

Full Autopilot for Small Electrical Helicopter

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Abstract. A small helicopter can be considered as a test bench of a highly nonlinear unstable multiconnected mechanical control system. An automatic stabilization system was designed on a standard computer for a small electrical helicopter. The sensor device consists of two Web cameras connected to the computer and four onboard diodes. The control signal is transmitted to actuators through a wireless control panel connected to the computer. Results of experiments are reported where the full autopilot system stabilizes a helicopter near a fixed 3-dimensional point that is not far from the ground. TV tracking is reproduced on the screen.

1 Device description.

Small helicopters with manual control by a wireless control panel are easily available in the market in various configurations. They are extremely unstable and require a skilled driver to prevent a downfall. The time constant in the lateral channel of a small electrical helicopter is much less than 1 s. There are four control actuators that are closely interconnected in a highly nonlinear system. The system can be described to various levels of accuracy from the second order time invariant plants to turbulence equations. It is shown in this paper, that a simple autopilot system can be successfully designed for a widely spread model of helicopter with TV observations and software installed on a standard computer.

An electrical helicopter Walkera X450 was chosen for experiments. Its height is 830 g. The main rotor has a diameter of 700 mm and a hub of the Hiller type with two blades and two servo blades. Each pair of blades is hard connected on a rod and the collective pitch is fixed. The rotor draft is controlled by rotation frequency at 27 - 29 Hz. The other control inputs are the rotation frequency of the tail rotor and two angles of the swash plate position.

The control signals are transmitted from computer directly through USB port to the control panel that sends a radio signal to the onboard actuators.

Dependence of the lateral, directional and lifting forces and torques from the controls was approximated in experiments on special test benches. Dependence of the main rotor speed from the control was identified in experiments by the stroboscope.

A full aerodynamic nonlinear model is derived. It contains both differential and algebraic equations with nonunique solutions. An effective computational algorithm is proposed that finds a unique admissible solution of the system. Balanced operational points are determined for arbitrary linear velocity of a helicopter. All algorithms were written on Matlab and C. Their performance was studied by simulations.

2 TV observation

2.1 Primary recognition

A particular measurement of the helicopter position is a single TV image that contains spots from diodes on the nonhomogeneous background. The image processing is traditionally divided into the primary stage where the grey image is transformed to the set of points, and the secondary stage that builds objects from the extracted points and eliminates alarms. Primary processing is made in the full image or in the tracking strobes. It must be quick since this part is the most time consuming.

The primary recognition stage consists of a search of spots from all 4 diodes on the nonhomogeneous background. The shape and amplitude of a spot depends on the diode foreshortening with respect to the camera. A diameter of a spot is normally between 4 and 10 pixels at the level of 70% amplitude for the distance 1.5 - 2 m between a camera and a helicopter. The spot shape may be distorted if the diode is directed outside or if it is located on a bright background or it is shaded. Images contain in addition specks of light, bright borders of objects and reflections from surfaces.

A search of the diode spots is made by the following algorithm. First, the average brightness level is eliminated by filtering that contracts the low frequency component. Then areas with sufficiently big remaining amplitude are extracted. Then the image is processed by a local moving average filter that calculates a discrete second derivative. Samples with positive or relatively small values of this derivative are eliminated.

The remaining samples are collected into connected areas. That is performed through one pass of the image. Then very small and very long objects are removed. The remaining spots are approximated by ellipsoids and the discrepancy is measured by consistent filtering.

As a result of primary recognition, a number of objects are obtained that have the best values of likelihood after filtering. For each spot the center is determined together with an approximating ellipsoid. The second order matrix describing the ellipsoid is considered as the covariance of the center estimate.

2.2 Three dimensional estimation

The helicopter position as well as the both cameras positions are not known at the stage of primary recognition. These objects are three solid bodies, and their positions are determined by coordinates of the center of gravity or focuses and by the Euler angles.

Human eyes have no problems with identification of three-dimensional object coordinates because the distance between eyes is fixed and known and the survey directions are the same. In the TV observation these parameters are to be estimated, and the only data for estimation are two TV shots that contain images of the same helicopter. Therefore, the first task is study of a single image from one camera and a restoration of the 3-dimensional coordinates of the helicopter in the coordinate system of the camera.

The 2-dimensional image of the helicopter on the camera screen is projective. An image on the screen of an arbitrary point in the space can be considered as an intersection of the screen plane with a line connecting this point and the camera focus. Thus, the image on the camera screen contains a projective image of the tetrahedron formed by the diodes.

The origin of the camera coordinate system is its focus. The axes OX and OY are the

natural coordinates on the screen. The axis OZ is directed forward and makes the right triple of coordinate axes. Denote the coordinate vector of the i -th diode in the camera coordinate system by

$$r_i = \begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix}, \quad i = 1, 2, 3, 4.$$

Projection of this vector to the screen is denoted by (ξ_i, η_i) and measured in pixels. It follows from geometry that

$$(\xi_i, \eta_i) = \rho \frac{(x_i, y_i)}{z_i}$$

where ρ is the coefficient of transformation from angle to pixels. For the cameras chosen it is equal to $\rho = 800$. Substitution of this equation results in the representation

$$r_i = z_i \begin{pmatrix} \rho^{-1}\xi_i \\ \rho^{-1}\eta_i \\ 1 \end{pmatrix}.$$

The image recognition is based on the distances d_{ij} between each pair of diodes that are considered to be known. The square distance is

$$d_{ij}^2 = \rho^{-2} [(z_i\xi_i - z_j\xi_j)^2 + (z_i\eta_i - z_j\eta_j)^2] + (z_i - z_j)^2.$$

Since the measurements (ξ, η) are known for each diode, this expression is a quadratic form with respect to the unknown pair (z_i, z_j) . A tetrahedron has totally 6 edges. Hence measurements of the vertices projections give 6 equations with respect to 4 unknown variables (z_1, z_2, z_3, z_4) :

$$A_{ij}z_i^2 + 2B_{ij}z_i z_j + C_{ij}z_j^2 = d_{ij}^2, \quad 1 \leq i < j \leq 4.$$

This is the overdetermined system of algebraic equations. An effective algorithm and corresponding software were developed to solve this system by the maximum likelihood method. The cost function in this method contains covariances of noises that are taken from ellipsoids describing the diode spots.

Experiments have shown that the estimate error for the coordinates (x_i, y_i) is less than 1 mm and that accuracy of the coordinate z_i is within 2% of the distance to the helicopter.

A considerable number of false points are extracted by the primary recognition procedure. For all combinations of the points the overdetermined algebraic system is solved and the corresponding value of likelihood is calculated. The biggest value of the likelihood index extracts one or several candidates for the helicopter image. Each selected image is provided with the likelihood value that is used later for the dynamic filtering and for estimation of the cameras positions.

The connected coordinate system is defined as a system that is attached to the helicopter as a solid body. Its origin and axes are defined from geometry of diodes positions. The coordinates of the diodes calculated in a camera coordinate system are transformed to the connected coordinate system. This procedure is made for each camera separately, but the result should be the same for both cameras. The discrepancy of estimates obtained is a good test for accuracy of the previous calculations. All experiments have shown the discrepancy less than 0.1 mm at the distance of 1.5 - 2 m.

2.3 Mutual camera position

Tracking of the moving body is made with respect to a fixed earth coordinate system. In this system estimates of the state variables of the cameras and the helicopter are calculated together with their joint covariance.

The earth coordinate system cannot be attached to the helicopter or to its initial position because this information will be lost after the motion starts. Therefore, the system must be attached to the immovable cameras.

Origin of the earth coordinate system is defined as the middle of the segment between the cameras focuses. The plane OYZ contains both focuses, the horizontal axis OX is directed to the helicopter and makes the right triple of axes. If the image contains a vertical plumb-line then the axis OY can be directed upward.

In the chosen coordinate system vectors of the cameras focuses have the coordinates $(0, Y_c, Z_c)$ and $(0, -Y_c, -Z_c)$, respectively, where the values of Y_c and Z_c are included into the total system state vector.

The Euler angles of the cameras with respect of the earth coordinate system are denoted by $(\gamma_{c1}, \Phi_{c1}, \theta_{c1})$ and $(\gamma_{c2}, \Phi_{c2}, \theta_{c2})$, respectively. Hereafter γ denotes roll, Φ denotes yaw and θ denotes pitch.

The full state vector of the dynamic system under investigation contains 20 variables:

$$S = (x, y, z, V_x, V_y, V_z, \Omega_x, \Omega_y, \Omega_z, \gamma, \Phi, \theta, \gamma_{c1}, \Phi_{c1}, \theta_{c1}, \gamma_{c2}, \Phi_{c2}, \theta_{c2}, Y_c, Z_c)^T,$$

where $r = (x, y, z)^T$ is the vector of the helicopter center of gravity, $V = (V_x, V_y, V_z)^T$ is the linear velocity vector, $\Omega = (\Omega_x, \Omega_y, \Omega_z)^T$ is the angular velocity vector, γ, Φ, θ are the Euler angles of the helicopter. All the vectors are written in the earth coordinate system.

Initial estimation of this vector is made by the maximum likelihood method. The estimate is calculated together with its covariance matrix. Thus, the step of initial estimation of the cameras and helicopter positions is completed with calculation of the 20-dimensional vector S and of the full covariance matrix P of this vector.

Under hard observation conditions a number of such pairs (S, P) can be obtained. After tracking the moving object all spurious pairs will show bad performance and therefore they will be extracted and removed.

3 Tracking of helicopter motion

The TV images are received asynchronously with the averaged speed of 30 shots per second. The tracking system calculates the predicted diodes positions after receiving a new TV image. Then a strobe is selected around the predicted position for each diode. A spot from a diode is searched in the strobe in a similar way to the primary recognition.

When all spots are found their centers and approximating ellipsoids are determined. They describe the results of position estimates on the screen and their expected accuracy.

The spot positions on the screen are nonlinear functions of the state vector. But this function can be linearized in the strobe of tracking to appropriate accuracy. Therefore, the linearized Kalman filter is used for the state vector correction. Experiments have shown that the measurement noises are small enough such that this filter provides good performance.

The observation system does not depend on the number of detected diode spots in the image. Some of spots can be shadowed or lost on the bright background. Even in the case of completely shadowed diodes for a couple of images the Kalman filter does not lose the helicopter unless it makes a new manoeuvre.

A search in the strobes is the most consuming part of the algorithm. Nevertheless, we did not meet speed limitations with the standard C program in the real time system.

4 Aerodynamic model of helicopter

Helicopter dynamics is described by the equation

$$\dot{S} = f(S, u),$$

where u is control containing cyclic and collective pitch control of the main rotor and directional control by the tail rotor. The function f is given by a system of nonlinear algebraic equations.

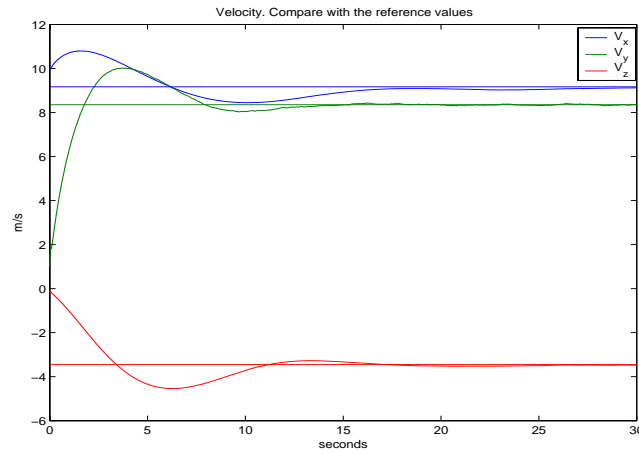


Рис. 1: Stabilization of the linear velocities

Positions of cameras do not change. Therefore, the last 8 components of the vector $f(S, u)$ are zeros. The equations

$$\begin{aligned}\dot{x} &= V_x, \\ \dot{y} &= V_y, \\ \dot{z} &= V_z\end{aligned}$$

are trivial. Dynamics of the Euler angles is determined by the angular velocities according to the equations

$$\begin{pmatrix} \dot{\gamma} \\ \dot{\Phi} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \frac{\cos \Phi}{\cos \theta} & 0 & -\frac{\sin \Phi}{\cos \theta} \\ -\tan \theta \cos \Phi & 1 & \tan \theta \sin \Phi \\ \sin \Phi & 0 & \cos \Phi \end{pmatrix} \begin{pmatrix} \Omega_x \\ \Omega_y \\ \Omega_z \end{pmatrix}.$$

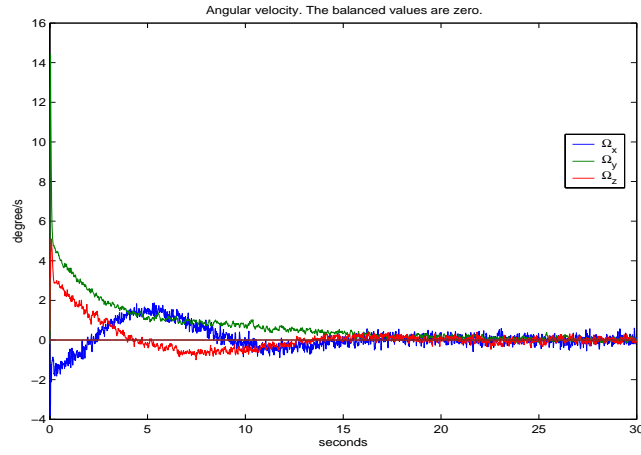


Рис. 2: Stabilization of the angular velocities

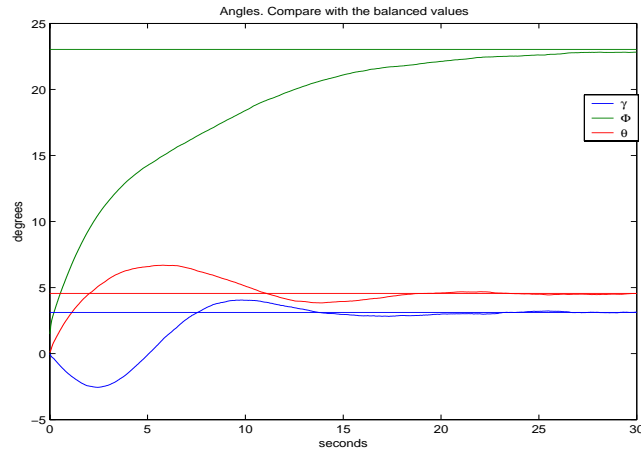


Рис. 3: The Euler angles

The most important part of the helicopter dynamics are the equations for linear and angular velocities that are determined by forces and torques. In the vector form they are

$$\begin{aligned}\dot{V} &= -\Omega \times V + \frac{1}{m}(F_{\text{rot}} + F_{\text{tail}}), \\ \bar{I}\dot{\Omega} &= M_{\text{rot}} + M_{\text{tail}},\end{aligned}$$

where $\Omega \times V$ is the vector product, m is the mass of helicopter, $\bar{I} = \text{diag}\{I_x, I_y, I_z\}$ is the diagonal matrix with moments of inertia, F_{rot} and F_{tail} are forces from the main rotor and the tail rotor, M_{rot} and M_{tail} are the torques from the same rotors.

The forces and torques are determined in the connected coordinate system and then they are transformed to the earth coordinate system by rotations defined by the Euler angles of helicopter. In the connected coordinates the force of the main rotor is $F_{\text{rot}} =$

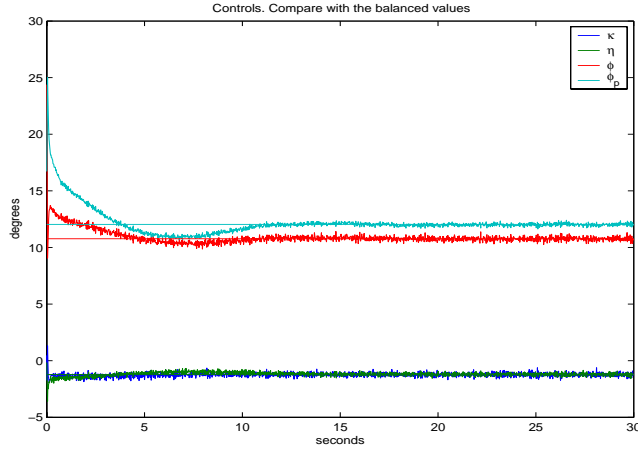


Рис. 4: Stabilizing control signals

$\text{col}(-H, T, -S)$ with

$$\begin{aligned} T &= \frac{1}{2} C_T \rho (\pi R^2) (\omega R)^2, \\ H &= T(a_1 - \phi_z), \\ S &= T b_1, \end{aligned}$$

where ω is the rotation speed, R is the blade radius, ρ is the density of air, ϕ_z is the angle between the OY axis and the main rotor shaft, C_T is the rod coefficient, a_1 and b_1 are the angles of the vertical periodic blade motion.

The torque $M_{\text{rot}} = (M_x, M_y, M_z)$ can be described by

$$\begin{aligned} M_x &= -T y_T b_1, \\ M_y &= \frac{1}{2} m_y R \rho (\pi R^2) (\omega R)^2, \\ M_z &= T x_T + T y_T a_1, \end{aligned}$$

where (x_T, y_T) are the coordinates of the hub with respect to the center of gravity and the coefficient of the rotating torque is defined by

$$m_y = -\frac{1}{B^2} C_T (\lambda + a_{1,\beta} \mu) + \frac{C_{x,p} \sigma}{4} (1 + 5\mu^2),$$

where B is the coefficient of the efficient radius,

$$\mu = \frac{1}{\omega R} \sqrt{V_{x,h}^2 + V_{z,h}^2}$$

is the normalized horizontal speed, $a_{1,\beta}$ is the blade motion coefficient in the speed coordinate system, $\sigma = 2b/(\pi R)$ is the coefficient of filling, b is the blade width, $C_{x,p}$ is the averaged resistance coefficient.

Similar formulas can be written for the force and torque of the tail rotor.

The parameters a_1 and b_1 of the blade motion and the coefficient C_T are the main variables that are influenced by control and that determine the helicopter motion.

The coefficient C_T can be taken from the impulse theory of the main rotor:

$$\bar{V}_y - \lambda = \frac{C_T}{4B^2\sqrt{\lambda^2 + \mu^2}},$$

where \bar{V}_y is the normalized vertical velocity, λ is the coefficient of the induced air flow.

Theory of the blade motion gives the following formula for the same variables:

$$C_T = a_\infty \sigma (C_\lambda(\mu)\lambda + C_\phi(\mu)\phi_k + C_\Theta(\mu)A_{\text{rot}}\Theta + C_\Omega(\mu)A_{\text{rot}}\bar{\Omega}_{xz})$$

where a_∞ is the derivative of the rod by angle of attack, ϕ_k is the effective angle of the blade, Θ is the normalized cyclic pitch control vector, A_{rot} is the rotation matrix to the air flow direction, $\bar{\Omega}_{xz}$ is the 2-dimensional projection on the plane OXZ of the angular velocity Ω normalized by ω , the coefficients $C_\lambda(\mu)$, $C_\phi(\mu)$, $C_\Theta(\mu)$, $C_\Omega(\mu)$ are the functions of the squared parameter μ^2 that is differentiable at zero.

Effective numerical algorithms were developed for solution of the nonlinear differential/algebraic system of the helicopter dynamics. The corresponding software in C was implemented in the real time system.

5 Control design

A balanced mode is defined as a motion with a constant vector of linear velocity and with zero angular velocity. For any vector $V = \text{col}(V_x, V_y, V_z)$ from a natural admissible domain there exists a unique state vector S and the unique control vector u that realize the balanced mode with the velocity V .

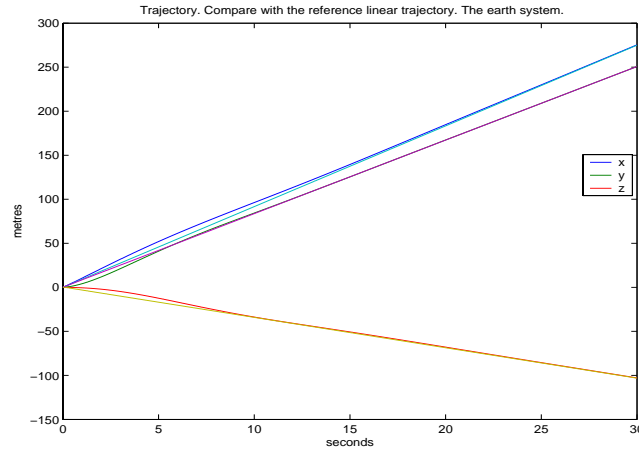


Рис. 5: Tracking the trajectory

Effective numerical algorithms and software were developed that calculate the vectors S and u by V .

Since the trajectory for the balanced mode is a stationary point of the state vector excluding position (x, y, z) , the system of dynamical equations can be linearized around this trajectory. The linearization means calculation of derivatives by all components of the

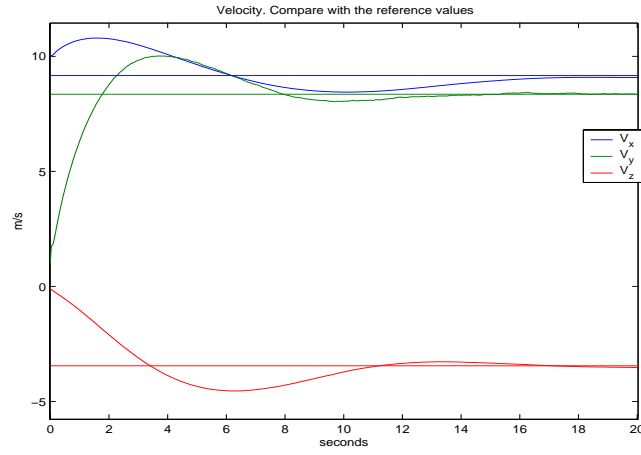


Рис. 6: Transient of linear velocities after manoeuvre

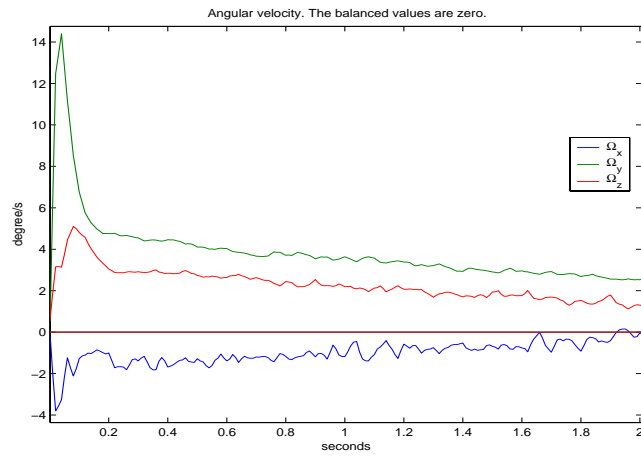


Рис. 7: Angular velocities with manoeuvre

state vector. The deviations \tilde{S} of the state vector S from the nominal balanced trajectory can be described by the linear system

$$\dot{\tilde{S}} = A\tilde{S} + B\tilde{u}$$

where A and B contain partial derivatives in the balanced state vector.

The LQ optimal control system was developed for the continuous-time plant with discrete asynchronous measurements from two cameras.

Results of simulations are shown in Fig. 1 - Fig. 5. The system was integrated according to the full nonlinear description of the aerodynamic model. Controller was designed by the LQ approach in the neighbourhood of the chosen balanced mode.

The same simulations with manoeuvre is shown on Fig. 6 - Fig. 10.

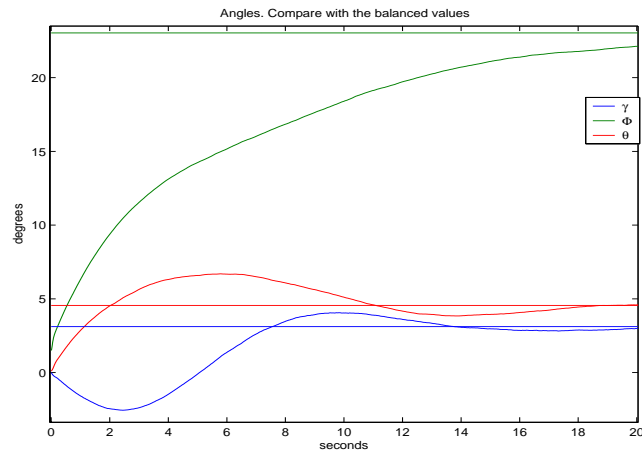


Рис. 8: Euler angles under manoeuvring

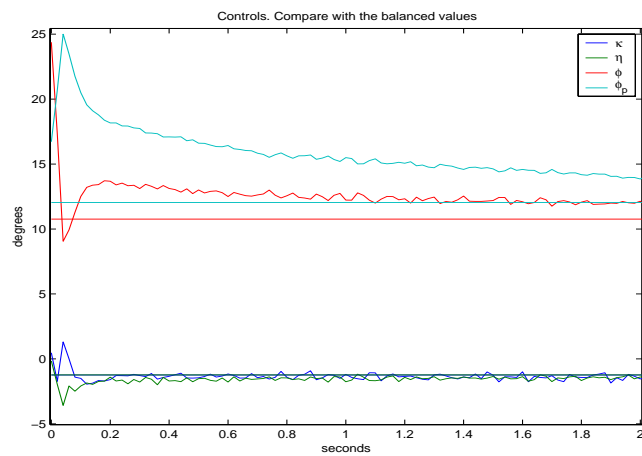


Рис. 9: Control signals with manoeuvre

6 Embedded system and real time experiments

A TV film is shot with successful stabilization of the electrical helicopter model with the TV tracking system and PI regulators for each of three control channels. It will be shown in the presentation.

The controller design was preceded by experiments for identification of the main dynamic parameters. These parameters include all forces and torques as functions of the control. The overall delays appeared to be important for the design. In the most unstable roll channel the delay is around 0.2 s.

Special test benches were designed for identification of forces, torques and the balanced values of the Euler angles and controls. The periodic solutions under the relay feedback gave important information about delays and gains.

Experiments have shown essential nonlinearity of the system dynamics. Small oscillations appeared when the control feedback gain is increased. The closed loop system shows better quality of stabilization but the closed loop system is unstable near origin and dissipative outside small neighbourhood of the origin. If the gain is increased more than oscillations

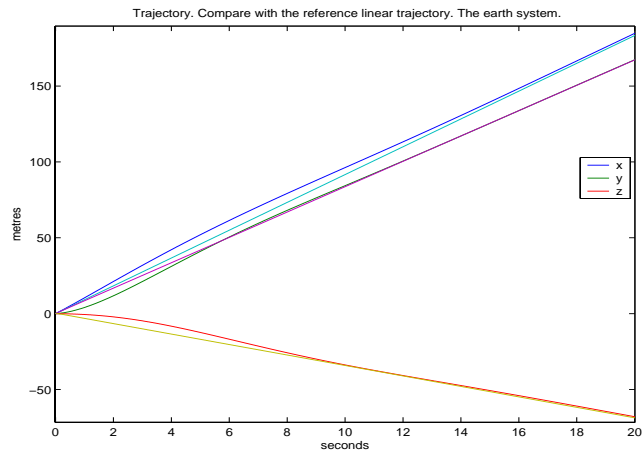


Рис. 10: Transient of position of the center of gravity

can destroy the system up to instability.

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