AUTOMATIC LANDING AND TAKE OFF AT CONSTANT SLOPE WITHOUT TERRESTRIAL AIDS

Franck Ruffier, Julien Serres and Nicolas Franceschini

Biorobotics Department, Movement and Perception Lab.,
CNRS / Univ. de la Méditerranée
31, chemin Joseph Aiguier, 13402 Marseille Cedex 20, FRANCE
{ruffier, serres, franceschini}@laps.univ-mrs.fr

Abstract

We have developed an automatic autopilot called OCTAVE (Optic flow based Control sysTem for Aerial VEhicles) that enables a rotorcraft to take off and land automatically at a constant ascent or descent angle without requiring any Terrestrial Aids.
In previous studies, we described how a rotorcraft equipped with the OCTAVE visually based AFCS reacts suitably when encountering shallow terrain and wind disturbances.
The OCTAVE autopilot involves a feedback loop designed to keep the downward *optic flow* (i.e., the ratio between groundspeed and ground height) constant by adjusting the rotor thrust. The downward optic flow is measured with an optronic angular velocity sensor that operates in the present version in the near IR but could work with millimetre waves as well.
The electronic processing system required is so lightweight (it weighs only a few grams) that it can be mounted onboard Unmanned Air Vehicles and even on Micro-Air Vehicles. On the other hand, the OCTAVE autopilot could also provide guidance and / or warning signals to help pilots keep a constant ascent or descent slope in the absence of any off-board navigation aids.

Keywords – Operations, Takeoff and Landing Manoeuvres, Constant descent angle, AFCS (Automatic Flight Control Systems), UAV (Unmanned Aerial Vehicle), MAV (Micro-Air Vehicle), Vision, Optic Flow (OF), Biorobotics, Biomimetics

Abbreviations :

AFCS*: Automatic Flight Control Systems*
EMD*: Elementary Motion Detector*
ILS*: Instrument Landing Systems*
IR*: Infra-Red*
MAV*: Micro-Air Vehicle*
MLS*: Microwave Landing Systems*
OCTAVE*: Optic flow based Control sysTem for Aerial VEhicules*
OF*: Optical Flow*
UAV*: Unmanned Aerial Vehicle*

1. Introduction

Winged insects are valuable model systems for building dynamic stabilization and visual guidance systems into UAVs (Ref 1-5).

During the last few years, we have developed a visually based autopilot called OCTAVE (Optic flow based Control sysTem for Aerial VEhicles), with which a miniature rotorcraft can be made to automatically cruise, follow terrain (Ref 3), and control risky manoeuvres such as take-off and landing, while reacting appropriately to wind disturbances and rejecting relief disturbances (Ref 4-6) (see Figure 1A). We built a proof-of-concept, tethered robot that flies indoors over an environment composed of contrasting features randomly arranged on the floor. We showed the feasibility of a visuomotor feedback loop that controls the rotor thrust $\mathbf{F_N}$ (see Figure 1B) so as to keep the downward optic flow (OF) ω constant. The latter parameter is equal to the ratio between the groundspeed $\mathbf{v_x}$ and the ground height $\mathbf{h}$:

$$\omega = \mathbf{v_x}/\mathbf{h} \quad [\text{rad.s}^{-1}] \qquad \text{Eq.1}$$

The OF sensor is an Elementary Motion Detector (EMD) (Ref 7-8), the functional structure of which was analysed by performing electrophysiological recordings on a motion sensitive neuron in the housefly's compound eye while applying optical microstimuli to single photoreceptor cells (Ref 9).
We recently described how a micro-aerial robot equipped with the OCTAVE autopilot can take off and land autonomously without being provided with any specific information about its altitude, height of flight, ground speed, air speed, wind speed or the ground relief (Ref 5). We concluded that a downward *OF regulator* is the key to rotorcraft automatic guidance because it frees a rotorcraft from the usual bulky, costly avionics (avionic equipment).

Based on their own detailed behavioural studies on honeybees' landing behaviour, Srinivasan et al. suggested that the landing strategy used by these animals consisted in maintaining a constant OF (Ref 10). This strategy requires the bee to: (i) maintain a constant descent angle, (ii) adjust its forward flight speed so as to keep the image angular velocity constant throughout the descent phase (Ref 11). The latter authors implemented this strategy on a robotic gantry devoid of dynamics (Ref 11). Chahl et al. recently attempted to meet the above two requirements while controlling the descent of a free-flying model rotorcraft from altitudes of 60 m to 30 m (Ref 12).

In the present study, we developed a downward optic flow regulation procedure with which a rotorcraft can be made to take off and land safely *at a constant angle* without requiring any inputs specifying the ascent/descent or ground speed.
We first explain why performing takeoff and landing manoeuvres at a constant angle can be so useful.
We then describe modelling and simulation studies focusing on the landing manoeuvres/phase and report on actual landing tests carried out on a real miniature rotorcraft equipped with the OCTAVE autopilot.
Lastly, we suggest a constant ascent angle take-off procedure based on the results of our modelling and simulation studies.

2. Reasons for implementing landing and takeoff manoeuvres at a constant angle

Many avionic systems were originally developed to provide pilots with reliable data under all-weather conditions. When landing without good visibility, most rotorcrafts manage to keep a constant descent angle thanks to the use of terrestrial aids.
Keeping a steady descent angle means keeping the descent speed (v_z=dh/dt) proportional to the ground speed v_x, which gives the safest trajectory until touching down at zero speed. These descent speed requirements are particularly appropriate in cases where the relief changes sharply or where the landing platform is moving (decking). Conversely, keeping a constant climbing angle, i.e., keeping the climbing speed proportional to the groundspeed, is the safest way to take off.

Recent video recordings have shown that honeybees land at a constant angle and seem to maintain their ventral optic flow constant during their final approach (Ref 11). This landing strategy makes insects' and birds' landings safe despite any unforeseen changes in the relief or wind conditions. As far as we know, honeybees perform their landing glide without receiving any data from terrestrial aids: when landing, they seem to rely solely on the optic flow.

Terrestrial aids such as ILS or MLS are expensive, bulky and highly power-consuming systems. Equipment of this kind is available only at airports and therefore cannot be used on unprepared terrain.

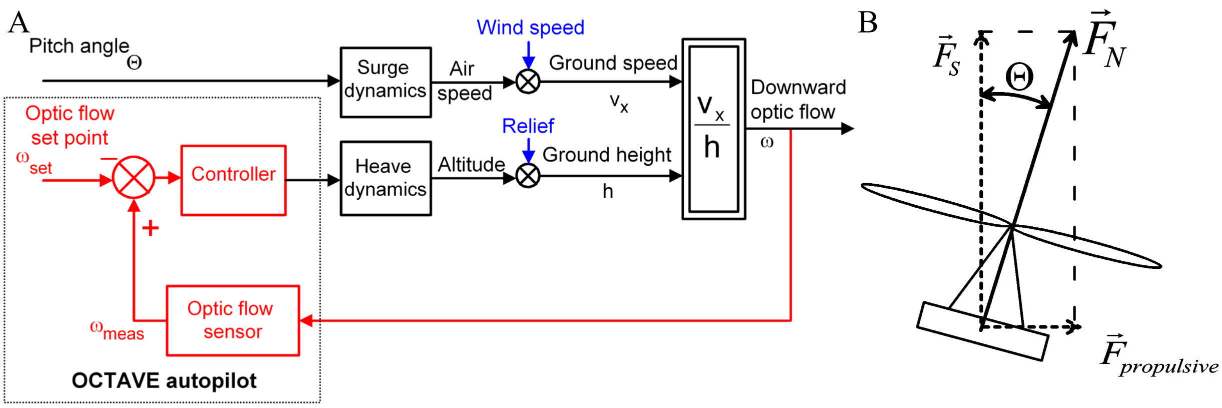


Figure 1 : **(A)** The OCTAVE autopilot (Ref 3-5) controls two of the rotorcraft's dynamic inputs:

- the pitch angle Θ of the craft, which sets the *direction* of the rotor thrust vector $\mathbf{F_N}$ which in turn will eventually determine the groundspeed $\mathbf{v_x}$,
- the magnitude of the rotor thrust vector $\mathbf{F_N}$, which will eventually determine the altitude **z**.

Throughout the automatic takeoff and landing manoeuvres, the OCTAVE autopilot adjusts the rotor thrust $\mathbf{F_N}$ so as to keep the downward *optic flow* (the ratio between ground speed and ground height $\omega = \mathbf{v_x/h}$) constant.
(B) The rotor thrust can be decomposed into the lift $\mathbf{F_S}$ and the forward propulsive force $\mathbf{F_P}$. In a first approximation, the pitch angle Θ and the magnitude of the rotor thrust $\mathbf{F_N}$ independently control the propulsive force and the lift, respectively, provided the pitch angle remains small (Θ<15°).

3. Landing at a constant descent angle without Terrestrial Aids

Model of an automated landing trajectory

Here we focus on the landing trajectory profile in the vertical plane, over an optically contrasting terrain.

We started by adopting two hypotheses:

1. The surge dynamics between pitch angle Θ and groundspeed $\mathbf{v_x}$ are described by a simple first order transfer function :

$$\frac{\Delta v_x(s)}{\Delta\Theta(s)} = G_v(s) = \frac{H_0}{1+\tau\cdot s} \qquad \text{Eq. 2}$$

2. The optic flow is regulated perfectly throughout the landing manoeuvre. It is maintained at the constant optic flow set-point ω_{set} (Fig 1A).

$$\omega(t) = \frac{v_x(t)}{h(t)} = \omega_{set} \qquad \text{Eq. 3}$$

Assuming the initial conditions to be cruise flight at constant ground speed, we pitch the aerial vehicle backward by $\Delta\Theta = -\Theta_0$, reaching a zero-degree pitch attitude at time $\mathbf{t = t_0}$.

$$\begin{aligned} \Theta(t_0) &= 0 \\ v_x(t_0) &= v'_0 \end{aligned} \qquad \text{Eq. 4}$$

We now change the reference point : $\mathbf{t' = t - t_0}$, and study the rotorcraft trajectory from $\mathbf{t' = 0}$; i.e., starting from the time when the system is no longer subjected to pitch variations. The groundspeed $\mathbf{v_x}(t)$ and the descent speed $\mathbf{v_z}(t)$ are calculated as follows :

$$\begin{cases} \Delta v_x(s) = G_v(s)\cdot\Delta\Theta(s) \\ \quad with \quad v_x(t'=0) = v'_0 \\ \quad and \quad \Theta(t'\geq 0) = 0 \\ \\ v_z(t') = \dfrac{d\,h(t')}{dt'} \\ with \quad h(t') = \dfrac{1}{\omega_{set}}\cdot v_x(t') \end{cases} \Rightarrow \begin{cases} v_x(t') = v'_0 \; e^{-\frac{t'}{\tau}} \\ v_z(t') = -\dfrac{v'_0}{\omega_{set}\tau}\; e^{-\frac{t'}{\tau}} \end{cases}$$

Eq. 5

Eq. 6

The groundspeed $\mathbf{v_x(t)}$ is the free response of the first order system $\mathbf{G_v(s)}$. The groundspeed response therefore decreases exponentially with time (Eq. 5).

The altitude is driven by the OCTAVE autopilot so as to keep the ratio between the groundspeed and the ground height (v_x/h) constant. The ground height therefore also decreases exponentially with time and so does its time derivative, the descent speed (Eq. 6).
The descent angle α is calculated as follows :

$$\tan\alpha = \frac{v_z(t)}{v_x(t)} = -\frac{1}{\omega_{set}\tau} \qquad \text{Eq. 7}$$

where ω_{set} is the optic flow set point (the angular velocity of the ground image), which is kept constant, and τ the surge time-constant of the first-order transfer function linking groundspeed to pitch angle.

Eq. 7 means that the rotorcraft equipped with the OCTAVE autopilot will automatically keep a constant descent angle whenever the rotorcraft is raised back towards the vertical. This occurs although the rotorcraft has no means of measuring its own descent slope.
The height of flight **h** of the rotorcraft at any time depends on the ground speed, which gradually decreases as the result of the drag.

Simulated trajectory of the automatic rotorcraft landing approach

Figure 2 shows a simulated rotorcraft trajectory in the longitudinal plane according to Eq. 2, 3 and 4, before, during and after a rampwise decrease in pitch angle that eventually raises the rotor axis back to the vertical. The shape of the trajectory profile as a function of the horizontal distance travelled confirms that the rotorcraft lands at a constant descent slope $-1/\tau\omega_{set}$ as predicted by Eq. 7.
This occurs in the final approach that starts at time $t=t_0=5$s, when the rotorcraft pitch value is back to zero degrees (fig. 2e). The simulation therefore shows that a downward *optic flow regulator* makes the rotorcraft land with a constant slope without having to measure this slope. Adjusting the optic flow set point $\boldsymbol{\omega_{set}}$ to another value will result in a corresponding change in the descent slope.

As we have shown here, a rotorcraft equipped with an *optic flow regulator* whose surge dynamics is a first order system will adopt a landing trajectory that necessarily shows a constant slope equal to $\mathbf{-1/\tau\omega_{set}}$ in the reference plane (x, h).
The descent slope is given by the product of the aero-mechanical time constant of the rotorcraft and the optic flow set point (i.e., the imposed angular speed of the ground image).

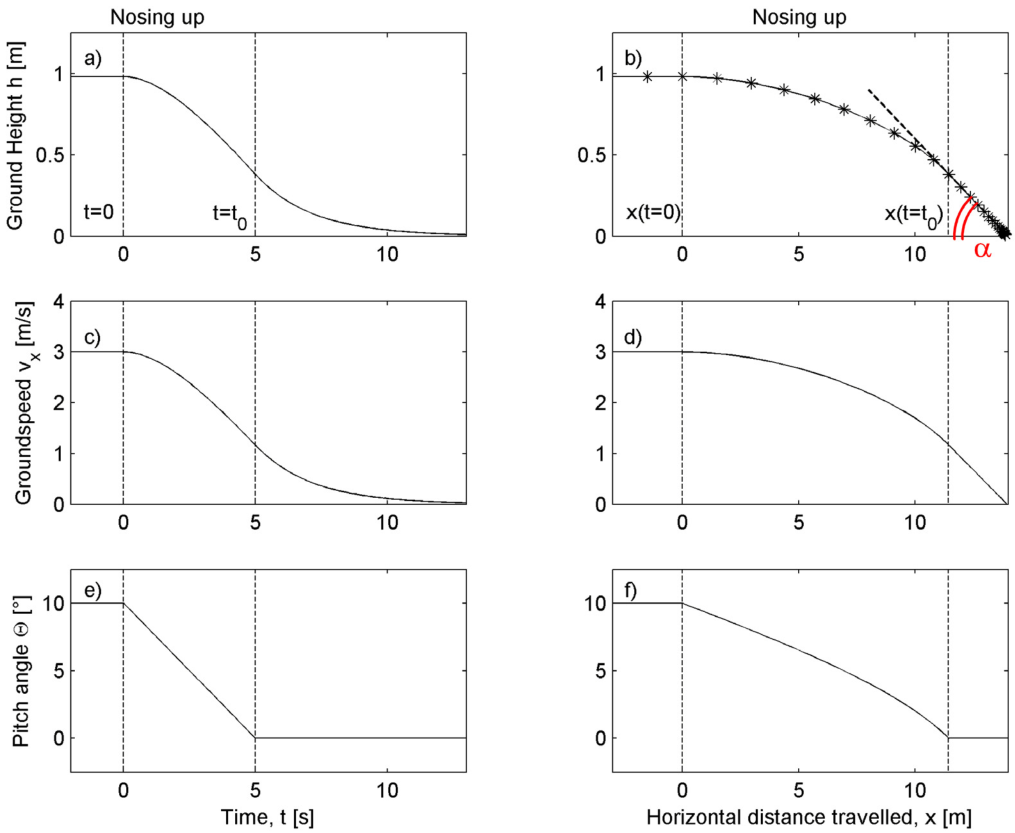


Figure 2 : a-b) Simulated rotorcraft landing trajectory in the longitudinal plane as a function of both time (a) and horizontal distance (b) travelled according to Eq. 2, 3 and 4 as a result of a 5-second ramp-wise backward pitch (τ=2.15s).
c-d) From t=t_0=5s, the groundspeed decreases exponentially with time and proportionally to the ground height.
e-f) Landing is triggered by the remote-controlled rampwise backward pitch. The rotorcraft lands, giving a constant slope from the end of the backward pitch zone, as predicted by Eq. 7.
The vertical dotted lines show the start and end of the backward pitch zone.

Holding a constant descent angle simply results from the surge dynamics and the particular control procedure used, which consists in regulating the downward optic flow. There is therefore no need to measure the descent speed $\mathbf{v_z}$ and the groundspeed $\mathbf{v_x}$ to be able to achieve landing at a constant descent slope **tan** α (Eq. 7). Both the variometer and the Doppler-radar can henceforth be dispensed with.

Real trajectory of an automatic rotorcraft landing approach

Figure 3 shows the temporal decay profile of the groundspeed measured after the rotorcraft axis was raised back to the vertical. The exponential decay observed shows that the aerial robot's surge dynamics can be described by a first order system.

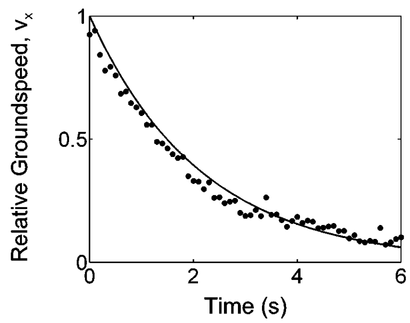
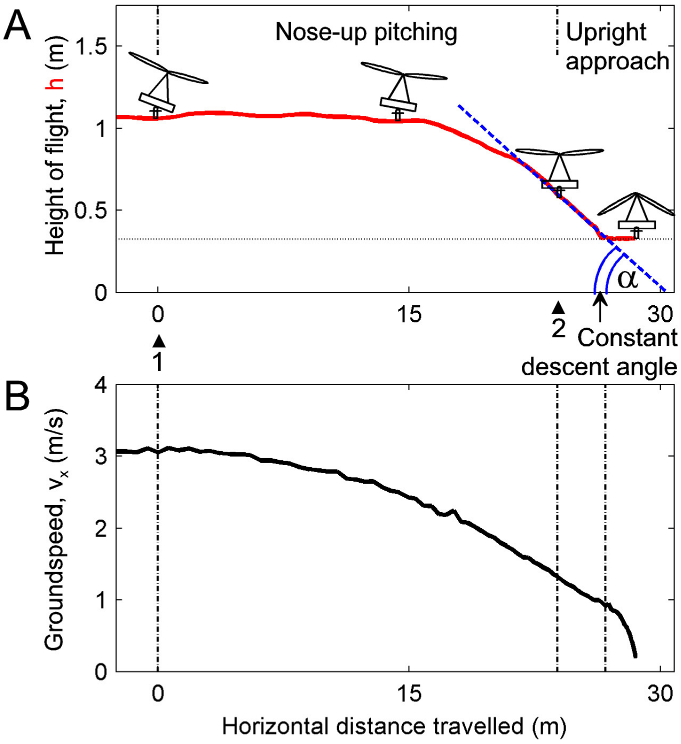


Figure 3: Free response of the aerial robot's groundspeed. The surge dynamics time constant has been identified as τ=2.15s.

Figure 4 shows the real trajectory of the OCTAVE aerial robot equipped with the OCTAVE autopilot. This trajectory in the vertical plane shows that the experimental robot performs automatic landing at a constant descent angle in its final approach.



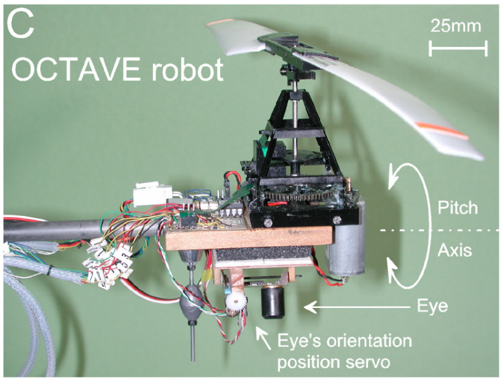


Figure 4 (A-B) After performing cruise flight (arrowhead 1), the rotorcraft was simply pitched nose-up (from 10° to 0° rampwise). This immediately initiated automatic landing. The final approach occurred when the rotorcraft was completely upright (arrowhead 2). A constant descent angle (α=-8.8° as calculated from Eq. 7 and -6.7° as measured in Fig.4A) results from the feedback loop and the first order surge dynamics (see text). The dotted line shows the height of the landing gear. **(C)** The 100-gram proof-of-concept rotorcraft is equipped with an eye that is kept pointing vertically downwards by means of a micro actuator.

In Figure 4A (left), the miniature rotorcraft is pitched nose-down by 10° and is flying at constant height (1.2 m) and constant speed (3m/s). At the point indicated by arrowhead 1, the rotorcraft is gradually raised nose up by remote control. This automatically initiates landing. Both height and groundspeed decrease gradually (Fig.4A, B). Once the rotorcraft is fully upright (final approach, starting at arrowhead 2, Fig.4A), it continues to descend under visual closed loop conditions. The groundspeed decreases exponentially with time (free response of a first order system with time constant τ). The autopilot maintains the ratio between groundspeed and ground height (**v_x/h**) constant, the height **h** also decreases exponentially with the same time constant τ, and so does its time derivative, the descent speed. The ratio between descent speed and groundspeed (**v_z/v_x**) is therefore constant, which means that the descent angle is constant.

4. Take off at a constant climbing angle without any Terrestrial Aids

Here it is proposed to define a specific pitch angle law governing the take off of a rotorcraft equipped with OCTAVE at a constant climbing angle. The results of this study show that an exponential law over time can account for this manoeuvre.

Model for optic flow based autonomous take off

Here we adopt the same hypotheses as in the previous section, namely that:

1. a simple first order function defines the surge dynamics between pitch angle **Θ** and groundspeed **v_x** (see Eq. 2);
2. the optic flow is perfectly regulated and maintained constant at ω_{set} throughout the take off phase (see Eq. 3).

It is assumed in addition that in the initial position, the rotorcraft's rotor thrust **F_N** compensates perfectly for its weight as long as the landing gear remains on the ground.
We then gradually vary the pitch angle so that it obeys an increasing exponential law from $t=t_1$ to $t=t_2$, at which time it reaches $\Theta(t_2)=\Theta_0$:

$$\Theta(t) = k \cdot e^{at} \quad with \quad \Theta(t = t_2) = \Theta_0$$
$$v_x(t \le t_1) = 0 \qquad \text{Eq. 8}$$

In Appendix I, we establish that under these conditions, the groundspeed also increases exponentially with time, concomitantly with the pitch angle.

To keep the ratio v_x/h constant, the OCTAVE autopilot adjusts the heave dynamics so that the height, and therefore its time derivative, the climbing speed v_z, increase exponentially.

$$\begin{cases} \Delta v_x(s) = G_v(s) \cdot \Delta\Theta(s) \\ \quad with \quad v_x(t \le t_1) = 0 \\ \quad and \quad \Theta(t = t_2) = \Theta_0 \\ \quad v_z(t') = \dfrac{d\,h(t')}{dt'} \\ with \quad h(t') = \dfrac{1}{\omega_{set}} \cdot v_x(t') \end{cases} \Rightarrow \begin{cases} v_x(t') = v_0'' \cdot k \cdot e^{at} & \text{Eq. 9} \\ (See\ Appendix\ I) \\ v_z(t') = \dfrac{a}{\omega_{set}} \cdot v_0'' \cdot k \cdot e^{at} & \text{Eq. 10} \end{cases}$$

It can therefore be predicted that during the exponential increase in pitch angle, the rotorcraft will climb at a constant angle, because the ascent speed turns out to be proportional to the groundspeed, as shown by Eq. 11.

$$v_z(t) = \frac{a}{\omega_{set}} v_x(t) \qquad \text{Eq. 11}$$

Simulated trajectory of the automated take off

Figure 5b shows a simulated rotorcraft trajectory according to Eq. 2, 3 and 8 before, during and after an exponential increase in pitch attitude. The simulated trajectory as a function of the horizontal distance travelled confirms that the rotorcraft takes off with a constant climbing slope a/ω_{set} as predicted by Eq. 11.
An OCTAVE optic flow regulator can therefore make a rotorcraft take off at a constant ascent angle without requiring any inputs specifying the climbing slope, the current height of flight, airspeed, groundspeed or ascent speed. This situation differs considerably from what occurs in conventional aviation, where a host of avionic sensors are required and where information about the climbing slope is provided by off-board navigation aids.

A rotorcraft using the takeoff procedure described here would automatically climb at a constant angle in the reference plane (x, h).
The climbing slope depends on the ratio between ***a*** (the constant of the increasing exponential function) and the preset optic flow ω_{set} (i.e., the angular velocity set point of the ground image).

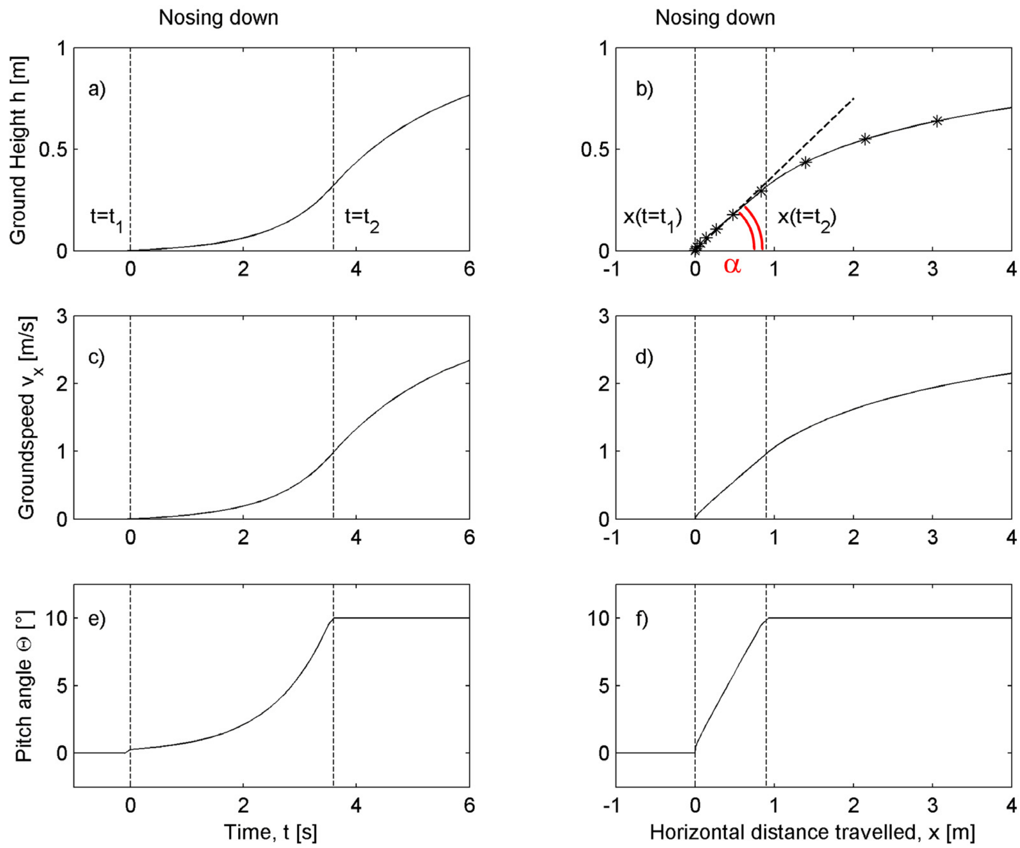


Figure 5 : a-b) Simulated take off trajectory as a function of time (a) and horizontal distance (b) travelled according to Eq. 2, 3 and 8 for a 5-second exponential forward pitch (τ=2.15s).
c-d) Groundspeed increases exponentially with time, proportionally to the ground height from $t=t_1$ to $t=t_2$.
e-f) Take off is initiated by the forward pitch. The rotorcraft takes off with constant slope from the start of the forward pitch zone, as predicted by Eq. 11.
The vertical dotted lines show the start and end of the forward pitch zone.

The two parameters ***a*** and ω_{set} can be selected on-board the rotorcraft to impose a given ascent slope. The possibility of holding a constant angle during the ascent phase results from the surge dynamics and the particular control procedure used, which consists in regulating the downward optic flow. It is therefore not necessary to measure the ascent speed $\mathbf{v_z}$ or the groundspeed $\mathbf{v_x}$ (using a variometer and a Doppler-radar, respectively) to keep the climbing slope equal to a/ω_{set}.

Interestingly, the OF regulator used here makes the required rotor thrust F_N remain constant during take off at a constant angle, as described in detail in Appendix II. The power required for a rotorcraft to be able to take off therefore makes this approach particularly suitable for helicopter applications, since helicopters' collective pitch can be set at a constant value.

5. Conclusion

Unlike other methods of controlling autonomous rotorcraft landing and takeoff at a constant descent angle, which would require bulky Terrestrial Aids (Ref 13), our OCTAVE autopilot is a proper "see and avoid" system capable of performing these demanding tasks without requiring any off-board navigation aids. The key to this remarkable achievement is the *optic flow regulator* on which the unique visuomotor feedback loop is based. Rotorcrafts equipped with this device can land automatically at a constant angle without receiving any inputs specifying the groundspeed, air speed, descent speed or height over the terrain. The onboard non-emissive sensor and the simple autopilot are particularly suitable for use on MAVs with an avionic payload of only a few grams. However, the OCTAVE autopilot could also be used to relieve remote operators performing the painstaking and difficult task of continuously piloting and guiding larger UAVs.

As soon as the human pilot of a manned rotorcraft flying at cruising speed reduces the airspeed, the OCTAVE autopilot will do the rest: it will take care of the descent speed at any time so as to make the rotorcraft land safely at a constant descent angle. The human pilot can easily select the descent/ascent angle by selecting a given reference optic flow ω_{set} (Eq. 2). The downward optic flow sensor, which is the key to the *optic flow regulator*, can provide the human pilot with guidance and/or useful warning signals, since the pilot's visual field can be dramatically impaired in the heading direction when the helicopter is nosed up.

The optic flow sensor used in the OCTAVE autopilot design (fig 1) is compatible with the far infrared (IR) and millimetre-wave ranges, and could therefore be used to assist pilots performing demanding tasks such as autonomous landing under all-weather conditions. A further advantage of the OCTAVE autopilot is the fact that it requires no airspeed measurements – especially at the low airspeeds at which landing occurs, which are hard to measure because of the rotor downwash disturbances.

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<u>Appendix I :</u> Autonomous take off :

Proof that the groundspeed increases exponentially with the pitch angle

From the surge transfer function, we obtain the convolution function :

$$\frac{\Delta v_x}{\Delta\Theta} = \frac{H_0}{1+\tau s} \Rightarrow g(t) = \frac{H_0}{\tau} e^{-\frac{t}{\tau}}$$

We write the convolution product as follows:

$$\Delta v_x(t) = (g * \Delta\Theta)(t) = \int_{t_1}^{t} g(t-t') \cdot \Delta\Theta(t') \cdot dt'$$

Substituting the pitch variation law from equation 8 into the convolution product gives:

$$\Delta v_x(t) = \int_{t_1}^{t} \frac{H_0}{\tau} \cdot e^{-\frac{t-t'}{\tau}} \cdot k e^{at'} dt'$$

$$\Leftrightarrow \Delta v_x(t) = \frac{H_0}{\tau} \cdot k \cdot e^{-\frac{t}{\tau}} \cdot \int_{t_1}^{t} e^{\left(\frac{1}{\tau}+a\right)t'} dt'$$

$$\Leftrightarrow v_x(t) - v_x(t_1) = \frac{H_0}{\tau} \cdot \frac{1}{\frac{1}{\tau}+a} \cdot k \cdot e^{-\frac{t}{\tau}} \cdot e^{\left(\frac{1}{\tau}+a\right)(t-t_1)}$$

$$\Leftrightarrow v_x(t) - v_x(t_1) = \frac{H_0}{1+a\tau} \cdot k \cdot e^{-\left(\frac{1}{\tau}+a\right)t_1} \cdot e^{\left(\frac{-1}{\tau}+\frac{1}{\tau}+a\right)t}$$

Lastly, using the initial condition, we obtain the groundspeed equation during the increasing exponential pitch phase:

$$t_1 = 0, \quad v_x(t_1) = 0$$

$$v_x(t) = \frac{H_0}{1 + a\tau} \cdot k \cdot e^{at} = \frac{H_0}{1 + a\tau} \cdot \Theta(t)$$

The groundspeed equation is a linear function of the pitch angle, which is an exponential function of time.

Appendix II: Autonomous take off :

Proof that the rotor thrust remains fairly constant during take off at a constant angle

At small pitch angles (Θ<15°), we can approximate both propulsive and lift forces as follows:

$$\begin{cases} F_{propulsive} \cong F_N \cdot \Theta \\ F_S \cong F_N \end{cases}$$

From the surge dynamics transfer function (identified as a first order transfer function), we have:

$$\frac{\Delta v_x(s)}{\Delta\Theta(s)} = \frac{H_0}{1 + \tau s}$$

Using the inverse Laplace transform, we then obtain :

$$\frac{1}{\tau}\frac{d\Delta v_x(t)}{dt} = H_0 \Delta\Theta(t) - \Delta v_x(t)$$

From the initial takeoff manoeuvre conditions, we obtain:

$$\frac{1}{\tau}\frac{dv_x(t)}{dt} = H_0 \Theta(t) - v_x(t)$$

$$with \quad v_x(t=0) = 0 \quad and \quad \Theta(t=0) = 0$$

The Newton equation gives:

$$m\frac{dv_x(t)}{dt} = F_{propulsive} + F_{Drag}$$

$$with \quad F_{Drag} = -D_x v_x(t)$$

$$\text{as identified at slow speed}$$

Finally, we identify the following 2 equations:

$$\begin{cases} \frac{1}{\tau}\frac{dv_x(t)}{dt} = H_0 \Theta(t) - v_x(t) \\ m\frac{dv_x(t)}{dt} = F_N(t) \cdot \Theta(t) - D_x v_x(t) \end{cases}$$

We can identify $\tau = m/D_x$, the aero-mechanical time constant of the surge dynamics along the x-axis, and the rotor thrust $F_N = D_x H_0$, which happens to be constant during this takeoff procedure.

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