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APPLICATION OF FAST FREE WAKE ANALYSIS TECHNIQUES TO ROTORS

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Abstract

The fast free wake analysis technique has been applied using various models for the near and far wake, including two-dimensional and three-dimensional configurations, and lifting surface and lifting line treatments of the blade. The technique has also been applied to the case of a wind turbine, operating in the turbulent wake/vortex ring state.

Introduction

It has been evident for some time that a consistent aerodynamic theory applicable to rotors of arbitrary planform and twist, yet simple enough for heuristic, possibly even formal, optimization would be a valuable design tool for the aerodynamic analysis of rotors. Extensive experimental and analytical work, for example references 1, 2, 3, and 4, have shown that rotor performance is critically dependent on wake geometry and this geometry must be carefully modelled if performance and blade loads are to be correctly predicted. These conclusions suggest the need for free wake analytical techniques in which the wake geometry is not established a priori, but allowed to assume any position determined by the velocity field induced by both the rotor and its wake. References 4 through 8 develop such techniques. In references 5-7 the entire wake is modelled, whereas in reference 8 a combination of prescribed and free wake modeling is used resulting in a considerable shortening in the required computational time.

Reference 9 proposed a simplified free wake representation of the complete rotor wake as a useful tool for identifying the basic problems inherent in rotor aerodynamic analysis and as a guide to the development of more complete solutions, including real fluids effects. These points were discussed further in references 10 and 11. This paper will present the results of additional research, using the suggested simplified wake models, in order to assist in clarifying some of the basic factors which appear to influence rotor performance.

The free wake modeling technique is of primary importance for establishing the location of the tip vortex at its first encounter with a following blade. But it is also important for estimating the inner wake characteristics and to determine the effects of far wake modeling. While both these latter effects are probably secondary in their influence on blade airloads, they may be important for such measures as the "induced" or "ideal" figure of merit, IFM, a sensitive measure of rotor performance.

This paper will discuss the influence of the intermediate and far wake, the effect of varying assumptions in modeling the near wake, the results using a modified lifting surface representation of the blade and, finally, an extension of the technique to wind turbine performance in the turbulent wake/vortex ring condition.

Influence of Intermediate and Far Wake

In reference 9 the intermediate and far wakes were represented by tip, mid and root vortices, rolled up from the near wake according to the Betz criteria of conservation of linear and angular momenta. Two models were suggested for representing the intermediate and far wakes, one a three-dimensional model (3D) in which the vortices in the wake are represented by a series of vortex rings and cylinders (Fig. 1) and the other a quasi-two-dimensional model (2D) in which the vortex rings and cylinders are replaced by pairs of doubly infinite line vortices and sheets (Fig. 2).

The essence of the simplification in the case of both models results from considering the velocity only at an azimuth in the wake below the blade in question. Vortices are located at these points as a result of previous blade passages. Wake displacements are determined from the average of the velocities at two vortices acting over a time increment corresponding to the time between the individual blade passages which produced them. The blade bound circulation distributions are determined from the velocities at the blade induced by the vortex system in the wake whose geometry was determined by this velocity averaging technique.

It was found that the 3D model required very little additional computational time as compared to the 2D case when the elliptic integrals involved were expanded using Cayley's logarithmic series. The conceptually simpler 2D model however is useful in clarifying some of the physics of the problem. Both models gave similar results as shown in Fig. 3. As discussed in references 9 and 10, the effects of the root vortex shown in Fig. 3 were found to be negligible and the computations appreciably simplified by its neglect, leaving a tip and mid vortex. In reference 11, the wake was represented by two rolled up mid-vortices and a tip vortex giving a slightly better distribution of circulation compared to the experimental results, as shown in Fig. 4, and a more reasonable IFM of .95 as compared to 1.02 for the two vortices. However in the case of the 2D model the IFM was always slightly greater than 1 due to the limitations of the far wake modeling by vortex sheets. Doubly infinite vortex sheets tend to overestimate the contribution of the far wake to the blade induced velocities as compared to a vortex cylinder representation.

Fig. 5 shows the results for both the case of an intermediate wake extending to 4π radians (4 vortices below the blade) and also for the case where the wake is allowed to extend to 18π radians or 18 vortices below the blade, in the latter case without introducing the constraints of a far wake vortex cylinder. There is little change in blade load distribution or thrust for either case. The unconstrained extended wake, however, shows an expansion of the wake after 2 rotor radii resulting from intermingling between the inner and outer vortices.

As might be expected, representation of the wake by discrete singular vortex filaments occasionally results in convergence problems in the extended computations, which are alleviated by introducing a nonsingular vortex core. The two bladed rotor results are insensitive to vortex core size, but this is not the case for four and more blades, as discussed in reference 11.

Influence of Near Wake

As mentioned above the wake displacements are determined by taking the average of

1) the velocities in the wake behind the blade in question at the computed spanwise location of its rolled up trailing vortices and

2) the velocities in the wake behind the following blade (thus including the contribution from its trailed rolled up vortices) at the locations of the displaced rolled up vortices from the first blade.

While this approach has the merit of consistency and results in good agreement with test data, both as regards geometry and bound circulation distribution, there is some question as to how well it models the near wake displacements during the roll up process and before encounter with the following blade. For example, an alternative approach would be to compute the velocities in the wake immediately ahead of, rather than behind, the following blade neglecting the contribution in that region of the vortices trailed by the following blade.

Fig. 6 shows geometries and circulation computed using such an assumption for velocity averaging. The thrust coefficient is lower, and both the circulation distribution and wake displacement at first encounter do not agree with the observed results. Evidently the original assumption of velocity averaging after blade passage results in a better representation of the complex roll up process occurring in the near wake. A preliminary analytical investigation of wake roll up is contained in reference 12, based on the techniques discussed in reference 13 in which the wake velocities are determined from the Euler rather than the Biot-Savart relationships.

Modified Lifting Surface Solution

Because of the proximity of the blade to the vortex at first encounter, it is logical to consider the use of a lifting surface rather than a lifting line representation of the blade. In reference 14 it is shown that, for the case of a perpendicular intersection between vortex and blade, the Weissinger approximation to a lifting surface solution gives almost exact agreement with a more complex five panel solution. In this approach the control point is placed at the 3/4 chord location, and the velocity is determined from the vortex system consisting of a bound vortex at the 1/4 chord position and associated trailing vortices. Since the free wake analysis uses an iterative technique, it is readily adaptable to the Weissinger approximation without appreciable increase in computer time. The results are compared in Fig. 4 with the lifting line solution. It is evident that the circulation peaks are somewhat smoothed out and C_T slightly lower, although the wake displacement at first encounter remains essentially the same.

To explore this point further, solutions were obtained for the multi-bladed rotors for which experimental results were presented in reference 2. Fig. 7 shows comparisons for these rotors between test data and both the lifting surface and lifting line solutions. The same core size of .03 of the blade radius as postulated in reference 11 was used. Evidently, for the greater number of blades, the lifting surface solution gives better results.

Application to Wind Turbines

A logical application of the simplified free wake analytical technique is to wind turbines where effects such as tower shadow and reflection, and of operation in the turbulent wake/vortex ring state, require free wake analysis techniques in order to obtain meaningful numerical results. In fact the fast free wake analysis technique discussed here was originally developed with such applications in mind (reference 9).

In reference 15, test results were obtained for a model rotor operating under various conditions, including well into the turbulent wake/vortex ring state where, under certain conditions of low inflow the simple momentum or rigid wake vortex models would predict the existence of a backflow as the induced velocities in the fully developed wake become greater than the wind velocities. Fig. 8 compares the analytical results with the experimental data. The fast free wake technique is apparently a useful tool for determining performance under all operating conditions. It is also interesting to observe the close agreement when momentum theory is modified by the experimentally determined corrections suggested in reference 16.

Conclusions

The aerodynamic characteristics of rotors and wind turbines have been analyzed using various versions of the fast free wake analytical technique. It has been shown that, in the case of 2-bladed rotors, performance, blade load distribution and wake geometry are adequately predicted using either the 2D or 3D wake models and either lifting surface or lifting line blade representation. For the case of 4 or more blades better results were obtained using the lifting surface solution.

Although the velocity averaging technique used in the fast free wake method gives reasonable results, a further definition of the near wake may be desirable in order to determine the vortex roll up schedule and possible core growth.

The fast free wake technique is capable of predicting the performance of wind turbines throughout their operating regimes, including the turbulent wake/vortex ring state.

Nomenclature

C_T Thrust coefficient - Thrust/$\rho S \Omega^2 R^2$

C_{Q_i} Induced torque coefficient - Induced Torque/$\rho S \Omega^3 R^3$

IFM Ideal Figure of Merit - $\sqrt{\frac{C_T^3}{2}} / C_{Q_i}$

R Rotor Radius

S Rotor disc area

T_C Thrust coefficient - Thrust/$\frac{1}{2}\rho S V^2$

V Ambient wind velocity

Z Vertical distance below rotor blade

Γ Blade bound circulation

σ Rotor solidity - blade area/S

Ω Blade rotational speed

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Figures

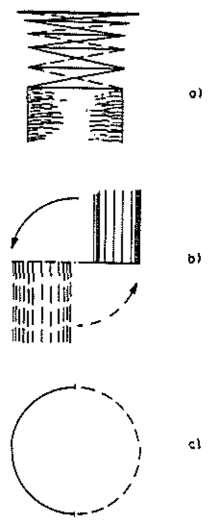

FIG. 1: GEOMETRY OF MODEL USING VORTEX RINGS AND CYLINDERS TO REPRESENT THE WAKE

a) SIDE VIEW OF ROTOR WAKE MODEL SHOWING INTERMEDIATE AND FAR WAKES FORMED FROM VORTEX SPIRAL - 2 BLADES. TIP VORTEX ONLY SHOWN.
——— BLADE ONE — — — BLADE TWO

b) PLAN VIEW SHOWING NEAR WAKE

c) FORMATION OF INTERMEDIATE WAKE

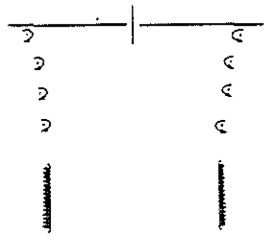

FIG. 2: GEOMETRY OF MODEL USING LINE VORTICES AND VORTEX SHEETS TO REPRESENT THE WAKE.

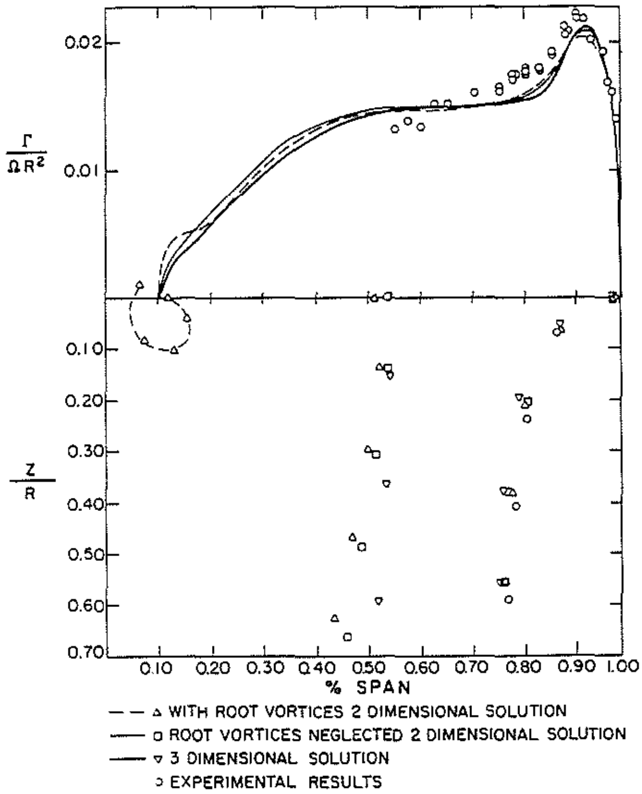

FIG. 3: BLADE BOUND CIRCULATION DISTRIBUTION AND LOCATION OF VORTICES IN WAKE FOR TWO BLADED ROTOR OF REF. 17.

— — C_T = .00456

—— C_T = .00460

▬▬ C_T = .00454

O C_T = .00459[17]

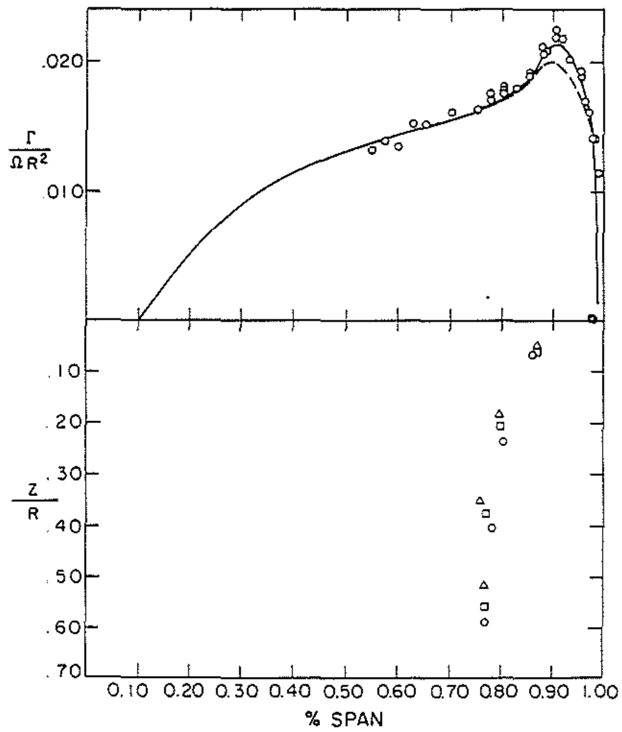

FIG. 4: BLADE BOUND CIRCULATION DISTRIBUTION AND LOCATION OF VORTICES IN WAKE FOR TWO BLADED ROTOR OF REF. 17.

——— LIFTING LINE REPRESENTATION OF BLADE C_T = .00456

— — LIFTING SURFACE REPRESENTATION OF BLADE C_T = .0044

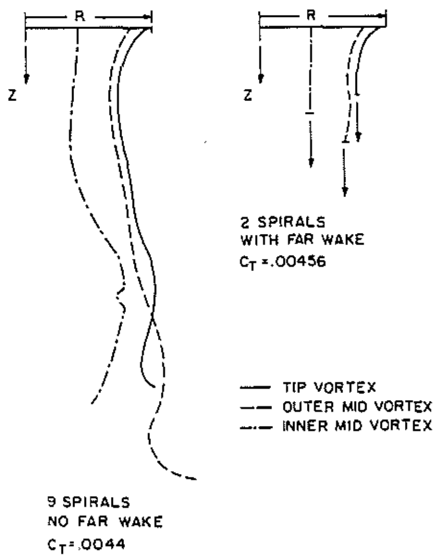

FIG. 5: EXTENDED INTERMEDIATE WAKE FOR TWO BLADED ROTOR OF FIG. 4.

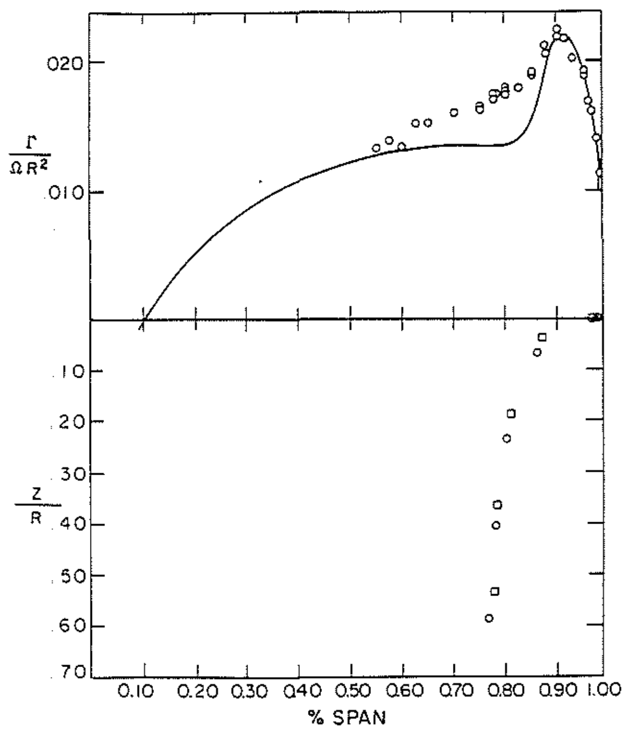

FIG. 6: BLADE BOUND CIRCULATION DISTRIBUTION AND LOCATION OF VORTICES IN WAKE FOR ROTOR OF FIG. 4, BUT WITH EFFECT OF FOLLOWING BLADE TRAILED VORTEX NEGLECTED.

$C_T = .00418$

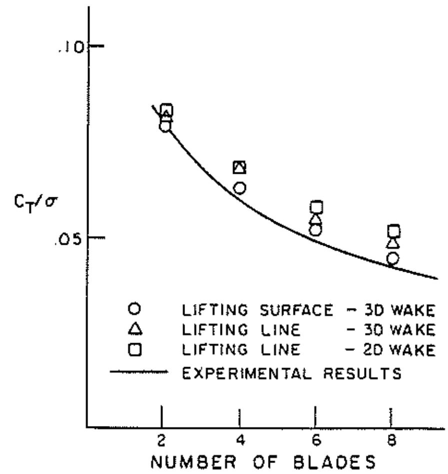

FIG. 7: EFFECT OF BLADE AND WAKE MODELING WITH INCREASING NUMBER OF BLADES AND COMPARISON WITH EXPERIMENTAL RESULTS OF REF. 2, FIG. 16b, $\theta_{75} = 8°$.

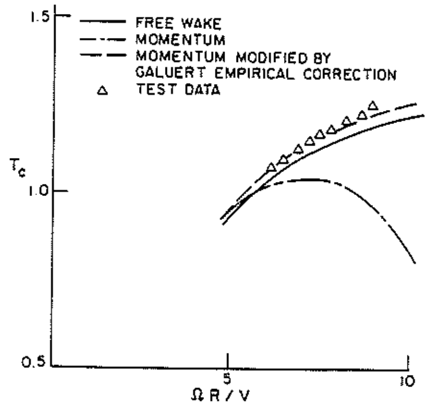

FIG. 8: WIND TURBINE PERFORMANCE PREDICTIONS IN TURBULENT WAKE/VORTEX RING STATE AND COMPARISON WITH EXPERIMENTAL RESULTS OF REF. 15.