AUTOGYRO INVERSE SIMULATION FOR HANDLING QUALITIES ASSESSMENT

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Abstract

This paper describes progress made at Glasgow towards developing handling qualities standards for autogyros. The first ever attempt to assess autogyro handling qualities using inverse simulation has been made. The paper breifly reviews the development and applications of inverse simulation at Glasgow, and reveals primary aspects of the algorithm used in this research. In the initial stage of the research an autogyro mathematical model has been developed for use in the inverse simulation. Then, the autogyro model has been validated by flight/simulation comparisons in trimmed flight. An acceleration and deceleration manoeuvre from the ADS-33 specifications was modified to suit light autogyros, such as Glasgow University research autogyro used in this work. Finally, the paper focuses on the first results obtained from inverse simulation with the aim of investigating the possibility of assessing autogyro handling qualities.

Nomenclature

$[J]$	Jacobian matrix
p, q, r	aircraft roll, pitch and yaw rates; (rad/sec)
q_{pk}	peak pitch rate; (rad/sec)
t	time; (sec)
$\mathbf{u}$	control vector
$\mathbf{u}_{error}$	control error vector
u, v, w	aircraft velocity components; (knots)
V_f	airspeed; (knots)
$\mathbf{x}$	state vector
x_e, y_e, z_e	displacements relative to an Earth fixed inertial frame; (m)
$\mathbf{y}$	output vector
$\mathbf{y}_{des}$, $\mathbf{y}_{error}$	desired and error output vectors
$\Delta\Theta_{tilt}$	area under a discrete pulse of longitudinal tilt; (deg sec)
$\Delta\theta_{pk}$	peak pitch displacement from trim; (deg)
δ_r	rudder angle; (deg)
ϕ, θ, ψ	aircraft attitude angles; (deg)
θ_{tilt}, ϕ_{tilt}	longitudinal and lateral rotor tilt angles; (deg)
$\theta_{tilt\ pk}$	longitudinal tilt peak; (deg)
T	propeller thrust; (N)
Ω	rotorspeed; (rpm)

ADS	Aeronautical Design Standard
CAA	Civil Aviation Authority
Genisa	Generic inverse simulation algorithm
HGS	Helicopter Generic Simulation
Helinv	Helicopter inverse simulation algorithm
Hibrom	Helicopter individual blade rotor model
ISA	International Standard Atmosphere
MTE	Mission Task Element
NACA	National Advisory Committee for Aeronautics

Introduction

Autogyros (or autogiros, or gyroplanes) are attracting more attention from general aviation amateurs due to their low operating cost and easy maintenance. In most countries, there are no special design and handling qualities standards for autogyros as they are usually categorised as ultralight or experimental aircraft. This is a possible contributory factor in an increasing accident rate, particularly in the UK. For example, between 1989 and 1991, the gyroplane fatal accident rate in the UK was 6 per 1000 flying hours, whereas the overall general aviation rate during 1990 was 0.015 per 1000 flying hours (Ref 1). According to the CAA “Aviation Safety Review 1992 – 2001” (Ref 2) for this decade there have been 29 reportable accidents to UK autogyros, of which 5 were fatal. These reportable accidents resulted in 5 fatalities and 2 serious injuries. The average rate of fatal accidents per million hours flown is 109. As example, the fatal accident rate for the same period for public transport helicopters is 1.8, for airline aeroplanes (maximum takeoff weight < 5700 kg) 36.1, and for airline aeroplanes (maximum takeoff weight > 5700 kg) it is zero. For last year statistics show that it was one fatality per 150 thousand flying hours for general aviation, and one fatality per 1.85 thousand flying hours for autogyros in UK.

To address this problem the UK Civil Aviation Authority has developed a new design standard for light autogyros: "British Civil Airworthiness Requirements, Section T, Light Gyroplane Design Requirements" (Ref 3), and its superseding, "British Civil Airworthiness Requirements, Section T, Light Gyroplanes" (Ref 4). The University of Glasgow has been involved in this process since 1993 conducting wind tunnel tests, simulations and flight trials. The aircraft used in this research were the VPM M16 and Montgomerie-Parsons autogyros. Unfortunately, these CAA airworthiness requirements are limited in terms of detailed technical content, and have no requirements relating to handling qualities, which are evidently one of the primary objectives of the design of modern rotary-wing aircraft, where improved handling qualities reduce pilot workload and increase mission effectiveness and safety.

Currently, inverse simulation has become a very useful tool in estimating rotorcraft handling qualities. Such an approach was first used by Thomson (Ref 5) to study helicopter agility. In the US Military ADS-33E-PRF handling qualities requirements (Ref 6) flight test manoeuvres are provided in the form of precisely defined Mission Task Elements (MTEs). Mathematical representation of the MTEs (Ref 7, 8) can be used as an input for the inverse simulation algorithm to calculate the pilot control inputs, which allows an estimate of handling qualities. Using this technique, Thomson and Bradley (Ref 8) proposed the inverse simulation tool as a preliminary assessment of helicopter handling qualities. In this work they made an important conclusion that validity of inverse simulation is equivalent to validity of conventional simulation based on the same helicopter model. Another way of looking for this problem was proposed by Celi (Ref 9). The inverse simulation algorithm developed at this work was based on numerical optimisation. This methodology operates on a family of possible trajectories and control inputs. Using special criteria the proper ones can be selected. The method was applied to the slalom manoeuvre from the ADS-33 specifications.

The basic premise of the research presented in this paper is that the ADS-33 handling qualities requirements (Ref 6) can be modified to suit a light rotorcraft such as an autogyro. To test this assertion the aim of the project is to develop MTEs suitable for autogyro operations using inverse simulation techniques, then to test fly them using the Department's fully instrumented Montgomerie-Parsons autogyro, G-UNIV (Fig 1), to demonstrate their use. This autogyro is equipped with a range of sensors (accelerometers, rate gyros, angle indicators, air data probe and position transducers) and a main instrumentation pallet, which includes laptop PC and signal conditioning units. Digital on-board recording system operates at 50 Hz.

Fig 1. Glasgow University research autogyro

Inverse Simulation

Background

The inverse simulation algorithm calculates the pilot control inputs that will force a vehicle fly a specified manoeuvre. Inverse simulation can be solved using one of two different methods, numerical differentiation or numerical integration. The differentiation method was first successfully used by Thomson (Ref 5) to quantify helicopter agility. Since this time, the University of Glasgow has become a centre of excellence in the development and research of the inverse simulation problem. The first inverse simulation algorithm was called Helinv (Ref 10), which used the Royal Aerospace Establishment's helicopter mathematical model Helistab (Ref 11). A new model, helicopter generic simulation, HGS was incorporated into the Helinv algorithm by Thomson and Bradley (Ref 12). The HGS model is nonlinear with seven degrees of freedom (six body modes and rotor-speed). This model has a disc representation of the main and tail rotors, and includes a multiblade description of main rotor flapping, dynamic inflow and lookup tables for helicopter fuselage aerodynamics.

The numerical integration technique for helicopter inverse simulation was proposed by Hess *et al* (Ref 13). Dividing the initial flight trajectory into small intervals, the equations of motion are integrated and compared with desired trajectories. A Newton-Raphson iterative scheme was applied to minimise the error vector. Rutherford and Thomson (Ref 14) used the same approach in the numerical integration algorithm called Genisa (Generic inverse simulation algorithm). Results show that the two methods of numerical differentiation and numerical integration compare favourably, the only significant difference being that the Genisa algorithm is an order of magnitude slower than Helinv. In contrast, Genisa demonstrated flexibility and scope for simulating different type of flying vehicles. In this way, the Genisa algorithm was chosen as a basis for this research.

Algorithm Structure

The detailed description of the Genisa algorithm including the problem of numerical stability is given in the Ref 14, therefore in this paper only primary aspects of the algorithm will be considered. In general, the aircraft dynamics may be described by the nonlinear equations of motion in the following standard form

$$\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u})\,; \quad \mathbf{x}(0) = \mathbf{x}_0 \tag{1}$$

$$\mathbf{y} = g(\mathbf{x}) \tag{2}$$

In particular, for autogyro case the state and control vectors are

$$\mathbf{x} = \begin{bmatrix} u & v & w & p & q & r & \phi & \theta \end{bmatrix}^T, \tag{3}$$

$$\mathbf{u} = \begin{bmatrix} \theta_{tilt} & \phi_{tilt} & \delta_r & \mathrm{T} \end{bmatrix}^T, \tag{4}$$

where θ_{tilt} and ϕ_{tilt} are the longitudinal and lateral rotor tilt angles, δ_r is the rudder angle, and T is the propeller thrust. The autogyro controls differ from those of the helicopter, the pilot controlling the direction of rotor thrust by tilting the rotor shaft.

The initial flight trajectory is divided into small time intervals, forming the series of time points t_k. Integrating the given system at the time point t_k, the estimates of state and output vectors can be calculated at the next time point

$$\mathbf{x}(t_{k+1}) = \int_{t_k}^{t_{k+1}} \dot{\mathbf{x}}(t_k)dt + \mathbf{x}(t_k) \tag{5}$$

$$\mathbf{y}(t_{k+1}) = g[\mathbf{x}(t_{k+1})] \tag{6}$$

The desired output vector obtained from the manoeuvre model (or from flight test data) is compared to the integrated equations of motion. Thus, the error function can be formed

$$\mathbf{y}_{error}(t_{k+1}) = \mathbf{y}(t_{k+1}) - \mathbf{y}_{desired}(t_{k+1}) \tag{7}$$

The Newton-Raphson method can be used to minimise the error vector and found the required control vector

$$\mathbf{u}(t_k)_{n+1} = \mathbf{u}(t_k)_n - [J^{-1}]\mathbf{y}_{error}(t_{k+1})_n\,, \tag{8}$$

where n indicates the n th iteration of the Newton-Raphson solver at the current time point, and $[J]$ is the Jacobian matrix

$$[J] = \left[\frac{\partial \mathbf{y}_{error\,i}(t_{k+1})_n}{\partial \mathbf{u}_j(t_k)_n}\right] \tag{9}$$

Jacobian is calculated numerically using central differencing scheme. When actual and desired outputs match to within given tolerance, the process repeated for the next time point.

To avoid inverting the Jacobian matrix the Genisa algorithm uses a modified form of the Newton-Raphson scheme

$$\mathbf{u}(t_k)_{n+1} = \mathbf{u}(t_k)_n - \mathbf{u}_{error}(t_k)_n\,, \tag{10}$$

where control error vector is evaluated by solving the system

$$[J]\mathbf{u}_{error}(t_k)_n = \mathbf{y}_{error}(t_{k+1})_n \tag{11}$$

This linear system can be solved using LU factorisation, or singular value decomposition algorithms. According to Ref 14, such an approach is more accurate and stable for a wider range of Jacobians.

Manoeuvre Definition

In the ADS-33E-PRF standard (Ref 6) flight test manoeuvres are provided in the form of precisely defined MTEs. To use an MTE as desired flight path for inverse simulation it is necessary to represent it mathematically. Thomson and Bradley (Ref 7, 8) proposed and described in details the appropriate techniques for modelling such helicopter manoeuvres. The acceleration and deceleration manoeuvre (Fig 2), according to the ADS-33E-PRF document, was defined mathematically using such an approach. As an autogyro cannot hover, it was decided to modify the manoeuvre, in other words to start this task in contrast to the original not at the hover, but at a specified airspeed, and fly the autogyro as fast as possible acquiring maximum acceleration. When the aircraft achieved an adequate longitudinal velocity an aggressive deceleration is initiated to achieve the initial airspeed at constant altitude.

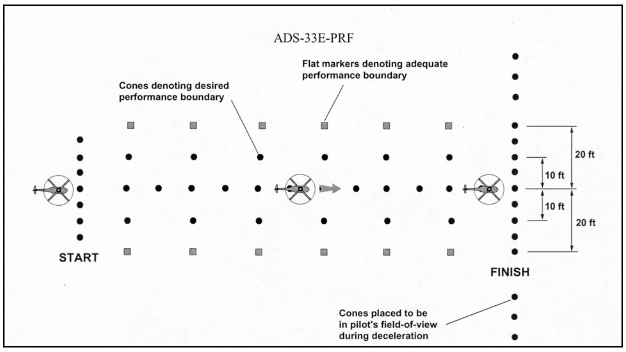

Fig 2. Acceleration and deceleration manoeuvre from the ADS-33E-PRF (reproduced from Ref 6)

The autogyro flight trajectory is defined in terms of the vehicle's Earth referenced accelerations $\ddot{x}_e(t_k)$, $\ddot{y}_e(t_k)$, $\ddot{z}_e(t_k)$, and heading angle $\psi(t_k)$, thus forming the desired output vector

$$\mathbf{y}_{des}(t_k) = \left[\ddot{x}_e(t_k) \quad \ddot{y}_e(t_k) \quad \ddot{z}_e(t_k) \quad \dot{\psi}(t_k)\right]^T \tag{12}$$

The longitudinal acceleration $\ddot{x}_e(t_k)$, or $\dot{V}_f(t_k)$ is specified as a piecewise polynomial function (Fig 3). The polynomials of degree three and five can be used to form the acceleration function. The rest of the desired vector components must be equal to zero.

The autogyro velocity can be evaluated numerically from

$$\ddot{x}_e(t_k) = \dot{V}_f(t_k), \tag{13}$$

the longitudinal displacement $x_e(t_k)$ in its turn can be evaluated by integrating

$$\dot{x}_e(t_k) = V_f(t_k) \tag{14}$$

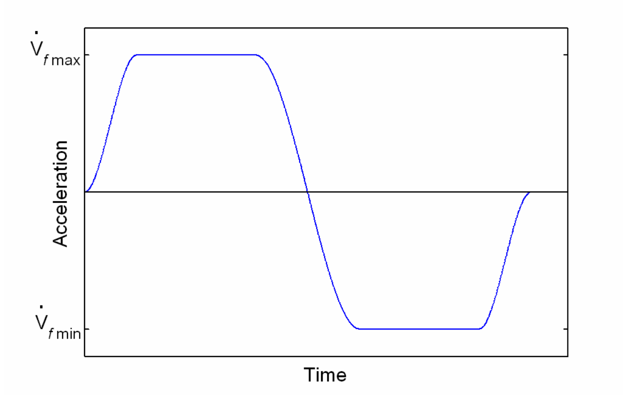

Fig 3. Piecewise polynomial representation of the acceleration and deceleration manoeuvre

Algorithm Modifications

A helicopter individual blade rotor model Hibrom (Ref. 15) was developed at Glasgow to incorporate into Genisa algorithm. This model in contrast to HGS model and disc models on the whole describes the helicopter blade dynamics separately giving higher fidelity and range of applicability. Unfortunately, the Genisa/Hibrom algorithm has a constant rotorspeed assumption, in other words time step for inverse simulation is equal to an integer number of main rotor revolutions.

Houston has had a considerable amount of success in investigating autogyro stability and controllability using conventional simulation (Ref 16-18). He states (Ref 17) that the rotorspeed degree of freedom is very significant for autogyro simulation. To achieve an autorotation conditions, rotorspeed must be adjusted to give a zero net torque. As the rotorspeed is not constant, the simulation time step is not fixed as it was in the initial Genisa/Hibrom algorithm. Hence, the manoeuvre time cannot be predicted before simulation. Doyle and Thomson (Ref 19) proposed a solution for this problem by adding an estimate of the next time point to the control vector

$$\mathbf{u}(t_k) = \left[\theta_{tilt}(t_k) \; \phi_{tilt}(t_k) \; \delta_r(t_k) \; \mathrm{T}(t_k) \; t_{k+1}\right]^T \tag{15}$$

Thus, the control time step is recalculated iteratively at each time point. To minimize the error between the actual and desired blade azimuth, the desired output vector is formed

$$\mathbf{y}_{des}(t_k) = \left[\ddot{x}_e(t_k) \; \ddot{y}_e(t_k) \; \ddot{z}_e(t_k) \; \dot{\psi}(t_k) \; \psi_{az}(t_k)\right]^T \tag{16}$$

Autogyro Model

Description

The initial stage of the research involves developing a suitable mathematical model of an autogyro for use in the inverse simulation. Use was made of an existing helicopter individual blade rotor model, Hibrom (Ref. 15), developed at Glasgow to incorporate into inverse simulation algorithm Genisa (Ref 14). The same approach was used to develop a new autogyro individual blade/blade element coupled rotor-fuselage model. Blade element theory was used to calculate the rotor forces and moments. The autogyro teetering rotor has two blades with the NACA 8H12 airfoil. The lift and drag characteristics for the airfoil section were obtained from NACA reports (Ref 20, 21) dated 1946 and 1949. The blades are attached to the hub without flap and lag hinges, untwisted and have a zero setting pitch angle. No cyclic pitch can be applied. It was assumed that autogyro blades are fully rigid.

The autogyro model uses the dynamic inflow model of Pitt and Peters (Ref 22) improved later by Peters and HaQuang (Ref 23). The Pitt-Peters model initially was written in the wind-axis reference system for zero hub motions, Peters and HaQuang have rewritten this model in a general rotor frame that allows for sideward flight. This model considers the effect of the rotor moments and the lag between application of the blade pitch and changes in the aerodynamic forces.

The lookup tables of force and moment coefficients obtained from wind-tunnel tests have been used for the fuselage, tailplane and fin aerodynamics. Key properties of the model are given in Table 1 in the Appendix.

Model Verification

Flight tests measurements taken in steady level flight were compared with model results to validate the autogyro model (Fig 4). The fuel mass could not be measured in flight, therefore the simulation results were calculated for two different conditions of the autogyro weight – maximum weight of 355 kg (full fuel) and minimum weight of 325 kg (zero fuel). The Montgomerie-Parsons autogyro leading configuration data are presented in Table 2 in the Appendix.

A partial periodic trim algorithm proposed by McVicar and Bradley (Ref 24) was used to trim the autogyro model. Fig 4 shows a favourable flight/simulation comparison for both rotor tilt angles over a wide airspeed range. The autogyro attitude angles and rotorspeed correlations are slightly less satisfactory.

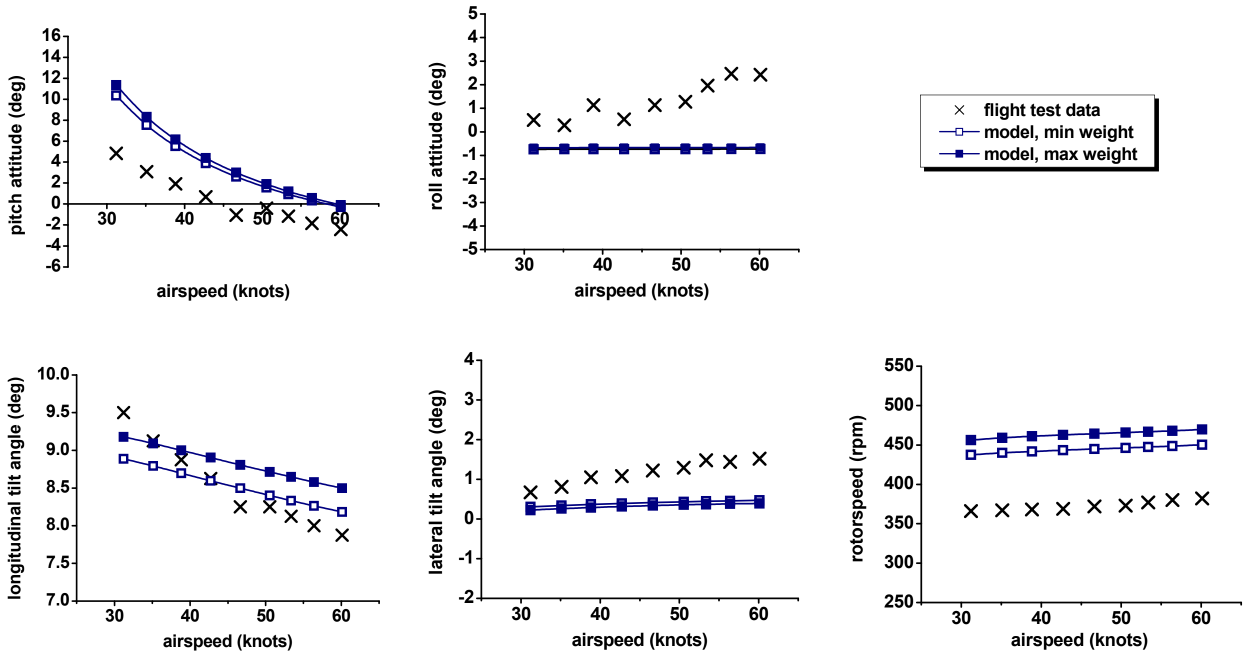

Fig 4. Autogyro model validation in trim

Pitch results for trim show the almost constant mismatch, but at least the trend is similar to flight data. It can be seen that model in general predicts the autogyro behaviour well, the discrepancy most likely caused by geometrical inaccuracies in the model. Trim roll angles do not agree well probably because of the lack of any sideslip indicator available to the pilot. Therefore, in equilibrium flight it is very difficult for the pilot to keep zero sideslip angle, affecting the roll. It is an incontestable fact that the rotorspeed is in inverse proportion to the blade drag. Probably lack of accurate initial data for blade drag causes the flight/simulation discrepancies in rotorspeed, while it should be noted that trend with the speed is similar. Other factors, such as complexity of autorotation conditions and shaft friction, possibly can be also a reason for this mismatch. Nonetheless, in general, trim results showed good global agreement between the flight data and simulation.

Results

In the preliminary simulation experiments acceleration and deceleration manoeuvre from ADS-33E-PRF specifications (Ref 6) was examined in order to evaluate inverse simulation results. This manoeuvre was defined mathematically and modified to suit light autogyros. The primary task objective was to assess the autogyro longitudinal handling qualities in the context of the ADS-33 criteria.

Fig 5 shows inverse simulation results for modified acceleration and deceleration manoeuvre. The manoeuvre is carried out over about 100 meters and takes 4.5 seconds with an initial velocity of 40 knots and maximum velocity of 50 knots, and maximum and minimum accelerations of ± 3 m/sec^2. The flying altitude was assumed to be sea level, but can be easily changed in case to compare simulated results with flight data.

For the good visual conditions the ADS-33 document is very strict about nose-up pitch attitude during the deceleration period of this manoeuvre. The pitch angle must be at least 30 degrees above the hover attitude for desired performance, and at least 10 degrees for adequate performance. It can be seen from Figs 5 and 6 that the autogyro behaves differently in this manoeuvre, using mainly a propeller thrust for fast acceleration and deceleration. Such behaviour resembles that of helicopter with thrust compounding. Thus, it is not necessary to use such a large pitch angles neither for acceleration nor for deceleration. Therefore pitch attitude has been not specified in this task.

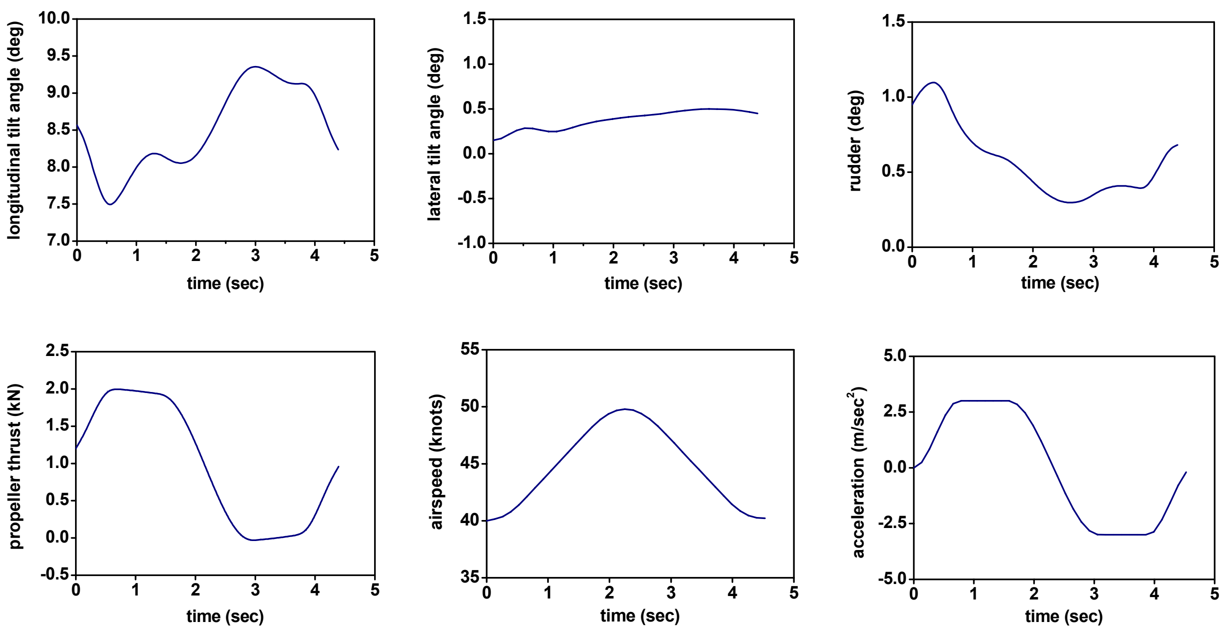

Fig 5. Inverse simulation results for acceleration and deceleration manoeuvre

It should also be noted that engine model used in this simulation was assumed linear, that is propeller thrust is proportional to throttle stick position. Therefore the inverse simulation results for thrust simply follow the trend of acceleration, and it can be argued that such fast thrust perturbations are unrealistic. Research is currently underway to incorporate enhanced engine and propeller model into the autogyro model to obtain more realistic behaviour of propeller thrust and autogyro as a whole.

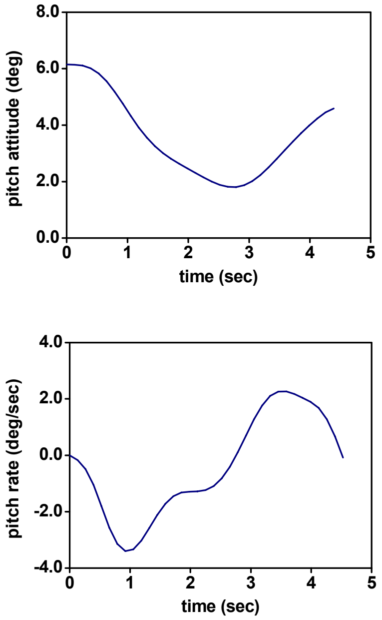

Fig 6. Pitch attitude and pitch rate during acceleration and deceleration manoeuvre

Thomson and Bradley (Ref 8) were the first to propose inverse simulation as a tool to evaluate helicopter quickness parameters specified in the ADS-33 document. This approach was adapted for autogyro case. Using inverse simulation results for pitch attitude and pitch rate (Fig 6) as responses of longitudinal tilt pulse, pitch attitude quickness has been estimated according to the ADS-33 definitions

$$\text{Pitch quickness}=\frac{q_{pk}}{\Delta\theta_{pk}},$$

where q_{pk} is the peak pitch rate and $\Delta\theta_{pk}$ is the peak pitch displacement from trim. Results corresponding to three different acceleration profiles are shown in Fig 7. It can be seen that results predict Level 2 handling qualities.

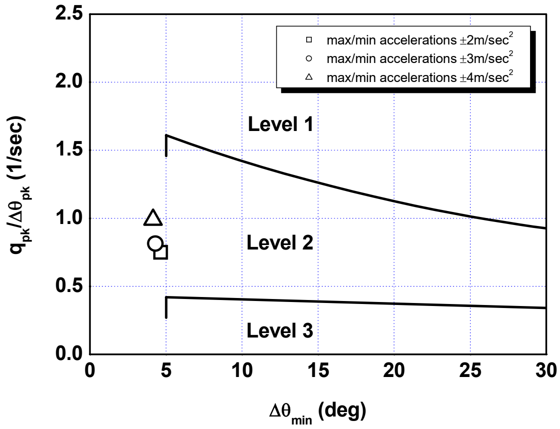

Fig 7. Pitch quickness chart for the different acceleration profiles

As was discussed above, the autogyro model uses mainly a propeller thrust for acceleration and deceleration, making only small changes in pitch attitude. Therefore, the pitch quickness chart from the ADS-33 specifications should be changed to suit autogyros, that is boundaries of handling qualities levels should be extended to the low pitch angles, and maximum border for $\Delta\theta_{min}$ should be fixed at lower angles. Here it would be very significant to have flight data to compare with inverse simulation predictions.

The control quickness parameter was proposed by Thomson and Bradley (Ref 8) in addition to quickness parameters from the ADS-33. The reason was that the attitude quickness parameters give very poor information about pilot workload to fly specified task. In autogyro case, the control quickness parameter defined by

$$\text{Longitudinal tilt quickness}=\frac{\theta_{tilt\ pk}}{\Delta\Theta_{tilt}},$$

where $\theta_{tilt\ pk}$ is the longitudinal tilt peak and $\Delta\Theta_{tilt}$ is the area under a discrete pulse of longitudinal tilt. Fig 8 shows results for longitudinal tilt quickness chart corresponding to the different acceleration profiles. The boundaries in the chart represent 25%, 50% and 100% of the longitudinal tilt control limits.

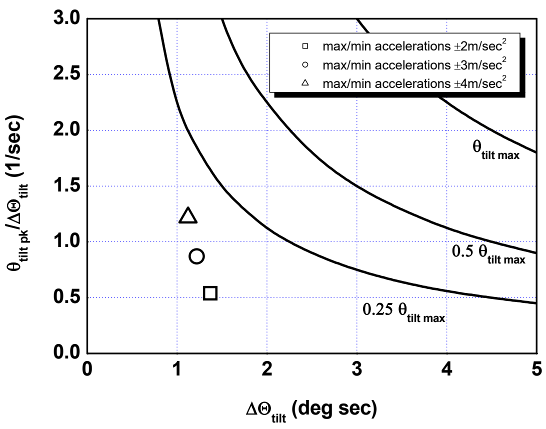

Fig 8. Longitudinal tilt quickness chart for the different acceleration profiles

It can be seen from Fig 8 that the more aggressive acceleration profile, the closer the results to control limits. Also, it should be noted that the control quickness results do not achieve even 25% of control limit during acceleration and deceleration manoeuvre. It confirms the inverse simulation results showed that autogyro uses mostly propeller thrust for acceleration and deceleration. This is significant difference between behaviours of helicopter and autogyro in this manoeuvre.

Concluding Remarks

The primary objective of this paper was to examine autogyro handling qualities using the modified acceleration and deceleration manoeuvre from the ADS-33 document. Inverse simulation has been proposed as a tool to evaluate the autogyro handling qualities. The autogyro model included individual blade dynamics, and a detailed representation of fuselage, tailplane and fin. Before incorporating into inverse simulation, the autogyro model has been validated by flight/simulation comparisons in trimmed flight. Finally, using developed technique the autogyro pitch quickness and longitudinal tilt quickness parameters were estimated. It was discovered that pilot control inputs obtained from inverse simulation can give an indication of pilot workload needed to fly the specified task. Research has also shown that MTEs as well as quickness requirements from ADS-33 specifications can be applied to autogyros with small reasonable changes.

The autogyro pitch attitude quickness parameter was found not so relevant in acceleration and deceleration manoeuvre, therefore additional quickness parameters and criteria must be designed to obtain more detailed information about handling qualities and pilot workload. While in the others manoeuvres such as slalom or transient turn the attitude quickness parameters can be more relevant.

It would be very significant to compare flight tests data with inverse simulation predictions. Flight tests program at Glasgow University is currently underway, further flight trials of Montgomerie-Parsons research autogyro are planned for autumn 2003. The acceleration and deceleration, slalom and transient turn manoeuvres from the ADS-33 document are modified to suit autogyro and prepared for test flights. Also, it would be very important to obtain subjective pilot's assessments of autogyro handling qualities using Cooper-Harper handling qualities rating scale, and then compare them with objective, or predicted ones obtained from inverse simulation. The authors know of no other instance of the Cooper-Harper rating scale being applied to an autogyro.

The results presented in this paper are unique and significant, and reveal the behaviour of an autogyro in terms of handling qualities. In addition, results in the area of autogyro handling qualities are timely, because of the bad autogyro accident statistics in UK. The research results can be contributed to the development of new design standards for autogyros. In spite of the fact that the first results obtained during this work are significant, autogyro model improvements and further research are required to obtain more accurate and reliable data.

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Appendix

Table 1. Autogyro mathematical model description

Model Item	Characteristics
Rotor dynamics	Rotor blades are fully rigid. Lead/lag freedom has been neglected. No hinge offset.
Rotor loads	Aerodynamics and inertial loads represented by 20 elements per blade.
Blade aerodynamics	NACA 8-H-12 airfoil. Lookup tables for lift and drag as functions of angle of attack and Mach number.
Wake model	Peters and HaQuang dynamic inflow model. Effect of the rotor moments and the lag between application of the blade pitch and changes in the aerodynamic forces.
Airframe	Lookup tables and polynomial functions for fuselage, tailplane and fin aerodynamics.
Atmosphere	ISA (International Standard Atmosphere).

Table 2. Autogyro leading data

Parameter	Data
Gross mass I_{xx} I_{yy} I_{zz} I_{xz}	355 kg 72.96 kgm^2 297.21 kgm^2 300.0 kgm^2 0.0 kgm^2
No of blades Blade radius Blade mass Blade chord Blade twist Flapping inertia Lift curve slope Airfoil section No of elements Rotor direction	2 3.81 m 17.255 kg 0.197 m 0.0 deg 83.492 kgm^2 5.75 rad^{-1} NACA 8H12 20 Anti-clockwise
Propeller blade radius Propeller blade chord	0.787 m 0.09 m
Rudder area Lift curve slope	0.368 m^2 3.5 rad^{-1}