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# HELICOPTER NONLINEAR FLIGHT CONTROL SYSTEM DEVELOPMENT

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## 1. ABSTRACT

This paper considers synthesis of a helicopter full authority flight controller using approximate inversion of the nonlinear model of the vehicle. Based on the natural time scale separation between position and attitude dynamics of the vehicle, the vehicle attitudes are treated as pseudo-command variables. In order to simplify the controller, approximations to the body axes forces are used in the controller calculations. The first approximation involves neglecting the cyclic and pedal control force terms and the second approximation involves neglecting the body x- and y-axis force components in the controller calculations. The adequacy of these approximations and the performance of the resulting controller in executing an elliptical turn maneuver are evaluated through nonlinear simulation.

## 2. BACKGROUND

Current and future combat rotorcraft are required to perform a variety of complex missions. Some of those missions, e.g., air-to-air combat, would require controlling of the vehicle through large attitude maneuvers. The use of an automatic flight control system simplifies the piloting task significantly, resulting in reduced pilot workload and increased mission effectiveness. Traditionally, helicopter flight control system design has been carried out using a linear representation of the vehicle dynamics about a set of pre-selected equilibrium conditions or trim points. The major advantage of such a design approach is that relatively minor on-line calculations are required. However, this approach suffers from several disadvantages. First, a linearized model is only approximate and does not embody the more complete information contained in a nonlinear model. Second, linearized models are valid only near specific points (equilibrium or trim points) and hence, some kind of gain scheduling is indispensable for acceptable performance of controllers based on these models over an entire flight regime. Third, assumptions inherent in a linear model, e.g., the small angle assumption, restrict the authority of the designed controller to executing commands close to the design flight condition. It is important to recognize that the perturbation equations used in the design process do not represent the plant dynamics adequately for a practical system design[1].

Control of nonlinear systems by inverting their nonlinearities is well known and has been applied to a wide variety of nonlinear systems. The main advantage is that the nonlinear dynamics are transformed to an equivalent linear system for which standard linear control theory techniques can be used. This results in a single controller valid throughout the flight envelope and it eliminates the need for gain scheduling. The necessary and sufficient conditions for decoupling a nonlinear system are given in Ref. 2. Also, Ref.2 applies the nonlinear decoupling theory to the aircraft control problem by using a simplified aircraft model to decouple the vertical and horizontal flight path angles. The inverse dynamics of a VSTOL aircraft are constructed in Refs. 3 and 4. However, due to the complexity of the equations involved, either the differentiations are carried out numerically or a linear approximation to the nonlinear system is made using a truncated Taylor series expansion. In Refs. 5 and 6, forced singular perturbation theory is used to simplify the linearizing transformations and the same is applied to aircraft flight control problem. Ref. 7 deals with the control problem of aircraft in extreme flight conditions accompanied by severe nonlinear effects arising from high angles of attack and high angular rates. Through the use of nonlinear inverse dynamics of a 12 state aircraft model, the controller decouples specific state variables that are of particular interest to the pilot. These so-called command variables are organized in sets that can be varied as functions of the flight phase, in order to provide the pilot with a maximum control of the aircraft with a minimum effort.The forced singular perturbation approach is used to develop a full authority controller for an autonomous helicopter in Ref. 8. Also, Ref. 8 presents a method for solving the inverse kinematic problem. Though a nonlinear controller offers several advantages over a linear controller, due to the complexity of the dynamic equations of motion of a helicopter, the implementation of a nonlinear controller involves intensive on-line computations.

This paper investigates the approximations that can be made in the controller calculations in order to simplify the controller. The first approximation involves neglecting the cyclic and pedal control force terms in the controller calculations. This approximation is similar to the one considered in Ref. 8. The second approximation involves neglecting the body x- and y- force components in the controller computations. The adequacy of these approximations and the performance of the resulting controllers are evaluated using the elliptic turn test, which is suggested as a controller robustness test in Ref. 1. The elliptical turn test involves maneuvering the vehicle through 360 degrees of yaw attitude changes while maintaining a reference velocity[1]. Simulations are carried out using the TMAN simulation model of the UH-60A Black Hawk helicopter. The TMAN simulation program was originally developed at NASA Ames for nap of earth one-on-one air combat simulation[9] and it was later modified to include a two-time scale nonlinear controller[10]. The paper is organized according to the following. First, the helicopter mathematical model used in this study is described. Next, the nonlinear controller synthesis and the approximations considered in the synthesis procedure are presented. Finally, using simulation results for an elliptical turn maneuver, the adequacy of the approximations used in the nonlinear controller synthesis are evaluated.

## 3. HELICOPTER DYNAMIC MODEL

A six-degree-of-freedom rigid body dynamic model of the helicopter is used in this study. The position dynamics of the vehicle is described by [11]

$$m\begin{Bmatrix} \ddot{X} \\ \ddot{Y} \\ \ddot{Z} - g \end{Bmatrix} = \Big[ [L_3(-\psi)][L_2(-\theta)][L_1(-\phi)] \Big] \begin{Bmatrix} F_x \\ F_y \\ F_z \end{Bmatrix} \tag{1}$$

where X,Y and Z represent the inertial position of the center of gravity of the vehicle and $F_x$, $F_y$ and $F_z$ are the body axes components of external force other than gravitational force. The attitude dynamics of the vehicle is given by[11]

$$
\begin{Bmatrix} \ddot{\phi} \\ \\ \ddot{\theta} \\ \\ \ddot{\psi} \end{Bmatrix} = \begin{Bmatrix} \dot{p} + (q\dot{\phi} + \dot{r})\cos\phi\tan\theta + (\dot{q} - r\dot{\phi})\sin\phi\tan\theta \\ \quad + (q\dot{\theta}\sin\phi + r\dot{\theta}\cos\phi)\sec^2\theta \\ (\dot{q} - r\dot{\phi})\cos\phi - (q\dot{\phi} + \dot{r})\sin\phi \\ (\dot{q} - r\dot{\phi})\sin\phi\sec\theta + (q\dot{\phi} + \dot{r})\cos\phi\sec\theta \\ \quad + \dot{\theta}(q\sin\phi + r\cos\phi)\tan\theta\sec\theta \end{Bmatrix} \tag{2}
$$

Assuming an xz-plane of symmetry, the body angular accelerations $\dot{p}, \dot{q}, \dot{r}$ are related to the external moments L, M, N by

$$
\begin{Bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{Bmatrix} = \begin{Bmatrix} (I_z L + I_{xz} N)/(I_x I_z - I_{xz}^2) \\ M/I_y \\ (I_{xz} L + I_x N)/(I_x I_z - I_{xz}^2) \end{Bmatrix} \tag{3}
$$

A quasi-static aerodynamic representation is used to model the aerodynamic forces and moments.

$$
\begin{bmatrix} F_x/m \\ F_y/m \\ F_z/m \end{bmatrix} = \begin{bmatrix} X_u u + X_{\delta_e} \delta_e \\ Y_v v + Y_{\delta_a} \delta_a \\ \bar{Z}_{trim} + Z_w w + Z_{\delta_c} \delta_c \end{bmatrix} \tag{4}
$$

$$
\begin{bmatrix} L \\ M \\ N \end{bmatrix} = \begin{bmatrix} L_v v + L_p p + L_{\delta_a} \delta_a \\ M_u u + M_q q + M_{\delta_e} \delta_e \\ N_v v + N_p p + N_r r + N_{\delta_p} \delta_p \end{bmatrix} \tag{5}
$$

In Eqs. (4) and (5), u, v and w represent the x-, y- and z-axis components of body velocity relative to atmosphere and $\delta_e$, $\delta_c$, $\delta_a$ and $\delta_p$ represent longitudinal cyclic, collective, lateral cyclic and pedal control displacements, respectively. The aerodynamic derivatives change with flight condition.

## 4. NONLINEAR CONTROLLER SYNTHESIS

In this section, the approximations used in the nonlinear controller synthesis are described. First, a general scheme for input-output linearization[12] of a nonlinear system is presented. The dynamic equations of motion of the helicopter described by Eqs. (1)-(5) can be rewritten as

$$
\ddot{\chi} = A(\chi, \dot{\chi}) + B(\chi, \dot{\chi})\eta \tag{6}
$$

where $\chi = \{X, Y, Z, \phi, \theta, \psi\}^T$ and $\eta = \{\delta_e, \delta_c, \delta_a, \delta_p\}^T$

The vector of output variables to be controlled, S, can be written as a linear combination of the state vector, $\chi$.

$$S = C\chi \tag{7}$$

Taking the second time derivative of Eq. (7) and substituting for $\ddot{\chi}$ from Eq. (6) results in

$$\ddot{S} = C(A + B\eta) \tag{8}$$

where the functional dependency of A and B are dropped to preserve clarity. Denoting the right hand side of the above equation by a pseudo-control vector, V, would result in

$$\ddot{S} = V \tag{9}$$

This system can be put into Brunovsky's Canonical Form[13]. PID (proportional, integral, and derivative) control laws can be formulated for each of the variables to be controlled in the S vector by imposing

$$V(\chi, \dot{\chi}) = \ddot{S}_c + K_P(S_c - S) + K_D(\dot{S}_c - \dot{S}) + K_I\int_0^t (S_c - S)\,dt \tag{10}$$

where $K_P$, $K_D$, and $K_I$ are diagonal matrices. The feedback control law can be determined by equating the right hand sides of Eqs. (8) and (10). If the number of output variables to be controlled is more than the number of available controls, a pseudo-inversion procedure[12] may be used to obtain the nonlinear control law from Eqs. (8) and (10). A block diagram representation of the nonlinear controller is shown in Fig. 1.

Typically, the output variables to be controlled for a helicopter are the three components of position and yaw attitude of the vehicle. In order to achieve input-output linearization of the vehicle dynamics, the second time derivatives of the four output variables to be controlled are equated to four pseudo-control variables. This results in

$$V_1 = \ddot{X} \qquad V_2 = \ddot{Y} \qquad V_3 = \ddot{Z} \tag{11}$$

$$V_4 = \ddot{\psi} \tag{12}$$

For conventional helicopter systems, changes in body pitch and roll attitudes play a significant role in contributing to the body longitudinal and lateral accelerations[14]. This becomes evident if one looks at the simplified linearized model of the vehicle about an equilibrium point. For example, if we consider the approximate linearized X-force and pitching moment equations, which are valid about the hover condition, we get

$$\ddot{X} = X_u\dot{X} + \frac{F_{z0}}{m}\theta + X_{\delta_e}\delta_e \tag{13}$$

$$\ddot{\theta} = M_q\dot{\theta} + M_{\delta_e}\delta_e \tag{14}$$

where $F_{z0}$ is the equilibrium force component along the body z-axis. Eq. (13) suggests that it is possible to accelerate the vehicle using pitch attitude changes. The magnitude of the speed damping derivative, $X_u$, is typically very small compared to the magnitude of the pitch damping derivative, $M_q$. This results in a large time constant for position changes as compared to attitude changes. This time scale separation between position dynamics and attitude dynamics is the basis for the two-time scale controller considered in Ref. 8 and the same approach is used for the controller synthesis carried out in this study. The required pitch and roll attitudes $(\bar{\theta}, \bar{\phi})$ for achieving position command tracking are treated as pseudo-commands and the vehicle pitch and roll attitudes are controlled to follow these pseudo-commands. This is accomplished by defining two additional pseudo-controls as

$$V_5 = \ddot{\phi} \qquad V_6 = \ddot{\theta} \tag{15}$$

The actual controls required for achieving approximate tracking of the commanded values are computed by combining Eqs. (1), (2), (10), (11), (12) and (15). First, combining Eqs. (1) and (11) results in

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 - g \end{bmatrix} = \begin{bmatrix} [L_3(-\psi_c)][L_2(-\bar{\theta})][L_1(-\bar{\phi})] \end{bmatrix} \begin{bmatrix} F_x/m \\ F_y/m \\ F_z/m \end{bmatrix} \tag{16}$$

From Eq. (16), the following three relationships can be obtained.

$$\left(\frac{F_x}{m}\right)^2 + \left(\frac{F_y}{m}\right)^2 + \left(\frac{F_z}{m}\right)^2 = V_1^2 + V_2^2 + (V_3 - g)^2 \tag{17}$$

$$(V_1 \cos\psi_c + V_2 \sin\psi_c)\cos\bar{\theta} - (V_3 - g)\sin\bar{\theta} = \frac{F_x}{m} \tag{18}$$

$$\frac{F_y}{m}\cos\bar{\phi} - \frac{F_z}{m}\sin\bar{\phi} = -V_1 \sin\psi_c + V_2 \cos\psi_c \tag{19}$$

The unknown quantities in Eqs. (17)-(19) are $\bar{\phi}, \bar{\theta}, \delta_c, \delta_e, \delta_a$ and $\delta_p$. In order to be able to determine the pseudo-commands and the collective control from Eqs. (17)-(19), the cyclic and pedal control force terms are dropped from the expressions for the body axes forces. This approximation is similar to the assumption used in Ref. 8 that the cyclic and pedal controls are primarily moment generating controls. The controller that results from this approximation is denoted as Controller A.

$$\left(\frac{\tilde{F}_x}{m}\right)^2 + \left(\frac{\tilde{F}_y}{m}\right)^2 + \left(\frac{\tilde{F}_z}{m}\right)^2 = V_1^2 + V_2^2 + (V_3 - g)^2 \tag{20}$$

$$(V_1 \cos\psi_c + V_2 \sin\psi_c)\cos\bar{\theta} - (V_3 - g)\sin\bar{\theta} = \frac{\tilde{F}_x}{m} \tag{21}$$

$$\frac{\tilde{F}_y}{m}\cos\bar{\phi} - \frac{\tilde{F}_z}{m}\sin\bar{\phi} = -V_1 \sin\psi_c + V_2 \cos\psi_c \tag{22}$$

where $\bar{F}_x$, $\bar{F}_y$ and $\bar{F}_z$ are the body axes force components without cyclic and pedal control force terms.

The actual computation of $\delta_c$, $\bar{\phi}$, and $\bar{\theta}$ using this approximation involves solution to an algebraic equation in terms of $\delta_c$ (Eq. 20) and two transcendental equations in $\bar{\phi}$ and $\bar{\theta}$ (Eqs. 21 and 22). In order to simplify the controller, a second approximation is used in which it is assumed that the body z-force is very large compared to the x- and y-forces. This amounts to neglecting the body x- and y-force terms in Eqs. (20)-(22). The controller that results from the second approximation is denoted as Controller B.

$$(\frac{\bar{F}_z}{m})^2 = V_1^2 + V_2^2 + (V_3 - g)^2 \tag{23}$$

$$\bar{\theta} = \tan^{-1}(\frac{V_1 \cos\psi_c + V_2 \sin\psi_c}{V_3 - g}) \tag{24}$$

$$\bar{\phi} = \sin^{-1}(\frac{V_1 \sin\psi_c - V_2 \cos\psi_c}{\bar{F}_z/m}) \tag{25}$$

Note that the second approximation greatly simplifies the controller calculations. Using $\bar{\phi}$ and $\bar{\theta}$ computed from these approximations along with $\psi_c$, the cyclic and pedal controls are computed by combining Eqs. (2), (10), (12) and (15).

## 5. CONTROLLER EVALUATION

The adequacy of the approximations used in the nonlinear controller synthesis is evaluated using the TMAN simulation program.The aerodynamic data required for modeling the UH-60 helicopter is taken from Ref.15. With integral gains set to zero, the proportional and derivative gains in Eq. (10) are chosen to obtain a bandwidth of 0.5 rad/sec in the X- and Y-position loops, 1.0 rad/sec in the Z-position loop, 2 rad/sec in pitch, roll and yaw attitude loops and a damping ratio of 0.7 in all the loops. The vehicle is trimmed at 40 knots forward speed and the following commands are used to simulate an elliptical turn maneuver.

$$\dot{X}_c = 40 \text{ knots}$$
$$\dot{Y}_c = 0$$
$$\dot{Z}_c = 0$$
$$\psi_c = \begin{cases} 0 & t \le 1\,\text{sec} \\ (t-1) & 1 \le t \le (1+6\pi)\,\text{sec} \\ 6\pi & t \ge (1+6\pi)\,\text{sec} \end{cases}$$

The simulation results obtained using the two controllers are shown in Figs. 2 through 11. In all the figures, the solid line corresponds to simulation results obtained using controller A, which is based on neglecting the cyclic and pedal control forces in the controller calculations, and the dotted line corresponds to results obtained using Controller B, which is based on neglecting the body x- and y-force components in the controller calculations. The vehicle flight speed versus time is shown in Fig. 2. Both controllers result in vehicle flight speed maintained within roughly ±4% of the reference speed of 40 knots. The body yaw attitude time history is shown in Fig. 3. The yaw attitude command involves three rotations about the Z-axis at a rate of 1 rad/sec and both controllers are able to achieve the commanded yaw attitude with a slight over-shoot towards the end of the maneuver. The body pitch and roll attitude time histories are shown in Figs. 4 and 5 and the time histories of the body x-, y- and z-axis components of velocity are shown in Figs. 6, 7 and 8, respectively. As the vehicle rotates about the vertical to follow the yaw attitude command while maintaining the reference straight and level flight path, the magnitude of the body x-axis component of velocity decreases where as the magnitude of the y-axis component of velocity increases. This results in decreased drag along the x-axis and increased drag along the y-axis. In order to balance the changes in drag along the x- and y- axes, the controller forces the body to pitch up and roll to the left. The pitch and roll attitudes change cyclically as the vehicle goes through three complete rotations about the vertical. Though the body x- and y-axis components of velocity are almost identical for the two controllers, the pitch and roll attitude excursions are different. The magnitude of pitch attitude response is more for Controller B as compared to that for Controller A. Also, there is a significant phase difference between the two responses. The magnitude of roll attitude response is more for Controller A as compared to that for Controller B. The roll attitude responses are nearly in phase for the two controllers. It is felt that the differences in pitch and roll attitude responses for the two controllers are a consequence of the different approximations used in the controller calculations. The changes in longitudinal cyclic, lateral cyclic and pedal controls from trim are shown in Figs. 9, 10 and 11, respectively. From Fig. 9, it is noticed that the magnitude of the longitudinal cyclic control required for Controller A is more than that for Controller B. The lateral cyclic and pedal controls required for both controllers are nearly the same. Though there are differences in pitch and roll attitude responses, the performance of Controller B in achieving output command tracking is very nearly same as that of Controller A. However, based on the approximations used, the on-line computations for Controller B are less and simple as compared to Controller A. The trends in these results compare favorably with those of Ref. 1 baseline design subjected to an elliptical turn test. The design procedure followed in Ref. 1 is based on converting the multiple input, multiple output problem into a series of single input, single output, relatively decoupled problems. The approach taken here is a direct synthesis of a helicopter full authority flight controller using approximate inversion of the nonlinear model of the vehicle.

## 6. CONCLUSIONS

The adequacy of approximations that can be made in the nonlinear controller synthesis using inversion of the vehicle dynamic model is investigated in this study. Using simulation results obtained for an elliptical turn maneuver, it is shown that neglecting the body x- and y-axis force components in the controller calculations simplifies the controller and it is adequate for achieving reasonably good output command tracking. Based on these results, it is concluded that it is possible to approximate the model inversion process and hence, to simplify the nonlinear controller while retaining reasonably good output command tracking performance.

## 7. ACKNOWLEDGEMENTS

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## 8. REFERENCES

1. S. Osder, "Practical Robustness Testing for Helicopter Flight Control Systems," **Proceedings of the American Helicopter Society Annual Forum**, Arizona, 1991.
2. S.J. Asseo, "Decoupling of a Class of Nonlinear Systems and its Application to an Aircraft Control Problem," **Journal of Aircraft**, Vol. 10, No. 12, 1973.
3. G. Meyer and L. Cicolani, "Application of Nonlinear System Inverses to Automatic Flight Control Designs - System Concepts and Flight Evaluations," Theory and Application of Optimal Control in Aerospace Systems, AGARD-AG251, 1981.
4. G. Meyer, R. Su and L.R. Hunt,"Application of Nonlinear Transformations to Automatic Flight Control," **Automatica**, Vol. 20, 1984.
5. P.K.A. Menon, M.E. Badgett, R.A. Walker and E.L. Duke, "Nonlinear Flight Test Trajectory Controllers for Aircraft," **Journal of Guidance, Control and Dynamics**, Vol. 10, No. 1, 1987.
6. S.N. Singh, "Control of Nearly Singular Decoupling Systems and Nonlinear Aircraft Maneuver," **IEEE Transactions on Aerospace and Electronic Systems**, Vol. 24, No. 6, 1988.
7. S.H. Lane and R.F. Stengel, "Flight Control Design using Nonlinear Inverse Dynamics," **Automatica**, Vol. 24, No. 4, 1988.
8. M.W. Heiges, P.K. Menon and D.P. Schrage, "Synthesis of a Helicopter Full Authority Controller," **AIAA Guidance, Navigation and Control Conference**, 1989.
9. M.S. Lewis and E.W. Aiken, "Piloted Simulation of One-on-One Helicopter Air Combat at NOE Flight Levels," NASA TM 86686, April 1985.
10. M.W. Heiges, " A Helicopter Flight Path Controller Design via Nonlinear Transformation Technique," **Ph.D. Thesis**, Georgia Institute of Technology, Atlanta, 1989.
11. B. Etkin: Dynamics of Atmospheric Flight, **John Wiley & Sons, Inc.**, New York, 1972.
12. J.V.R. Prasad, M. Mittal and D.P. Schrage, "Control of a Twin Lift Helicopter System using Nonlinear State Feedback," **AIAA Guidance, Navigation and Control Conference**, 1990.
13. A. Isadori: Nonlinear Control Systems, An Introduction. **Springer-Velag**, 2nd Edition, 1989.
14. E. Seckel, Stability and Control of Airplanes and Helicopters, **Academic Press**, 1968.
15. K.B. Hilbert,"A Mathematical Model of the UH-60 Helicopter," NASA TM 85890, April 1984.

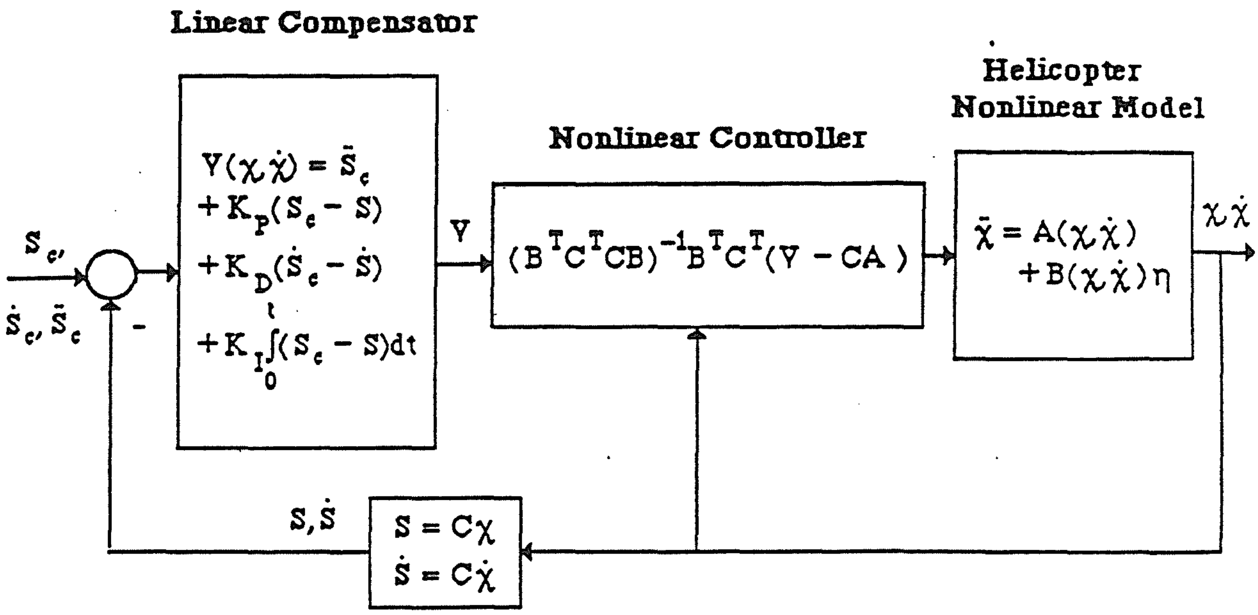


Figure 12. Helicopter nonlinear controller.

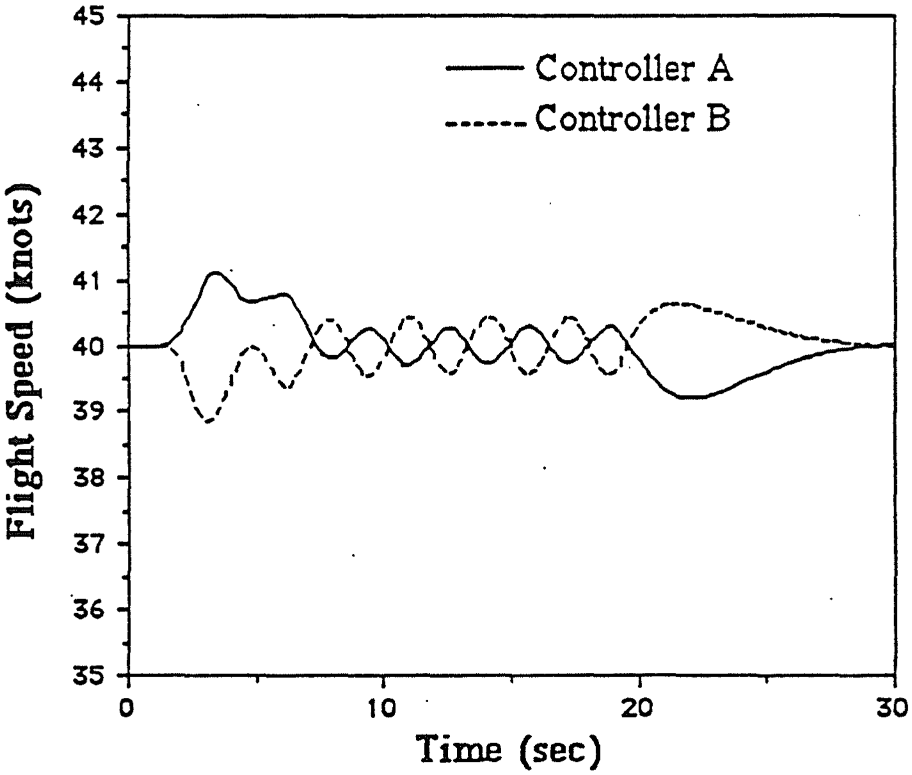


Figure 2. Helicopter flight speed variation in an elliptical turn maneuver.

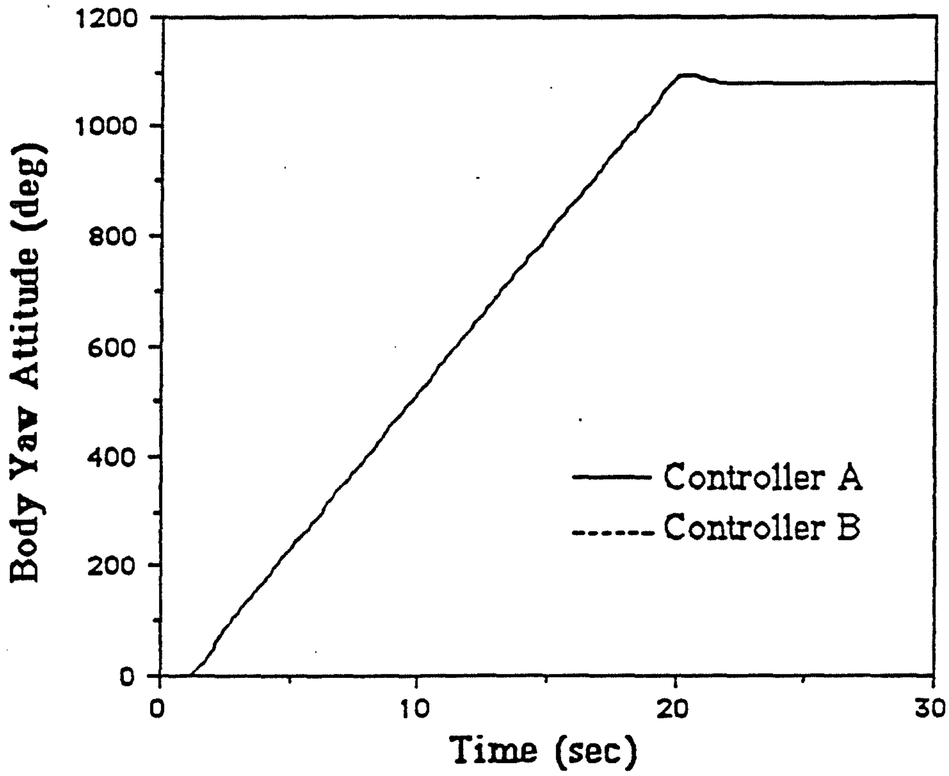


Figure 3. Helicopter yaw attitude response in an elliptical turn maneuver.

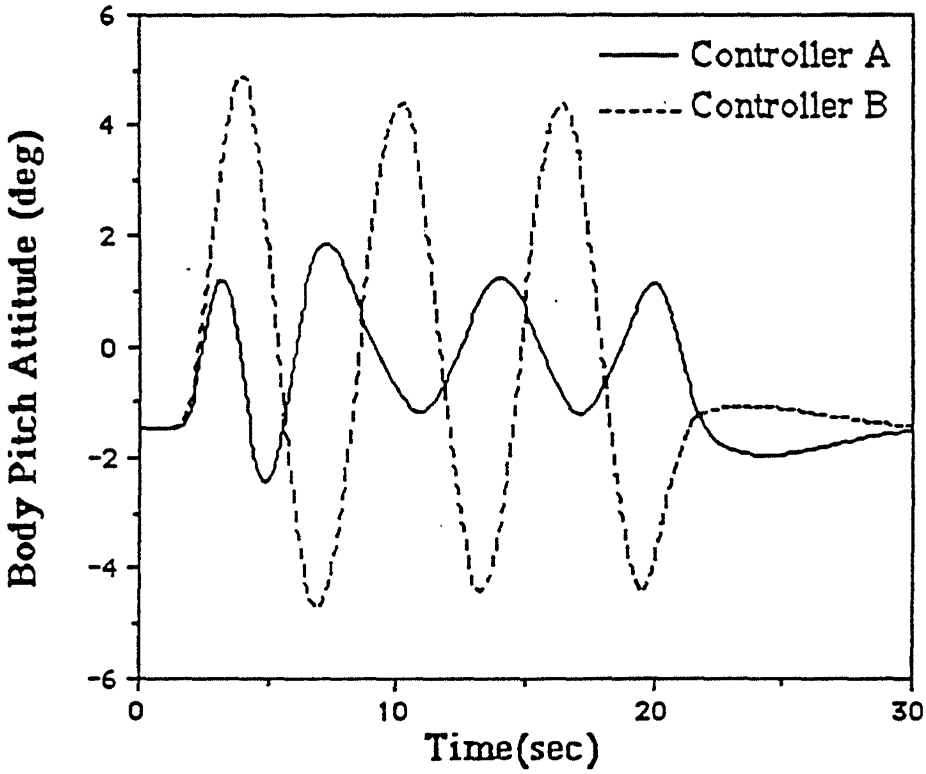


Figure 4. Helicopter pitch attitude response in an elliptical turn maneuver.

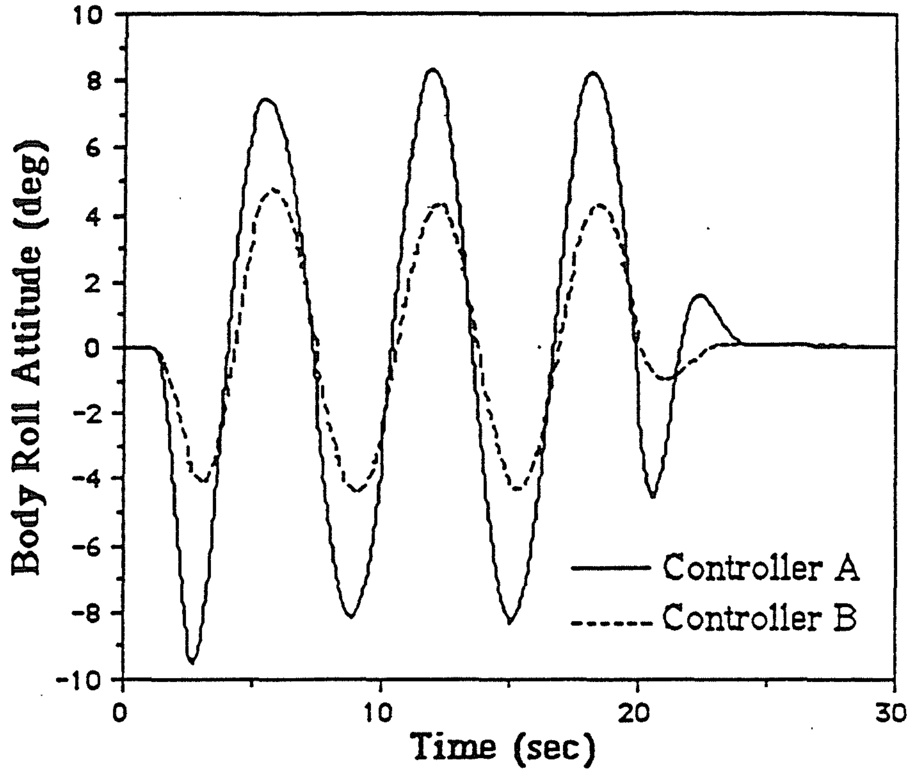


Figure 5. Helicopter roll attitude response in an elliptical turn maneuver.

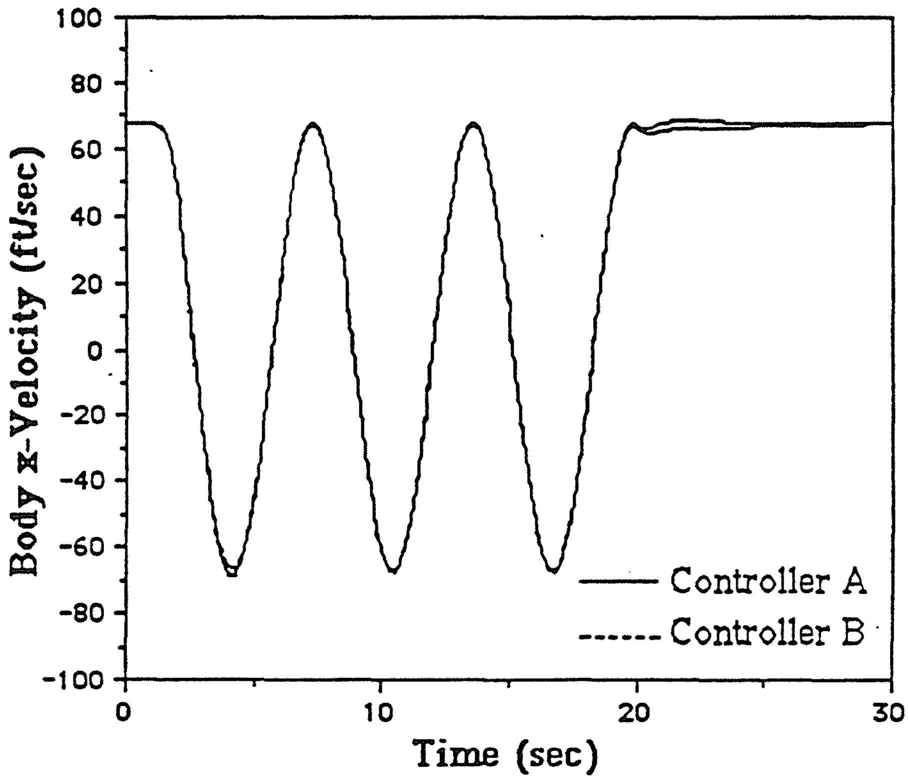


Figure 6. Helicopter body x-velocity response in an elliptical turn maneuver.

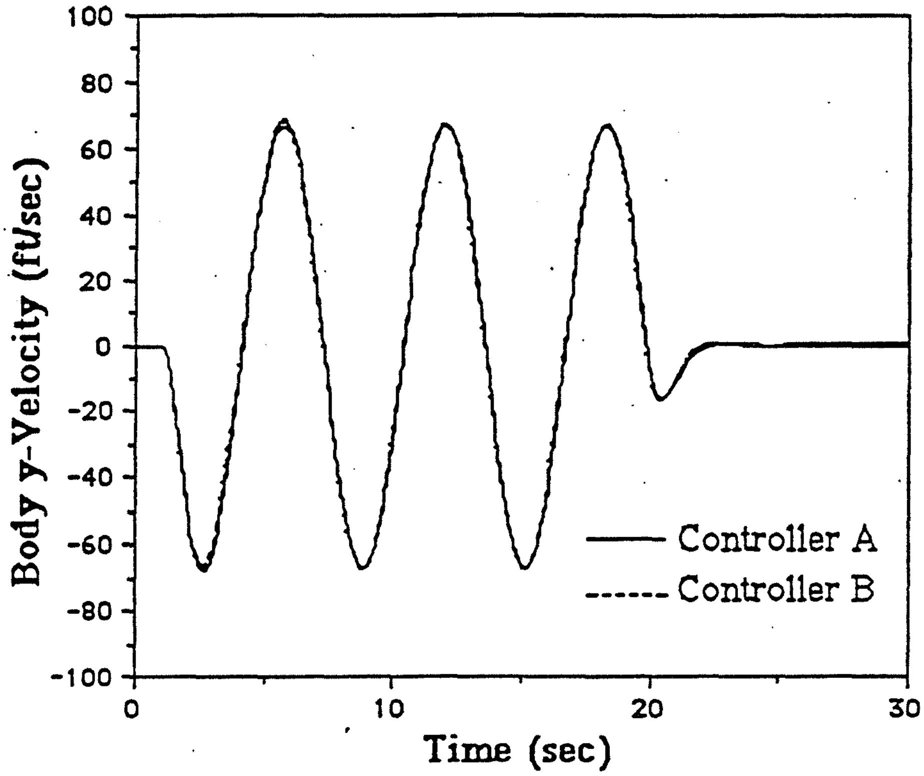


Figure 7. Helicopter body y-velocity response in an elliptical turn maneuver.

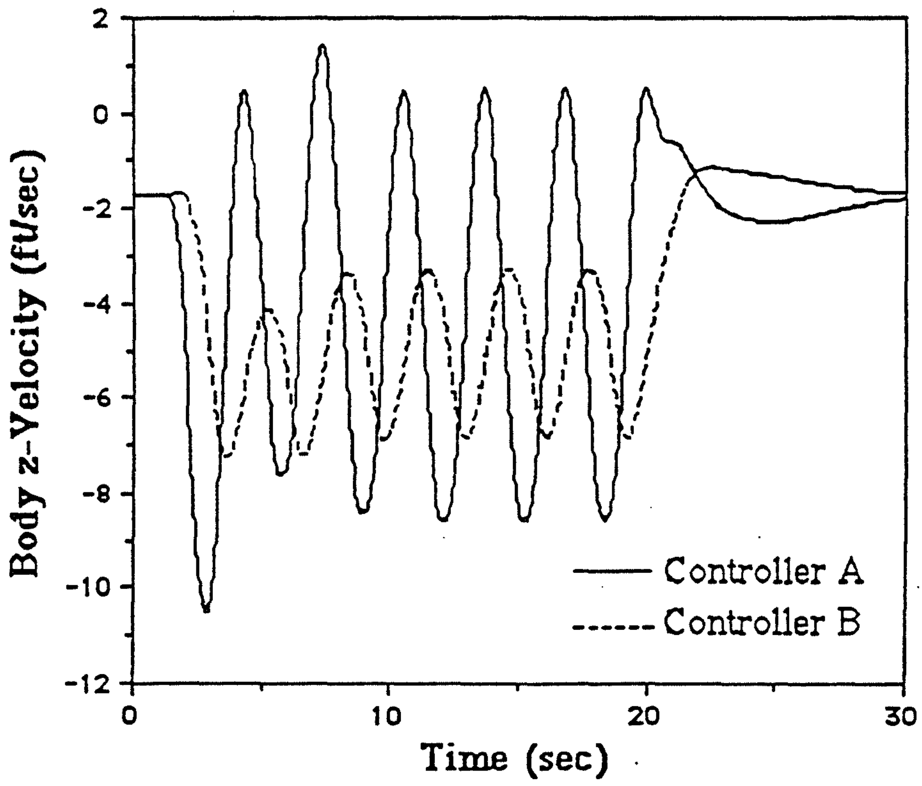


Figure 8. Helicopter body z-velocity response in an elliptical turn maneuver.

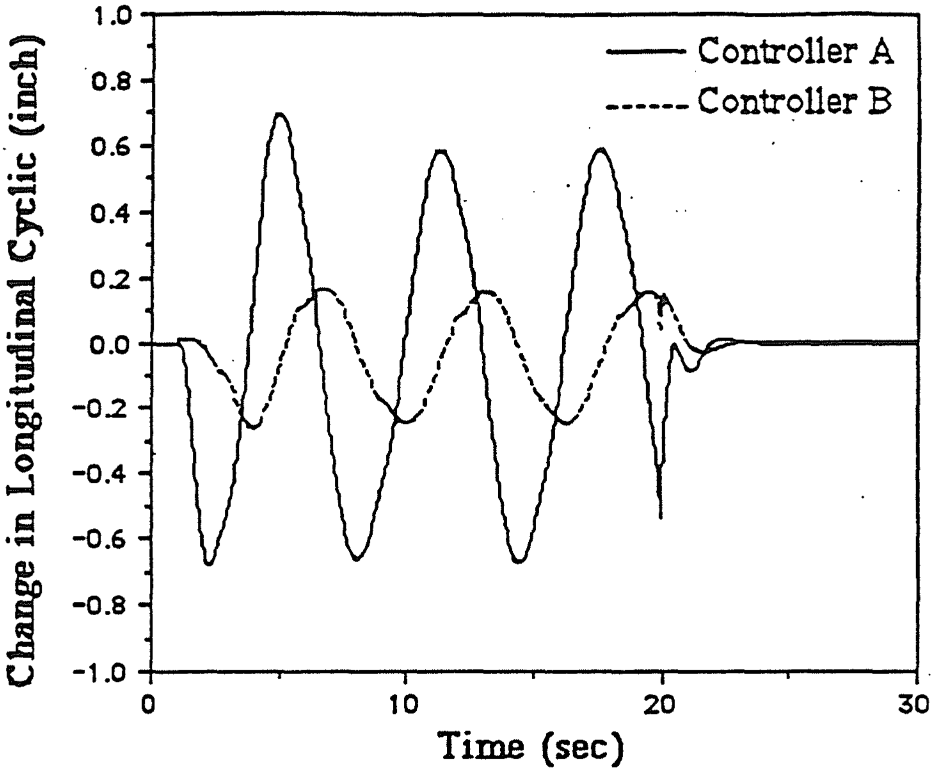


Figure 9. Variation of longitudinal cyclic control in an elliptical turn maneuver.

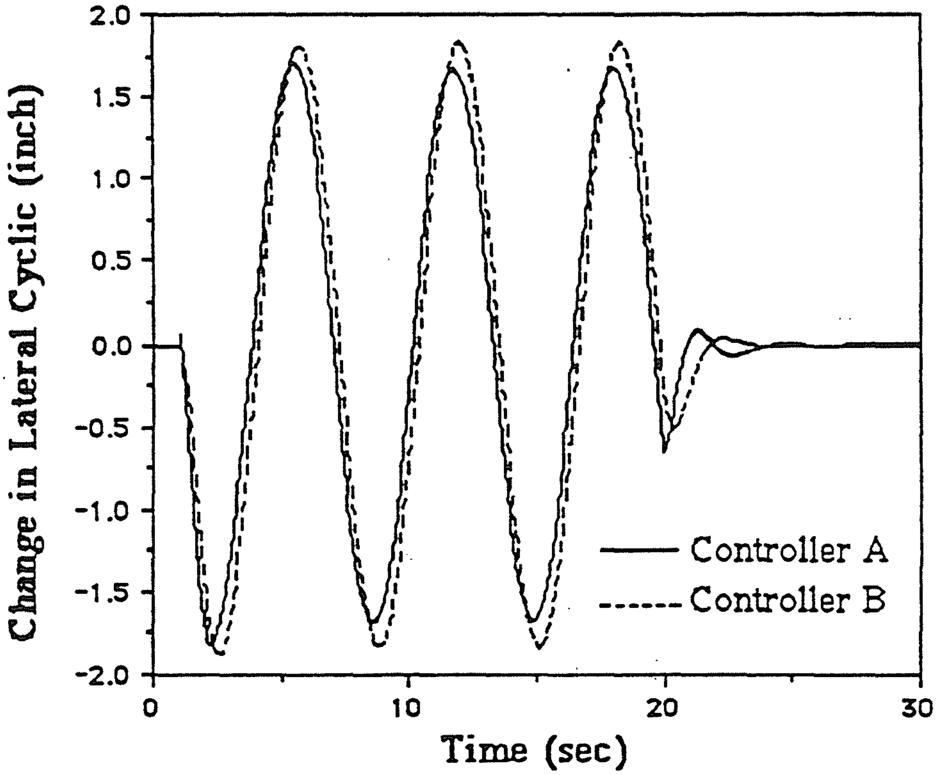


Figure 10. Variation of lateral cyclic control in an elliptical turn maneuver.

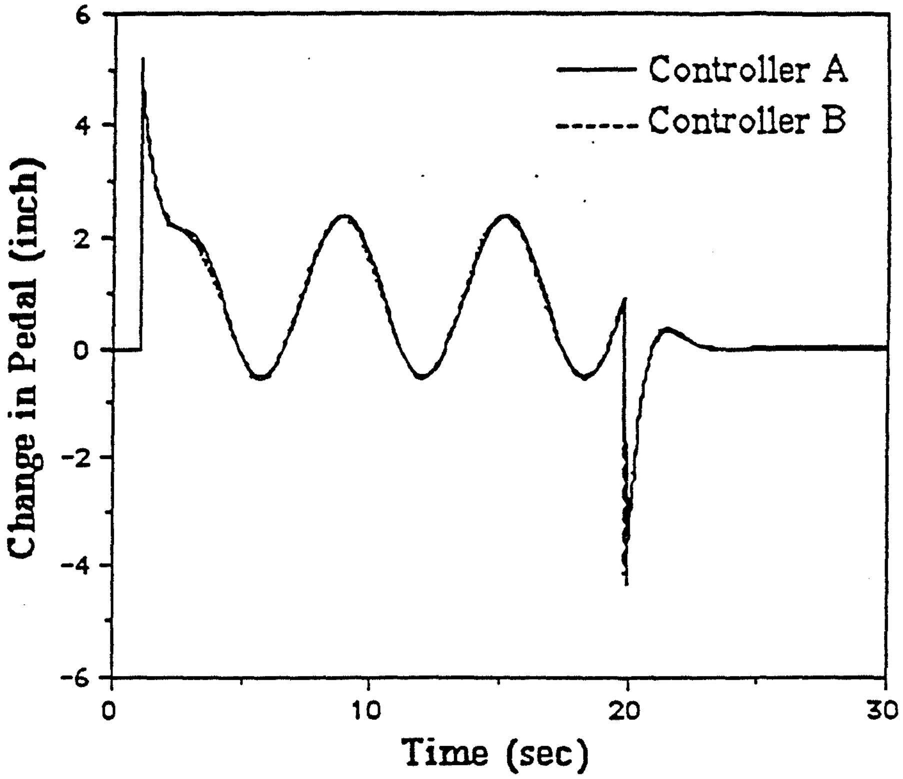

Figure 11. Variation of pedal control in an elliptical turn maneuver.