Back to Session Subjects

METHODS FOR REFINEMENT OF STRUCTURAL FINITE ELEMENT MODELS: SUMMARY OF THE GARTEUR AG14 COLLABORATIVE PROGRAMME

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Abstract: The finite element model is the main tool used by helicopter manufacturers for the analysis of helicopter structures and, in particular, for the prediction of vibration. High fidelity models that can accurately represent the structural dynamics are the key to producing effective low vibration designs. Helicopters with 'jet' smooth comfort are demanded by discriminating customers who also require ownership of vehicles with high reliability, low maintenance and reduced through life costs.

The finite element model is an important tool in the assessment of aircraft modifications after the initial design and production when adverse vibration levels may become apparent. At this stage, a structure is available and measured dynamic data from a shake test may be used to validate and improve the initial finite element model. The model derived from test data is not, by itself, comprehensive enough to allow the study and manipulation of the structural dynamics, but its role in conjunction with the finite element model is a vital step towards improving the helicopter structural design.

The main purpose of this GARTEUR collaboration was to explore methods and procedures for improving finite element models through the use of dynamic testing. For the foreseeable future it is expected that shake tests combined with finite element models will be the major tool for improving the dynamic characteristics of the helicopter structural design. It is therefore of great importance to all participants that the procedure of validating and updating helicopter finite element models is robust, rigorous and effective in delivering the best match based on realistic engineering adjustments to the finite element model.

The industry need for finite element models, the variety of update procedures and their advantages are discussed in this paper together with some requirements for dynamic testing. The results of a systematic study on the model updating of a Lynx Mk7 airframe are presented and conclusions drawn. Recommendations are made with regard to performing subsequent dynamic tests, model updating and for future collaborative study.

1 INTRODUCTION

The primary aim of the GARTEUR Action Group (HC AG-14) was to explore methods and procedures for improving finite element models of helicopter type structures through the use of error location, model updating and dynamic testing. The dynamic tests and analyses, together with application of several different techniques for reconciliation of test and finite element models are presented in this paper. Advantages and disadvantages of the approaches are given and future developments of the procedures for localising areas of the models causing the discrepancies and for improving the updating process are presented.

The main source of helicopter vibration excitation is the rotor system itself, which generates complex aerodynamic and dynamic loads. These vibratory loads are transmitted via the rotor hub to the airframe and occur, predominantly, at the blade passing frequencies. Similar vibrations are introduced by the tail rotor. Vibration reduction on helicopters is traditionally a challenging task. Although much progress has been achieved, the vibration problem remains to be improved due to increased requirements for future helicopters embodied in an EU directive.

There are three general approaches to achieve the 'jet smooth' ride of helicopters. One approach is to minimise the hub vibrating forces through good design of the rotor system, incorporating features such as aeroelastic tailored blades and active blade technology. Another approach is the reduction of the vibratory loads by means of passive or active vibration suppression systems (anti-resonant systems, Active Control of Structural Response (ACSR) or Higher Harmonic Control (HHC), etc.). Such systems have been developed for many modern helicopters. The final approach is the reduction of the dynamic response of the airframe by means of careful structural design to avoid fuselage modes which coincide with blade passing frequencies. Finite element models of helicopter structures are satisfactory for static load calculations but there is still a need to improve the predictive capabilities of such models for vibration analyses. This is due to the fact that helicopter structures, even more so than fixed-wing aircraft, are complex three-dimensional systems.

Shake tests, combined with finite element models, are the major approach for improving the dynamic characteristics of the helicopter structural design. These are used in combination with a variety of tools for identifying and updating modelling discrepancies and optimisation techniques for determining a low vibration airframe.

As a test structure for this GARTEUR investigation, a time-expired Lynx Mk7 helicopter airframe was chosen. All extraneous components and materials were removed from the airframe and from the finite element model, with the objective of simplifying the structure to a fundamental build state and removing any ‘rattles’ that might have impaired the quality of the dynamic measurements.

2 FINITE ELEMENT MODELS: NEED

It is essential that modern helicopters have low airframe vibration. Reduced cost of ownership is a major requirement for both civil and military operators and it is generally accepted that reducing vibration will help to lower unscheduled maintenance and lead to better equipment and airframe life. This in turn leads to other benefits from improved aircraft reliability, availability and a better environment for cabin crew making them more effective. In the civil market, customer perception is all important and high vibration aircraft do not meet with customer expectations resulting in potential loss of sales. Recent EU directives, [1], on noise and vibration are important for operators and these have now been implemented into UK law, [2], and employers have a responsibility to ensure their employees do not exceed specific vibration dosage limits.

High fidelity mathematical models capable of simulating the airframe vibratory response are essential to producing good low vibration designs. The mathematical model, usually a finite element model, must be capable of indicating the sensitivity of the response due to parameter changes as well as the prediction of absolute vibration levels. Predictions for new designs are notoriously difficult and as a general rule the airframe dynamic design process takes a lower priority to other requirements (eg rotor performance/vibratory forcing and structural strength/stiffness design) until prototype structures are available and dynamic performance can be assessed. It is at this stage that the modelling tools are essential to ensure a good dynamic performance through the application of structural modifications and mitigation techniques.

The models are also essential for assessing airframe upgrades, the installation of equipment and external stores. The objective of ensuring the vibration response is not degraded and the equipment similarly obtains a smooth ride is best done with the aid of a well matched finite element model. This reduces the dependency on testing and substitutes a trial and error approach for a systematic design process.

The objective, therefore, is to obtain a well correlated dynamic finite element model for the airframe which can be used to predict the vibratory response in assessment of new designs, upgrades and modifications. This work is complementary to that done on rotor hub vibratory load prediction which is required to complete the capability for vibration response prediction. [1]

3 AG14 FINITE ELEMENT MODEL: APPROACH

A time-expired Lynx Mk7 airframe used at QinetiQ as a dynamic test structure was available for use within the GARTEUR programme. The Lynx was primarily used to support QinetiQ's dynamics research programme and was available on a long term basis. A programme for the research group AG14 was defined within the GARTEUR exploratory group EG19. The programme made use of a series of tests proposed for the airframe. The baseline structure consisted of the basic airframe with engines and gearboxes removed. Other internal fitments such as seats and avionics were also removed. The baseline structure was essentially a skin stringer vehicle with a reduced number of modelling complications. Achieving a good correlation with this structure was the aim of the study, providing an indication of the most suitable approaches for modelling the structure. Figure 1 illustrates the baseline structure used in the AG14 programme.

A finite element model of a Lynx Mk7 was obtained from AgustaWestland Helicopters. In parallel to the physical removal of components from the airframe, the finite element model was suitably modified to establish as close a match as possible. The initial finite element model of the Lynx XZ649 helicopter baseline configuration comprised 63792 degrees of freedom and its total mass was 808kg. As an indication of the difficulty of aligning the finite element model with the actual structure there was a mass deviation of 6.2% between the two. The discrepancies being due to the omission of items in the model, for example the rivets fixings, brackets, flight control rods and actuators (Figure 2) and the fuel system. This discrepancy was compounded due to the adoption of a density 'smeared' mass approach to modelling rather than a lumped mass approach of the original model. This was necessary as there was no record of the mass changes to the baseline structure and hence the model was constructed through a combination of measurement and estimation. The aim was to establish a close mass match between structure and model with the updating process used to improve the final mass correlation. In the frequency range up to 75 Hz, 15 modes were extracted from the finite element analysis in contrast to the experimental results, where only 11 modes were identified in this region.

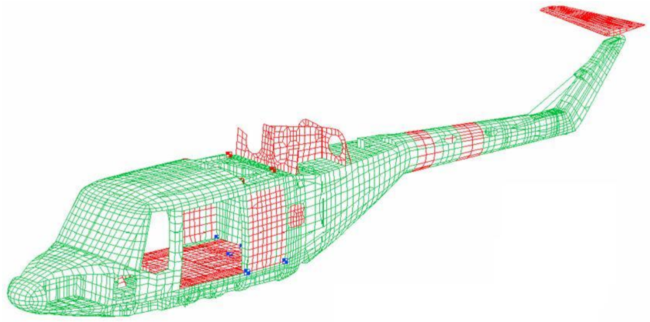
Baseline Structure The basic airframe structure consisting of the cabin, tail boom and tailplane. The baseline mass was approximately 800 kg with dimensions of 12m in length, 2m in width and 2.5m high.	

Figure 1: XZ649 baseline structure for AG14 programme

Inevitably, as part of the process of attempting to produce an accurate finite element model of the physical airframe there were certain restrictions that prevented this from being achieved. The most significant restriction was limited access to CAD data. Uncertainties and engineering judgements also had to be made as part of the process of developing the finite element model, which inevitably clouds the fidelity of the final FE model and is the reason for using test data to help improve the model usefulness.

The representation of external panels, especially in the vicinity of the intermediate gearbox, raised some concern and potential modelling uncertainties. Laminations in this area have been introduced, changing both the mass and stiffness. Comparisons made with the finite element mesh geometry quite clearly show that this does not match the contours of the actual laminations, suggesting that approximations and hence uncertainties have been introduced. Figure 3 illustrates a typical region on the tail-boom with regions of extra material not originally represented in the finite element model.

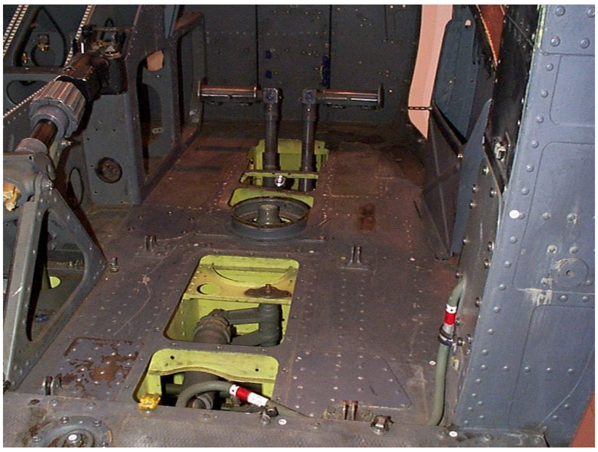


Figure 2: Control rods in cabin	*Figure 3: Example of laminations/local reinforcing*

4 DYNAMIC TESTING

The dynamic characteristics of the structure are established through the process of modal testing. The resulting information is used in a correlation and updating exercise to improve the finite element model. It is important that the data collected in this process is of good quality and fit for the purpose of updating the finite element model.

The baseline modal test comprised 29 response points and 4 excitation attachments as shown in Figure 4. Translational acceleration responses in the x-, y- and z-axis directions were measured at each of the response points.

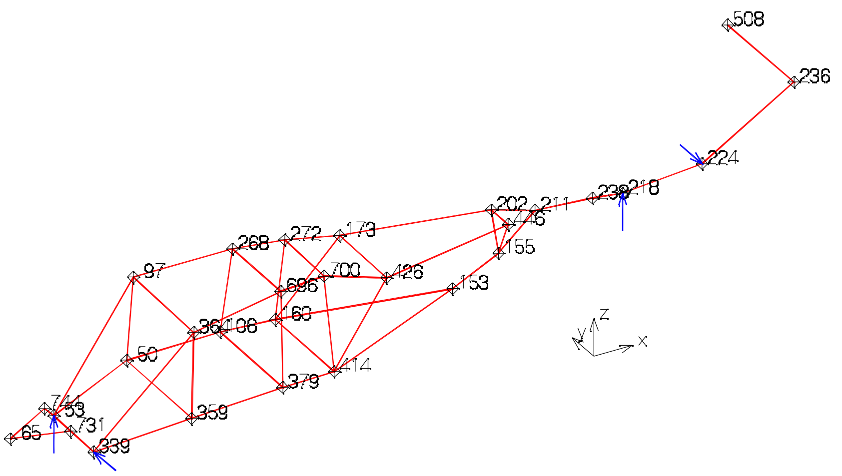


Figure 4: Location of response and excitation points (blue arrows) for baseline test

Figure 5: Baseline structure under test

The suspension arrangement for XZ649 in the baseline condition can be seen in Figure 5 Compression springs at the top of the 2m strop provide a low-frequency suspension in the vertical direction and the pendulum length provides a low-frequency suspension for the airframe in the lateral and longitudinal directions.

Some Frequency Response Function, FRF, results from the baseline tests on XZ649 can be seen in Figure 6 and Figure **7**. It can be seen from one of the drive point FRFs shown in Figure 6 that the resonance peaks occur in groups of two for the first 4 modes and that they are all well defined and separated by anti-resonances. The Nyquist view of the same data is presented in Figure **7** and it is clear that there is adequate definition of the modal circles indicating that the frequency resolution for these measurements is satisfactory.

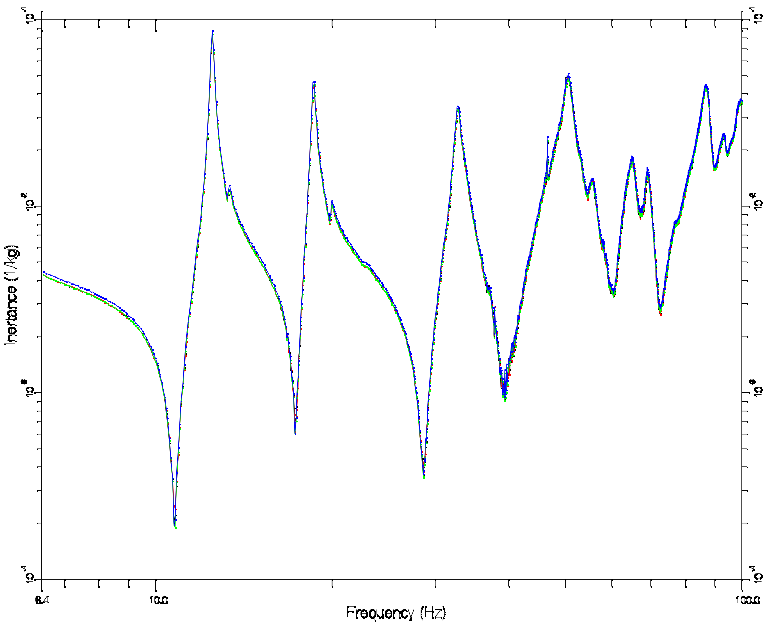


*Figure **6**: Example of Point FRF (Drive point 53z)*

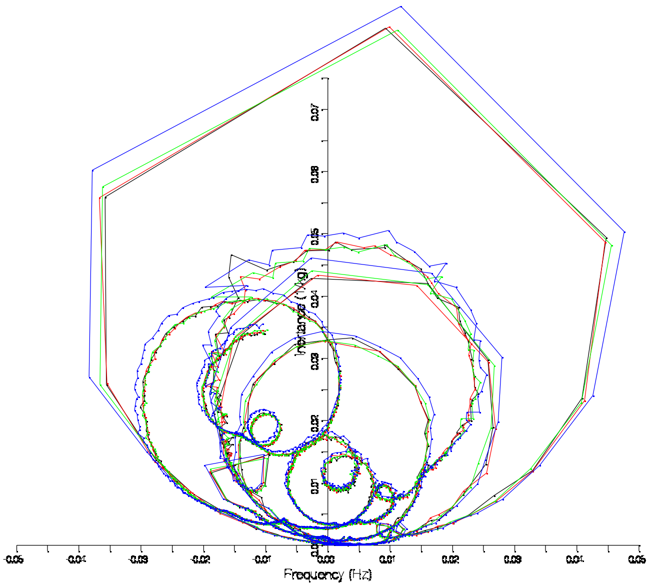


Figure 7: Nyquist view of drive point FRF (repeatability check)

5 CORRELATION AND ERROR LOCATION

5.1 Introduction

For the purposes of this discussion, error localisation is the process of identifying the parameters of a finite element model giving rise to discrepancies with test data, and in particular, the degrees of freedom in which the most significant errors are manifest. It should be noted that this is not necessarily at the degrees of freedom with the largest mis-match in shape between test and FE. A number of methods are listed in [4]. Here, the use of modal deflection scatter plots, the Coordinate Modal Assurance Criterion (COMAC) and Error Matrix Method (EMM) are discussed and applied.

5.2 Manual Inspection

There can be no substitute for manual inspection of the structure alongside study of the finite element model, although it is to a great extent facilitated by the indication methods described in the following sections. While the region and nature of errors may have been narrowed down, operator inspection alone may offer an understanding of the precise cause of the discrepancy (e.g. “structure features an unrepresented modification”) thereby allowing a targeted update (e.g. “incorporate the modification in the model” as opposed to “adjust regional mass and stiffness”); changes to the model structure require such insight. Greater understanding may also indicate how errors may be avoided in a future design process (e.g. “keep the finite element model current after introducing production modifications”).

5.3 Scatter plots

When comparing the overall degree of correlation between pairs of modal vectors, the Modal Assurance Criterion (MAC) is typically applied, but does not offer any insight as to which degrees of freedom are correlating well or poorly. To examine the individual freedoms in more detail, it is instructive to plot the modal displacements against each other in a scatter plot format. The relative Modal Scale Factor (MSF) is the gradient of the line of best fit passing through the origin. The determination of the best fit line is typically via a minimum quadratic error approach, but more sophisticated algorithms or operator judgement may be required if outlier points are corrupting an otherwise sound fit. An example is provided in Figure 8 showing 4 correlated mode pairs. Clear outlier points may indicate invalid measurement (for example due to a loose accelerometer) or local errors in the finite element model.

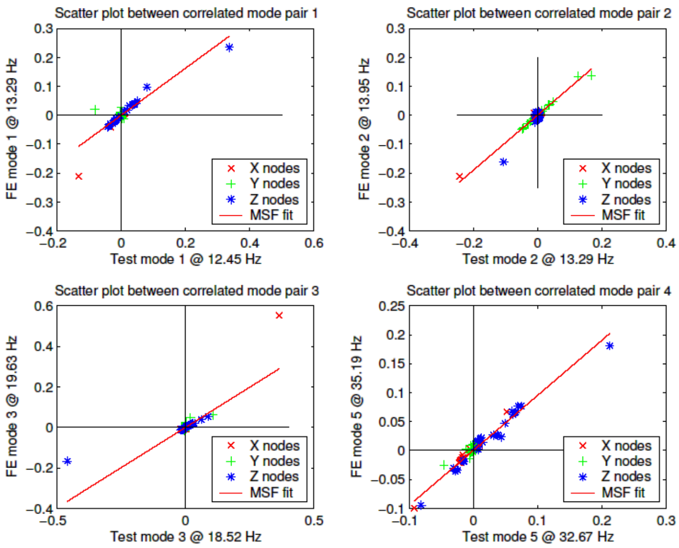


Figure 8: Example scatter plots; first 4 correlated modes of original fuselage baseline model

5.4 COMAC

It may be the case that measurements at particular degrees of freedom result in scatter plot points showing consistent disagreement across multiple modes.

A number of definitions exist for similar measures of such lack of fit.

The nominal COMAC for the l^{th} freedom across N_m mode shapes, ϕ, is defined as

$$COMAC_l = 1 - \frac{\left\{\sum_m^{N_m} \left|\phi_{1ml}\phi_{2ml}\right|\right\}^2}{\sum_m^{N_m}\left\{\phi_{1ml}^{\ 2}\right\}\sum_m^{N_m}\left\{\phi_{2ml}^{\ 2}\right\}} \qquad \text{Equation 1:}$$

such that larger values indicate larger errors (this is 1 minus the original definition found in [5]. This measure implicitly requires that modes are similarly scaled. When this is not necessarily the case, it may be preferable to use the scaled COMAC ('sCOMAC') in which the second set of mode shapes is pre-scaled by the modal scale factor (MSF) before being processed as for the COMAC:

$$\tilde{\phi}_2 = \phi_2 MSF_2 = \phi_2 \frac{\phi_2^{\ T}\phi_1}{\phi_2^{\ T}\phi_2} \qquad \text{Equation 2:}$$

In this case, the scaled COMAC represents the typical deviation of each measurement from the MSF best-fit line.

A further metric exists; the enhanced COMAC ('eCOMAC') [6] first unity-normalises each modal response:

$$eCOMAC_l = \frac{\sum_m^{N_m}\left\|\tilde{\phi}_{1lm} - \tilde{\phi}_{2lm}\right\|}{2N_m} \qquad \text{Equation 3:}$$

$$\tilde{\phi} = \frac{\phi}{\|\phi\|}$$

Finally, it must be borne in mind that if comparisons are desired between the COMACs of two different comparisons of correlated dataset pairs, the absolute value of the COMAC depends on the mode scaling as well as the number of modes. If responses are considered to be a normally distributed random variable, the central limit theorem indicates that the variance of a set of samples is inversely proportional to the number of samples. Therefore, for comparisons, it is preferable to use the eCOMAC multiplied by the number of modes [7]

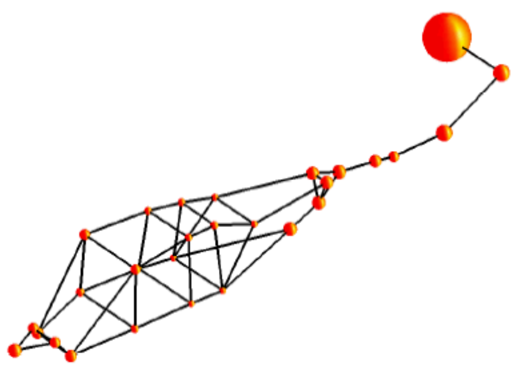


Figure 9: Comac for baseline FE model and test data

For visualising the location of COMAC results, it is useful to plot the values on the test wireframe as the radii of spheres at each node, taking the norm or maximum where multiple freedoms are measured. As an example, Figure 9 shows the COMAC for the original baseline finite element model with comparable test data and, it is clear that the rear portion of the tail boom is showing poor correlation. This model was subsequently updated to correct tailplane spar offset inconsistencies..

5.5 Eigenvalue equation balancing

The eigenvalue equation can be stated with reference to analytical mass and stiffness matrices (K and M respectively, with the '*a*' subscript) alongside experimental frequencies and mode shapes (Ω and Φ respectively, with *m* subscript), giving:

$$[\mathbf{K}_a + \Delta\mathbf{K}]\Phi_m - [\mathbf{M}_a + \Delta\mathbf{M}]\Phi_m\Omega_m = \mathbf{0} \qquad \textit{Equation 4:}$$

The $\Delta\mathbf{K}$, $\Delta\mathbf{M}$ are typically required to balance the equation, and can be used to generate a localisation matrix [8]

$$L = \Delta\mathbf{M}\Phi_m\Omega_m - \Delta\mathbf{K}\Phi_m = \mathbf{K}_a\Phi_m - \mathbf{M}_a\Phi_m\Omega_m \qquad \textit{Equation 5:}$$

This requires the expansion of experimental modeshapes to the order of the finite element model, or reduction of the analytical matrices, achievable via a number of methods such as Guyan [9] or SEREP [10] reduction/expansion.

5.6 Error Matrix Method

The Error Matrix Method (EMM) is an error localisation and updating method to determine the regions and magnitudes of discrepancies between the experimental and finite element models, described in [11] and [12].

This method is applied to yield a stiffness or mass error matrix, following finite element model reduction or test mode expansion as mentioned in the previous section:

$$\begin{aligned} \Delta\mathbf{K} &\approx \mathbf{K}_a\left[\Phi_a\omega_a^{-2}\Phi_a^T - \Phi_m\omega_m^{-2}\Phi_m^T\right]\mathbf{K}_a \\ \Delta\mathbf{M} &\approx \mathbf{M}_a\left[\Phi_a\Phi_a^T - \Phi_m\Phi_m^T\right]\mathbf{M}_a \end{aligned} \qquad \textit{Equation 6:}$$

The error matrices may be used to highlight regions of error, with reference to the row-wise norm which can be plotted as a scaled marker on the test wireframe. They may also be used to update the analytical mass and stiffness matrices:

$$\begin{aligned} \mathbf{K}_{a\,updated} &= \mathbf{K}_a + c\Delta\mathbf{K} \\ \mathbf{M}_{a\,updated} &= \mathbf{M}_a + c\Delta\mathbf{M} \end{aligned} \qquad \textit{Equation 7:}$$

with c as a constant chosen to control the rate of updating which may be carried out iteratively. It may be desirable to use a subset of the available modes for the updating, in order to assess the effect on the remaining modes as a means of validation.

Examples of practical applications of the method are [13], [14], [15]; accuracy and convergence are potentially sensitive to modal or DOF incompleteness and measurement noise [16].

Figure 10 shows the stiffness error matrix visually located at the appropriate nodes of the structure, with the radius proportional to the error magnitude. In this case, the test dataset in question relates to the Baseline build, and the model is the original model before updating. The equivalent result for the mass error matrix is shown in Figure 11. These show two key error regions; the cabin roof and horizontal tailplane. The finite element model was subsequently found to be modelling the tailplane root spar incorrectly, and the effect of the lifting frame attached to the cabin roof had not been accounted for in this particular model either. These modifications subsequently featured in an updated model.

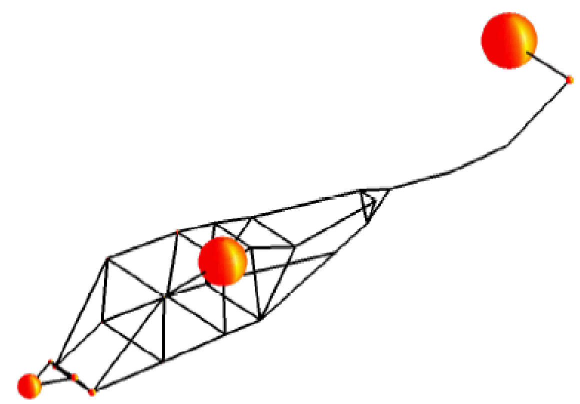


Figure 10 Visual representation of EMM stiffness error locations, original baseline model (error as nodal sphere radius)

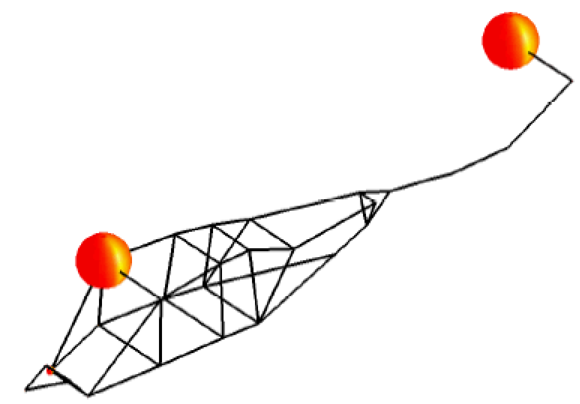


Figure 11 Visual representation of EMM mass error locations, original baseline model (error as nodal sphere radius)

6 SENSITIVITY STUDIES AND UPDATING

6.1 Industrial Updating Methodology

6.1.1 Model Preparation

Before using an initial finite element model for model updating the user must be aware of the modelling uncertainties which may be grouped in the following three categories:

Category 1: Erroneous assumptions for model parameters

The updating procedures used in this programme corrected only those parameters identified in i and ii below:

i. material parameters like Young's modulus, shear modulus, mass density, mass moments of inertia,

ii. local design parameters (e.g. beam area moments of inertia, cross section areas, torsional constants, plate thicknesses and spring stiffnesses).

These parameters can affect one element or a group of elements with the same properties, i.e. a single parameter may be assigned to a substructure. Generally speaking all parameters can be used for updating which are related linearly to the stiffness matrix and the mass matrix according to

$$\Delta\mathbf{K} = \sum_i \alpha_i \mathbf{K}_i \ , \ \Delta\mathbf{M} = \sum_j \beta_j \mathbf{M}_j \qquad \textit{Equation 8:}$$

where: $\mathbf{K}_i$ and $\mathbf{M}_j$ represent the i^{th} stiffness and the j^{th} mass substructure (element or group of elements) to be updated.

The corresponding correction parameters α_i and β_j are calculated by the updating algorithm whereas the sub-matrices containing the assumptions for the type and the location of the modelling error have to be defined by the user. These assumptions are important for the success of the updating. If these assumptions are ***not consistent*** with the real error source and location, the parameters may well minimise the difference between analytical and experimental data, but the parameters may lose their physical significance and represent pure mathematical optimisation variables.

The following error types occurring in finite element modelling ***cannot*** be updated using current update procedures. These errors are introduced either by inaccurate assumptions for the structure of the analytical model (category 2) or by numerical errors (category 3).

Category 2: Erroneous assumptions related to the structure of the analytical model:

i. Erroneous simplification of the structure, for example, when a plate is treated like a beam, or when a thick volumetric structure exhibiting a 3D non-uniform stress state is simplified by a 2D model with a uniform stress distribution in one direction.
ii. Inaccurate assignment of mass properties, for example when distributed masses are modelled with too few lumped masses, or when an existing eccentricity of a lumped mass is disregarded.
iii. When the finite element formulation neglects particular properties, for example, when the influence of transverse shear deformation or warping due to torsion in beam elements is neglected.
iv. Errors in the connectivity of the mesh i.e. some elements are not connected or connected to a wrong node.
v. Erroneous modelling of boundary conditions, for example when an elastic foundation is assumed to be rigid.
vi. Erroneous modelling of joints, for example when an elastic connection is assumed to be rigid (clamped) or when an eccentricity of a beam or a plate connection possibly existing in the real structure is disregarded.
vii. When a non-linear structure is assumed to behave linearly.

Category 3: Errors introduced by numerical methods like:

i. Truncation errors in order reduction methods like static condensation.
ii. Incomplete integration order used to calculate element matrices.
iii. The finite element model mesh is too coarse (not fully converged modal data in the frequency range of interest).

Models which include such errors are also referred to as ***inconsistent*** and may be updated in the sense that the deviations between test and analysis are minimised but, as always, in the case of inconsistent models the parameters may lose their physical meaning after updating. A typical result of updating such inconsistent models will be that they may be capable of reproducing the test data but may not be useful to predict the system behaviour beyond the frequency range used in the updating, or to predict correctly the effects of structural modifications or to serve as a substructure model to be assembled with an overall structural model.

6.2 Engineering judgement and modelling uncertainties

Fundamental to any updating process is the application of engineering experience and knowledge of the structure. Differences between the laboratory structure and the production finite element model were noted through a painstaking audit of the model alongside the structure. Ideally, error location techniques should ease this burden by localising the regions of discrepancy and reducing the work load of the engineer. Several discrepancies were noted in the audit and the structure was subsequently modified. One modification that had a major impact on the results was located in the tail-plane region. A misalignment of a revised tail-plane model and a missing elastomeric bush influenced several mode shapes and frequencies (Figure 12)

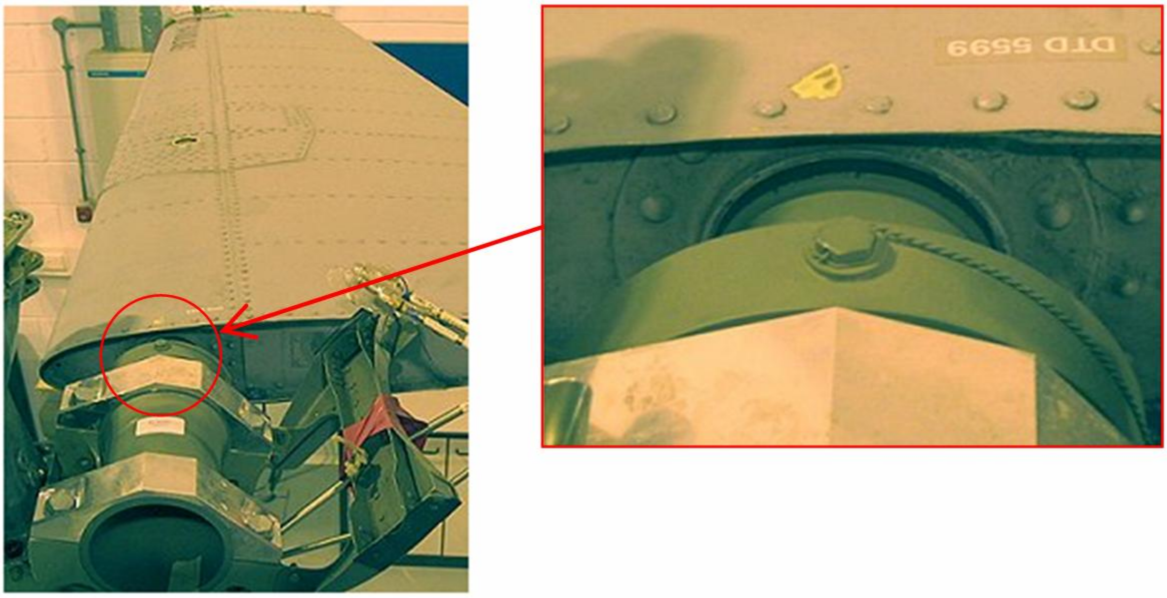


Figure 12: Region of tail-plane leading to modelling errors.

6.3 Sensitivity Analysis

The purpose of sensitivity analysis is to determine the parameters of the finite element model that when changed by small amounts have a large effect on the finite element model predictions of dynamic behaviour (usually natural frequencies and mode shapes). Engineering judgement must be used to select the parameters used in updating from the set of sensitive parameters. In general, the changes in the natural frequencies (and mode shapes) may be expressed nonlinearly in terms of the parameters. The sensitivity method assumes a linearization based on the finite element model and model updating is carried out iteratively by correcting the model and determining new sensitivities at each step. The initial sensitivity analysis, used to select the updating parameters, is based on the initial finite element model. This means that the finite element model should not be too different from the physical test structure.

The sensitivity equation,

$$S\,\boldsymbol{\delta\theta} = \boldsymbol{\delta\lambda} \qquad \textit{Equation 9:}$$

(where $\delta\theta$ and $\delta\lambda$ denote the changes in the parameters and eigenvalues respectively and **S** is the matrix of eigenvalue sensitivities) should be scaled to take account of the different numerical values of the natural frequencies and different nominal parameter values,

$$\begin{bmatrix} \theta_1 s_{11}/\lambda_1 & \theta_2 s_{12}/\lambda_1 & \cdots & \theta_m s_{1m}/\lambda_1 \\ \theta_1 s_{21}/\lambda_2 & \theta_2 s_{22}/\lambda_2 & \cdots & \theta_m s_{2m}/\lambda_2 \\ \vdots & \vdots & \ddots & \vdots \\ \theta_1 s_{n1}/\lambda_n & \theta_2 s_{n2}/\lambda_n & \cdots & \theta_m s_{nm}/\lambda_n \end{bmatrix} \begin{Bmatrix} \delta\theta_1/\theta_1 \\ \delta\theta_2/\theta_2 \\ \vdots \\ \delta\theta_m/\theta_m \end{Bmatrix} = \begin{Bmatrix} \delta\lambda_1/\lambda_1 \\ \delta\lambda_2/\lambda_2 \\ \vdots \\ \delta\lambda_n/\lambda_n \end{Bmatrix}$$

Equation 10:

In the helicopter airframe, the complex connectivity and interaction between different parts of the finite element model precludes a physically meaningful parameterization. The material properties of substructures or groups of elements may however be used to correct the natural frequency predictions. In complicated, large-scale, systems this is the only practical approach because realistically it would be too difficult and too time consuming to attempt to understand every aspect of the model, for example every joint, in detail. Engineering judgment is used to inform the choice of substructures and generally this leads to an equivalent model after updating.

6.3.1 Initial sensitivity study

An initial sensitivity study was carried out using MSC NASTRAN (Solution 200) based on 109 different material properties specified in the original finite element model. The natural frequencies in the range 0-75 Hz were found to be most sensitive to nine element groups. Some modes are strongly affected (high values) while other modes are hardly affected at all. The tail boom and tail plane dominate the first six modes so that it is straightforward to identify the parameters having the most influence upon them. For example, Young's modulus changes to material in the tail plane (ID 1250000) has a dominant effect on the 4th mode, vertical bending of the tail plane.

The major groups were divided into subgroups, which enabled Young's modulus to be changed independently in the subgroups without changing the entire group. Part of the reason for doing this was that all of these nine groups were made up of many parts distributed to a greater or lesser extent throughout the whole airframe. Other sensitive groups (not included in the nine) were much more localised to particular regions, so it was deemed unnecessary to split them into subgroups. The nine groups were divided into smaller subgroups mainly in sections along the x-axis. For each subgroup, new shell or beam element properties and new material properties based on the original group were defined. After parameterisation of the subgroups, the sensitivity analysis result revealed detailed information about the sensitive regions inside each group.

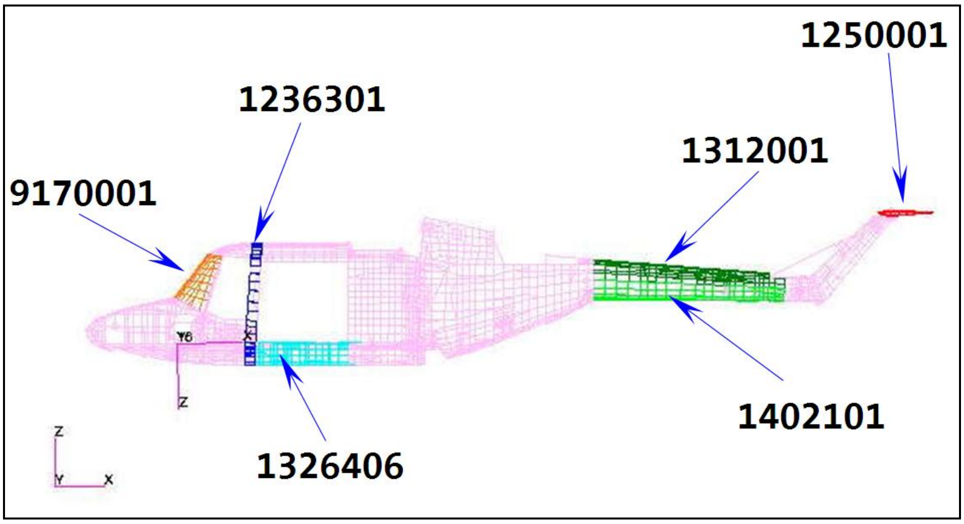


Figure 13: The most sensitive areas after dividing the major groups

6.3.2 Sensitivity study of the complete airframe

The original 109 groups, with the most sensitive nine split into subgroups, produced a total number of 147 candidate parameters. Figure 13 shows the 6 most sensitive groups amongst the 147 as follows: the spar of the tail plane (12500001), the windshield (9170001), the tail skin (1312001 and 1402101), the floor composite layer (132406), and the cabin main frame (1236301).

The results of the eigenvalue sensitivity analysis with respect to mass density were also considered. This analysis showed the most sensitive groups: the floor composite layer, the roof beams, the spar of the tail plane, the beams in the cockpit (roof), 2nd cabin main frame and the tail skin.

In a large and complex structure, such as a helicopter finite element model, it is difficult to find parameters that strongly affect all of the modes. The tail modes are generally at low frequencies appearing in pairs. The cabin modes are more complex and contain torsion and bending modes in the roof and the floor at higher frequencies. Investigating the structure in the roof region where the cabin showed great sensitivity revealed some mechanical linkages that had not been modelled. These linkages accounted for a lack of mass in the model, which were corrected using lumped masses. The stiffness effect of the linkages was deemed to be slight.

6.4 Clustering Parameter Sensitivities

When large and complicated structures are to be updated the number of potential updating parameters may be very large and the problem of discriminating between them extremely difficult. One formalised way of selecting parameters based on sensitivity data is the 'clustering' approach [17]. The columns of the sensitivity matrix are considered as vectors in multi-dimensional space. If, for example, two columns of the sensitivity matrix are identical (within a scalar multiplier) then they appear as collinear vectors. This means that the two parameters of the two columns affect the modes proportionately in exactly the same way. Of course this is an extreme and most undesirable situation because the rank of the sensitivity matrix is less than its number of columns. Nevertheless, because of insufficient information, the sensitivity columns of many parameters are in practice often found to be very close to one another, leading to ill-conditioning of the model-updating problem. The clustering method provides a way of identifying similar columns, so that for example their parameters may be combined and the conditioning of the updating problem improved. Clustering also allows one to direct the correction to chosen natural frequencies (and modes) while leaving other natural frequencies that are already closely predicted, unchanged. In small, fairly simple structures it is usually quite straightforward to work out which parameters might have a similar effect, but this is not so when finite element models of large and complicated structures are to be corrected.

The model updating problem is usually written as:

$$\boldsymbol{S}\,\boldsymbol{\delta\theta} = \boldsymbol{\delta\lambda}$$
$$\boldsymbol{S} \in \Re^{m\times n}, \quad \boldsymbol{\delta\lambda} \in \Re^{m\times 1}, \quad \boldsymbol{\delta\theta} \in \Re^{n\times 1}, \quad m > n \qquad \textit{Equation 11:}$$

where S is the sensitivity matrix containing first order derivatives of the model eigenvalues (or eigenvectors) with respect to modelling parameters, $\boldsymbol{\delta\theta}$ is the vector of parameters considered for updating and $\delta\lambda$ is the vector containing the relative errors between predicted and test natural frequencies. The mode shape sensitivities can be used although mode-shape measurements are generally less accurate than natural frequencies.

The simplest approach used within the updating loop, is to select the columns of the sensitivity matrix closest to the data [18]. The methods avoid ill-conditioning by eliminating the less sensitive parameters, which on their own would result in large changes not justified by the physics of the structure. A more complex method is based on minimising the angles between different subspaces of the sensitivity matrix and the data [19].

In the present context, clustering can help the analyst ensure that the parameterisation represents the many different parts of the model by adding together the most similar columns of the sensitivity matrix. This results in grouping together all those parameters with similar effects on the predicted natural frequencies that can be justified from an engineering understanding of the structure. The parameters in a cluster are linked together in order to impose the same variations to all members and the information contained in the measurements may then be used in a way that improves the model in an overall way by a small change to the updating parameters.

The clusters are defined based on a similarity measure and the i^{th} and j^{th} columns of the sensitivity matrix (α) are compared to each other by using the cosine of the angle between them, the so called 'cosine distance', having values ranging from zero to unity:

$$d_{ij} = 1 - \sum_{k=1}^{m} a_{ki}\, a_{kj} \Bigg/ \left(\sum_{k=1}^{m} a_{ki}^{2} \sum_{k=1}^{m} a_{kj}^{2} \right)^{1/2} \qquad \textit{Equation 12:}$$

The classification of the parameters is carried out hierarchically producing a series of successive fusions of the initial parameters into groups [17]. With this method, once the fusions of the clusters are made, they are permanent, placement within a group excluding the membership of another one. A two-dimensional diagram known as a dendrogram, can show the hierarchical clustering and the unifications made at each step. The dendrogram is used to present the similarity between the clusters at each step and the most sensitive columns can be used as centres of the cluster space that contain similar clusters within a tolerance angle.

6.4.1 Clustering of Airframe Parameters

There are many different ways of joining or 'linking' clusters as outlined for example by Everitt [17]. In this exercise, the Un-weighted Pair Group Method with Arithmetic Mean (UPGMA) was used to cluster the columns of the sensitivity matrix. The previous section on Sensitivity Analysis describes the parameterisation of the airframe resulting in a sensitivity matrix with 147 columns. Column 1 is the column of the least sensitive parameter and column 147 the most sensitive.

In the dendrogram, shown in Figure 14, each vertical line at the bottom of the graph with no further objects beneath is known as a leaf. It represents a column of the sensitivity matrix corresponding to a single parameter. Due to the number of the lines (147 in this case) it is not possible to show the column numbers. A selected group at the bottom right of Figure 14 are shown again in Figure 15, where the column numbers and their positions in the model may be seen.

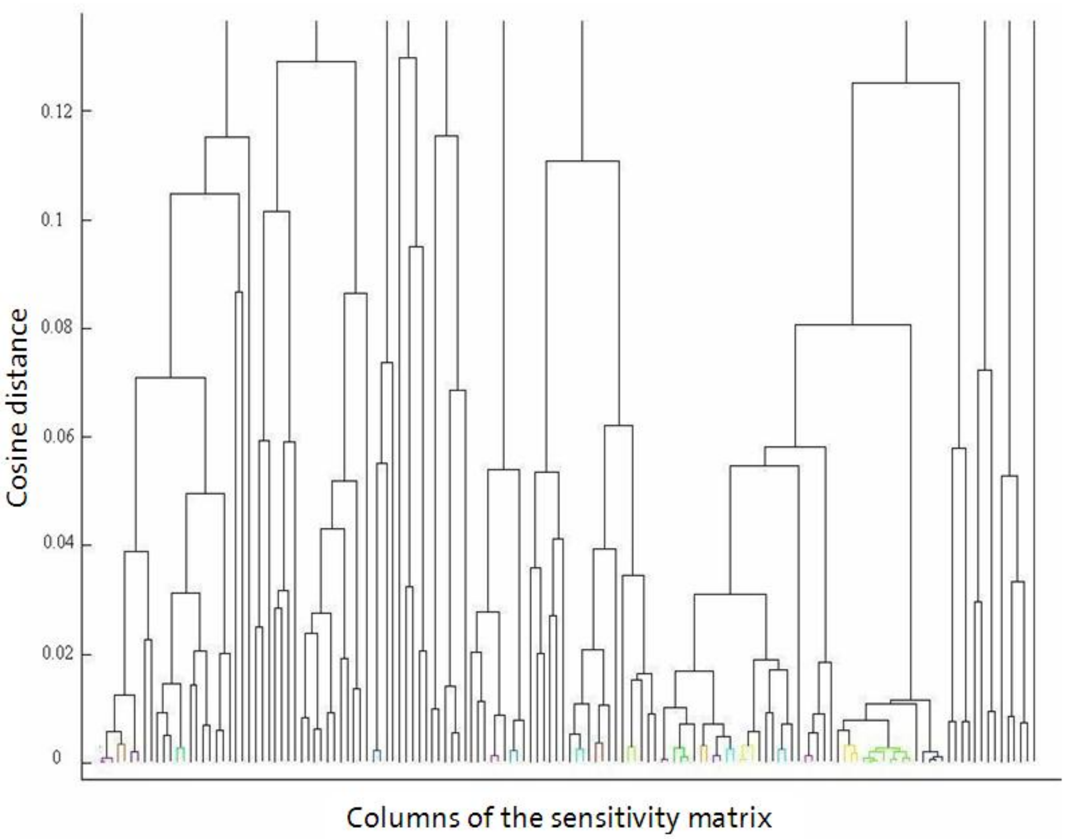
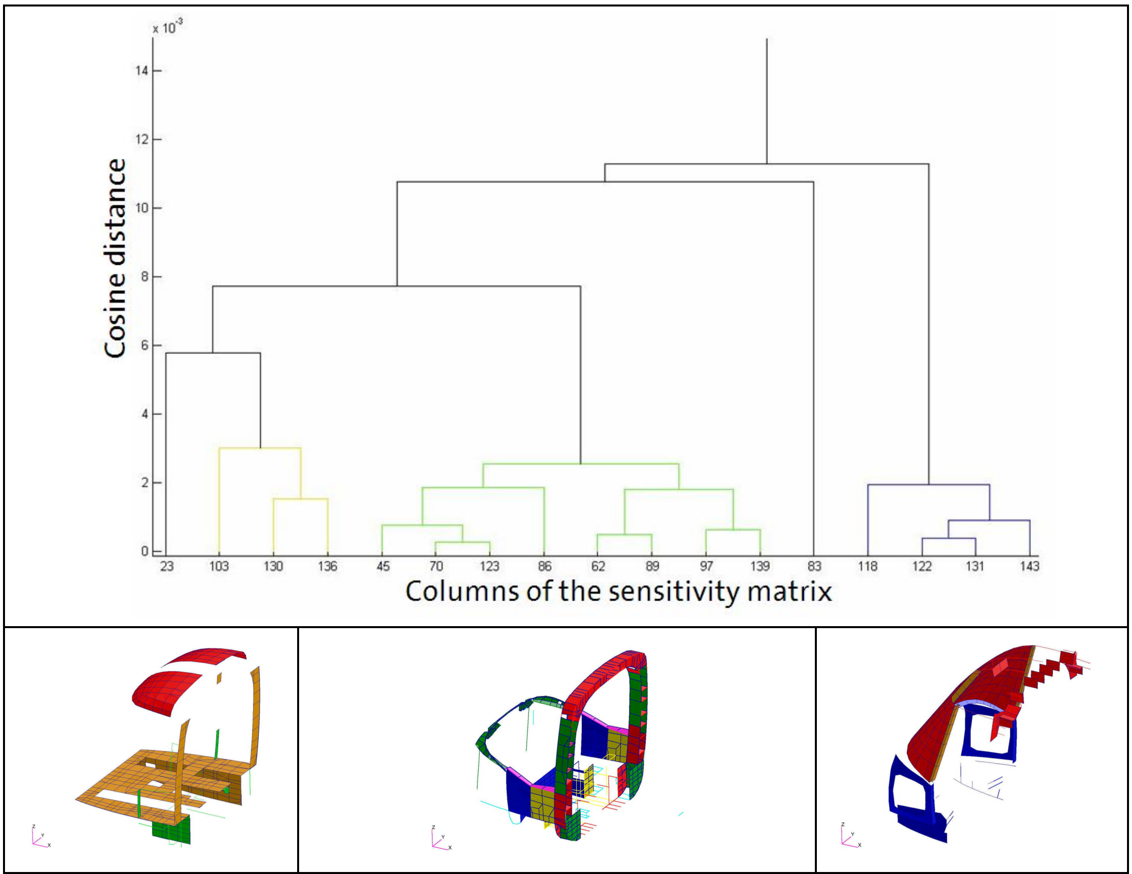


*Figure **14**: Hierarchical clustering of the helicopter sensitivity matrix Similarity measure: cosine distance, Linkage: UPGMA.*

In general, if there are n columns in the sensitivity matrix, the method proceeds by computing an $n \times n$ matrix of distances between all the parameters. At the first step, the smallest term in the distance matrix represents the closest two vectors which may be joined together if deemed to be sufficiently close according to a linkage criterion. The 'height' of the link in the dendrogram is the cosine distance between the two vectors. In the next step, a new distance matrix is determined after adding the two columns together to form a new single column. Subsequently from among the new groups of vectors, the most similar pair will be identified and the process repeated. The distance matrix becomes smaller at each step. The cut-off level or the number of the clusters required can be assigned. It is therefore possible either to choose a certain number of clusters to be formed or to place an upper limit on the distance between clusters. In the latter case, an imaginary horizontal line is drawn across the dendrogram to divide the sample data according to the request. For example the arbitrary cut-off level of the hierarchical clustering of the helicopter sensitivity matrix shown in Figure 14 is 0.13 and eleven clusters have emerged. The clusters at this level are (at least) 30° apart.

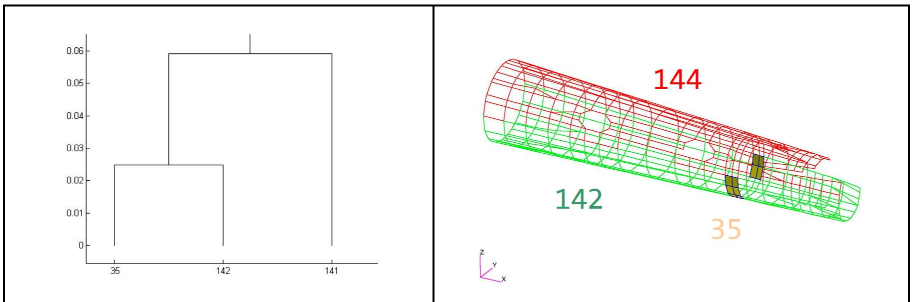
Figure 15 shows three clusters with an angle of less than 5° between them. The numbers shown are the column numbers in ascending order of sensitivity. For example, the main cabin frame (column 139), one of the most sensitive regions, is joined with other parts having similar sensitivity columns within the 5° angle criterion. It is a common practice to filter out insensitive parameters, such as the columns numbered 45 or 62 in the dendrogram. But, by using hierarchical clustering, these are joined together with the cabin main frame to form a single, more powerful updating parameter.



(130,136) and 103	(((123,70), 45), 86) and ((62,89), (97,139)	(((122,131), 143), 118)
a) 3 columns:	b) 8 columns	c) 4 columns

Figure 15: Clustering of three groups in the cabin region: 5° angle criterion

The top and bottom layer of the tail-boom are shown in Figure 16. Both are sensitive terms having columns numbered 144 and 142 in the dendrogram. They are separated by an angle of almost 20° with no other column close to them (column 35 is part of the bottom layer). This seems to indicate that the top and bottom skins should be updated separately.



((142,35), 144)

Figure 16: Angular separation: 12~20 deg

6.5 Parameter Updating of Lynx Baseline Model

Starting from the existing baseline test database of the Lynx Mk7 helicopter and the Finite Element models (Figure 17 & Figure 18) provided by QinetiQ updating techniques were exercised and compared and conclusions have been drawn on the best approach.

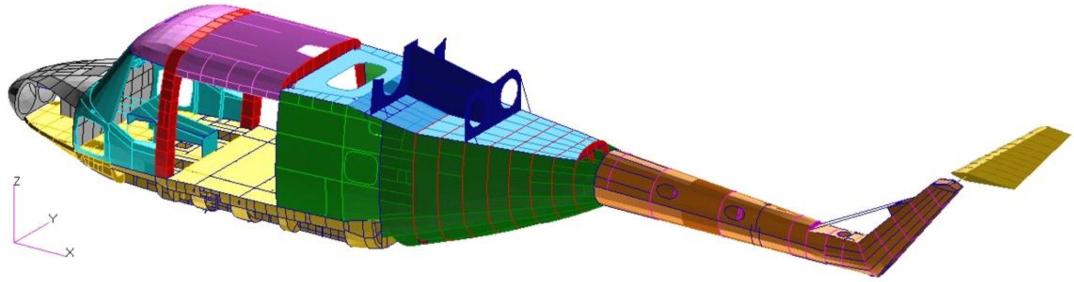


Figure 17: Overall View of Baseline Finite Element Model

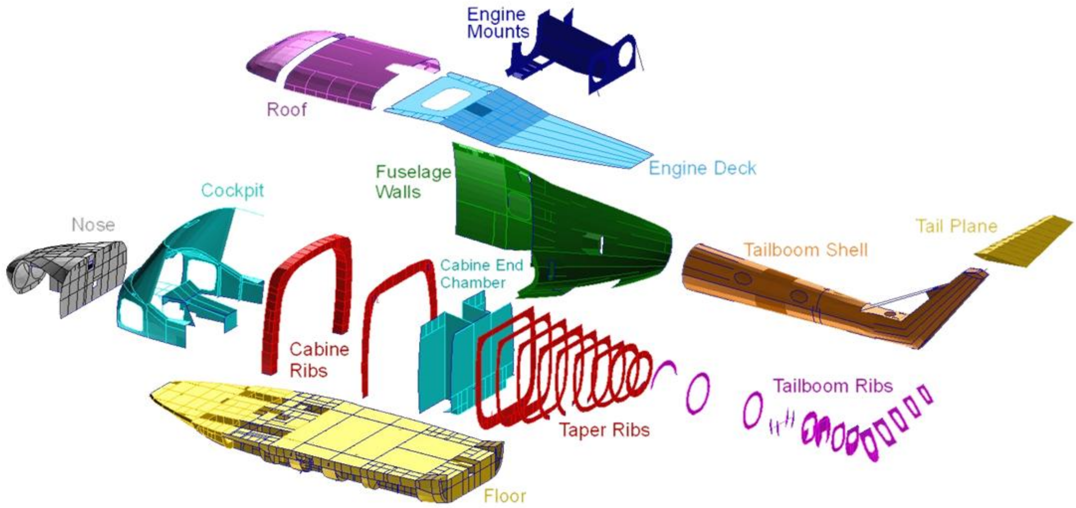


Figure 18: Exploded View of Main Substructures

The task was started by investigating the correlation of existing initial test and analysis data. The test data of the baseline structure (without engine and gearbox) consist of FRFs measured at 29 measurement points in x-, y- and z- direction and 12 modes under free/free conditions using single point harmonic excitation with an electrodynamic shaker. The correlation study was not only aimed at finding the source and the location of the modelling errors but also to assess the suitability of the existing test data sets with respect to model updating requirements.

#	EMA	EMA [Hz]	FEA initial	FEA [Hz] initial	Dev. [%]	MAC [%]	FEM updated	FEM [Hz] updated	Dev. [%]	MAC [%]	type
1	1	12.45	7	13.33	7.01	82.05	7	12.34	-0.89	72.71	1st vertical bending
2	2	13.29	8	13.97	5.10	94.22	8	12.93	-2.70	85.91	1st lateral bending
3	3	18.52	9	19.64	6.06	67.21	9	17.73	-4.26	55.17	tailplane vertical bending
4	4	19.84	10	22.84	15.13	58.55	10	20.63	3.97	48.12	tailplane fore/aft bending
5	5	32.67	11	35.27	7.94	95.93	11	34.46	5.47	95.66	2nd vertical bending
6	6	36.91	12	38.52	4.38	63.89	12	37.07	0.43	73.48	2nd lateral bending
7	7	50.36	13	47.06	-6.56	93.40	13	46.85	-6.98	94.45	fuselage torsion
8	8	52.36	14	56.24	7.41	66.73	14	54.44	3.97	58.78	
9	10	64.76	16	63.99	-1.19	83.39	16	63.37	-2.16	82.77	
10	12	98.49	25	84.29	-14.42	86.92	21	73.75	-25.12	85.24	

Initial mean frequency deviation [%]:	5.58	mean frequency deviation for updated model [%]:	-0.71
std dev of initial frequency deviation [%]:	6.43	std dev of frequency deviation for updated model [%]:	4.42

Table 1: Correlation table before and after updating

Ten analytical modes of the initial model could be correlated in the frequency range up to 85 Hz with an average eigenfrequency deviation of 5.6 % and the maximum deviation below 15.1 % (see MAC and eigenfrequency correlation Table 1). Experimental modes 9 and 11 could not be correlated.

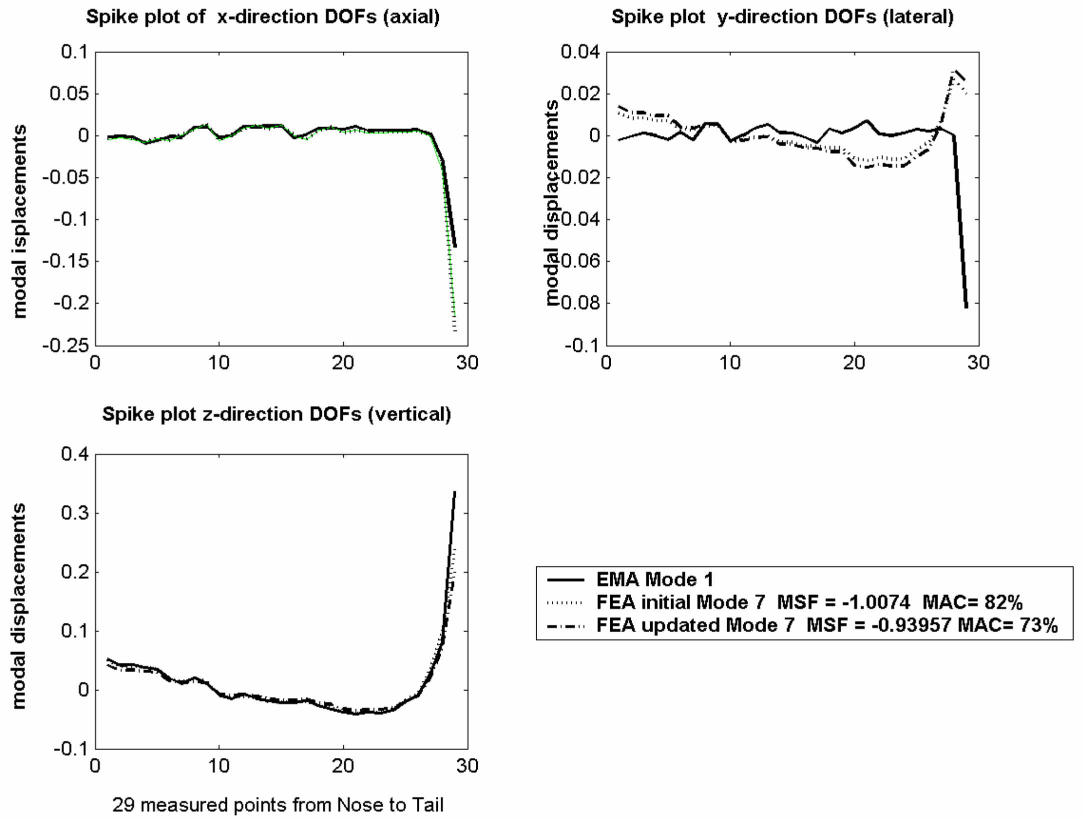


*Figure **19**: Spike plots of first elastic mode*

The comparison of the so-called spike plots (plots of the modal displacement components in x-, y- and z- direction ordered from the nose to the tail of the helicopter) like that for mode no.1 in Figure 19 revealed the dominant influence of the tail plane DOFs exhibiting the largest displacements for all modes and masking the displacements of the other DOFs so that the MAC was significantly altered when only one or two tail plane DOFs were excluded from the modal displacement vectors. Therefore, special attention was given to the accuracy of the experimental modes in that area.

The selection of the updating parameters forms an important step prior to computational updating. Apart from physical reasoning this selection is primarily based on studying the sensitivity of the analytical data used in the objective function with respect to a set of candidate parameters. In the present application we investigated the sensitivities of about 15 natural frequencies and modes with respect to 76 candidate parameters contained in 41 substructures like the tail boom shell, the side walls of the rear fuselage cone or the tail boom / fuselage connection described by stiffness parameters like Young's modulus and moments of inertia and mass parameters like density or concentrated masses. Only those FEA modes exhibiting a significant correlation with the experimental modes were considered for this study since it makes no sense to include an analytical mode which was not observed in the test.

The parameters were ranked according to their largest sensitivity separately for each mode and natural frequency to ensure that there was at least one sensitive parameter retained for each mode. It is well known that the robustness of the parameter estimates decreases when increasing the numbers of parameters. As a rule of thumb, the number of update parameters should not exceed the number of eigenfrequencies if only eigenfrequencies are used in the residual vector and not more than twice that number if mode shapes are also included. In a first try, a selection of 6 parameters related to the stiffness and mass of the substructures was made and is shown in Figure 20 (p1: Young's modulus of tail fin shell without ribs, p2: Young's modulus of tail boom shell without ribs, p3: tail plane density, p4: composite properties of cabin floor, p5: Young's modulus of the stiffening ribs of the fuselage cone. p6: concentrated masses on the cabin floor). The software ICS.Sysval[1] was applied for updating the initial parameter estimates. In this software the inverse sensitivity approach based on minimizing the differences of analytical and experimental natural frequencies and modes is implemented as described in ref.[20].

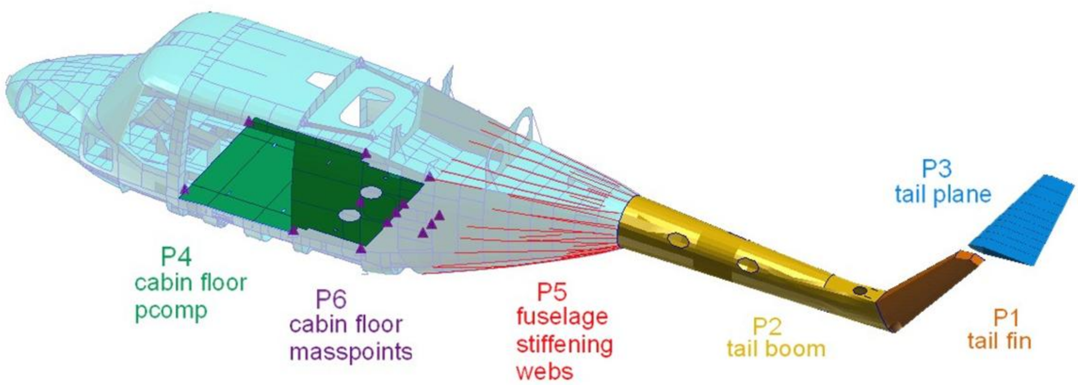


*Figure **20**: Substructures used for updating*

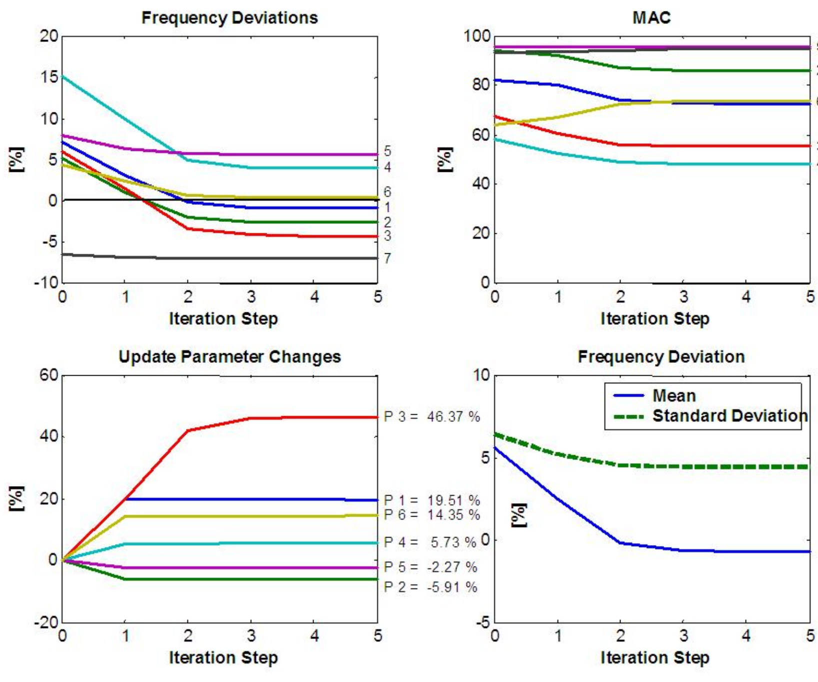


*Figure **21**: Evolution of updating over iteration steps*

[1] ICS.Sysval is a software package developed by the University of Kassel and ICS, Langen, Germany, designed for test planning and computational parameter updating of MSC.Nastran™ finite element models

The result of updating the six selected parameters using 7 natural frequencies and modes in the objective function is shown in Table 1 and Figure 21. Convergence after a few iteration steps is observed. At iteration start the natural frequency deviations scattered between +15.1% and –7% around a mean of +5.6% . At iteration end the scatter was between +5% and -7% about a mean value of only -0.7% Two MAC values improved, one remained unchanged and the others decreased. Initial parameter estimates changed within reasonable limits.

The success of parameter updating cannot only be assessed by comparing how well the modal data of the updated model modes were fitted to their experimental counterparts. Due to the non-uniqueness of the initial model and the parameter selection a perfect fit can not be expected for industrial applications. Even if a good fit was achieved the prediction capability of the updated model is not necessarily ensured. Therefore, as an additional criterion the prediction quality of some higher natural frequencies and modes which were measured but which were not included in the residual for computational updating (passive modes) were checked. The comparison of the passive modes 8-10 in Table 1 shows no significant improvement with respect to this criterion. Another relatively independent check of the model's predictive capability is obtained when measured and analytical FRFs are compared after updating, because in the present application these data were not included in the objective function.

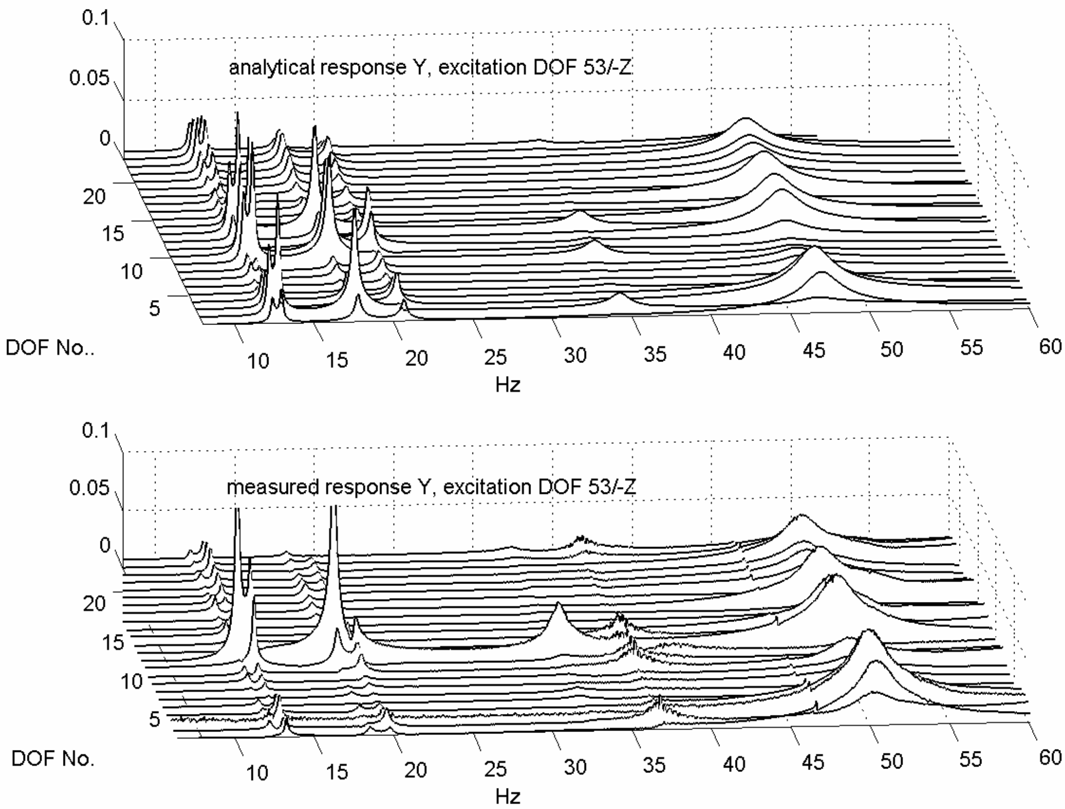


Figure 22: Comparison of analytical and experimental response at 24 DOFs

Figure 22 compares a subset of 24 FRFs measured in lateral y-direction due to excitation in vertical z-direction near the helicopter nose with the corresponding analytical FRFs after updating. From this and the corresponding plots of the other DOFs and the responses from 3 more excitation cases it could be concluded that the experimental response behaviour could be approximated by the analytical model reasonably well in a global sense but not in detail.

Figure 23 shows one of the measured FRFs of Figure 22 in detail together with the analytical FRF before and after updating. Experimental modal damping values were used to calculate the analytical responses. The comparison shows some improvement of the updated model response since the resonance peaks were shifted closer to the experimental peaks.

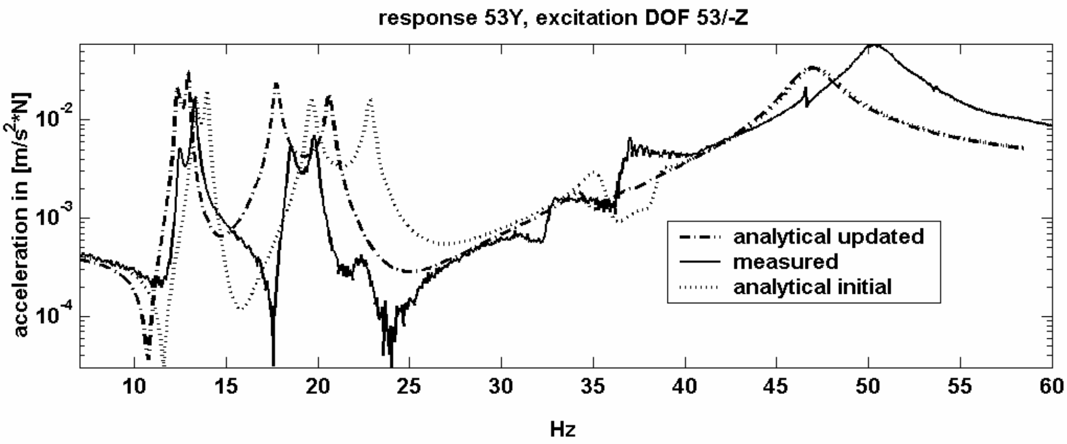


Figure 23: Comparison of experimental and analytical response before and after updating

However, the need became apparent for further improving the analytical model with respect to the response predictions in particular in the frequency range of the first four modes up to 25 Hz, so that after this first model updating step the model could not yet be considered as fully validated.

The study also revealed the difficulties in what could be called “blind” model updating when relying only on an analytical model and test data without including additional knowledge of design and hardware modifications which might have been applied in the time from when the structure was designed and modelled and the time when the structure was tested. Automated model parameter updating cannot be expected to reveal such kind of systematic structural modifications.

An underlying challenge affecting the prediction capability of the models after updating is related to the validity of the introduced modelling assumptions like the assumption of linear structural behaviour. Applying linear model updating techniques in such situations will restrict the applicability of the updated model to loading configurations similar to those of the test. Techniques for updating non-linear models, as investigated in ref.[21], could be more appropriate in such situations.

7 DISCUSSION

7.1 Measured Data

7.1.1 Quantity of Data

For each configuration of the structure tested, there were approximately 425 inertance FRFs measured in a number of sequential tests (strain based FRFs were also measured, but these have not been included in any analysis performed so far). By consolidation of the data into one measured database for analysis, there were FRFs for approximately 91 responses for 4 excitation points (the other FRFs being repeat measurements). This is considered to be a fairly large scale modal test. Nevertheless, the measured database is still many orders of magnitude smaller than a typical finite element model of the same structure.

7.1.2 Quality of Data

Great efforts were made to ensure the quality of the measured data and this was reflected in the clarity of the FRFs and levels of repeatability and reciprocity achieved for all the tests conducted. In all the data quality checks that were performed, the measured data were, apparently, very good. However, despite all the effort and the data checks, above about 50 Hz the data proved to be very problematic to analyse. Furthermore, and perhaps more fundamentally, it has not been possible to resolve some of the issues with the modal results for the first vertical and lateral bending modes of the structure. 'Simplification' of the baseline structure by splitting it into two components (the cabin and the tail boom) did not overcome the difficulties with analysis of the measured data above 50 Hz.

7.1.3 Fitness for Purpose

Structural dynamic measurement is almost always undertaken for a specific purpose. It is rare for the measured FRFs themselves to constitute the conclusion of the work, but rather the starting point for analysis and theoretical predictive investigations. As such, the success of the measurement process must be judged by the ease and accuracy with which the subsequent investigations can be performed. Two uses for the measured data have been considered in the following sections.

7.1.3.1 Characterisation of the structures

Consolidated measured FRF databases have been analysed using standard multi-degree-of-freedom frequency domain techniques. Results for the first 4 modes were obtained relatively easily but, thereafter, the modal extraction was not straightforward and the scatter and complexity of the mode shapes increased. Confidence in the modal results generally decreased with increasing mode number above 50 Hz.

7.1.3.2 'Error' Localisation

The ability to pinpoint the source of discrepancies between the measured model of a structure and its finite element model representation is compromised by several fundamental deficiencies;

i. Insufficient spatial coverage of the measurements (locations and degrees-of-freedom);

ii. Insufficient frequency range of measurement to capture the effects of all modes having an influence in the frequency range of interest, and;

iii. Insufficient confidence in the modal results obtained from the measured data.

As an illustration of the present (in)ability to locate the source of discrepancies between a measured model of XZ649 and its finite element model representation, two build states of this structure have been used; baseline and intermediate 1. Intermediate 1 build of the structure differs from the baseline structure in the addition of an 11 kg intermediate gearbox at the base of the tail fin and a 50 kg tail rotor gearbox at the top of the fin. By comparison of either;

- measured baseline with intermediate 1 finite element model, or;
- baseline finite element model with measured intermediate 1;

it was not possible to locate accurately and reliably the precise positions of the known differences between the models. Since the total mass modification was approximately 10% of the baseline mass this is a significant deficiency in the current capability with the data and techniques available at present. Much of the problem may be related to the limited spatial and frequency coverage of the measurements and not to the inadequacy of localisation techniques *per se*. In this respect there is probably little alternative to increasing the size of the measured model (both spatial and frequency coverage). Absolutely perfect measured data will still not enable pinpoint error localisation without sufficient spatial coverage.

In order to achieve this at a realistic cost, there is a need to develop new transducers and techniques of measurement – possibly capitalising upon the availability of cheap accelerometers originally developed for automobile airbag systems or computer games consoles. Furthermore, there is a possibility that laser response measurement equipment available to a number of the GARTEUR participants could be pooled to enable much better spatial coverage for measurements of specific areas of the baseline structure.

7.2 Validation and Adjustment of Finite Element Models

For the baseline structure, the initial correlation between test and finite element modes in the frequency region up to 50 Hz is relatively good. However, in the frequency range above 50 Hz there is almost no correlation between experimental and finite element model data because of local modes which were not measured.

A new ground vibration test on the Lynx XZ649 baseline configuration using an improved measurement plan is proposed in order to obtain sufficient reliable experimental data for computational finite element model updating.

Without a reliable and proven method for indication of the source of discrepancies between measured and finite element models, the automated adjustment of finite element models to improve the correlation of results will degenerate to a mathematical exercise with limited physical justification for the changes invoked.

It is very interesting to note that the consistently most successful technique for the adjustment of finite element models throughout this study has been by engineering judgement. Careful study of the physical structure and comparison with the interpretation in finite elements has resulted in remodelling in specific areas with commensurate improvements in the correlation of modal properties. Incorrect or inappropriate connectivity in finite element model updating (as it is sometimes known) is rarely taken into consideration – usually it is just the physical properties of the constituent elements (mass and stiffness properties) that are taken into account. This leads to problems when working with an updated model based solely on parameter adjustment rather than correcting an underlying connectivity problem. Such models usually end up needing unrealistic parameter changes to obtain a fit to the test data. However, the model will prove inadequate for engineering use such as assessment of structural modification or design of passive and active vibration mitigation systems.

Table 2 shows the correlation before and after the GARTEUR exercise. The final update reflects the changes made through engineering judgement. A direct comparison of the FE model and the physical structure resulted in remodelling of certain components in the tail region and on the roof. Interestingly, several of the sensitivity and error methods had indicated problems in these areas which helped to home in quickly to issues within the model. A good knowledge and experience of the structure is a powerful tool and, when combined with automated updating procedures, can yield major improvements to model correlation. Improving the higher order modes is more difficult, and issues raised earlier, such as improved spatial resolution of measurement points, need to be addressed if this is to be successfully achieved.

		Initial FE model		Final FE model	
Mode description	Test(Hz))	FE(Hz)	MAC	FE(Hz)	MAC
1st Vertical Bending	12.5	13.3	0.78	13.0	0.96
1st Lateral Bending	13.3	13.9	0.93	13.9	0.98
Tail plane F/A	18.5	19.2	0.63	19.8	0.97
Tail plane vertical	19.8	22.7	0.54	21.4	0.97
2nd Vertical Bending	32.7	35.2	0.96	35.8	0.96
2nd Lateral Bending	36.9	38.5	0.64	37.7	0.83
1st Torsion	50.4	47.1	0.93	46.7	0.91
3rd Vertical bending	52.4	56.2	0.66	54.1	0.66

Table 2: Correlation before and after the exercise

8 CONCLUSIONS

The aim of GARTEUR Action Group HC AG-14 'Methods for Refinement of Structural Dynamics Finite Element Models' was to review and explore methods and procedures for improving finite element models by means of model updating and dynamic shake testing. As a result of this collaborative exercise and the focussed discussions at the meetings, the following observations have been made:

i. In the particular case of Lynx XZ649, the lumped mass model had an initial 6% mass error when compared with the actual structure. Industrial experience indicated that a 1% mass error is now typical. The dynamic characteristics of any structure are governed by both the mass and stiffness distributions. As has been shown, certain aspects of the mass distribution can be checked relatively easily but it is very difficult to verify the stiffness properties directly.

ii. The number of measured degrees-of-freedom must be increased to allow error localisation techniques to resolve discrepancies between test and finite element models much better than is currently possible. The use of optical measurement methods is seen as one possible way in which the quantity of data measured could be increased significantly. It will be necessary to implement different testing techniques (such as 'stepped sine') to those used in the programme so far.

iii. Measurement of a selection of additional point FRFs distributed about the cabin and tail. These FRFs would not be included in the modal analysis for the complete structure (as only a subset of responses would be measured). Information from these measurements will provide independent data for assessment of the performance of the modal analysis and FRF synthesis. Furthermore, these additional point FRFs will enable more simple modification studies to be performed as further confidence building activities.

iv. Attention has been focussed on obtaining a good match for both the frequencies and the mode shapes between the test and finite element models. Ultimately though, it is prediction of the forced response of the structure that is important. For this, the modal masses and the damping values must also be correct, in addition to the mode frequencies and shapes.

v. Improvements to the finite element model for the baseline configuration have been achieved. The changes giving rise to the improvements were obtained largely through ‘engineering judgement’ and meticulous attention to detail. Subsequently, the use of automated methods for sensitivity analysis or model updating is beneficial. Set-up of automated methods requires a detailed examination of the structure and the model and this promotes attention to detail. Neither manual nor automated methods on their own are sufficient to resolve discrepancies between test and finite element models.

vi. Clarification of the industrial requirement for finite element / test models of a structure is required. The nebulous definition as ‘fit for purpose’ has been refined such that a model (test or finite element) ‘should enable accurate prediction and control of vibratory response’, given a prescribed excitation input.

Alignment of test and finite element representations of a helicopter structure is the problem which has been addressed throughout this study. Lessons for the construction of a finite element model of a helicopter when there is no hardware available (the ab initio case) have been learnt through experience rather than specific research.

9 RECOMMENDATIONS

The collaborative effort within the GARTEUR HC-AG14 group has broadened the understanding of error localisation and finite element model updating as applied to helicopter structures. Furthermore, this work has revealed specific ways forward for the research.

i. Use optical measurement equipment and the capability within the GARTEUR group in a collaborative measurement exercise of the Lynx baseline structure to gather a much greater quantity of measured data at very fine spatial resolution. An understanding of the data spatial resolution and type (translation/rotation) is required for identification of modelling discrepancies.

ii. A better understanding of structural variability and how this affects the updating procedure is required. A characterisation and understanding of the variability should provide a guide as to the required fidelity of a finite element model and the level of correlation that is needed.

iii. The measurement of additional drive point FRFs with and without simple mass modifications can be used to compare predictions and measurements for simple modifications to the model as a check on its suitability.

iv. Continue with the refinement of automated methods for error localisation and parameter identification. Embodiment of an ‘engineering judgement’ in the automated process is desirable.

v. Measure forced responses throughout the baseline structure for a number of different inputs and levels for direct comparison with predictions from the finite element models.

vi. Consider greater use of in-flight measurements for FE model refinement.

Ultimately, the ability to predict vibration levels for new and existing aircraft is the main purpose for a dynamic finite element model. As mentioned earlier, the model response is one half of the equation and knowledge of the vibratory forcing from the rotor is just as significant. A European Framework 6 programme, HeliResp [22], was proposed to tie the two halves of the equation together to put into practice the best efforts of European industry for predicting and minimising vibratory response. A systematic work programme was devised involving flight testing, vibratory loads prediction, hub load measurement, FE modelling, modal testing and optimisation to bring together the component parts into a comprehensive capability for the design of a low vibration helicopter. The proposal narrowly missed the funding target, however, the significance and importance of the programme is not diminished and it is essential that support for a programme of this sort is found if major inroads are to be made into this challenging problem.

10 ACKNOWLEDGMENTS

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