A METHODOLOGY FOR THE DERIVATION OF A ROTOR BLADE FLUTTER DETERMINANT IN FORWARD FLIGHT AND ITS APPLICATION TO STABILITY ANALYSIS OF ROTOR BLADE 3D-MOTION

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Abstract: Presented is a method for the derivation of a flutter determinant by using a model of coupled blade motions in forward flight. The convenient algorithm for flutter parametric investigation based on the derived determinant has been developed. Possibilities are given for simple and quick estimation of different rotor design modifications. The algorithm is successfully applied to obtain flutter critical angular velocities of rotor rotation and investigate the influence of blade parameters on flutter stability margin for Russian helicopter *Ansat*. Results of numerical calculations are presented.

INTRODUCTION

Flutter is generally understood as aeroelastic dynamic instability of lifting surfaces. For a helicopter it may be self-excited violent coupled blade flap-lag-torsion oscillation that occurs due to the interaction of inertial, aerodynamic and elastic forces of main rotor (MR) blades [1, 2]. Oscillating motion can become unstable and lead to undesired consequences at some regimes of a helicopter flight. For newly designed stiff main rotor helicopters with blades on the elastic element the problem of the stability of coupled blade motion is of current importance.

Due to rotor rotation helicopter blade flutter is far different from that of a fixed wing aircraft mainly because of the periodicity of rotor dynamics in helicopter forward flight through periodic variations of aerodynamic forces and moments. The set of differential equations governing coupled vibrations of a rotor blade in forward flight consists of non-linear differential equations with periodic coefficients. Numerical implementation of stability analysis of periodic equations even in linear case, for example, by using the well-known method based on the Floquet-Lyapunov theorem [3, 4], is time consuming that creates difficulties in its application for parametric analysis of rotor alternative designs.

While rotor system engineering development continuous control of MR parameters is necessary in order to maintain them within specified limits that provide the required flutter stability margin. Various structural and technological modifications made during design and factory test stages as well as during helicopter full-scale production may force blade parameters to change. So, development of engineering approaches to flutter analysis, which would enable to carry out simple and quick estimation of different design modifications, is of big importance.

The method for flutter determinant construction based on a well known harmonic balance technique [51] is proposed to make stability analysis of flap-lag-torsion coupled motion of a helicopter rotor blade in forward flight. The method is tested on a three-degree-of-freedom non-linear mathematical model, which approximates the blade motion in both hover and forward flight. Stability analysis of the developed linear perturbation equations enables us to obtain flutter critical angular velocities of rotor rotation and investigate the influence of blade parameters on flutter stability margin which is demonstrated for Russian helicopter Ansat. The developed method has been verified by numerical integration using the Runge-Kutta method for linear equations of perturbed motion of helicopter blade.

1. BLADE DESIGN MODEL

We consider hingeless MR with blades attached to the rotor hub via torsion. In the model the torsion is represented by equivalent hinges with concentrated stiffness. The stiffness is determined by dynamic similarity between blade attached by the torsion and the hinged blade. Blade natural flapping and lag frequencies can determine this stiffness and can be obtained whether from calculations, or from experiments. The system of differential equations governing oscillation of the elastically attached rigid blade during helicopter forward flight is derived using the hub-fixed, rotating coordinate system (definition of the frame is shown in Figure 1).

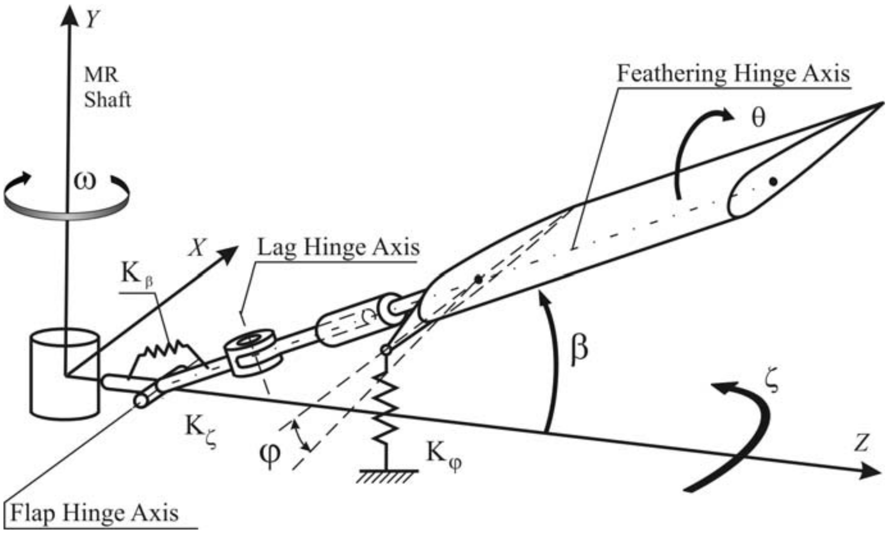

Figure 1. The rotation-plane-fixed coordinate system

The following assumptions provide practically satisfying accuracy for flutter analysis and are used in the derivation of blade motion:

– The blade is assumed to be flexural rigid. Due to the compliance of the springing element, blade swings in rotation and flap planes as a rigid body around equivalent flap and lag hinges.

– The blade is also assumed to be rigid in torsion. Swinging as a rigid body around the feathering hinge, it executes a torsional vibrations due to flap and lag compensators as well as to deformations of control links and swash plate.

– The blade axis is straight and coincides with the feathering hinge axis.

2. EQUATIONS OF COUPLED BLADE MOTION

While considering the equilibrium of moments about the equivalent flap hinge, lead/lag hinge and feathering hinge axes the following designations are used in the derived below equilibrium equations: R – blade radius, β – blade flapping angle, ζ – blade lag angle; m – mass per unit blade of rotor blade; F'_x –aerodynamic force per unit length of rotor blade parallel to disk plane; F'_y – aerodynamic force per unit length of rotor blade normal to disk plane; M_a – aerodynamic moment per unit length of rotor blade; x_m – the distance from blade longitudinal axis (passing through elastic center of blade section) to section center-of-mass; $y=\eta_\beta\beta$; $x=\eta_\zeta\zeta$; $z=u-r$; $\eta_\beta=\dfrac{r-r_\beta}{R-r_\beta}R$; $\eta_\zeta=\dfrac{r-r_\varsigma}{R-r_\varsigma}R$; $\dot z=\dot u$; $y'=\eta'_\beta\beta$; $x'=\eta'_\zeta\zeta$; $u=-\dfrac{1}{2}\int\limits_{r_\beta}^{r}\left[(x')^2+(y')^2\right]dr$.

A dot over a variable indicates differentiation with respect to time t and primed variables indicate differentiation with respect to radial distance r.

The following moments and forces per unit length of rotor blade, which produce the flap moments, are acting on the blade section:

1. Centrifugal force $\omega^2 mz$ acting at a distance y about the flap hinge.
2. Inertia force $m\left(\ddot y+x_m\ddot\theta\cos\theta\right)$ and aerodynamic force F'_y acting at a distance $\left(r-r_\beta\right)$ from the flap hinge.
3. Centrifugal moment $mx_m\omega^2 z\sin\theta$.
4. Because of a spring in flap hinge the moment $K_\beta(\beta-\beta_{\text{constr}})$ is acting on the blade element.

Integrating the moments along the blade radius gives the following equilibrium condition about the flap hinge axis:

$$\begin{aligned}&\int\limits_{r_\beta}^{R} m\omega^2 zy\,dr+\int\limits_{r_\beta}^{R} m\ddot y\left(r-r_\beta\right)dr+\int\limits_{r_\beta}^{R} mx_m\ddot\theta\cos\theta\left(r-r_\beta\right)dr+\\&+\int\limits_{r_\beta}^{R} mx_m\sin\theta\,\omega^2 z\,dr+K_\beta(\beta-\beta_{\text{constr}})=\int\limits_{r_\beta}^{R}(r-r_\beta)F'_y\,dr.\end{aligned}\tag{1}$$

For the lag motion the forces and moments per unit length of rotor blade acting on two-dimensional blade section are the following:

1. Centrifugal force $\omega^2 mz$ acting at a distance x about the lag hinge.
2. The forces acting at a distance $\left(z-r_\varsigma\right)$ about the lag hinge: inertia force $m\ddot x$, centrifugal force $-\omega^2 m\left(x_m\cos\theta+x\right)$, *Coriolis* force $2\omega m\dot z$ and aerodynamic force F'_x.
3. Centrifugal moment $mx_m\omega^2 z\cos\theta$.
4. Due to the spring in the equivalent lag hinge the moment $K_\zeta\zeta$ is acting on the blade element.

Taking moments about the lead/lag hinge and integrating the moments along the blade radius and collecting similar terms give the following equilibrium condition of moments about the lead/lag hinge axis:

$$\int\limits_{r_\beta}^{R} m\ddot x\left(z-r_\zeta\right)dr-\int\limits_{r_\beta}^{R}\omega^2 mxr_\zeta\,dr+\int\limits_{r_\beta}^{R}2\omega m\dot z\left(z-r_\zeta\right)dr+C_\zeta\dot\zeta+K_\zeta\zeta=\int\limits_{r_\beta}^{R}(z-r_\zeta)F'_x\,dr.\tag{2}$$

Hub of the hinge rotor has a mechanical damper in the lag hinge. Therefore, the term $C_{\zeta}\dot{\zeta}$ is added into the lag motion equation, where C_{ζ} is a derivative of the damper moment with respect to angular rate of blade rotation about the lag hinge.

The following moments and forces per unit length of rotor blade, which produce the moments about the feathering hinge, are acting on the blade section:

1. Centrifugal force $\omega^2 m x_m \cos\theta$ acting at a distance y about the feathering hinge.

2. Linear torsion moments: inertia moment $m x_m \ddot{y}\cos\theta + J_m\ddot{\theta}$, where $J_{\theta} = J_m + m x_m^2$, propeller moment $J_{\theta}\omega^2 \sin\theta\cos\theta$ and centrifugal moment $-\omega^2 m x_m x \sin\theta$.

3. Restoring moment $M_{CS} = C_{CS}\varphi$ about the feathering hinge axis conditioned by control system torsional stiffness. C_{CS} – control system stiffness coefficient.

4. An aerodynamic moment M_a, acting around the feathering hinge axis, which is positive when acts to increase the pitch angle.

Integrating the moments along the blade radius gives the following equilibrium equation of all the moments acting about the feathering hinge axis:

$$\int_{r_{\varphi}}^{R} \omega^2 m x_m \cos\theta y\,dr + \int_{r_{\varphi}}^{R} m x_m \cos\theta \ddot{y}\,dr + \int_{r_{\varphi}}^{R} J_m\ddot{\theta}\,dr + \int_{r_{\varphi}}^{R} \omega^2 J_m \sin\theta\cos\theta\,dr + C_{CS}\varphi = \int_{0}^{R} M_a\,dr. \quad (3)$$

The blade pitch angle θ equal to

$$\theta = \varphi + \Delta\theta(r) + \theta_0 + \theta_{1c}\cos(\omega t) + \theta_{1s}\sin(\omega t) - K_{P\beta}\beta - K_{P\zeta}\zeta \quad (4)$$

is the most complicated angle function. It consists of so-called control system compliance angle φ – the angle of blade torsion about the feathering hinge caused by deformability of blade control links, blade collective pitch θ_0, lateral cyclic pitch θ_{1c} and longitudinal cyclic pitch θ_{1s}, $\Delta\theta(r)$ – linear blade twist rate; as well as of angles induced by the operation of flap compensator $-K_{P\beta}\beta$ and lead/lag compensator $-K_{P\zeta}\zeta$. All of these angles except for blade collective pitch θ_0 are small over flight envelope and their sum is not more than 10°. Hence, following simplifying assumption can be used:

$$\begin{aligned} \sin\theta &\approx \sin\theta_0 + \left[\varphi + \Delta\theta(r) + \theta_{1c}\cos(\omega t) + \theta_{1s}\sin(\omega t) - K_{P\beta}\beta - K_{P\zeta}\zeta\right]\cos\theta_0, \\ \cos\theta &\approx \cos\theta_0 - \left[\varphi + \Delta\theta(r) + \theta_{1c}\cos(\omega t) + \theta_{1s}\sin(\omega t) - K_{P\beta}\beta - K_{P\zeta}\zeta\right]\sin\theta_0. \end{aligned} \quad (5)$$

Substitution of expressions (4) and (5) in equations (1), (2) and (3) permits us to express their coefficients explicitly in terms of compliance angle φ, and of collective pitch θ_0, which is important for parametric analysis of the stability margin of blade oscillating motion in three mutually perpendicular planes. Values of blade collective pitch θ_0, lateral cyclic pitch θ_{1c}, longitudinal cyclic pitch θ_{1s} are determined from helicopter trim calculation.

3. FLUTTER ANALYSIS METHOD

3.1. Linearization of blade motion equations

The representation of blade motion equations after the above substitution is clumsy and they are not presented in the paper. The equations are written in the unknowns β, ζ, φ, $\dot{\beta}$, $\dot{\zeta}$, $\dot{\varphi}, \ddot{\beta}, \ddot{\zeta}, \ddot{\varphi}$ and their products and are non-linear differential equations with periodic coefficients, which are functions dependent on blade design and flight parameters and trim characteristics. It can be shown [5] that the expressions

$$\begin{aligned}
\beta^* &= \beta_0^* + \sum_{k=1}^{n^*} (\beta_{ck}^* \cos k\omega t + \beta_{sk} \sin k\omega t);\\
\zeta^* &= \zeta_0^* + \sum_{k=1}^{n^*} (\zeta_{ck}^* \cos k\omega t + \zeta_{sk}^* \sin k\omega t);\\
\varphi^* &= \varphi_0^* + \sum_{k=1}^{n^*} (\varphi_{ck}^* \cos k\omega t + \varphi_{sk}^* \sin k\omega t)
\end{aligned} \tag{6}$$

are the particular solution of system (1), (2), (3) and describe undisturbed blade motion.

It is assumed that during the dynamic process variables β, φ and ζ change so that their displacements from steady-state values β^*, ζ^* and φ^* remain sufficiently small during the whole process. The perturbed angles $\beta = \beta^* + \Delta\beta$, $\zeta = \zeta^* + \Delta\zeta$ and $\varphi = \varphi^* + \Delta\varphi$ are introduced into the equations of blade motion (1), (2), (3). Here $\Delta\beta$, $\Delta\zeta$ and $\Delta\varphi$ are perturbation angles, that are the angles of displacement from steady-state blade motion. The angles β^*, ζ^* and φ^* satisfy blade motion equations (1), (2) and (3). Retaining the perturbation terms and discarding the terms of high order smallness, that are products and powers of perturbation angles $\Delta\beta$, $\Delta\zeta$, $\Delta\varphi$ and of their derivatives, will give the linear perturbation equations about the steady-state motion of the blade. Thereinafter symbol Δ is omitted to simplify expression form.

3.2. Flutter analysis

For determination of flutter critical rotor angular velocity it is necessary to analyze linearized system, derived from equations (1), (2), (3) for stability of trivial solution [6], that is to show that $\beta \to 0$, $\zeta \to 0$, $\varphi \to 0$ at $t \to +\infty$. The solution of the system of linear differential equations with periodic coefficients can be written down in the form [5]:

$$\begin{aligned}
\beta &= e^{\lambda_R t}\left(\beta_0 + \sum_{k=1}^{n} (\beta_{ck} \cos k\omega t + \beta_{sk} \sin k\omega t)\right);\\
\zeta &= e^{\lambda_R t}\left(\zeta_0 + \sum_{k=1}^{n} (\zeta_{ck} \cos k\omega t + \zeta_{sk} \sin k\omega t)\right);\\
\varphi &= e^{\lambda_R t}\left(\varphi_0 + \sum_{k=1}^{n} (\varphi_{ck} \cos k\omega t + \varphi_{sk} \sin k\omega t)\right).
\end{aligned} \tag{7}$$

Here n is constant number equal to the number of harmonic components retained in the solution of linearized equations. It is obvious that trivial solution will be stable only at $\mathrm{Re}(\lambda_R) < 0$. The case is considered when the coefficient vector $\left[b_0\ b_{c1}\ b_{s1} ... b_{cn}\ b_{sn}\ z_0\ z_{c1}\ z_{s1} ... z_{cn}\ z_{sn}\right]^T$ is not zero.

General algorithm for stability analysis of blade motion in forward flight, that is definition of flutter conditions, is presented [7].

1. Under the given flight and design parameters linearization of the system of non-linear differential equations (1), (2), (3) is carried out. As a result, the linear differential equations are obtained presenting the system of perturbation equations of the blade in the forward flight.
2. The value of n – the number of harmonic components retained in the linearized equation solution is assigned. Expressions (7) and their derivatives are substituted into the linear system and algebraic system is formed linear in $m = 6n + 3$ expansion coefficients of blade oscillation angles $\beta_0, \beta_{c1}, ..., \beta_{cn}, \beta_{sn},\ \zeta_0, \zeta_{c1}, ..., \zeta_{cn}, \zeta_{sn},\ \varphi_0, \varphi_{c1}, ..., \varphi_{cn}, \varphi_{sn}$. Common multiplier $e^{\lambda_R t} \neq 0$ is discarded.
3. The value of n^*, the number of harmonic components taken into account in steady-state motion, is assigned. Expressions (6) and their derivatives are substituted into the system of algebraic equations. The values of $\beta_0^*, \beta_{c1}^*, ..., \beta_{sn}^*,\ \zeta_0^*, \zeta_{c1}^*, ..., \zeta_{sn}^*,\ \varphi_0^*, \varphi_{c1}^*, ..., \varphi_{sn}^*$ are determined from helicopter trim calculation [7]. Standard trigonometric transformations are applied to the system coefficients.
4. The coefficients at the similar harmonic components and at the terms free from harmonics are grouped together for each of the system equation and then are equated to zero.
5. The equations for free terms and n low harmonics are selected from the equations obtained above and their coefficients are grouped according to the powers of the characteristic number λ_R.
6. The set of homogeneous algebraic equations derived above is written in the matrix form

$$(\mathbf{A}_2 \lambda_R^2 + \mathbf{A}_1 \lambda_R + \mathbf{A}_0) \times \mathbf{K} = 0 .$$

Here $\mathbf{A}_2$, $\mathbf{A}_1$, $\mathbf{A}_0$ are matrices of order $m \times m$, which elements depend on main rotor angular velocity ω, flight parameters and blade design parameters. and $\mathbf{K}$ here is a coefficient vector

$$\mathbf{K}^T = \left[\beta_0\ \beta_{c1} ... \beta_{sn}\ \zeta_0\ \zeta_{c1} ... \zeta_{sn}\ \varphi_0\ \varphi_{c1} ... \varphi_{sn}\right] \neq 0 .$$

7. Roots of characteristic equation $\det(\mathbf{A}_2 \lambda_R^2 + \mathbf{A}_1 \lambda_R + \mathbf{A}_0) = 0$ are determined. The characteristic number λ_R defines areas of stability and instability for the linearized system, depending on system parameters [5].
8. The criterion function of the algorithm is the quantity $R_\lambda^{\max} = \max\limits_k \mathrm{Re}(\lambda_{Rk}),\quad (k = \overline{1, 2 \times m})$: $R_\lambda^{\max} < 0$ means stability; $R_\lambda^{\max} > 0$ means instability.

The presented method is accurate enough and can be applied to MR flutter analysis in both hover and helicopter forward flight. The high speed of the algorithm allows its effective application in engineering practice.

4. CRITICAL MAIN ROTOR ROTATIONAL SPEED IN THE *ANSAT* FORWARD FLIGHT

The above method for defining stability of disturbed blade flap-lag-torsion motion in forward flight was taken as a basis for calculation of flutter critical angular velocity for Russian helicopter *Ansat* during its first test flights. To carry it out and make numerical calculations Maple 6 and MATLAB software systems were used. The results of calculations are presented in Figs. 2-4. Fig. 4 shows critical values of main rotor angular velocity evaluated for different rotor parameters. Stability region is located under the curve and instability region is located above the curve.

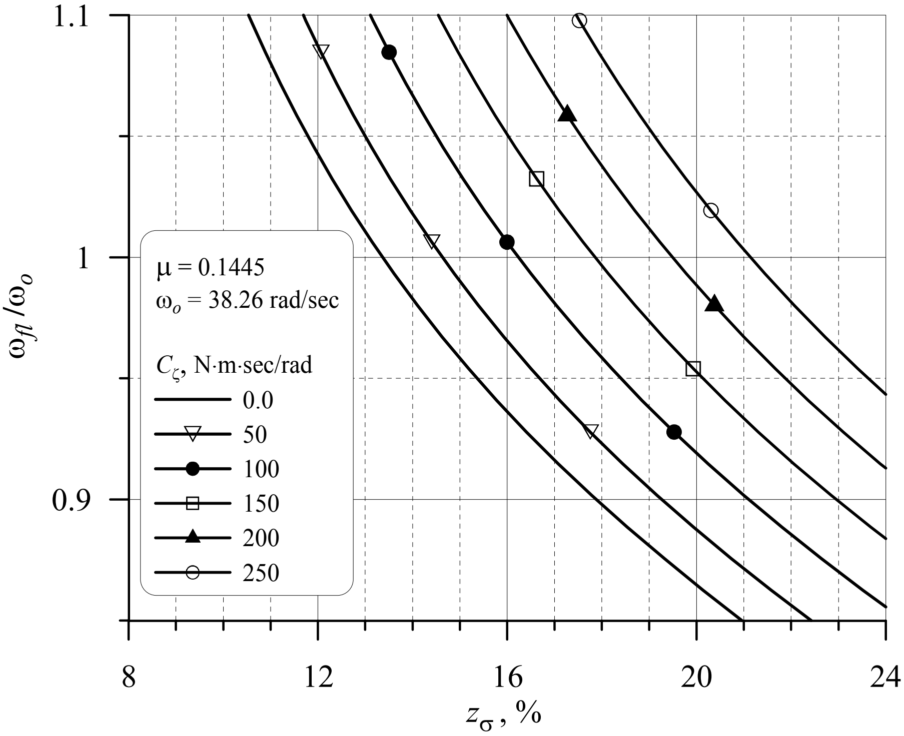

Figure 2. Effect of lag hinge damper coefficient on the blade motion stability

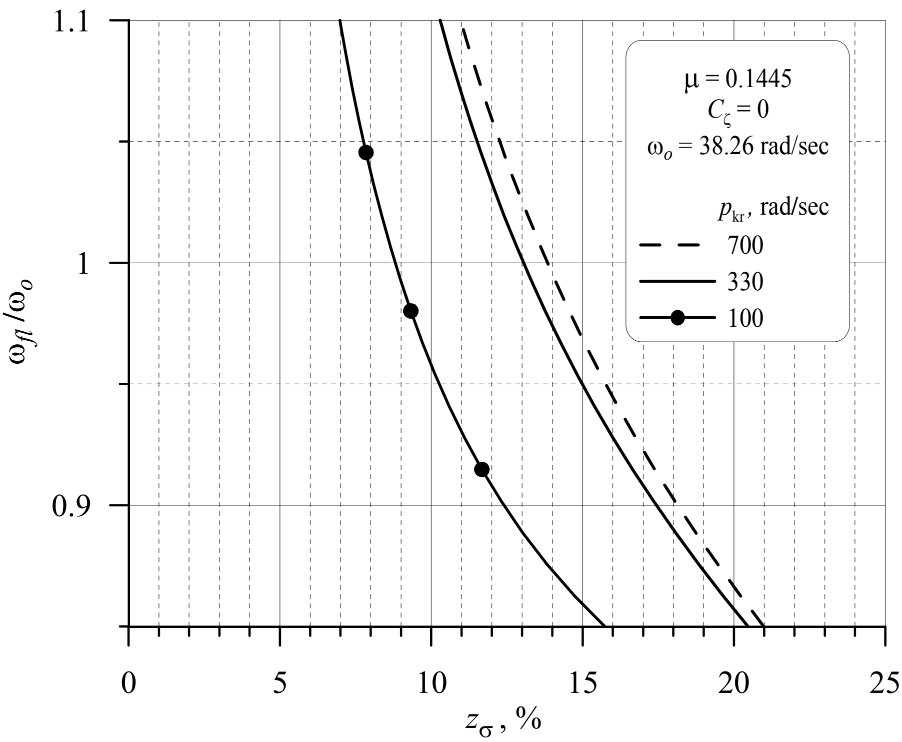

Figure 3. Effect of blade control links stiffness on the blade motion stability

For rotating-wing aircraft, it is the convention to nondimensionalize all velocities by the the blade tip speed in hover ωR. Nondimensional rotor advance ratio $\mu = \dfrac{V \cos\alpha}{\omega R}$ (where V is the magnitude of free-stream velocity, α is the rotor disk angle of attack) is a measure of forward flight speed and its values from 0 to 0.4 cover the entire speed range for the *Ansat* helicopter. Nominal operating rotor angular velocity is denoted by ωo. For the *Ansa*t helicopter $\omega o.$ = 38.26 rad/sec. The critical angular velocities were defined as functions of the so-called "center-of-gravity margin" $z_\sigma = \Delta\sigma / b\, 100\,\%$, where $\Delta\sigma$ represents the center-of-gravity shift, b stands for the blade chord length. Torsional frequency p_{kr} is a measure of control system torsional stiffness $p_{\mathrm{kr}} = \sqrt{\dfrac{C_{\mathrm{CS}}}{J_{\mathrm{FH}}}}$, here J_{FH} stands for blade moment of inertia about the feathering hinge.

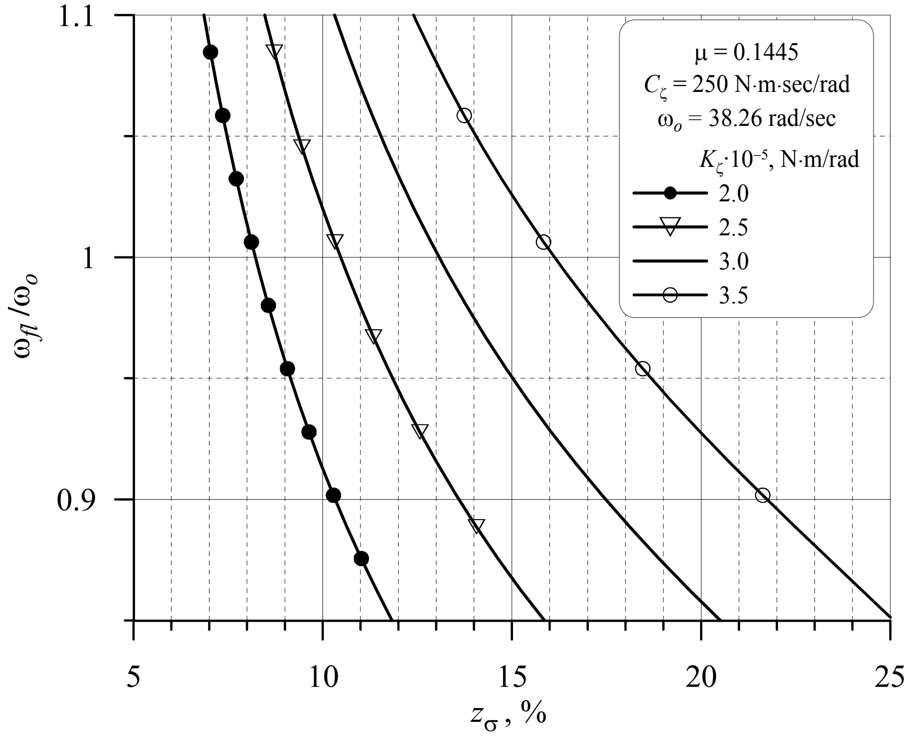

Figure 4. Effect of lag hinge spring rate on the blade motion stability

The developed method has been verified by numerical integration using the Runge-Kutta method for linear equations of perturbed motion of helicopter blade. Results of numerical modeling of the perturbed blade motion in forward flight are shown in Fig. 5. The plot (a) shows the blade perturbations at the boundary of stability. The method gives that such a point for flight speed $\mu = 0.2$ and operational angular velocity ω_o = 38.26 rad/sec is $\Delta\sigma = 0.04226\,\mathrm{m}$. The plots (b) and (c) demonstrate unstable flight regimes when angular velocity and center-of-gravity shift exceed the boundary values; the plot (d) presents stable flight regimes under the boundary angular velocity value.

The presented method is accurate enough and can be applied to flutter analysis in helicopter forward flight. The high speed of the algorithm allows its effective application in engineering practice.

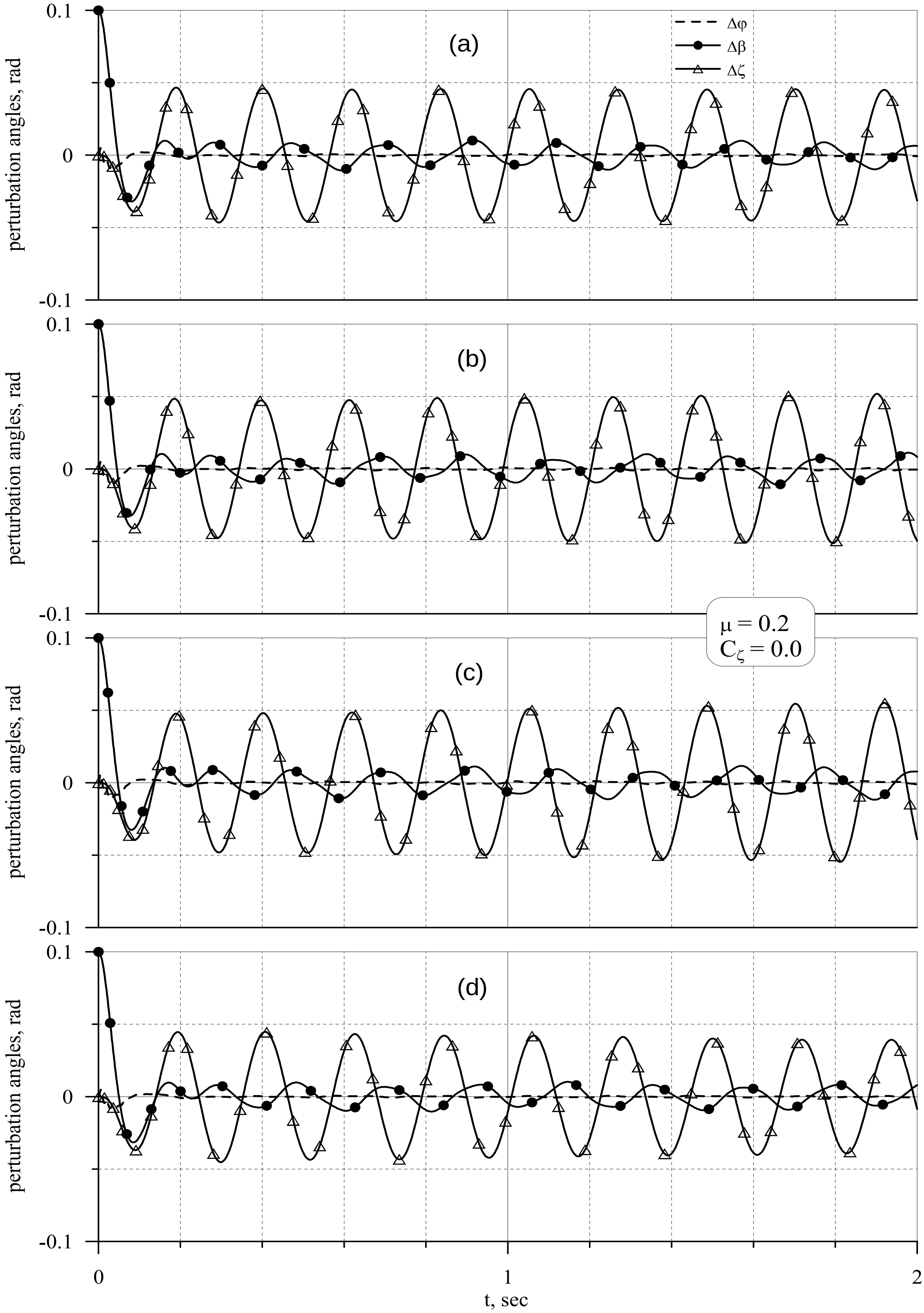

Fig. 5. Perturbations about the steady-state motion of the rotor blade; .
(a) $\Delta\sigma$ = 0.04226 m; $\omega = \omega_o$ = 38.26 rad/sec; (b) $\Delta\sigma$ = 0.04226 m; ω = 40.17 rad/sec;
(c) $\Delta\sigma$ = 0.05120 m; $\omega = \omega_o$ = 38.26 rad/sec; (d) $\Delta\sigma$ = 0.04226 m; ω = 36.35 rad/sec

5. CONCLUSIONS

The method for determination of the main rotor flutter critical parameters in helicopter's forward flight is presented. A detailed flutter parametric investigation for the MR blade of Russian helicopter *Ansat* has been developed using the method. Because of lack of space only few numerical results are presented in the paper. The following conclusions are made based upon research results:

1. The equations governing the coupled flap-lag-torsion motion for stiff blade attached by torsion have been derived.
2. The accuracy of the presented harmonic method is good and decreases only slightly with the increase of the center-of-gravity shift.
3. The high speed of developed method allows its effective application in engineering practice for blade parametric investigation.
4. No flutter effect exists at operating angular velocities of the *Ansat* helicopter MR over the entire flight speed range. These results are in agreement with experimental data.
5. Helicopter flight speed has a significant influence on flap-lag-torsion stability in forward flight. Stability margin decreases with increasing advance ratio μ.
6. Increasing lag hinge spring rate K_ζ has a strong stabilizing effect on a blade motion.
7. Increasing mechanical damper qualities of the lag hinge has a strong stabilizing effect on a blade motion.
8. It is important to include harmonic components of blade pitch angle in mathematical model; otherwise the model overpredicts the blade stability.
9. The deformability of blade control links has a grand effect on blade flutter characteristics. But too much increase of torsional stiffness does not bring further stability increase as saturation is stepping up.

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