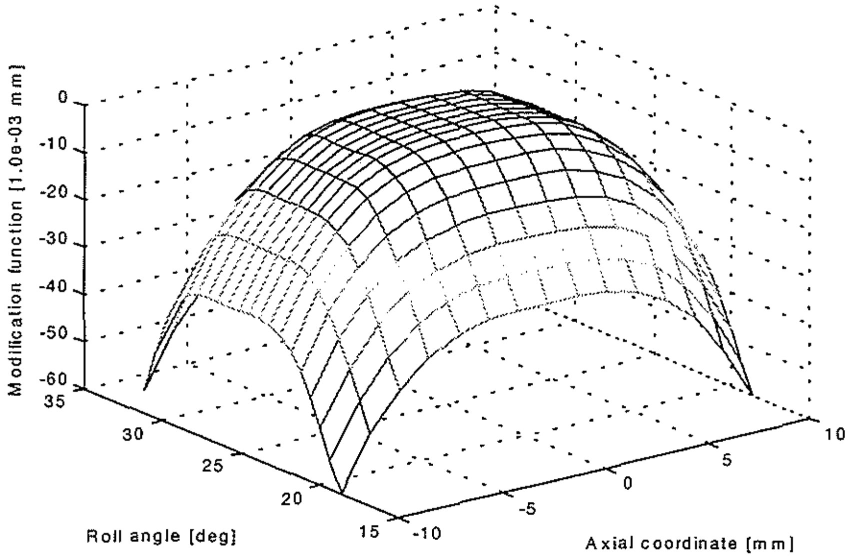


Figure 2: Modification function

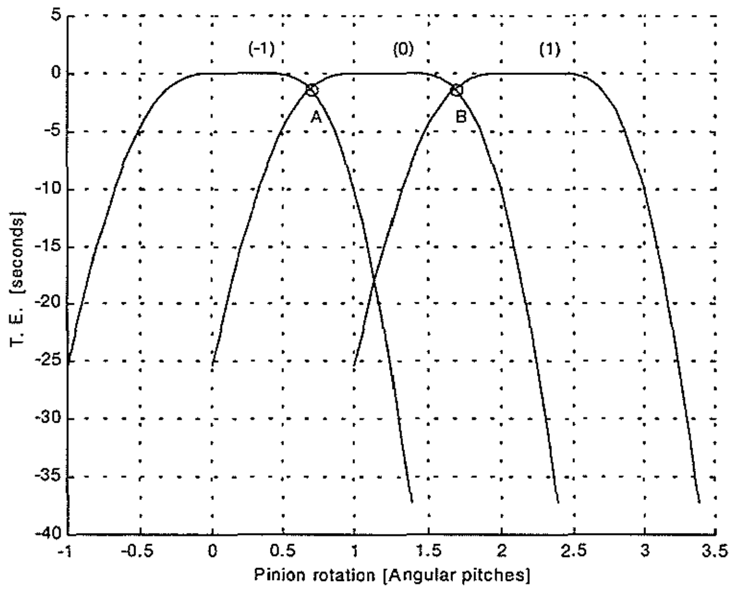
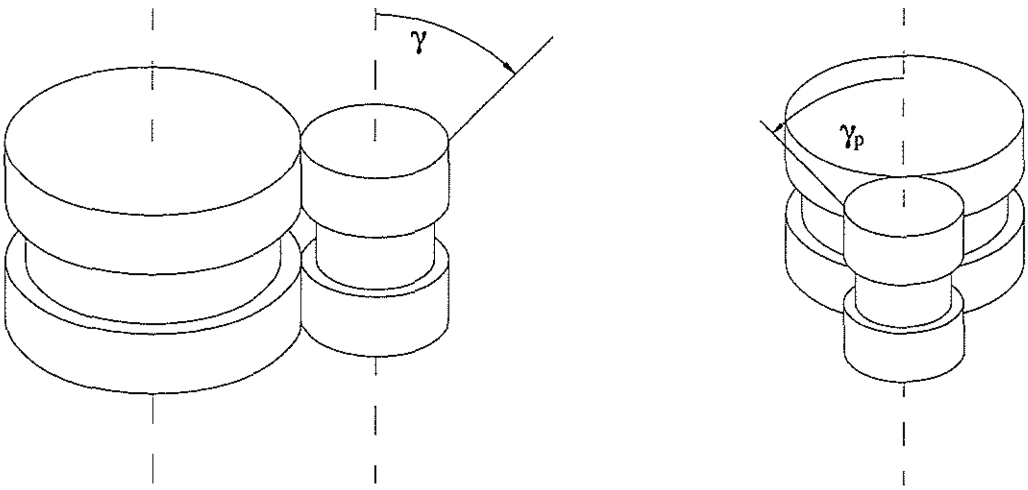


Figure 3: Transfer points



(a) In plane misalignment *(b) Orthogonal misalignment*

Figure (4): Convention for misalignment

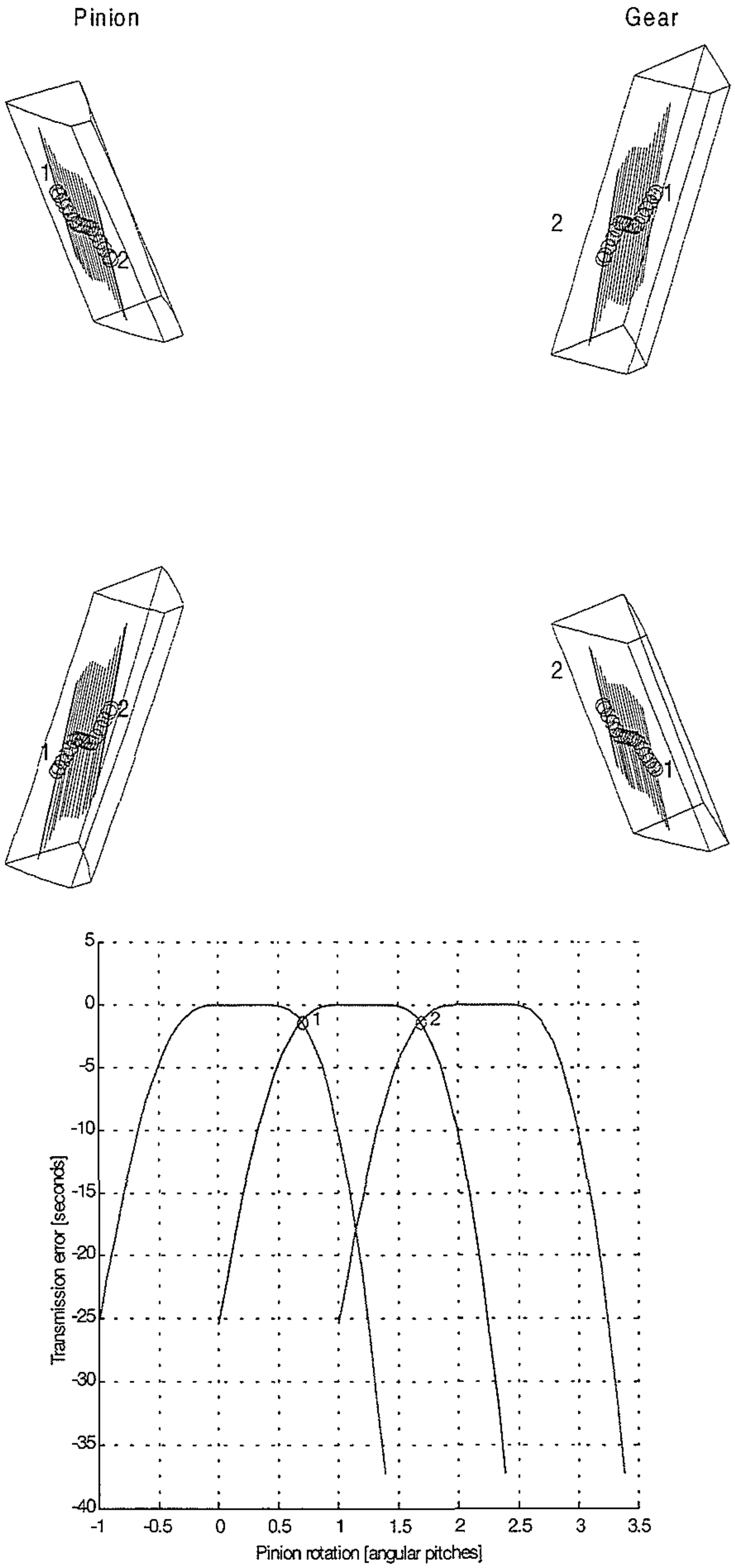


Figure 5: Path of contact and T.E. for a perfectly aligned gear

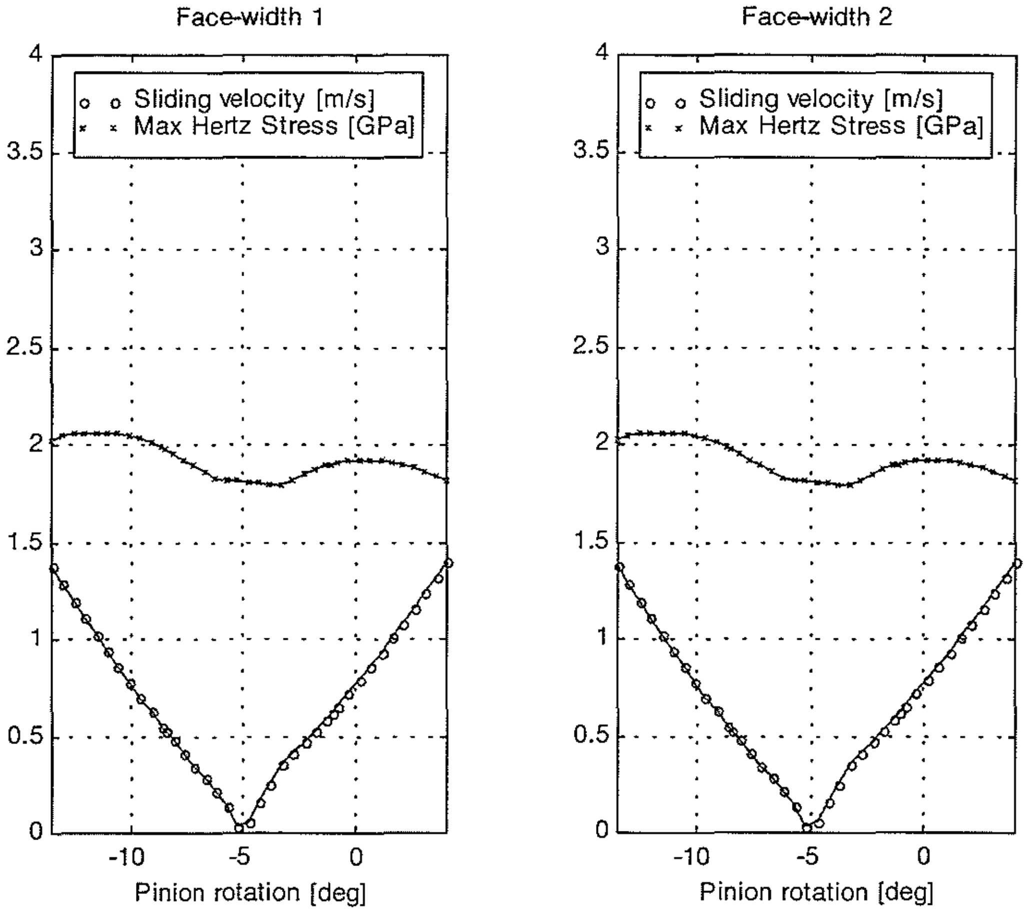


Figure 6: Maximum Hertzian stress and sliding velocity for a perfectly aligned gear

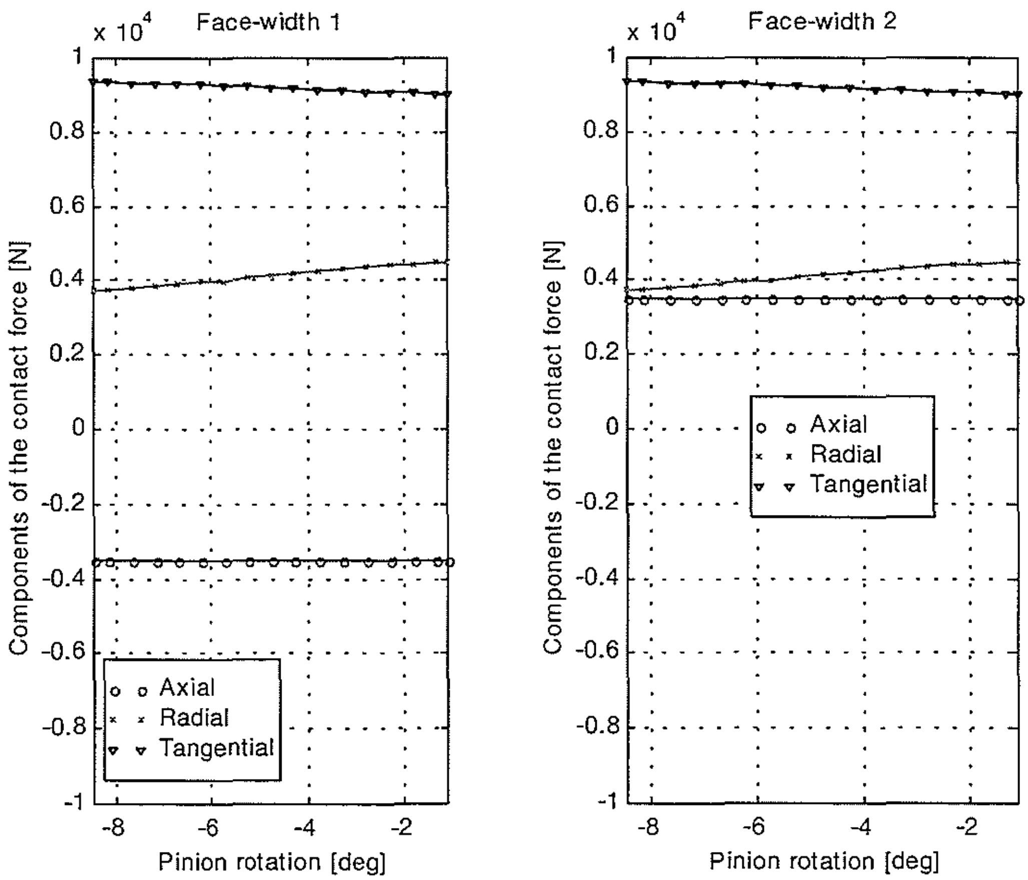


Figure 7: Components of the contact force for a perfectly aligned gear

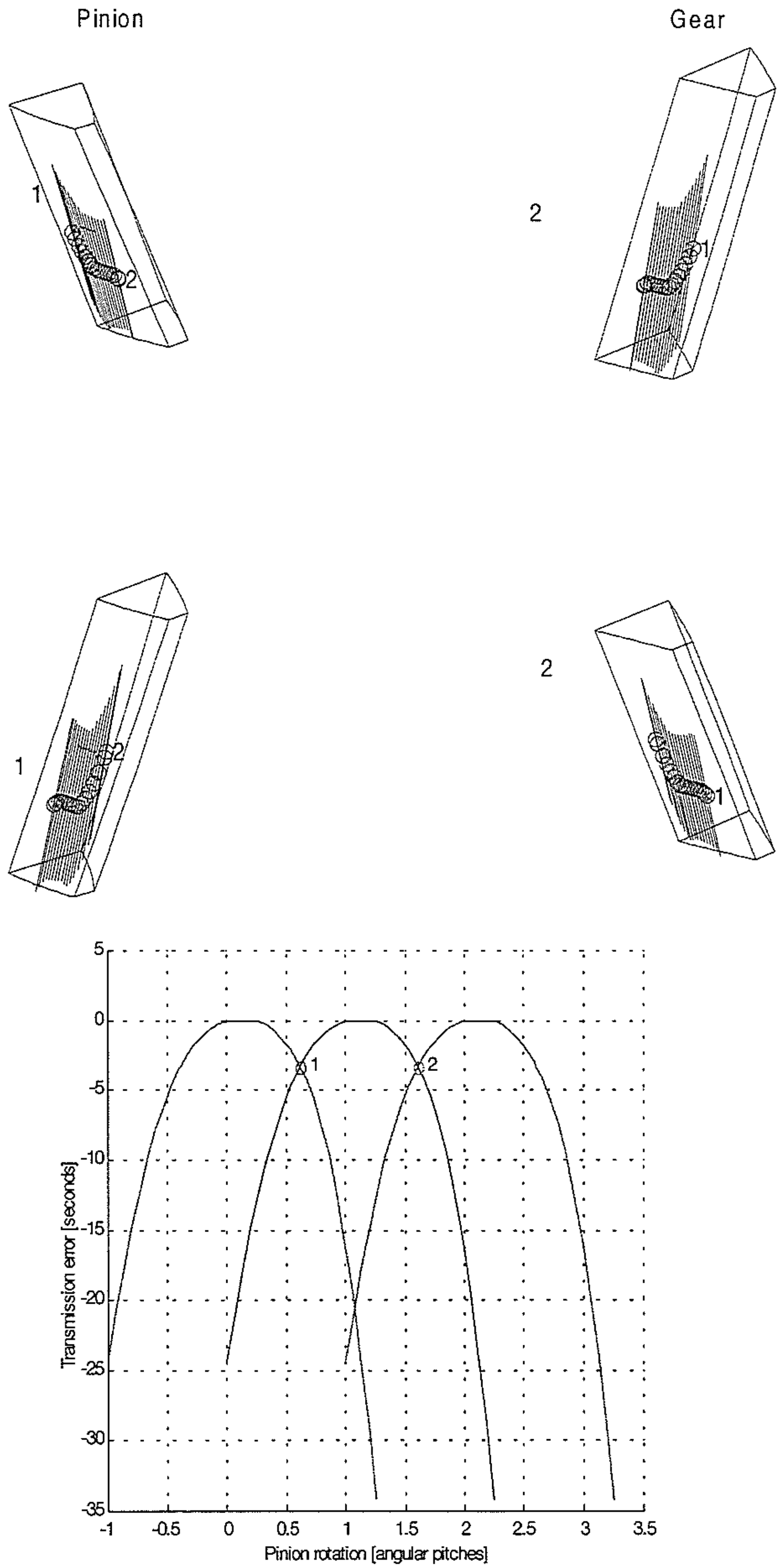


Figure 8: Path of contact and T.E. for a gear with $\gamma_p = 5'$

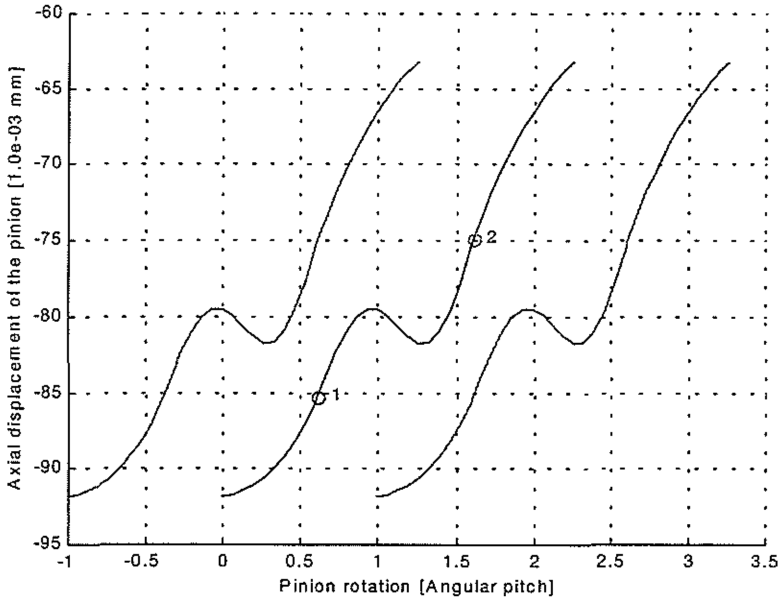


Figure 9: Pinion axial displacement for a gear with $\gamma_p = 5'$

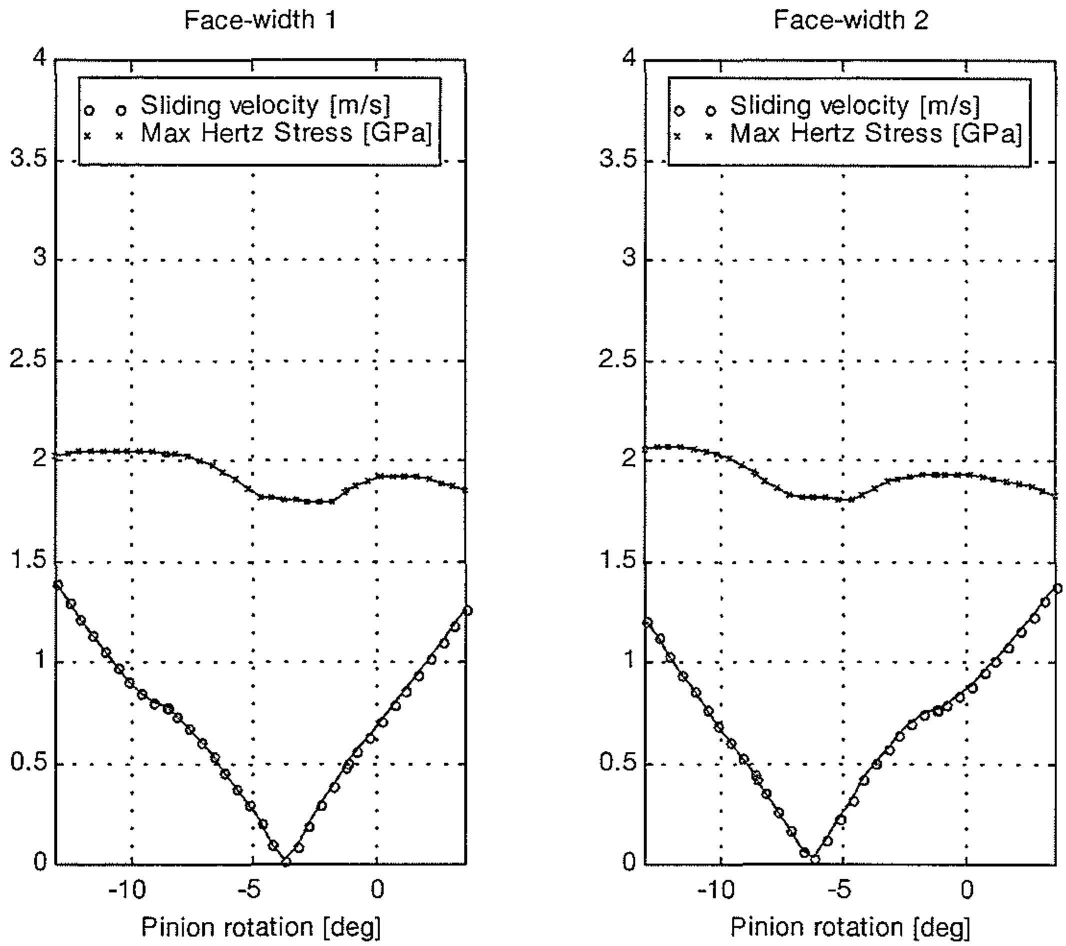


Figure 10: Maximum Hertzian stress and sliding velocity for a gear with $\gamma_p = 5'$

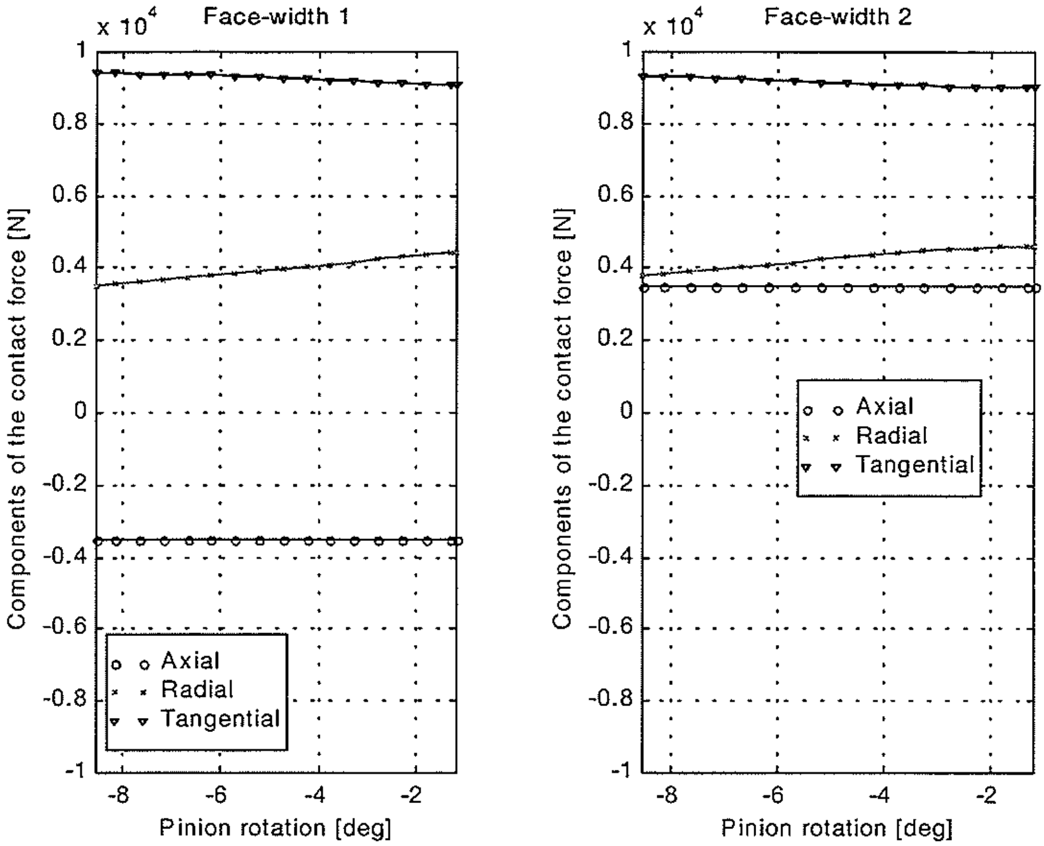


Figure 11: Components of the contact force for a gear with $\gamma_p = 5'$

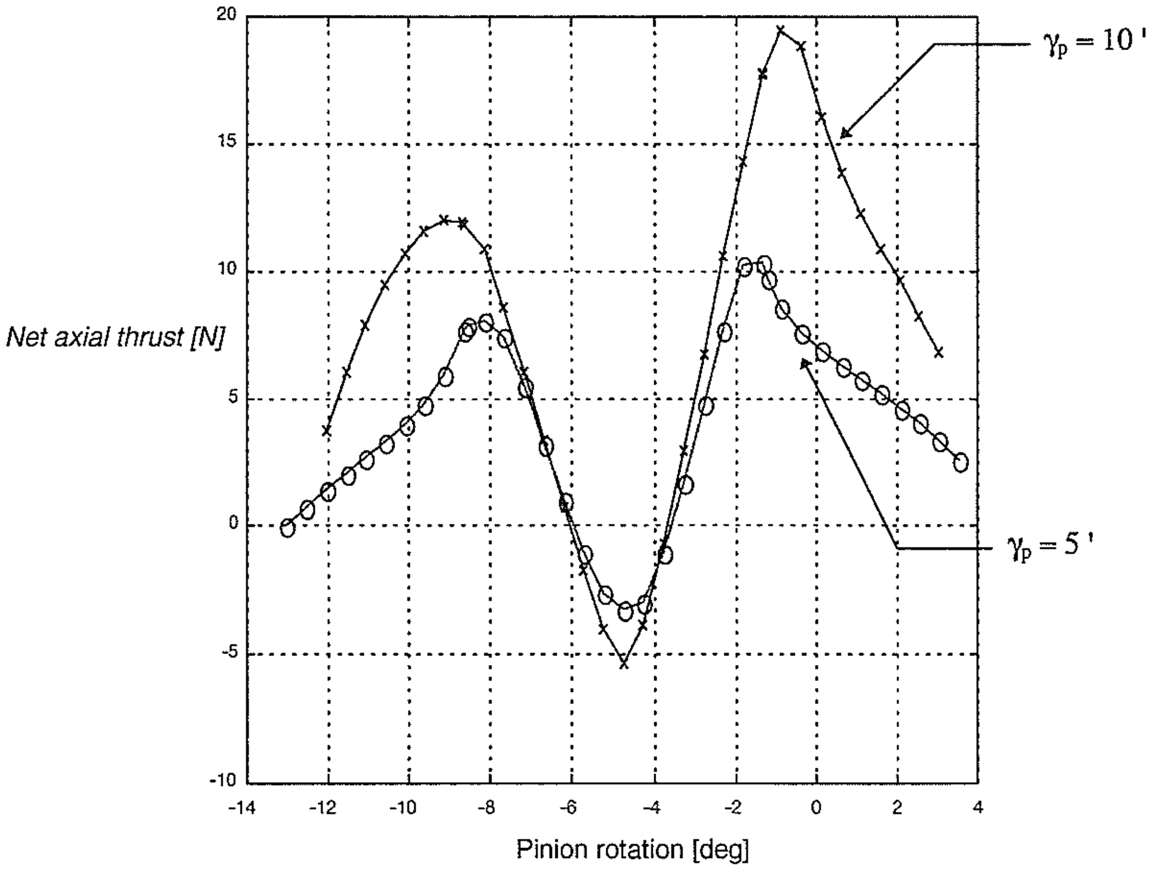


Figure 12: Net axial thrust

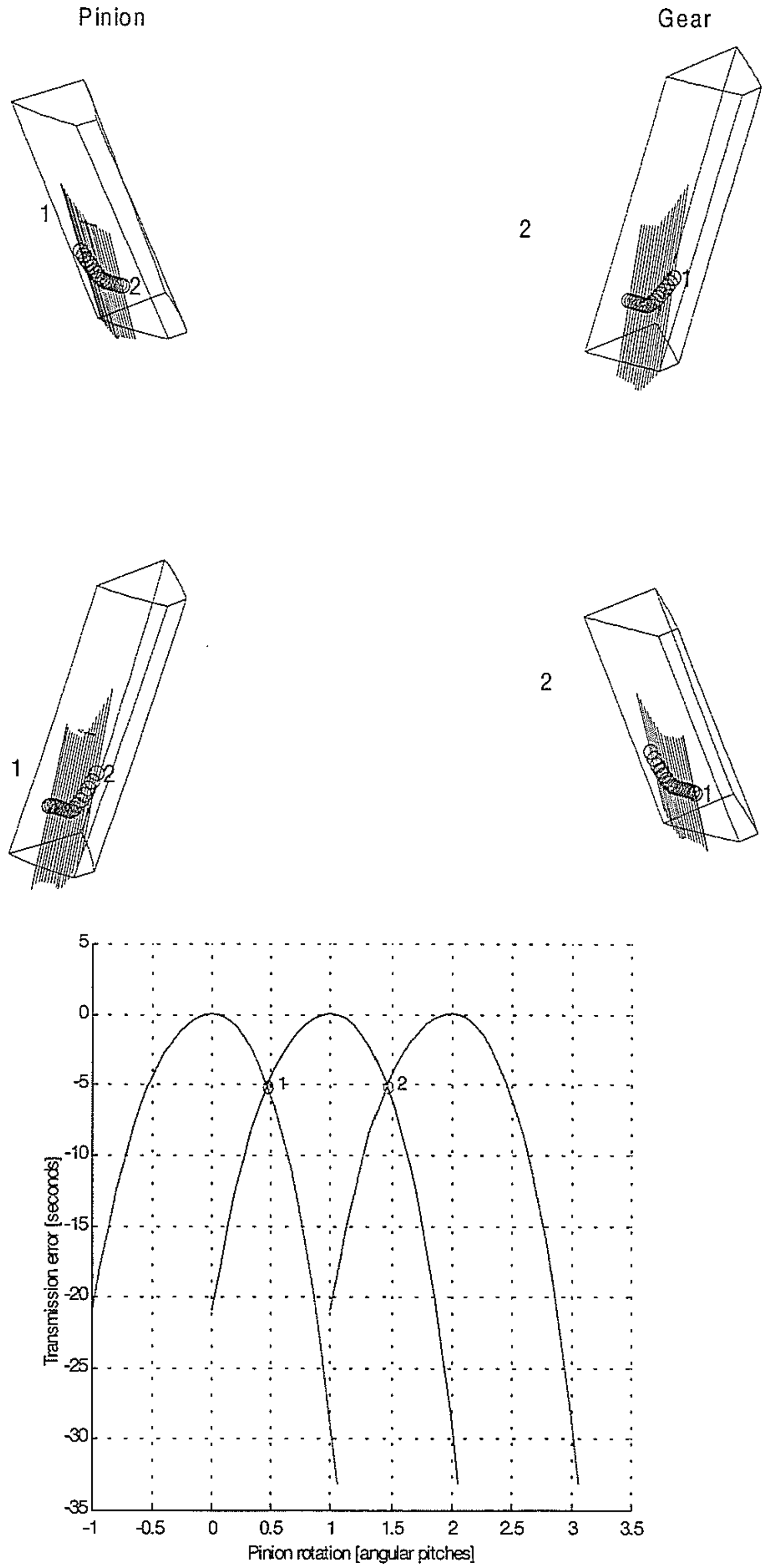


Figure 13: Path of contact and T.E. for a gear with $\gamma_p = 10'$