APPLICATIONS OF ENERGY CONCEPTS TO THE DETERMINATION OF HELICOPTER FLIGHT PATHS.

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Abstract

This paper discusses the application of energy principles to the determination of helicopter performance in maneuvering flight.

Multilinear regression techniques are applied to the classical energy model in level unaccelerated flight and result in good correlation with flight test performance data.

The model is then extended to maneuvering flight. Aircraft kinetic and potential energy variations are included in the basic equation and rotor induced and profile power are modified to account for changes in rotor thrust and RPM.

Several applications of the method to current helicopter performance problems are reviewed. Particular emphasis is placed on the prediction of flight paths in the event of engine failure during take-off.

At the design stage, the method allows the designer to select the engine power levels which guarantee good maneuverability essential to military aircraft.

It is suggested that this approach, which is shown to be simple and reliable, could significantly reduce the amount of flight tests required to show compliance with official regulations. This is of particular interest in the case of some of the more hazardous tests, such as those involved in the demonstration of Height-velocity diagrams, where crew safety is of prime concern.

List of symbols

$C_T = \dfrac{T}{\rho S U^2}$ Main rotor thrust coefficient

$C_{X_f} = \dfrac{D}{\frac{1}{2}\rho S V^2}$ Fuselage parasite drag coefficient

$C_{X_p} = \dfrac{8 P_p}{\rho S \sigma U^3}$ Equivalent profile drag coefficient

$C_{Z_m} = \dfrac{6 C_T}{\sigma}$ Mean blade lift coefficient

C'_Z Rate of change of section lift coefficient with angle of attack

$\vec{D}$ Fuselage drag

g Acceleration due to gravity

H	Aircraft skid height above ground
I	Mass moment of inertia of main rotor about axis of rotation
K_i	Induced power efficiency factor
M	Aircraft mass
N	Main rotor load factor
P_E	Engine shaft power supplied
$P_f = \frac{1}{2} \rho C_{X_f} S V^3$	Fuselage parasite power
$P_{io} = T v_{io}$	Theoretical induced power in hover
$P_i = T v_i$	Main rotor induced power
P_m	Main rotor momentum power
P_p	Main rotor profile power
$P_R = P_m + P_p$	Total power required on main rotor shaft
P_S	Engine power supplied on main rotor shaft
$Q_S = \frac{P_S}{\Omega}$	Engine torque supplied on main rotor shaft
R	Rotor radius
$S = \pi R^2$	Rotor disk area
T	Rotor thrust

$U = \Omega R$	Rotor tip speed
$\vec{V}$	A/C velocity vector (opposite to free stream velocity in zero wind)
V_X, V_Z	Horizontal and vertical components of velocity positive forward and up
$v_{io} = \sqrt{\dfrac{T}{2\rho S}}$	Theoretical induced velocity in hover
v_i	Induced velocity
α_R	Main rotor angle of attack, positive when the external flow is from below to above the rotor
$\lambda_{TPP} = \dfrac{v_i - V\sin\alpha_R}{v_{io}}$	Rotor inflow ratio
μ	Rotor advance ratio
Ω	Rotor angular velocity
σ	Rotor solidity

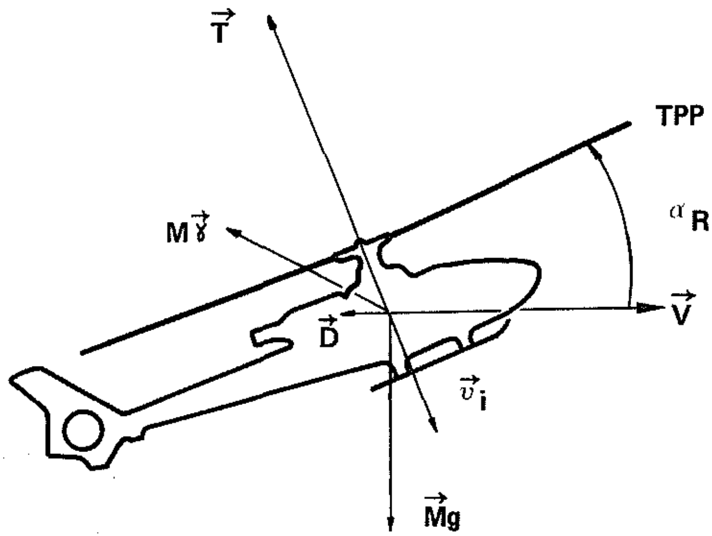


FIG 1 FORCE DIAGRAM
$M\vec{\gamma} = \vec{T} + \vec{Mg} + \vec{D}$

1 - INTRODUCTION

Aircraft agility and maneuverability are major factors of survival in the hostile environment encountered by modern attack helicopters. Large excess power supplied by the engines or internally stored as rotor kinetic energy is required to perform fast evasive maneuvers and to operate safety at low altitudes in tactical flight where the risk of detection by the enemy can be minimized. High rotor kinetic energy is an additional factor of safety when operating close to the ground because it reduces the extent of the restricted area in the height velocity diagram.

Civil aircraft performance is also largely conditionned by energy maneuverability particularily in view of the increasing use of civil helicopters in category A operations.

Determination of the maximum weight authorized at take-off requires the manufacturer to demonstrate that the emergency procedure used in the event of an engine failure allows the pilot to land safely or to continue the flight on the remaining engines.

The need for an accurate and simple approach to predicting helicopter maneuvering flight paths in thus motivated by the increasing importance of maneuver requirements in design specifications. It is also extremely desirable whenever flight testing is impractical or particularily hazardous. Demonstration of H/V diagrams is one area where the application of energy methods could considerably reduce the number of test points required. At high altitudes, proper test sites may not always be available closebyand accurate prediction methods could be usefully developed to extrapolate test results beyond the 2000 feet presently authorized by the FAA.

Most of the analytic methods of helicopter rotor performance prediction use sophisticated wake modeling techniques to describe local rotor inflows and aerodynamic load distributions. Although reasonably accurate these methods are often too elaborate and impractical to use in maneuvering flight simulation. In contrast the semi-empirical model used in this analysis is a simple extension of the energy model traditionally used in hover and level flight performance calculations. Induced and propulsive power requirements are obtained from classical momentum theory. An induced power efficiency factor is introduced to account for distribution effects such as tip losses and non uniform inflow.

Rotor profile losses are described by assuming a limited power series expansions of the profile drag coefficient in terms of advance ratio and mean blade lift coefficient. Correlation with test results is satisfactory provided stall and compressibility effects are not very significant.

With the energy model established a multilinear regression analysis is performed on the available hover and level flight performance data. The induced power efficiency factor, the rotor profile drag coefficients and the equivalent fuselage flat plate area are obtained as coefficients of the regression. The assumption is then made, that the model resulting from the reduction of the stabilized test data can be extended to all maneuvering flight configurations. This assumption is acceptable on theoretical grounds if the test points include climb and descent performance data particularily at moderate airspeeds and high rotor angles of attack.

2 - THE ENERGY MODEL

One of the characteristics of a helicopter as opposed to a fixed wing aircraft is its ability to use kinetic energy stored in the main rotor to improve performance in maneuvering flight.

The main rotor will accelerate when the torque supplied by the engines exceeds the torque required on the main rotor shaft. The torque balance equation can also be expressed in terms of the excess engine power :

$$I \, \Omega \, \dot{\Omega} \; = \; P_S - P_R \tag{1}$$

Here P_S refers to power supplied by the engines and available on the main rotor shaft after losses associated with driving the tail rotor, gearboxes and accessories have been deduced. P_R is the power required on the main rotor shaft to generate lift and propulsive thrust and to overcome blade profile drag.

The induced and propulsive power requirements are obtained from momentum theory which will now be reviewed briefly.

2.1 - MOMENTUM THEORY

According to momentum theory, the flow influenced by a rotor is equivalent to the flow passing through an area equal to that swept by the blades with a velocity equal to the vector sum of the free stream and induced velocities and the total velocity imparted to this flow is twice the induced velocity. The thrust generated by the rotor is equal to the rate at which momentum is imparted to the flow :

$$\vec{T} = -\rho S \, | \vec{V} - \vec{v}_i | \, . \, 2 \, \vec{v}_i \tag{2}$$

The momentum power required is simply the rate at which kinetic energy is transmited to the air. It is given by the scalar product of the thrust and the resultant velocity of the flow through the actuator disk.

$$P_m = \vec{T} \, . \, (\vec{V} - \vec{v}_i) \tag{3}$$

The first term in the scalar product, $\vec{T}.\vec{V}$, represents propulsive power, that is, power required to overcome fuselage parasite drag, to accelerate or to climb. It may be either positive or negative, as in autorotative flight, depending on rotor angle of attack.

The second term $P_i = -\vec{T}.\,\vec{v}_i$ represents the induced power required to produce thrust. Note that this term is always positive since the induced velocity vector is always oriented in the direction opposite to the thrust generated.

Momentum theory does not account for induced power inefficiencies such as those due to non uniform inflow and tip losses. Depending on blade twist, thrust coefficient and flight configuration these distribution losses may represent as much as 20% of the ideal induced power. They are traditionally included in the induced power efficiency factor K_i

$$P_i = K_i \, . \, T \, v_i \tag{4}$$

Theoretically the induced power efficiency increases as the disk loading is reduced. However in an attempt to simplify the calculation of power requirements it is often assumed to remain constant. This assumption generally gives good results.

2.2 - PROFILE POWER

Following simple blade element theory, profile power is traditionally referred to an equivalent profile drag coefficient :

$$P_p = \frac{\rho}{8} S \sigma U^3 C_{X_p} \tag{5}$$

Accurate descriptions of the profile power usually require extensive wind tunnel tests to determine the effect of thrust coefficient, advance ratio, angle of attack and advancing blade tip Mach number on rotor performance. However in hover and level unaccelerated flight, a limited power series expansion of the profile drag coefficient in terms of mean blade lift and advance ratio offers a convenient although approximate description of the profile power requirements. Statistical data fitting techniques such as multilinear regression analysis are used to determine the experimental coefficients in the model from flight test performance data. The method generally gives good correlation with test results provided stall and compressibility effects are not very significant. This is the case of most of the applications considered which are generally restricted to moderate airspeeds where compressibility can be neglected.

However a more sophisticated model is required whenever the rotor is operating close to the stall and compressibility limits.

2.3 - THE ENERGY EQUATION

The total power required on the main rotor shaft is obtained by summing the momentum and profile power,

$$P_R = \vec{T} \cdot (\vec{V} - K_i \vec{v}_i) + P_p \tag{7}$$

At this stage the energy formulation of the torque balance equation may be introduced. First consider Newton's equations of motion in vector form :

$$M\vec{\gamma} = \vec{T} + M\vec{g} + \vec{D} \tag{8}$$

By suitably combining equations 1, 7 & 8 and introducing the fuselage parasite power

$$P_f = -\vec{D} \cdot \vec{V} = \frac{1}{2} \rho C_{X_f} S V^3 \tag{9}$$

the rate of change of total internal energy is obtained :

$$\frac{d}{dt} \left(MgH + \frac{1}{2} M V^2 + \frac{1}{2} I \Omega^2 \right) = P_S - (P_i + P_p + P_f) \tag{10}$$

This equation corresponds to the energy conservation principle according to which any excess power supplied by the engines which is not dissipated by the helicopter, is stored as internal potential, kinetic or rotational energy.

Examination of equation 10 shows that the internal energy level of the helicopter can only increase if the engine power supplied on the main rotor shaft exceeds the total power required to maintain thrust (induced power) and to overcome blade profile and fuselage parasite drag. This excess power may be used indifferently to climb to accelerate or to increase rotor speed.

In the event of a complete engine failure, power supplied to the main rotor is reduced to zero (shaft power supplied may even be negative due to mechanical losses or residual tail rotor profile losses) and total energy will decrease at a rate which depends on aircraft configuration as defined by altitude, airspeed and main rotor thrust, angle of attack and RPM. The pilot's task when entering power-off autorotative flight is to control the thrust vector during descent in order to land the aircraft at the correct level attitude with the smallest possible vertical speed compatible with the structural integrity of the landing gear. At the same time, during the deceleration phase, the pilot must prevent the main rotor from reaching dangerous overspeeds which would leed to unacceptable blade centrifugal stresses. A minimum RPM should also be maintained in order to avoid stall.

2.4 - INDUCED POWER

At moderate advance ratios the time averaged resultant in-plane component of blade drag forces is small and the rotor thrust is essentially perpendicular to the tip path plane (fig. 1).

Referring to the momentum equation and taking in-plane and normal components of airspeed relative to the tip path plane,

$$T = 2\rho S v_i \sqrt{(v_i - V\sin\alpha_R)^2 + V^2\cos^2\alpha_R} \tag{11}$$

If speeds are referenced to the ideal induced velocity in hover $V_{io} = \sqrt{T/2\rho S}$ the resulting non-dimensional momentum equation can be solved for the induced velocity ratio.

$$1 = \frac{v_i}{v_{io}} \sqrt{\left(\frac{v_i}{v_{io}} - \frac{V}{v_{io}}\sin\alpha_R\right)^2 + \left(\frac{V}{v_{io}}\cos\alpha_R\right)^2} \tag{12}$$

Depending on rotor angle of attack and airspeed, the momentum equation has either one or three positive real roots. These are represented on figure 2 in terms of airspeed ratio for values of the rotor angle of attack ranging from -90^o to $+90^o$.

A detailed examination of the momentum equation will show that multiple valued solutions are only obtained for large positive rotor angles of attack

$$\mathrm{Sin}^{-1}\frac{2\sqrt{2}}{3} = 70.5^\circ < \alpha_R < 90^\circ \tag{13}$$

A minimum airspeed is also required $V/v_{io} = \sqrt[4]{12} \simeq 1.86$. It corresponds to the triple solution at $\alpha_R = 70.5^o$ where the theoretical induced velocity ratio is equal to $\sqrt[4]{3} \simeq 1.32$.

The multiple valued solutions obtained for these large rotor angles of attack are evidence of the unsteady flow patterns characteristic of the vortex ring state. Since momentum theory implicitly assumes a continuous steady slipstream it cannot be applied to the vortex ring state but only to the normal working state and to the windmill brake state for which such a steady slipstream exists. In fact experimental data shows that momentum theory generally underestimates the induced velocity ratio and that when multiple roots are obtained only the smallest of these should be considered. At low and moderate speeds ($V < 2\ v_{io}$) the induced power variations with rotor angle of attack are very large and highly non linear. As the angle of attack is reduced the increase in mass flow through the rotor decreases the induced velocity required for a given level of thrust. The rotor is particularily efficient when operating at high negative angles of attack as in climb or acceleration configurations. At larger airspeeds ($V > 3\ v_{io}$) the induced power becomes independant of rotor attitude and inversely proportional to airspeed.

$$\frac{P_i}{P_{io}} \simeq \frac{1}{V/v_{io}} \qquad (14)$$

Figure 3 shows the momentum power ratio (induced and propulsive power) as a function of airspeed ratio for various rotor angles of attack.

$$\frac{P_m}{P_{io}} = \frac{v_i - V \sin \alpha_R}{v_{io}} \qquad (15)$$

For every positive angle of attack (for which the external flow is from below to above the rotor), there exists a minimum airspeed ($V/v_{io} = \sqrt{2/\sin 2\alpha_R}$) above which the momentum power required is negative. The rotor is then extracting sufficient energy from the wind to counter the induced torque. The minimum airspeed for ideal autorotation (zero momentum power) is $V = \sqrt{2} \cdot v_{io}$ and occurs when $\alpha_R = 45^o$.

2.5 - REGRESSION ANALYSIS OF FLIGHT TEST DATA

In the energy model described in the previous section, the total main rotor shaft power required is obtained by summing the different power requirements. Several experimental coefficients are included in the model to account for induced power inefficiencies and variations in profile power with advance ratio and mean blade lift coefficient. These factors are determined by performing a multilinear regression analysis on the available flight test performance data. In practice only a limited number of test points are available and the regression technique only yields statistical estimates of the regression coefficients. Analysis of variance is therefore carried out to check the significance of the regression. Confidence limits corresponding to a given probability of error are established for the experimental coefficients.

Figure 4 shows a comparison between power required in hover OGE for the SA 349 experimental "Gazelle" equiped with standard moderatly twisted blades, and blades with augmented twist . Statistically significant differences were shown to exist between the total power required for the two sets of blades.

As one would expect, the highly twisted blades produced a more uniform velocity distribution and resulted in smaller induced power losses as demonstrated by the regression analysis.

In order to obtain statistically significant results, the flight test data should adequately cover the whole range of thrust coefficients and advance ratios. Small variations of the rotor speed about its nominal value, if included in the test data, will improve the description of the profile power coefficient.

Also high rotor angle of attack configurations at low and moderate airspeeds (as achieved with high rates of climb and descent) are required if the model is to be extended to maneuvering flight.

3 - APPLICATIONS OF THE ENERGY METHOD

In this section several applications of the energy method to current helicopter performance problems are reviewed. They have been chosen among the following topics :

- Dead-man zone of a single engine helicopter.
- Aircraft recovery following complete loss of power during climb at maximum continuous power and optimum climb speed.
- Level flight acceleration capability from hover.
- Flight path of a twin engine helicopter in category A Civil procedure after failure of one engine during take-off.
- Influence of the emergency power level on the maximum admissible hover skid height of a twin engine military helicopter.

Two versions of the computer program were set up.

In the first version, the collective pitch, main rotor tilt and engine shaft power supplied were fed as inputs to the program. Rotor speed and aircraft velocity were then obtained by integrating the torque balance equation and Newton's equations of motion.

In the second version, the reverse procedure was used. Aircraft velocity or acceleration time histories and engine power supplied were the inputs to the program and the torque balance equation was solved for rotor speed. It was then verified that the control laws required to achieve a given flight path remained within acceptable limits.

Whenever flight test recordings were available, the measured data was compared to the results predicted by the computer simulation.

3.1 - AS 350 DEAD MAN ZONE (fig. 5)

The demonstration of height velocity diagrams at different weights and altitudes probably constitutes one of the most demanding aspects of helicopter certification. In an area where flight testing is often dangerous and costly, analytical prediction methods could contribute to reduce significantly the number of flight tests required.

In the case of the single engine AS 350 "Astar", the demonstration of the dead-man zone at sea level and maximum design gross weight of 1900 kg was established with 15 test points mostly concentrated in the lower take-off portion and around the critical knee of the curve (fig. 5).

In order to illustrate the application of the energy method to this problem, the three characteristic points of the diagram, namely the two hover points and the knee, were determined using the flight path simulation program.

3.1.1 - Low hover point (fig. 7)

Flight tests have demonstrated that with proper action on the collective pitch a pilot could safely recover from an engine failure occuring while the helicopter was hovering 3 meters above the ground.

A simulation was carried out by reproducing the pilot action on the collective pitch lever and the decay in shaft power supplied following engine failure.

From figure 7 it can be seen that the predicted time histories of sink speed (in particular its value at impact), rotor RPM and load factor are in good agreement with the measured data.

Touchdown occured 2 seconds after engine failure at a vertical impact velocity of − 0.9 m/s. The helicopter had lost 3 m altitude and the rotor RPM had dropped down to 70% of its nominal value. Thus approximately half of the kinetic energy stored in the main rotor had been used up in the flare.

3.1.2 - Knee of the diagram (fig. 6)

In this case accurate tracking facilities were available on the test site to record the flight path along with the parameters recorded on board the aircraft.

The horizontal and vertical components of velocity were chosen as inputs to the program along with the decay in shaft power following engine failure.

As indicated in figure 6, the predicted variations in rotor RPM, collective pitch and aircraft attitude correlate well with those recorded in flight.

At the time of the engine failure the aircraft was flying at 55 Kts approximately 100 ft above the ground. The pilot initially lowered the collective pitch a few degrees in order to keep the rotor speed up. The cyclic stick was then slowly pulled back to reduce airspeed and, towards the end of the test (approximately 10 s after power loss), collective pitch was applied progressively and the helicopter landed horizontally at approximately 35 Km/h.

3.1.3 - High hover point (fig. 8)

The high hover point of the dead-man zone was demonstrated at 800 ft. However, owing to a failure of the recording equipment on board the aircraft, the only measured data available for this test was the recording of the flight path made by the ground tracking station. The horizontal and vertical components of velocity were fed into the simulation program and it was verified that variations in rotor RPM and control inputs (collective pitch and aircraft attitude) remained within acceptable limits.

The flight path can be analysed into three phases ;

- In the first phase the helicopter is tilted nose down and accelerates in a steep dive from hover to about 100 Km/h which corresponds to the airspeed at the knee of the H/V diagram.

With the collective pitch fully lowered a high rate of descent is reached (approximately 20 m/s) and the rotor RPM speeds up to 120% of nominal value.

- In the second phase the pilot pulls on the cyclic stick to return the aircraft to a level attitude thereby reducing sink speed to a value corresponding to the stabilized power-off rate of descent at knee airspeed (about 6 m/s).

  During this phase, collective pitch is progressively applied and the rotor overspeed is reduced.

- In the third and final phase, the flare is initiated by pulling the aircraft into a high nose-up attitude and progressively applying collective pitch in order to decelerate while continuing to reduce sink speed so as to land horizontally at approximately 35 Km/h.

3.2 - ENGINE FAILURE DURING CLIMB (fig. 9)

Various flight tests are commonly undertaken to evaluate the consequences of an engine failure on aircraft maneuverability. These tests cover extreme flight configurations such as descent at maximum authorized speed (VNE) or climb at maximum continuous power and optimum climb speed. This last configuration is one of the most severe to recover from in case of engine failure. The maneuver is reproduced on figure 9. A one second delay was observed before the pilot lowered the collective pitch causing the load factor to drop down to 0.15. During the maneuver, the aircraft was kept in a level pitch attitude so as to maintain airspeed at 100 Km/h.

Five seconds after engine failure the aircraft had reached a rate of descent of the order of 10 m/s with the RPM stabilized around 90% of nominal RPM. As seen on figure 9 correlation with test results is very good.

3.3 - LEVEL FLIGHT ACCELERATION FROM HOVER (fig. 10)

The SA 349 Z is an experimental helicopter derived from the SA 342 "Gazelle" with a modified main gearbox capable of 550 KW (100% torque) for one hour (in totalized time) and 440 KW (80% torque) in maximum continuous power. Figure 10 shows the acceleration obtained by applying 80% torque. At the start of the maneuver (0 – 70 Km/h) the acceleration is limited by the steep nose-down attitude of the aircraft (– 25°) and the pilot is incapable of applying maximum torque.

Subsequently, as power required increases, maximum torque is applied and the nose-down pitch attitude progressively reduced as airspeed builds up (100 Km/h are reached in 7.5 seconds and 200 Km/h in 23 seconds).

The aircraft velocity time history recorded by laser trajectography was fed into the simulation program. Since the engine fuel control system maintained a constant RPM during the acceleration, the engine power supplied was equated to the total power required for the maneuver. Collective pitch variations and engine shaft power supplied are in reasonably good agreement with measured data.

3.4 - CATEGORY A EMERGENCY PROCEDURE (fig. 11)

Figure 11 shows the flight path followed by a twin engine SA 330 "Puma" helicopter after failure of one engine during take-off from a platform. Failure of the engine occured 2 seconds after the pilot had initiated the acceleration into forward flight by tilting the A/C nose down in a standard take-off procedure.

As the power supplied by the remaining engine climbed to maximum contingency level, the pilot momentarily reduced collective pitch while continuing to accelerate towards the take-off safety speed (VTOSS). As this speed was reached the aircraft began a steady climb at approximately 3 m/s R/C. The recorded altitude loss during the acceleration was 25 m. The computed value is 23 m. The overall correlation between test and simulation is generally satisfactory although the predicted R/C at VTOSS is overestimated.

In FAR 29 Category A procedure, the maximum authorized weight at take-off is always less than the maximum weight in hover OGE.

It is determined to allow a safe recovery of the aircraft in the event one of the engines failed during take-off.

Helicopters usually operate from three different types of heliports : clear airfields, helipads and platforms. Depending on the type of heliport, the critical part of the take-off emergency procedure may be the loss of altitude during the acceleration to VTOSS at maximum contingency power, the R/C at optimum climb speed and intermediate contingency power or even the landing phase if the pilot is required to abort take-off.

As illustrated in the previous example the energy method can be applied to simulate with good accuracy, these emergency flight paths. It is thus a valuable tool in the prediction of civil helicopter performance in category A operations.

3.5 - EMERGENCY POWER LEVEL OF A TWIN ENGINE MILITARY HELICOPTER (fig. 12)

Modern antitank helicopters are intended to be operated in NOE flight and to hover undetected behind natural obstacles such as a screen of trees. As a result they will spend a substantial amount of time in the restricted portion of the H/V diagram. These are very critical flight configurations since, should one engine fail, the remaining engine will be required to deliver a large amount of additional power in a very short time.

The consequences of an engine failure occuring while the helicopter is hovering at low altitude have been examined as a function of the emergency power level (fig. 12).

- If, at the time of the engine failure, the helicopter is hovering high enough above the ground in an area free of obstacles, the pilot may accelerate into forward flight. The height required to clear the ground decreases as the emergency power level of the remaining engine increases.

– If the helicopter is too low or if the pilot is hindered by a screen of trees, a vertical landing is necessary. Rupture of the landing gear will occur and the helicopter will crash if the vertical velocity at impact exceeds 6 m/s. Between 2.5 m/s and 6 m/s impact velocity permanent skid damage will result preventing the helicopter from taking-off again. Below 2.5 m/s a safe landing is possible.

The emergency power level appears on figure 12 as a percentage of the power delivered by each engine in hover OGE before the engine failure. Note that above 170% the restricted area of the H/V diagram disappears.

4 - AUTOROTATION CRITERIA

Before concluding this survey of energy methods a few words will be said about autorotation criteria.

Several autorotation indices are commonly used by helicopter manufacturers to evaluate power-off autorotational characteristics (cf : T.L. WOOD, "High Energy Rotor System", presented at the 32nd Annual National Forum, of the American Helicopter Society, May 1976).

These indices, where rotor kinetic energy plays a major role, are well correlated with pilot ratings of the autorotational characteristics of various helicopters.

As will now be shown the autorotation index defined as "Rotor kinetic energy/Engine shaft power required in hover OGE" is directly related to the rotor speed time constant in hover. This constant determines the initial response of the rotor speed to a perturbation in the main rotor torque balance.

Rotor speed time constant in hover

In this section the acceleration characteristics of a rotor in response to a small perturbation in torque are derived. The analysis is greatly simplified if it is restricted to hover (rotor mounted on a test tower) and if the engine shaft torque supplied is assumed to remain constant at a value equal to the initial torque required before collective pitch was applied.

The rotor acceleration is governed by the torque balance equation :

$$I \mathring{\Omega} = Q_S - \rho S R^3 \Omega^2 C_p \tag{16}$$

If linearized for small perturbations :

$$I \delta \mathring{\Omega} = \delta Q_S - Q_S \left(2 \frac{\delta \Omega}{\Omega} + \frac{\delta C_p}{C_p}\right) \tag{17}$$

In hover the power coefficient is directly related to the thrust coefficient by equation

$$C_p = K_i C_T \sqrt{\frac{C_T}{2}} + \frac{\sigma C x_p}{8} \tag{18}$$

It can be shown using simple blade element theory that the thrust coefficient in hover is independant of rotor speed and that it is uniquely determined by the collective pitch setting. Therefore the power coefficient is also independant of rotor speed and

$$\delta C_p = \frac{\partial C_p}{\partial C_T} \cdot \frac{\partial C_T}{\partial \theta_o} \delta \theta o \tag{19}$$

Under the assumption of constant engine shaft torque, the small perturbation equation reduces to :

$$I \left(\delta \mathring{\Omega} + \frac{P_R}{\frac{1}{2} I \Omega^2} \delta \Omega \right) = - Q_S \frac{\delta C_p}{C_p} \tag{20}$$

This is characteristic of a first order system of time constant :

$$\tau = \frac{\frac{1}{2} I \Omega^2}{P_R} \tag{21}$$

Note that if a constant engine shaft power had been assumed a smaller rotor speed time constant would have been obtained (2/3 of the value corresponding to a constant engine shaft torque).

Figure 4 shows the initial sharp rise in load factor produced by the collective pitch input. The resulting increase in torque causes the rotational speed to decay exponentially to a new equilibrium value with a corresponding decay in rotor thrust. The rotor speed time constant which mesures the time to reach 63% of the steady state RPM value can be read on the rotor speed time history. It is found to be 3 s.

The power to be considered in the autorotation index is the total engine shaft power required when the helicopter is hovering out of ground effect. It therefore includes the losses associated with driving the tail rotor, gearboxes and accessories. For most helicopters these losses represent 15 % to 20 % of the engine shaft power supplied. The autorotation index is thus 80 % to 85 % of the rotor speed time constant in hover.

It is generally found that poorly rated helicopters experience a much more rapid decrease in rotational speed following engine failure than helicopters rated as having good autorotational characteristics. An autorotation index in excess of 2 s usually corresponds to a good pilot rating.

5 - CONCLUSION AND RECOMMANDATIONS

The energy method offers an accurate and simple approach to the prediction of helicopter performance in maneuvering flight. It is well suited to a wide variety of applications.

At the design stage the method can help in the selection of aircraft parameters to comply with maneuver requirements included in design specifications. During the development phase significant savings in the time and cost of flight test programs could be achieved by making use of predicted results.

Several improvements and extentions of the method are desirable. The calculation of rotor shaft power should be refined particularily at moderate airspeeds (between hover and best climb speed) and at high rotor angles of attack where, very often, only a few test points are available. Further investigation of the effects of stall and compressibility on the profile power requirements should also be undertaken.

Statistical data fitting techniques such as multilinear regression are very useful in this respect and should be developed to analyse wind tunnel and flight test data.

Finally it is suggested that helicopter performance in stabilized flight be obtained from maneuvering flight performance data by measuring the aircraft accelerations, rate of climb and RPM variations and determining the associated stabilized flight configuration.

Appendix A

Equations of motion

Rotor thrust

From simple blade element theory :

$$T = \frac{1}{2} \rho S \sigma U^2 C'_Z \left[(E_3 + \frac{1}{2} E_1 \mu^2) \Theta_o - E_2 \lambda_{NFP} \right] \qquad \text{(A1)}$$

λ_{NFP} is the inflow ratio relative to the no-feathering plane perpendicular to the control axis.

E_n are non-dimensional blade planform integrals defined as :

$$E_n = \int_{x_o}^{B} \frac{C}{C_{0.7}} \frac{x}{R}^{n-1} dx \qquad \text{(A2)}$$

X_o & B are the blade lift spanwise integration limits. The inflow ratio relative to the tip path plane is given by :

$$\lambda_{TPP} = \lambda_{NFP} - \mu a_1 \qquad \text{(A3)}$$

Where a_1 is the longitudinal (rearward) tilt of the TPP relative to the control axis due to forward speed.

$$a_1 = \frac{2 E_3 \Theta_o - E_2 \lambda_{NFP}}{E_4 - \frac{1}{4} E_2 \mu^2} \cdot \mu \qquad \text{(A4)}$$

Main rotor shaft power required

$$P_R = \rho S U^3 \left(C_T \lambda_{TPP} + \frac{\sigma C_{Xp}}{8}\right) \tag{A5}$$

$$\lambda_{TPP} = K_i \frac{v_i}{U} - \frac{V}{U} \sin \alpha_R \tag{A6}$$

If speeds are referenced to the theoretical induced velocity in hover v_{io}

$$P_R = T v_{io} \left[\frac{K_i v_i - V \sin \alpha_R}{v_{io}} + \frac{\sigma C_{Xp}}{16} \left(\frac{U}{v_{io}}\right)^3 \right] \tag{A7}$$

The theoretical induced velocity ratio is given by the momentum equation :

$$1 = \frac{v_i}{v_{io}} \sqrt{\left(\frac{v_i - V \sin \alpha_R}{v_{io}}\right)^2 + \frac{V^2 \cos^2 \alpha_R}{v_{io}^2}} \tag{A8}$$

The mean blade lift coefficient is defined by equation A9 :

$$C_{Zm} = \frac{6 C_T}{\sigma} = \frac{12}{\sigma} \left(\frac{v_{io}}{U}\right)^2 \tag{A9}$$

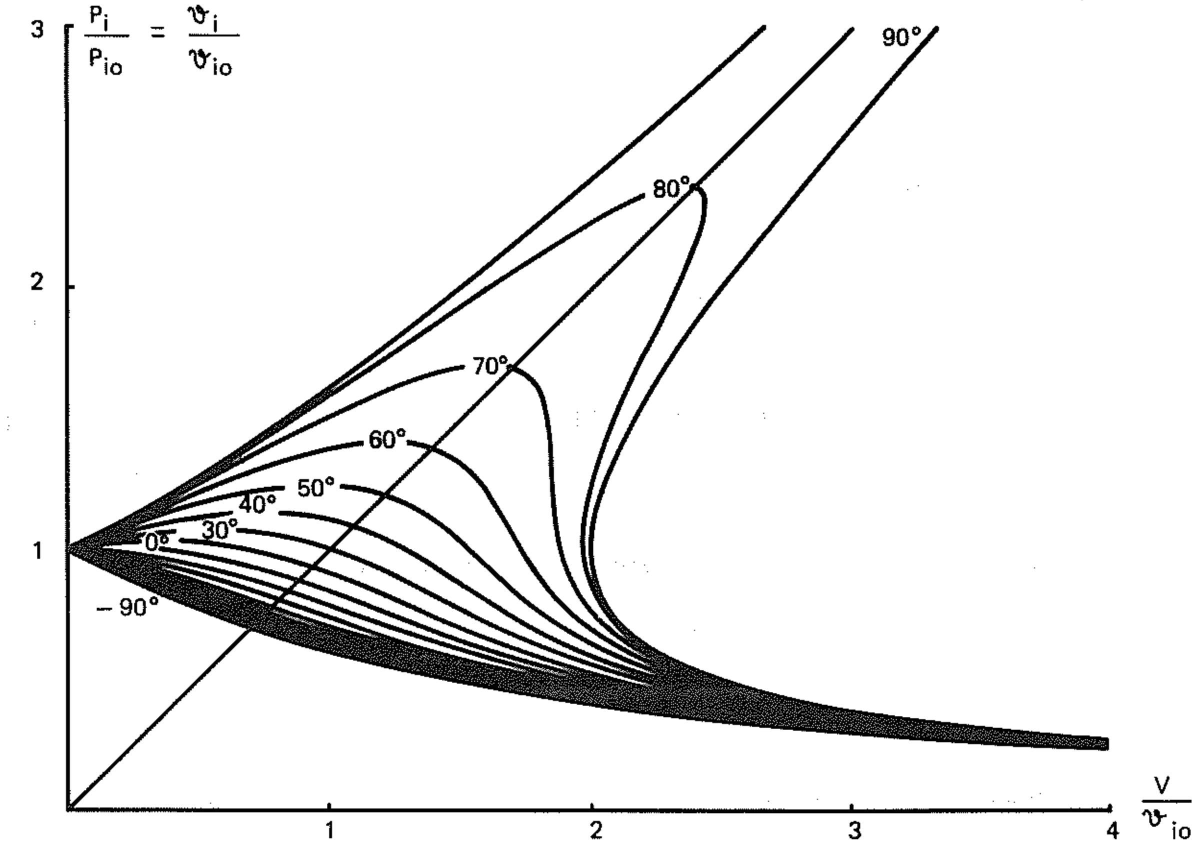


FIG 2 THEORETICAL INDUCED POWER RATIO VS AIRSPEED RATIO

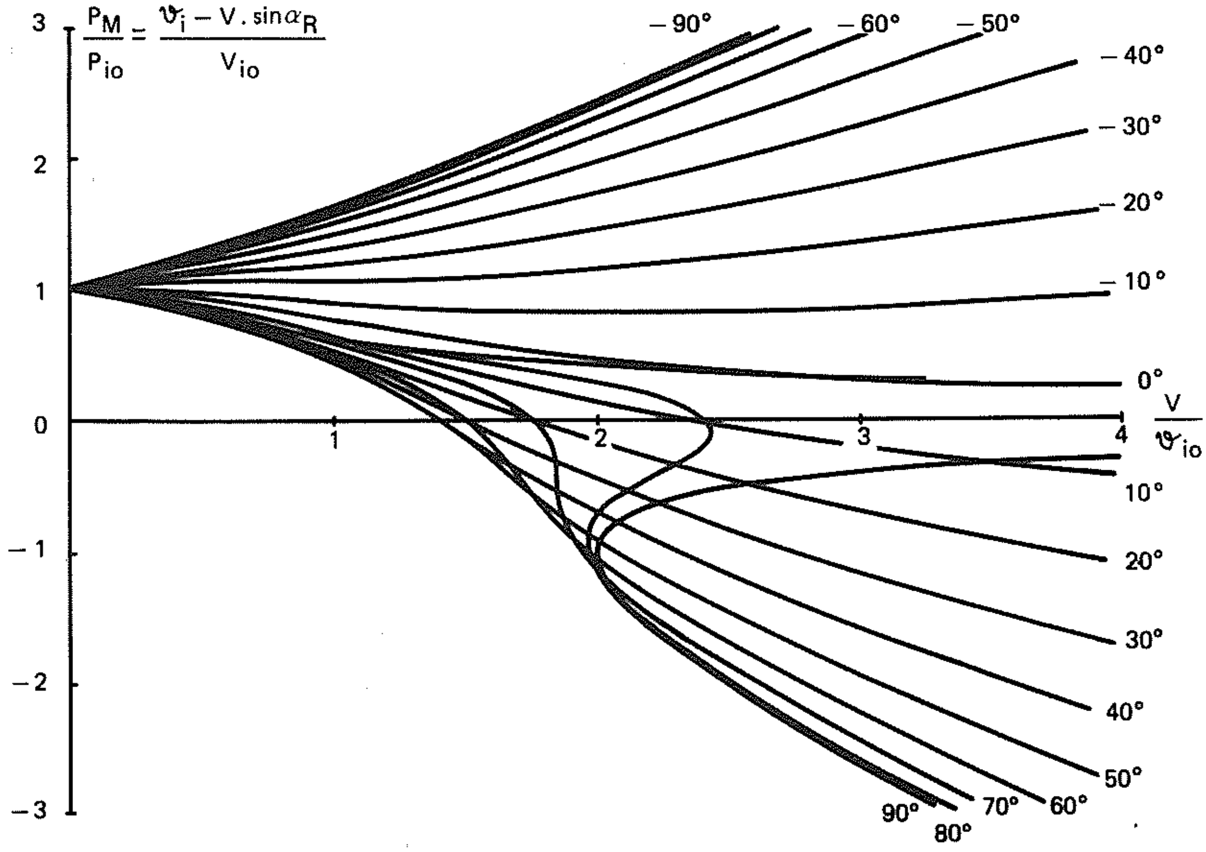
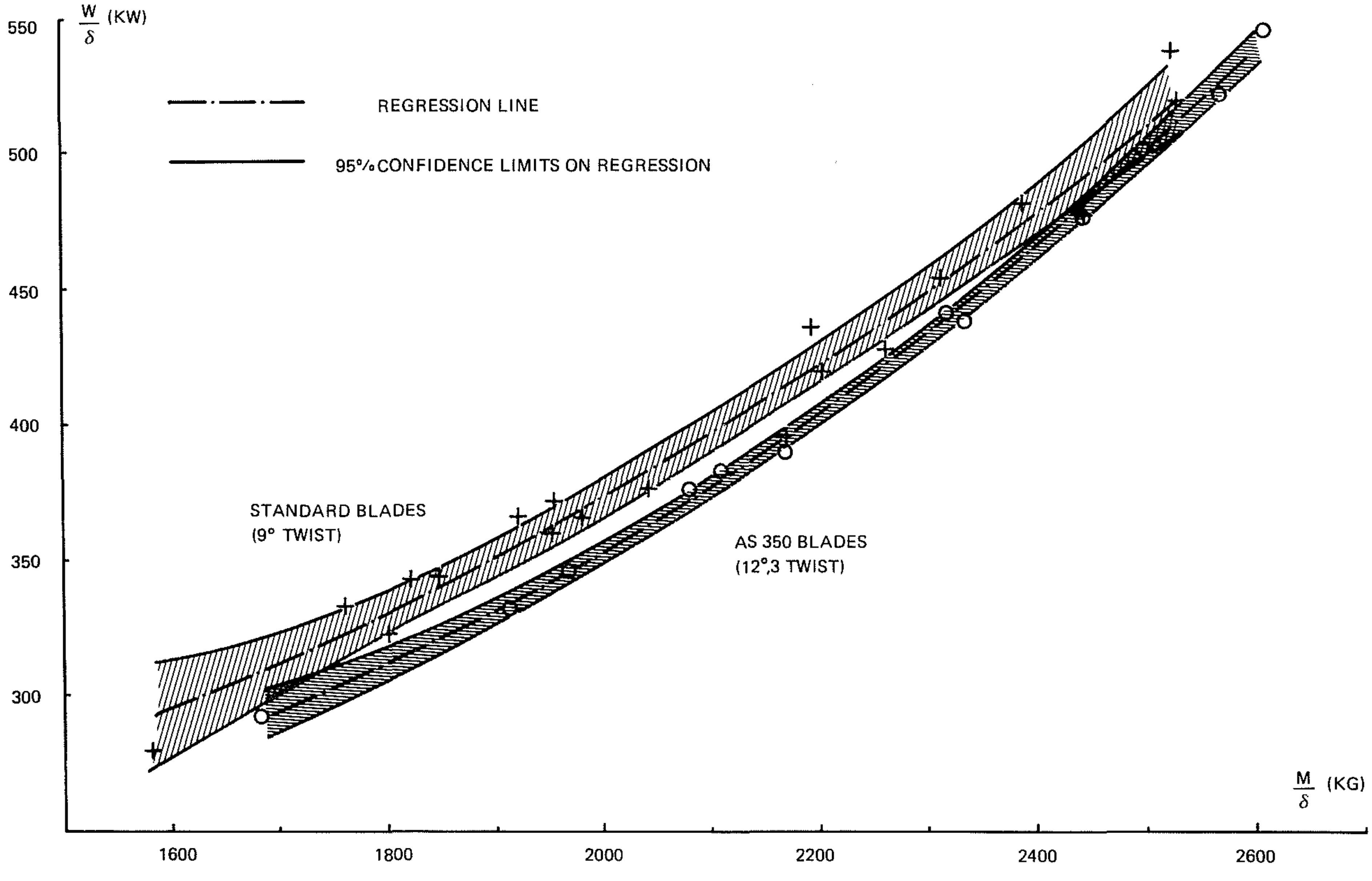


FIG 3 THEORETICAL MOMENTUM POWER RATIO VS AIRSPEED RATIO

FIG 4 SA 349 POWER REQUIRED IN HOVER-OGE VERSUS WEIGHT



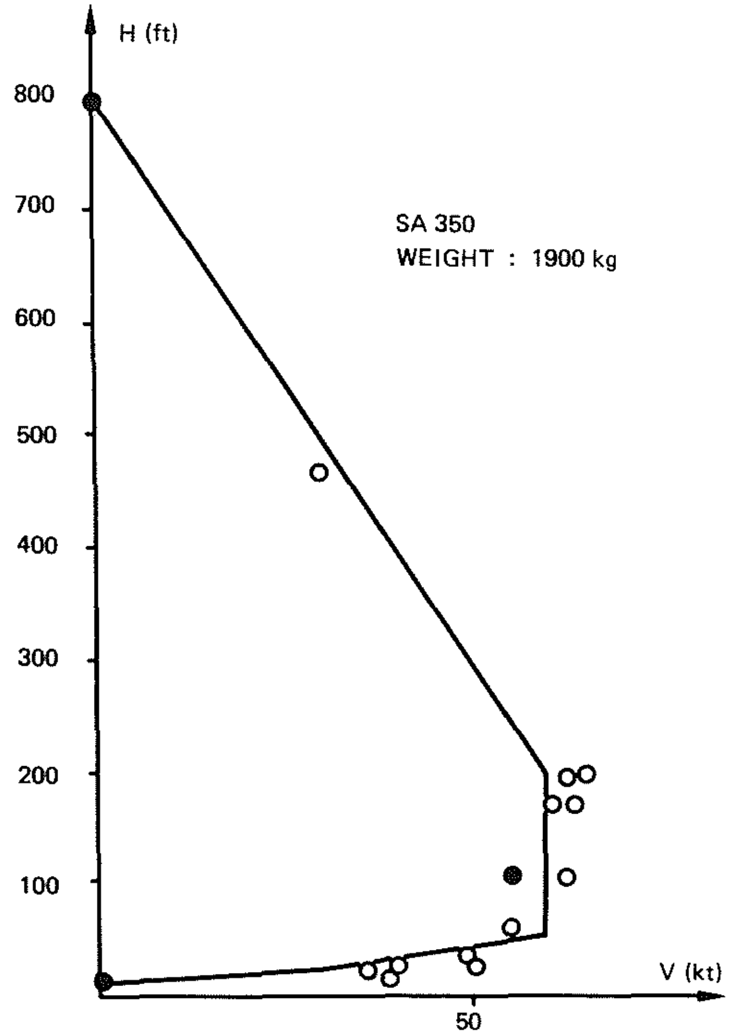


FIG 5 HEIGHT - VELOCITY DIAGRAM

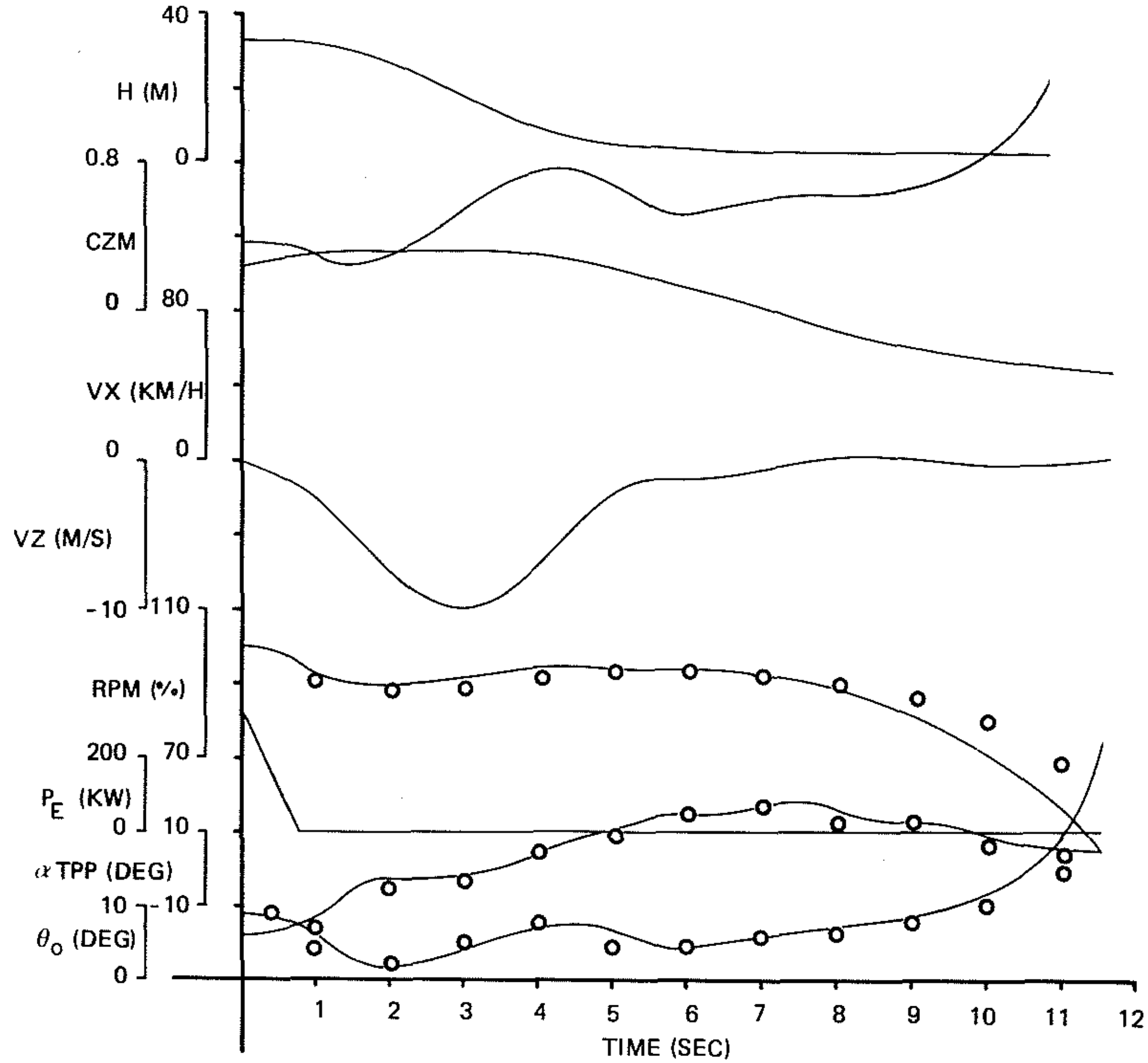


FIG 6 SA 350 - KNEE POINT

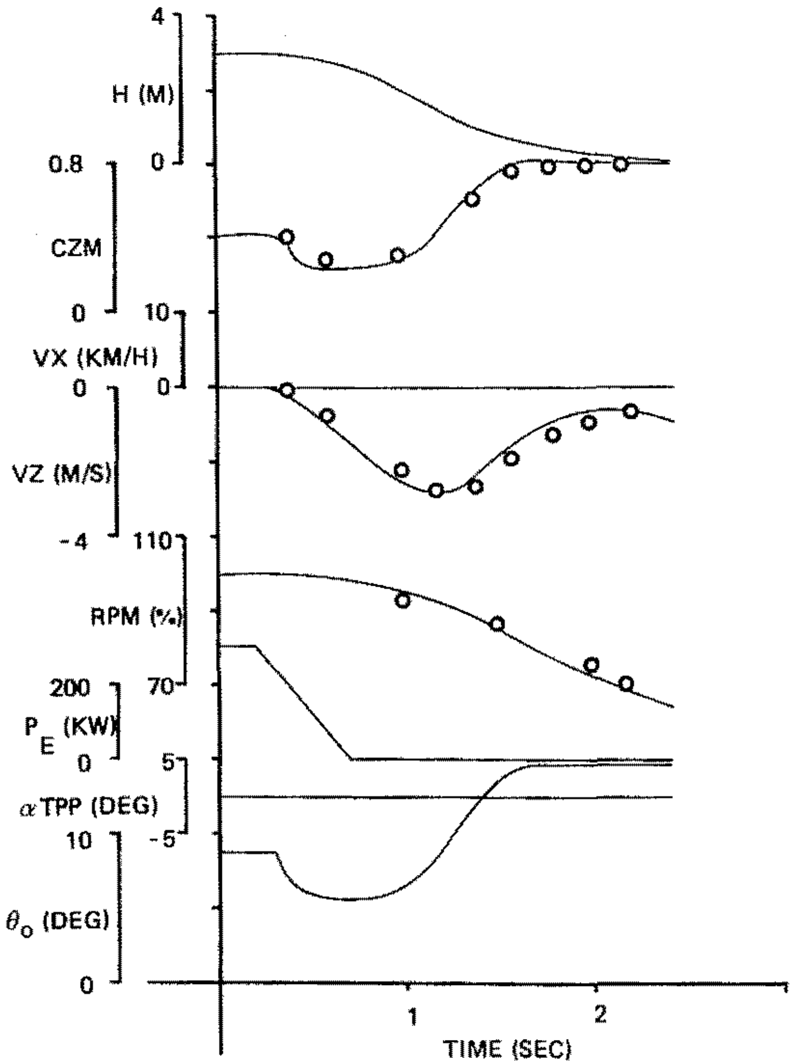


FIG 7 SA 350 - LOW HOVER POINT

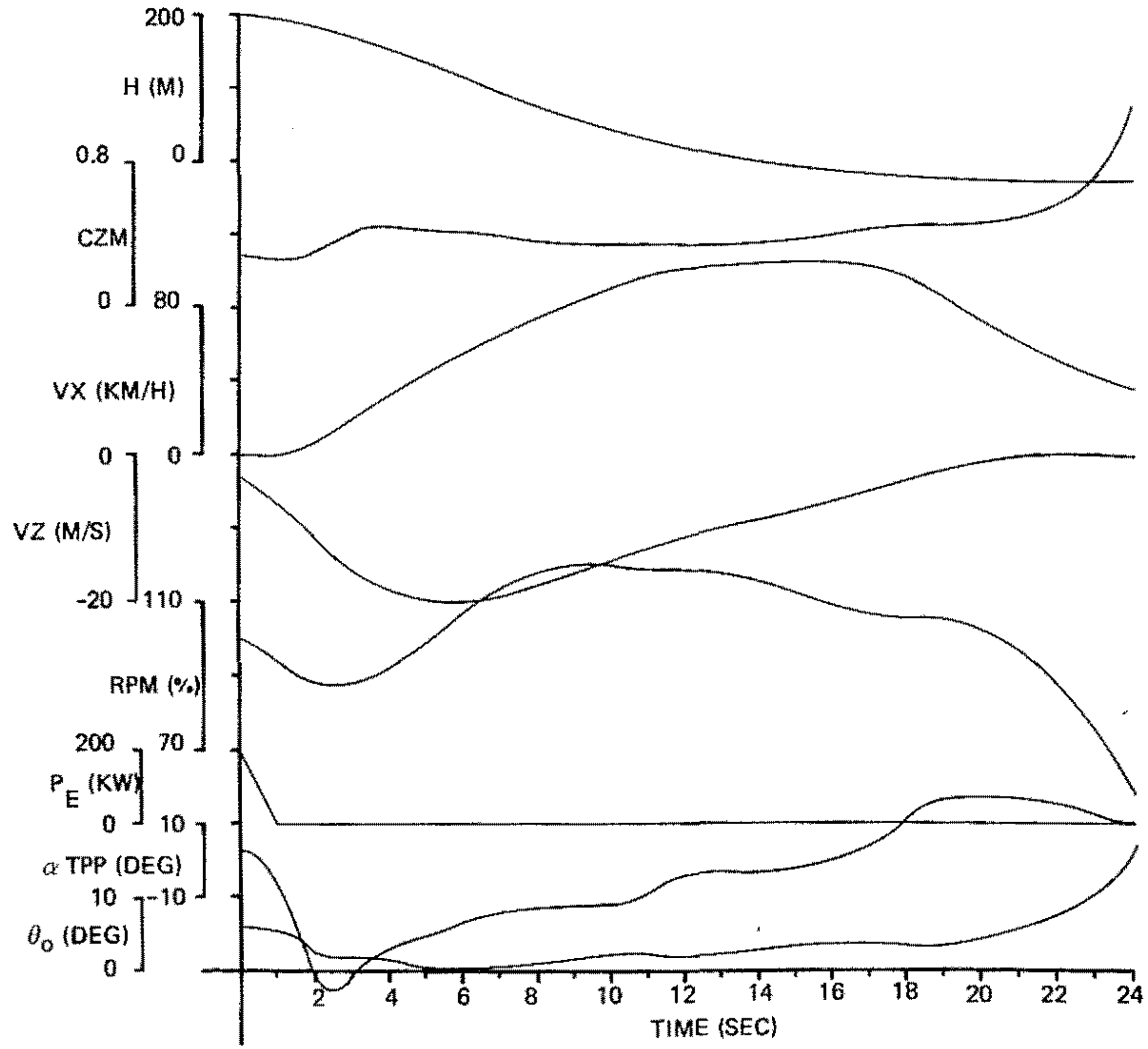


FIG 8 SA 350 - HIGH HOVER POINT

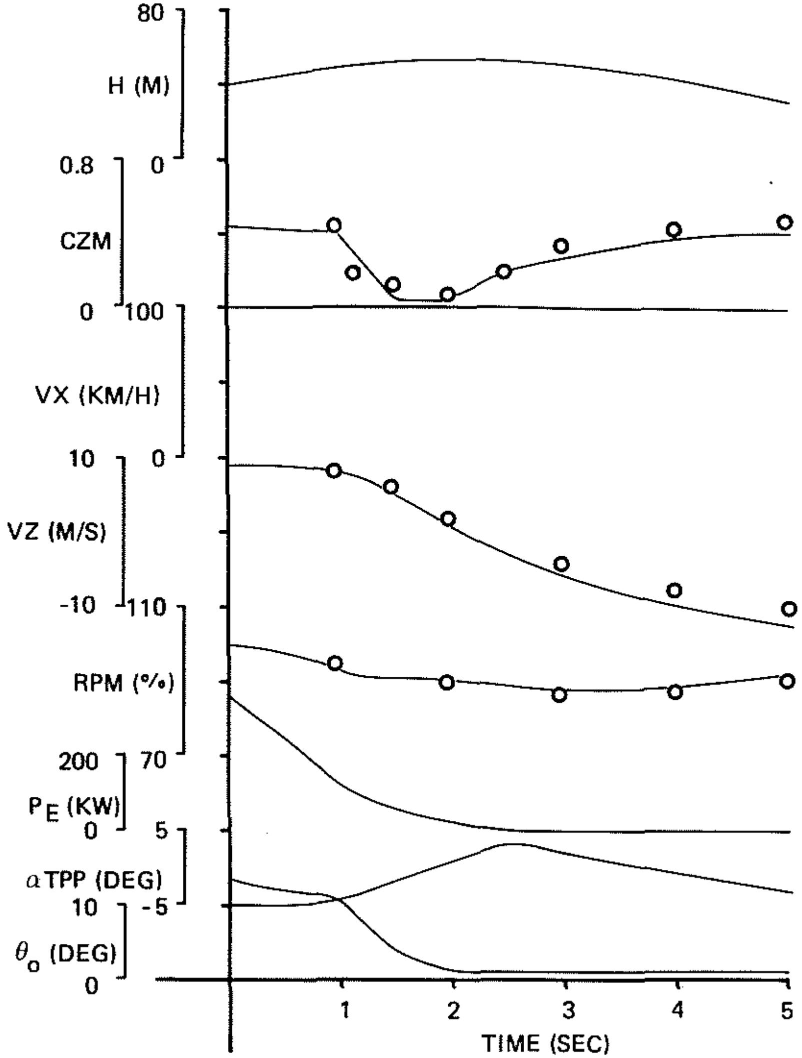


FIG 9 SA 350 - ENGINE FAILURE IN CLIMB

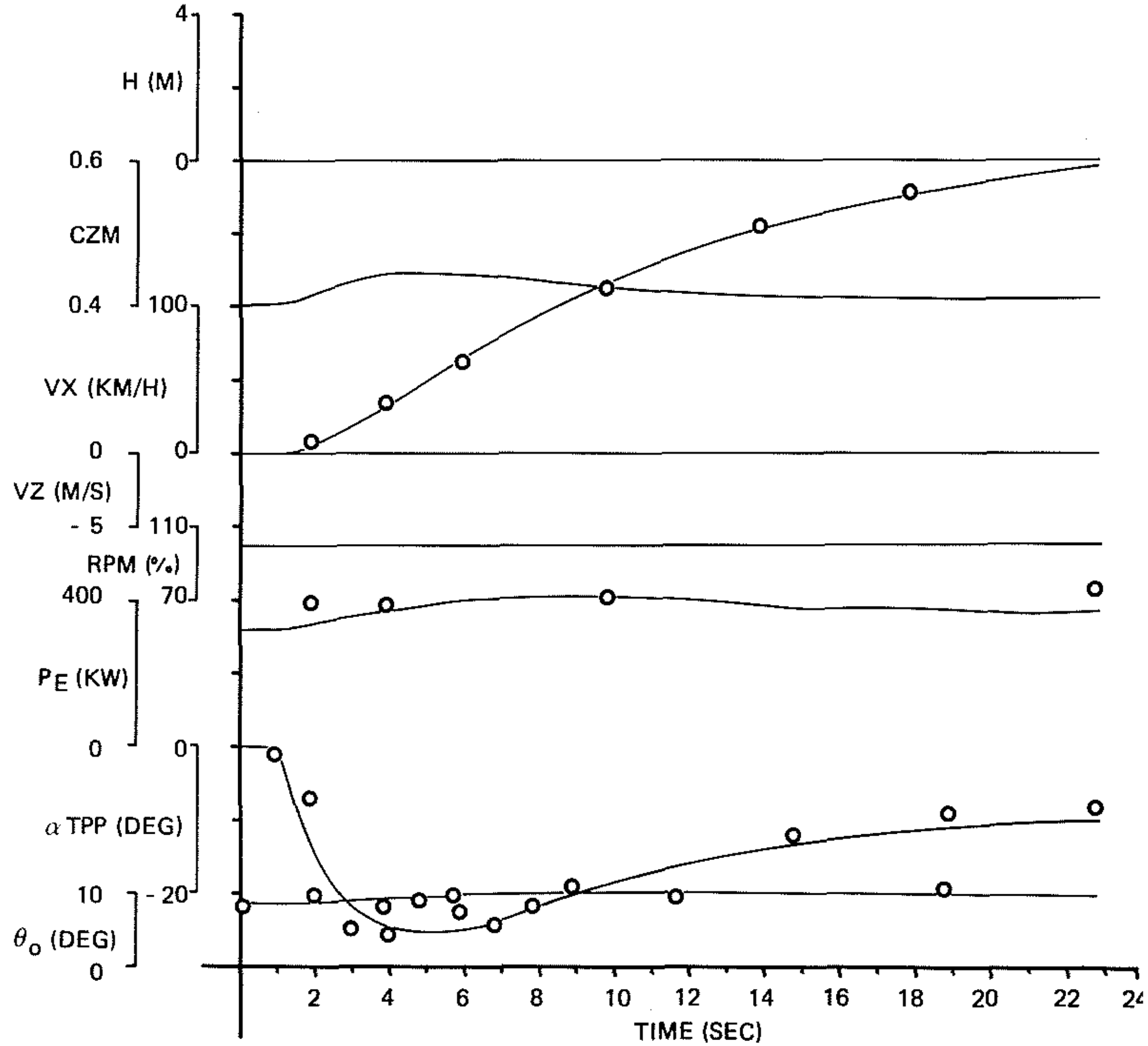


FIG 10 SA 349 - LEVEL FLIGHT ACCELERATION

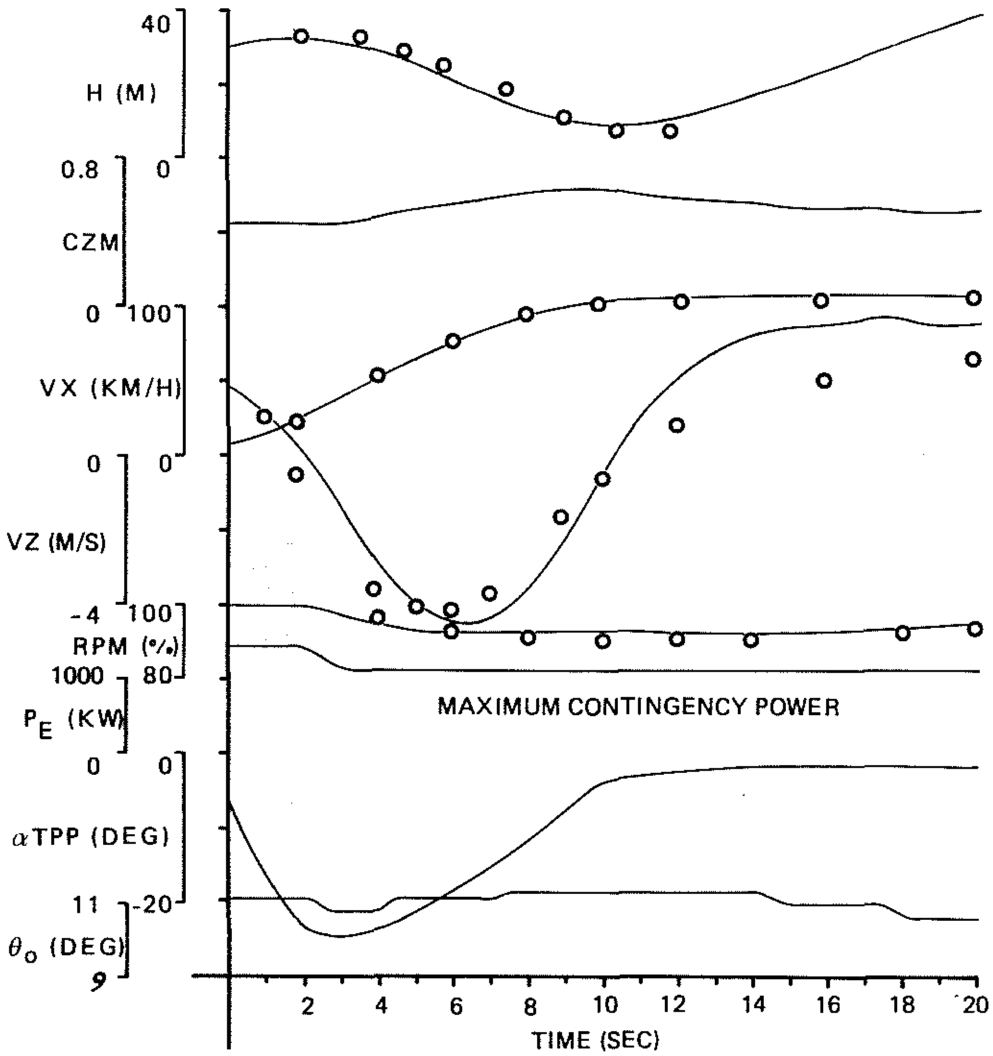


FIG 11 SA 330 - EMERGENCY PROCEDURE FAILURE OF ONE ENGINE DURING TAKE-OFF

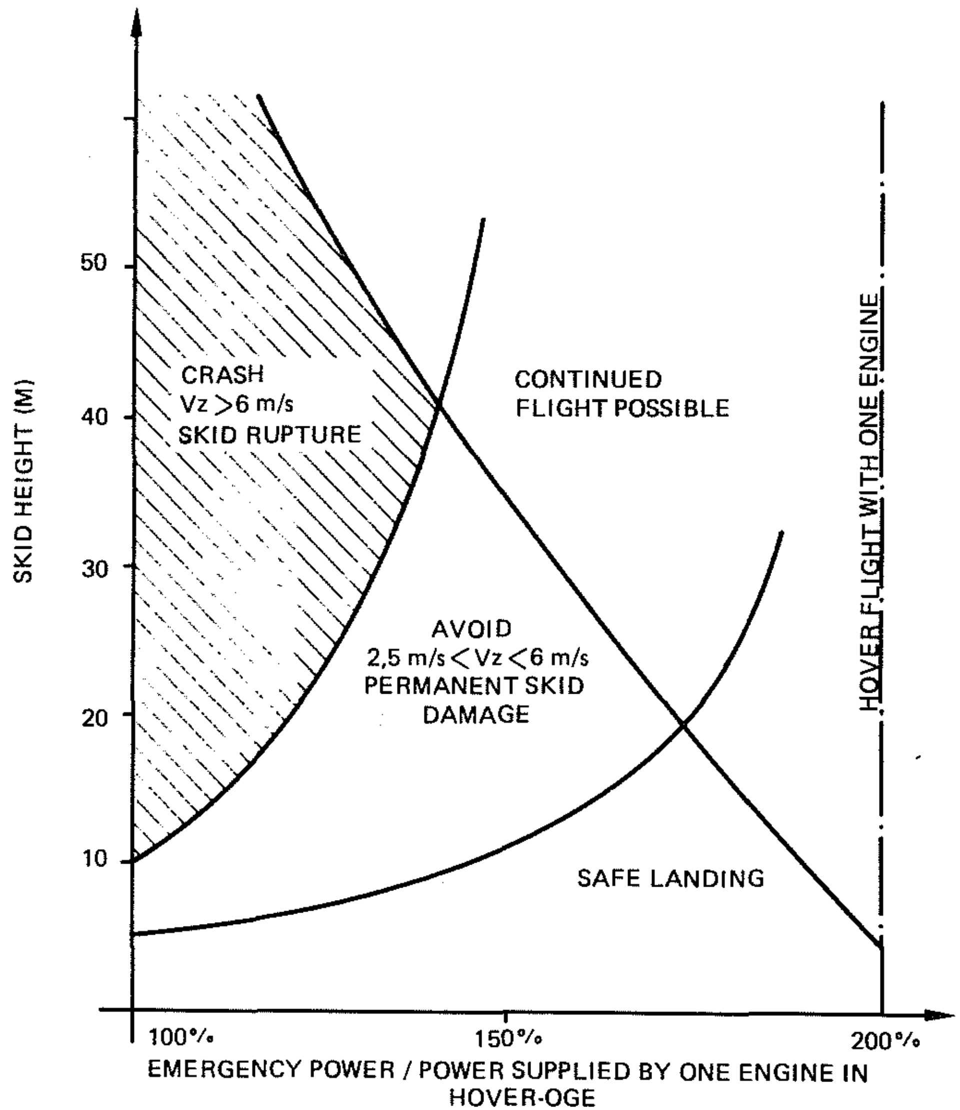


FIG 12 LEVEL OF EMERGENCY POWER

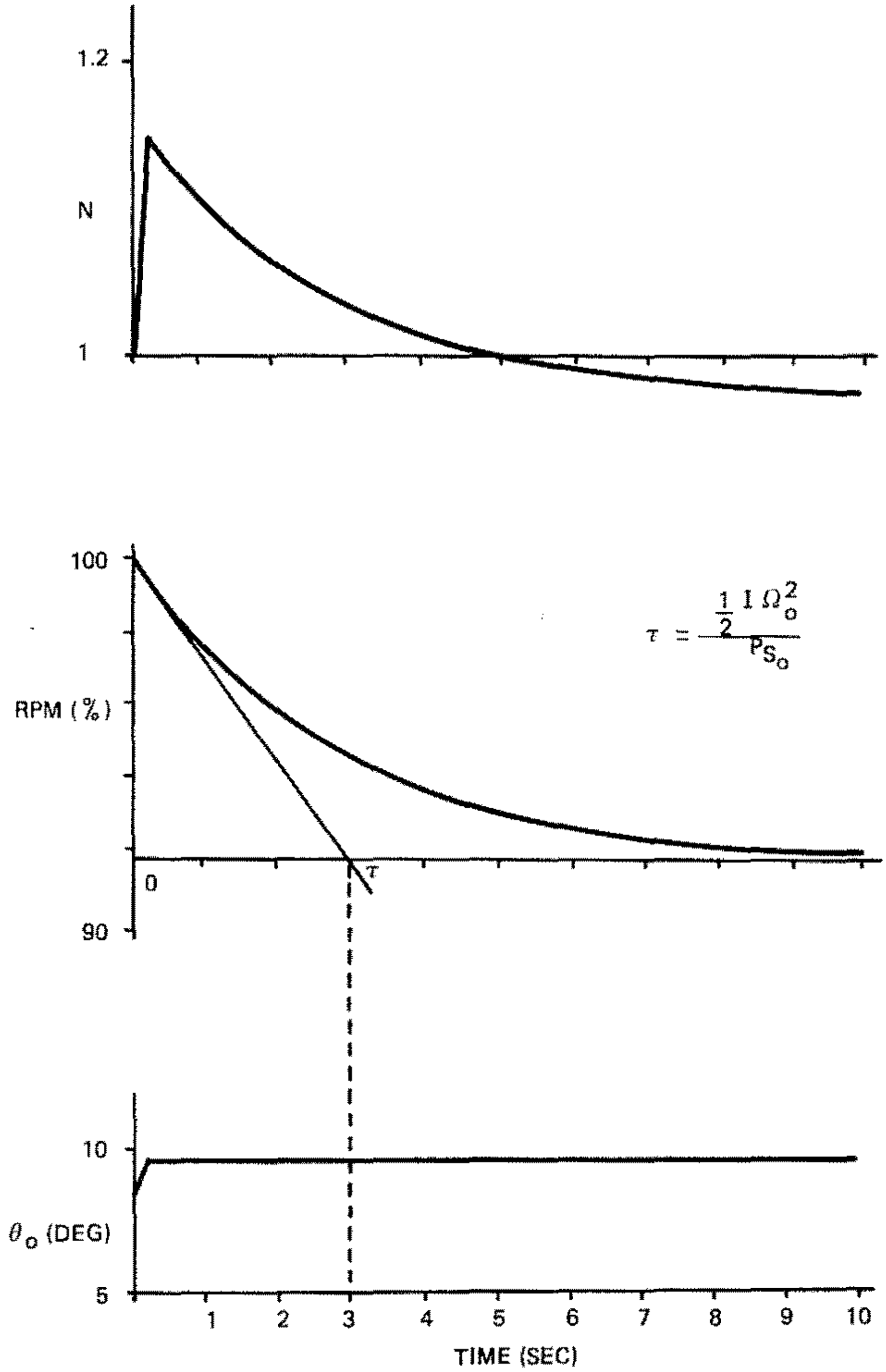


FIG 13 ROTOR SPEED TIME CONSTANT IN HOVER (CONSTANT ENGINE TORQUE SUPPLIED)