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INVESTIGATION OF HELICOPTER ROTOR BLADE MOTION STABILITY BY DIRECT LIAPUNOV METHOD

by

J. NARKIEWICZ and W. LUCJANEK
Warsaw Technical University
Warsaw, Poland

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INVESTIGATION OF HELICOPTER ROTOR BLADE MOTION STABILITY BY DIRECT LIAPUNOV METHOD

Janusz Narkiewicz and Wieslaw Lucjanek
Warsaw Technical University, Warsaw, Poland

Abstract

The direct Liapunov method was applied for helicopter rotor blade motion stability analysis. The algorithm and computer code are based on the theorem which gives sufficient conditions for boundedness of the solution for nonlinear ordinary differential equation system with periodic right hand sides. The computer program developed allows to determine the regions of instability for helicopter hover and forward flight. The stability of rigid, articulated rotor blade was considered. The obtained results show good qualitative agreement with phenomena observed in helicopter blade motion.

1. Introduction.

Problems which are concerned in the helicopter theory are usually of interdisciplinary type. This fact is particularly evident in rotor aeroelasticity where aerodynamic, dynamic and stiffness loadings act simultaneously [1]. If all these loadings were modeled in "the exact manner", the problem would be difficult to be solved, even numerically. This is the main reason for which simplifying assumptions are necessary.

The most general physical models lead to mathematical description in term of partial differential equations which are usually difficult to solve. Therefore the problem has to be simplified. Some way of discretization and the proper aerodynamic modeling allow to transform mathematical model to ordinary differential equations which are easier for analysis.

Usually in rotary wing problems, right hand sides of equations of motion are periodic with respect to time or azimuth angle due to the periodic excitation and cyclic blade pitch.

For these reasons rotor blade aeroelastic problems can be described by the system of equations of motion in the form:

$$\bar{z} = h(\psi, z), \tag{1}$$

where: $h(\psi+T,z) = h(\psi,z)$,
z - generalized coordinates,
ψ - azimuth angle,
T - period of excitation,
$(\bar{\ })$ - differentiation with respect to azimuth.

The solution of these equations usually can not be obtained in the closed form and some kind of numerical work is needed.

There are *two* kinds of analysis of the equations of physical system motion.

First one is *to solve* these equations by one of numerical codes, supplied by software companies. This analysis leads to approximate solution of equations, but only in the finite period of time and for limited number of initial conditions.

Second one is *qualitative analysis* of solution, aimed to obtain strictly defined information about solution properties without solving the equations [2]. The methods of this kind, commonly used in rotor analysis, are based on Floquet theorem. They require a linearization of system (1).

There are some disadvantages created by linearization. The steady motion for which linearization is done, should be known. Since equations of motion are linearized, some of their properties can disappear (for instance limited cycles). The linearized equations are valid only in the near neighborhood of the point where linearization has been done. The "large displacements and large velocities" are excluded from analysis.

These facts suggest looking for new methods of the system (1) analysis allowing to cancel some of the mentioned restrictions.

The method presented in this work is aimed to analysis of the boundedness of system (1) solution without linearization. This property of solution is called "technical stability" or "stability in Lagrangian sense". For stability considerations any steady motion, periodic with respect to azimuth ψ can be chosen. The analysis and the numerical algorithm is based on the direct Liapunov method.

The method was successfully applied to helicopter rotor blade stability analysis.

2. Theoretical background.

The solution of the system (1), stability of which is to be investigated, should be defined before starting the analysis. This solution we call "steady state motion".

Let $\xi(\psi)$ be the arbitrary chosen steady state solution periodic to ψ

$$\xi(\psi) = \xi(\psi + T). \tag{2}$$

The blade motion generalized coordinates $z(\psi)$ can be written in the form:

$$z(\psi) = x(\psi) + \xi(\psi), \tag{3}$$

where $x(\psi)$ are the coordinates of disturbed motion.
By putting (3) into (1), the system (1) can be transformed to the form:

$$x = f(\psi,x), \tag{4}$$

where right hand sides of (4), due to (2), are periodic with respect to ψ:

$$f(\psi+T,x) = f(\psi,x). \tag{5}$$

This transformation allows to investigate the stability of null solution (4) i.e.

$$x = 0, \tag{6}$$

instead of investigating stability of $\xi(\psi)$.

For ordinary differential equations there are several definitions of solution stability [3]. They are related to different properties of solution. The property of solution, we investigate in this work, is called "technical stability" or "stability in Lagrangian sense".

Def. The solution of the system is stable in Lagrangian sense, if:

$$\bigwedge \| x(\psi_0) \| < \lambda_0(\psi_0) \Rightarrow \| x(\psi) \| < A, \tag{7}$$

where λ_0, A - real, positive numbers, and $\lambda_0 \le A$.

It means (Fig.1), that for stable system if the solution starts from the appropriate neighborhood of initial conditions near the null solution $x=0$, it will stay close to $x = 0$ "forever".

The method and numerical algorithm for analysis stability of this kind is based on the following theorem [4]:

Theorem: If the system (4):

(a) has the right hand sides periodic in respect to ψ,
   i.e. there exists such $T > 0$ that: $f(\psi,x) = f(\psi+T,x)$,

(b) has the unique and continuous solution in the domain:

$$Q = \{ (\psi,x): \psi \in [0, \infty), \ \| x \| < H \}, \ H = \text{constant} > 0,$$

and there exists such Liapunov function $W(\psi,x)$ that in the Q domain it fulfills conditions:

(c) $W(\psi,x)$ is continuous and differentiable with respect to ψ and x,

(d) $W(\psi,x) > 0$ for $(\psi,x) \in Q$,

(e) $W(\psi,x)$ is periodic with respect to ψ,

(f) fist derivatives of $W(\psi,x)$ with respect to ψ and x are also periodic functions of ψ,

(g) if there exists $\lambda_0(\psi_0)$ such that $0 < \lambda_0(\psi_0) < A < H$ and

$$\sup_{\|x\| = \lambda_0(\psi_0)} W(\psi,x) < \inf_{\|x\| = A} W(\psi,x), \qquad \psi \in [0,T],$$

and in the domain:

$$Q_0 = \{ (\psi,x): \psi \in [0,T] \wedge 0 < \| x \| < A \}, \quad 0 < A < H,$$

(h) the derivative W' of Liapunov function W calculated along the system of equations (5) is not positive, i.e.:

$$W'(\psi,x) = \frac{\partial W}{\partial \psi} + \text{grad}\, [W(\psi,x)] \cdot f(\psi,x) \le 0,$$

then the solution $x(\psi)$ of the system (1) which fulfills condition:

$$\| x(\psi_0) \| < \lambda_0(\psi_0)$$

is stable i.e:

$$\| x(\psi) \| < A \quad \text{for each } \psi > \psi_0.$$

This theorem can be illustrated as it is shown in Fig.2 in the state space.

The methods of stability investigations based on properties of scalar function $W(\psi,x)$ are called "direct (or second) Liapunov methods" [5].

The assumptions (a) and (b) of the theorem concern the system of equations and are easily satisfied in practical applications. The condition (b) states, that there exists the solution of the system for all $\psi > \psi_o$ and $\| x \| < H$. The positive number H is the maximum value of the solution norm which can be taken into account.

The conditions (c) - (g) concern the Liapunov function $W(\psi,x)$. These are the main difficulties in the application of this theorem and Liapunov direct method.

There are no general rules for constructing Liapunov functions. It has to be assumed a priori and then its properties are investigated. The theory gives usually only sufficient conditions for solution stability.

Because of the problem periodicity, the conditions (g) and (h) may be checked only in one period of ψ. Thus the computations are limited only to one period of azimuth and the obtained results are valid for $\psi > \psi_o$.

3. Application of the theory.

The numerical algorithm and computer code were developed to analyze the stability of the system (4) based on the above given theorem. For each azimuth angle ψ_j the magnitude $\lambda_j(\psi_j)$ of stable solution initial conditions domain is calculated.

Before starting computations the conditions (a) and (b) of the system solutions existence and continuity should be checked. In practical problems these conditions are usually fulfilled.

The main steps of this algorithm are:

1. Assume the maximum value A of norm $\| x \|$. It means that the upper bound for displacements and velocities should be chosen. For greater flexibility of computations the norm of x is defined in the form:

   $$\| x \| = \sqrt{\sum (x_i/a_i)^2},$$

   i.e. the components of vector x are divided by assumed numbers a_i.
2. Chose the function $W(\psi,x)$ which fulfills the conditions (c)-(g).
3. Check the condition (h):

   $$W'(\psi,x) \le 0 \text{ for } \psi \in [0,T] \text{ and } \| x \| = A.$$

   If this condition is fulfilled, the motion *can be stable*; go to the step (6) of computation.
   If this condition is not fulfilled: compute the value A_1 such that the condition (h) is satisfied. If this value is not acceptable look for another Liapunov function W, otherwise put $A = A_1$ and go to next step of computations.
4. Compute the value:

   $$\alpha = \inf [W(\psi,x)] \quad \text{for } \psi \in [0,T] \quad \text{and} \quad \text{for } \| x \| = A.$$
5. For the set of azimuth values ψ_j (j=1,..N):

find the greatest value of λ_{oj} which satisfies the condition:

$$\sup_{||x|| = \lambda_o(\psi_{jo})} W(\psi_j,x) < \inf_{||x|| = A} W(\psi_j,x) = \alpha , \quad \psi \in [0,T].$$

If $\lambda_{oj} = 0$ *the motion is unstable* for ψ_j.

6. For the set of azimuth values ψ_j (j=1,..N) check if:

$$\dot{W}(\psi_j,x) \le 0 \text{ for } \lambda_{oj} < ||x|| = A.$$

If it is true, *the motion is stable.*

4. Application of the method to blade stability analysis.

The method was applied to the analysis of the motion of helicopter rotor blade in hover and in forward flight.

The main assumptions for physical model are (Fig.3):

1. A motion of isolated rotor blade is considered for helicopter insteady forward or vertical flight.
2. The rotor shaft angular velocity is constant.
3. The blade is stiff, fully articulated.
4. There are three hinges in the hub with perpendicular axis. These hinges are separated one from another by stiff elements. The hinges sequence is: flap, lag and pitch. In pitch hinge the blade collective and cyclic control is applied. Pitch - flap coupling is taken into account (Fig.3).
5. The equivalent blade stiffness, control stiffness and structural as well as artificial damping can be applied in each hinge.
6. The rigid blade is pretwisted, with different section airfoils and chord length along its span.
7. The aerodynamic loading is calculated with strip theory using two-dimensional, quasi-steady approximation. In each section drag, lift and aerodynamic moment are obtained using nonlinear steady aerodynamic coefficients for instant angle of attack.
8. The induced velocity is calculated using Glauert's formulae for hover and forward flight.

Three degrees of freedom: blade rotations in hinges were taken as generalized coordinates:

α_1 - flapping, α_2 - lagging, α_3 - pitching.

The equations of motion were derived with Lagrangian formulae using matrix calculus. All geometrical and aerodynamic nonlinearities were attained. The final system of equations consisted of three nonlinear second order ordinary differential equations with periodic right hand sides.

The sum of kinetic and potential energies was taken as Liapunov function. There were two reasons for it:

1) during the derivation of equations of motion both: kinetic and potential energies have to be determined; this fact simplified computer code constructing,
2) if the blade motion is unstable, its energy increases during the motion; the displacements and velocities increase too; this means, that assumption (h) of the theorem is not satisfied.

Although this function showed to be efficient in rotor blade applications, it should be understressed, that it is not the proof for the sum of kinetic and potential energies to be the best Liapunov function.

The angles of rotation in hinges for hover were taken into account as the rotor blade steady motion coordinates. This allowed to include into analysis also helicopter trim.

5. Sample results.

Only the limited amount of calculations was done for testing the method and proving its efficiency. The sample data concern medium size helicopter blade. The blade data are given in Table I.

Table I

Rotor radius	$R = 7.850$ m
Blade length	$R_1 = 6.772$ m
Lengths of hub elements (divided by blade length):	
$e_1 = 0.01$, $e_2 = -0.005$, $e_3 = 0.02$, $e_4 = 0.03$,	$e_5 = 0.0$
Blade chord (constant along the blade span)	$c = 0.4$ m
Pitch - flap coupling coefficient	$\tan(\delta_3) = -0.4$
Rotor shaft angular speed (constant)	$\Omega = 28.35$ rad/s
Blade twist	$\vartheta = 0^{0}$
Pitch hinge linear stiffness	$K_{\theta} = 6.304$ kNm/rad
Lag hinge linear damping coefficient	$\gamma_{\alpha} = 10.0$ kNms/rad
Aerodynamic center location (behind the leading edge)	0.235 c
Blade profile (constant along the span)	NACA 0012

There were two kinds of computation parameters:
- blade section center of gravity (C.G.) positions
  I - forward 0.02c, II - neutral, III - backward 0.02c;
- rotor advance ratio $\mu \in [\,0.0,\ 0.25\,]$.

At the beginning of computations the equilibrium blade angles of rotation in hinges were computed for helicopter trim in hover for three cross section C.G. positions (Fig.4). These angles were taken as steady blade angles for the all flight conditions considered. This part of computation was also utilized as one of the computer code tests.

The value of $W'(\psi,x)$ can be considered as the measure of instability. The instability occurs, when the value of $W'(\psi,x)$ is positive.

The value of $W'(\psi,x)$ for hover is shown in Fig.5 as a function of blade C.G. position. When the C.G. moves to the trailing edge of the blade, the stability decreases.

The magnitudes of $W'(\psi,x)$ for $\mu = 0.15$ and forward C.G. position are shown in Fig.6 as the function of azimuth angle and size of stable motion boundary A. When the size of A increases, the values of $W'(\psi,x)$ is decreasing.

The values of $W'(\psi,x)$ for forward flight are shown on Fig.7 and the regions of instability for this case are summarized in Fig.8. In hover the blade motion is stable for all ψ. When μ increases first region of instability arises at the retreating blade side (it suggests stall flutter) and then for μ=0.25 at advancing side.

The results obtained show good qualitative agreement with phenomena observed in helicopter blade motion.

6. Conclusions.

The new method for investigation helicopter rotor blade motion stability was developed. Two well known facts lie in the background of this method:

(1) analysis of rotor blade motion stability is an inherently nonlinear problem,

(2) blade motion can be described by the system of ordinary differential equations with periodic right hand sides.

The method is based on the theorem, which states sufficient conditions for boundedness of the solutions (stability in the Lagrangian sense) of the system of ordinary differential equations with periodic right hand sides. The proof of this theorem allows to construct algorithm and computer code for stability investigation using properties of Liapunov function. The regions of initial conditions of blade stable motion as well as the regions of instabilities can be determined. The steady motion, stability of which is determined, need not be the solution of the investigated system of equations.

The stability of single rotor blade in helicopter hover and forward flight was studied using this method. The blade was rigid, fully articulated. At each of three hinges the stiffness and damping could be applied. The steady two dimensional aerodynamic model was applied with Glauert's induced velocity distribution and static aerodynamic characteristics of blade airfoil sections. The final system of equations of motion was strongly nonlinear due to the fact that geometrical and aerodynamic nonlinearities were attained.

The sum of kinetic and stiffness potential energies was taken as the Liapunov function.The stability of periodic steady motion was investigated. The calculated regions of instabilities are in good qualitative agreement with the phenomena observed in helicopter flight.

7. References.

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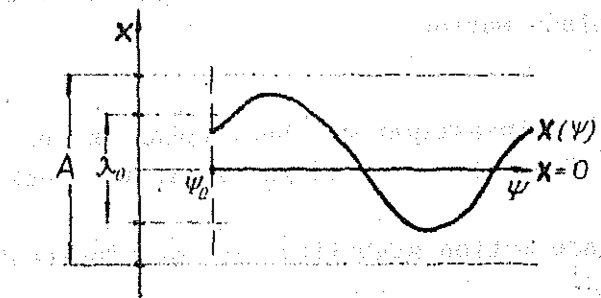

Fig. 1

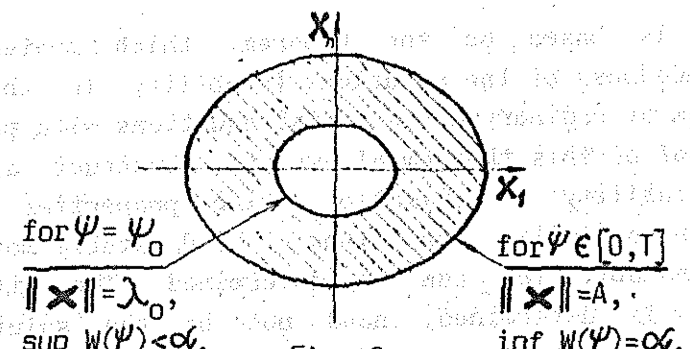

Fig. 2

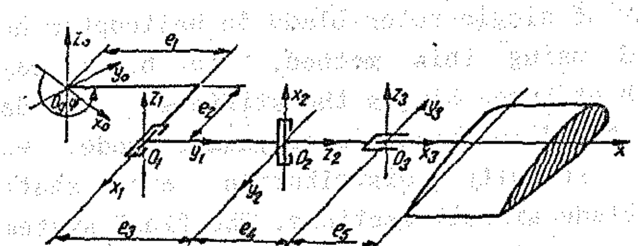

Fig. 3

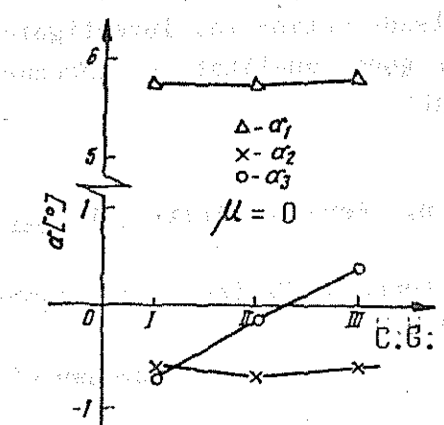

Fig. 4

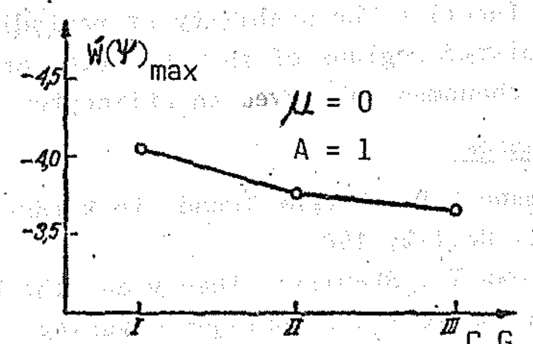

Fig. 5

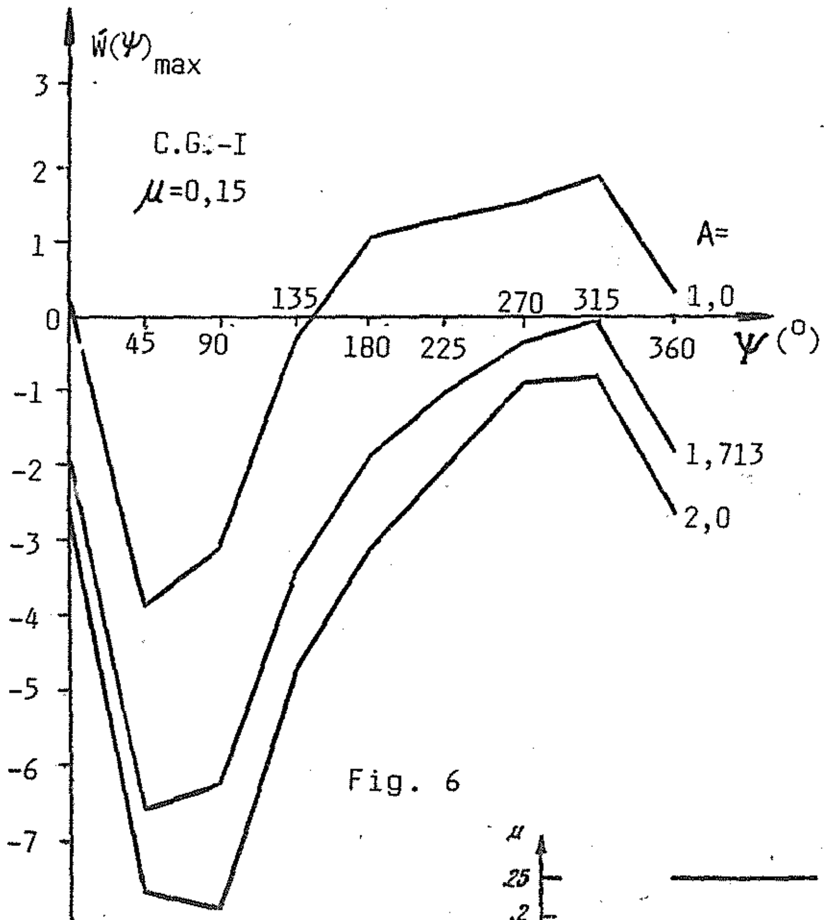

Fig. 6

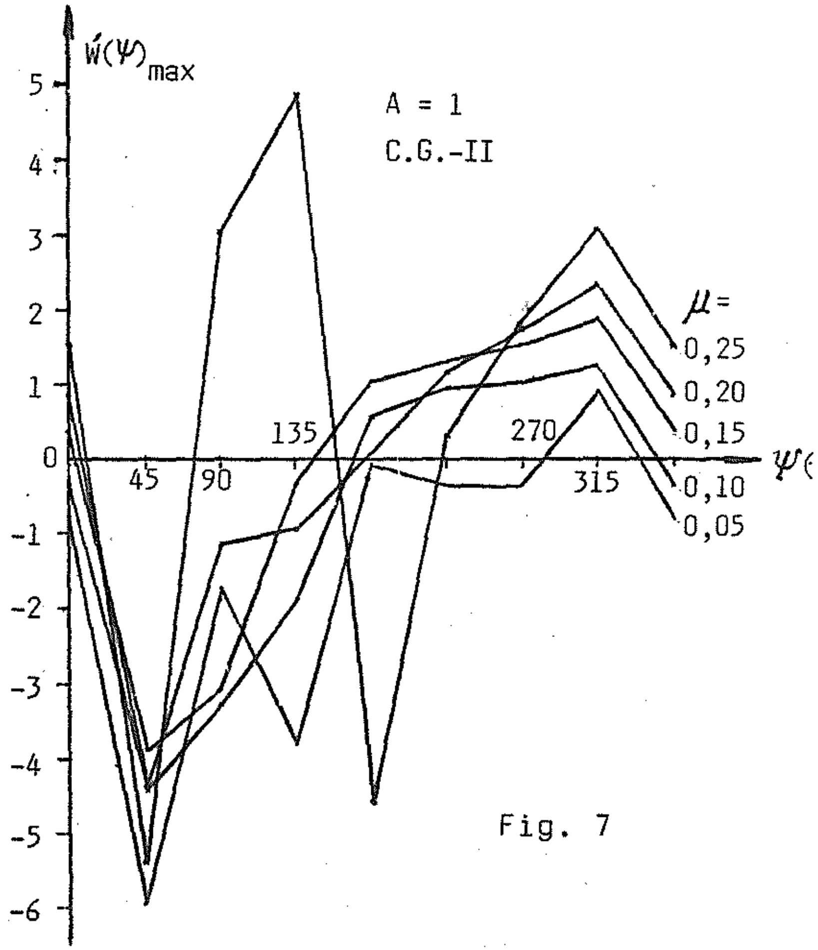

Fig. 7

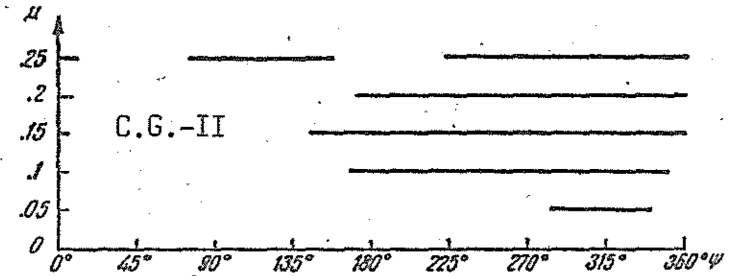

Fig. 8