Robust Optimal Design of Composite Helicopter Rotor Blade Cross Section

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Abstract

The effect of material uncertainty on the robust design of a composite helicopter rotor blade is demonstrated. A first-order reliability method is used to propagate the material uncertainty through a finite element code for both cross-sectional analysis and sensitivity analysis. First-order sensitivity derivatives obtained by central finite difference scheme are used in the uncertainty propagation. The statistical moments are then used to perform robust design of composite rotor blade cross section with constraints on the cross-sectional stiffness. The variation of uncertainty effect with stacking sequence of the rotor blade are shown. The robust design results show 12-23 percent reduction in the standard deviation of flap and lag stiffnesses when compared with the standard deviation of baseline design.

1 Introduction

Composites are inevitable in the aerospace industry because of their superior structural and tailorable characteristics compared to metallic materials. Manufacturing of composite for a specified structure is a complex process and depends on uncertain variables such as fibre and matrix material properties, fiber volume fraction and un-even temperature and pressure distributions in the autoclave during curing [1, 2]. The fluctuations in the micro-mechanical properties and fabrication process will reflect on the scattering of effective material properties, structural stiffness and consequently on the dynamic behavior of composite structures.

Some studies have addressed the problem of quantifying the uncertainty impacts on structural response and designing structural systems with minimal variability in the response to uncertainties in the input parameters. Oh and Librescu [3] studied the impact of randomness in layer thickness, elastic constants and ply angle on the free vibration of composite cantilever beam. The stacking sequence was optimized to minimize the impact of uncertainties on the natural frequencies of a composite cantilever beam. Noor et al. [4] studied the variability of non-linear response of stiffened composite panels due to randomness in geometric and material properties. Singh et al. [5] investigated the scatter in elastic stability of laminated composite panels with respect to the material uncertainties.

The helicopter rotor blade plays a dominant role in the overall vehicle performance and is typically made of composites. The predicted response of composite rotor blade based on deterministic material properties may not be reliable. Murugan et al. [10, 11, 12] have investigated the impact of random material properties on the blade cross-sectional stiffness, rotating natural frequencies, aeroelastic response and vibratory loads of helicopter. In Ref [10], the blade cross-sectional stiffnesses show around 20 percent scattering from their baseline values due to material uncertainty. One way to reduce these uncertainty effects is to account for the randomness in the preliminary stages of composite rotor blade design. This concept is called robust design in which the sensitivity of structural performance to the variations in design parameters is minimized [13]. In Ref [14], a reliability based design and optimization of rotor blade is studied. The rotor blade is optimized to match the cross-sectional properties with reliability constraints. However, no study has focused on the robust design of a composite rotor blade with respect to the randomness in the material properties.

A robust design problem is one in which a design is sought that is relatively insensitive to uncertain quantities [15]. In general, a robust design and optimization involves three steps. First, the input uncertainties are quantified. Second, the input uncertainties are propagated through the analysis code to quantify the uncertainties on output functions. Third, the output functions with uncertainties are used in the optimization objective and constraint functions to perform a robust design. The evaluation of statistical moments of the objective function at each design point needs additional simulations. This statistical analysis makes robust design and optimization computationally much more expensive than conventional

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deterministic optimization. Therefore, efficient methods to evaluate the statistical characteristics of the objective function with a less number of simulations are needed.

For uncertainty analysis of composite structures, the probabilistic methods are widely used by defining the uncertain variables with specified probability distributions. Monte Carlo simulation (MCS) method has been the most widely used probabilistic analysis method due to its generality, simplicity, and effectiveness for problems that are highly nonlinear with respect to the uncertainty parameters. In the MCS method, deterministic analysis is carried out for sample inputs generated according to probability distribution of the uncertain variables. The statistics of response such as mean and variance are then calculated based on the generated samples. However, the main disadvantage of MCS is the need for a large number of sampling points (analyses), which can be very costly if a time consuming computational analysis such as finite element analysis is involved. The non-statistical methods based on Taylor series expansion such as first-order reliability method (FORM) and second order reliability method (SORM) are computationally more efficient than MCS and have been used to study the uncertainty propagation in composite structures.

Other than the physical uncertainties, the fidelity of the mathematical model used for analysis is another source of uncertainty. Most works on composite rotor blade optimization have used analytical models for cross-sectional analysis. The analytical models have restrictions on the complexity of blade cross-section when compared to the detailed finite element analysis. A Finite Element Method (FEM) based on Variational Asymptotic Beam Sectional (VABS) analysis can handle complicated airfoil cross-section and structural inhomogeneity of a realistic helicopter rotor blade [16]. However, a robust design and optimization based on finite element methods will be computationally very expensive.

The focus of this study is robust design of a composite helicopter rotor blade subject to material uncertainty. The rotor blade is optimized to match the specified cross-sectional stiffness while its variation to randomness in material properties is minimized. A FORM statistical approximation method is used to calculate the statistical properties. First order sensitivity derivatives obtained by the central-difference scheme are used in the FORM to calculate statistical moments. These moments are then used to perform a robust design optimization of the rotor blade. The FEM based VABS is used for cross-sectional analysis of the rotor blade. The stacking sequence and geometrical dimensions of skin and spar of the airfoil cross-section are considered as design variables with the composite material properties as random variables. Real coded genetic algorithm is used as an optimization tool. The effect of material uncertainty with stacking sequence is studied. The robustness of the optimal design is shown by comparing with the baseline rotor blade. The paper is arranged as follows. First, the rotor blade cross section analysis is outlined. Then a brief introduction to uncertainty analysis is given. The real coded genetic algorithm used for optimization is introduced and finally numerical results and conclusions are presented.

2 Rotor Blade Cross-Sectional Analysis

A finite element method based on variational asymptotic beam sectional analysis is used to evaluate the blade cross-sectional stiffness. In the variational asymptotic procedure, the 3-D strain field is expressed in terms of 1-D strain measures and unknown cross-sectional warping functions (which account for the cross-sectional out-of-plane and cross-sectional in-plane deformations). The strain energy density of the beam is then minimized to determine the warping functions in terms of 1-D strain measures. The warping functions are determined asymptotically based on the orders of the small parameters involved [16]. Based on such an analysis, the classical stiffness model of a rotor blade turns out to be the most rudimentary, yet asymptotically correct result, and can be expressed as follows:

$$\left\{ \begin{array}{c} F_x \\ M_x \\ -M_y \\ M_z \end{array} \right\} = \left[\begin{array}{cccc} EA & K_{12} & K_{13} & K_{14} \\ K_{12} & GJ & K_{23} & K_{24} \\ K_{13} & K_{23} & EI_y & K_{34} \\ K_{14} & K_{24} & K_{34} & EI_z \end{array} \right] \left\{ \begin{array}{c} u' \\ \phi' \\ w'' \\ v'' \end{array} \right\} \quad (1)$$

where, u, v and w, corresponds to axial, lag (rotor in-plane) and flap (rotor out-of-plane) displacements, respectively, and ϕ corresponds to torsional displacement of the rotor blade as shown in Fig. 1. The detailed formulation and the asymptotic procedure for cross-sectional analysis are given in the VABS reference [16].

The macrolevel effective material properties $(E_1, E_2, G_{12}, G_{13}, v_{12}, v_{23})$ are considered as random variables with normal distribution. The coefficient of variations (c.o.v) for material properties are taken from a micromechanics study in which the fiber and matrix properties are considered as random variables [1]. The rotor blade cross section is meshed with 4-noded elements and the baseline stiffness is evaluated. A single cross sectional analysis of rotor blade with FEM based VABS takes about 10 minutes of CPU time. Now, a probabilistic cross-sectional analysis with 5000 MCS can take around 48 days of CPU time. Therefore, it is necessary to use more efficient methods for the cross-sectional analysis such as the FORM discussed in the next section.

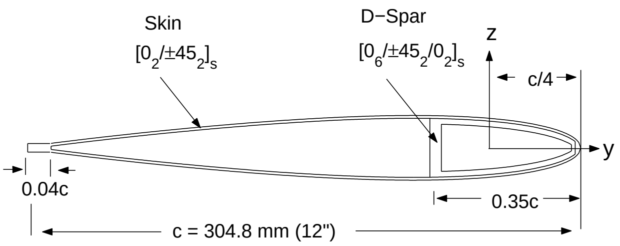

Figure 1: Elastic rotor blade

3 Uncertainty Analysis

In uncertainty analysis, the standard deviation or Coefficient of Variation (COV) of response measures the impact of randomness in the inputs on the response. The standard deviation (SD) of response can be calculated by statistical methods such as MCS and non-statistical methods such as FORM [21]. In FORM, the response or output is expressed as a first-order Taylor series expansion at the mean value point of random inputs. Assuming the random inputs (X) are statistically independent, the response $g(X)$ at the mean value (μ_X) can be expressed as

$$\tilde{g}(X) \approx g(X) + \nabla g(\mu_X)^T (X_i - \mu_x) \tag{2}$$

where ∇g is the gradient of g evaluated at μ_X. The mean (μ_g) and standard deviation σ_g of the response can be given as

$$\mu_g \approx E[g(\mu_X)] = g(\mu_X) \tag{3}$$

$$\sigma_{\tilde{g}} = \left[\sum_{i=1}^{n} \left(\frac{\partial g(\mu_X)}{\partial x_i} \right)^2 \sigma_{x_i}^{2} \right]^{\frac{1}{2}} \tag{4}$$

The above equations 3 and 4 need the sensitive derivatives which can be calculated analytically, if possible or by finite difference method. Therefore, the FORM can be used to evaluate the statistical properties of objective function in the robust design and optimization studies with less number of simulations.

4 Real Coded Genetic Algorithm

GAs are stochastic optimization techniques based on the Darwin's theory of survival of the fittest [17]. GA is a search algorithm based on the mechanism of natural selection that transforms a population (a set of solutions) into a new population (i.e., next generation) using genetic operators such as crossover, mutation and reproduction [17, 18]. A survival of the fittest strategy is adopted to identify the best strings and subsequently genetic operators are used to create a new population for the next generation. More details about how genetic algorithms work for a given problem can be found in the literature [17, 18]. A good representation scheme for the solution is very important in obtaining the best solution for a given problem using GA. The solution can be represented as the binary vector (0 and 1), integers and floating point numbers. In Real Coded Genetic Algorithm (RCGA), integers and floating point numbers are used in strings which is more efficient and produces better results than binary based GA.

The objective of the RCGA is to find out the optimal discrete values of ply angle variables for a given airfoil cross section. The various components of RCGA are described below.

4.1 String Representation

String representation is a process of encoding the discrete values of the angles assigned to each ply in the laminate wall of the airfoil section. Each string in a population represents a possible solution to the design problem. The string representation scheme depends on the structure of the problem in the GA framework and also depends on the genetic operators used in the algorithms. Each solution in a GA population consists of an array of integers. The values of integers determines the orientation of each ply. The use of real values instead of binary digits always produce valid chromosomes and therefore, increases the efficiency of the algorithm.

4.2 Population Initialization

Genetic algorithm starts with an initial population of solutions to the given problem. The most frequently used technique for population initialization is random generation based on the knowledge of a given problem. The initial population size and the method of population initialization will affect the rate of convergence of the solution. In this problem, random selection of ply angles from the given set of angles is used to initialize the stacking sequences.

4.3 Selection Function

The probability of a solution being selected to generate new solutions generally depends on the fitness of the solution. Usually, a solution with better fitness in a population has a higher probability of being selected more than once. In the literature, several schemes such as roulette wheel selection and its extensions, scaling techniques, tournament and ranking methods are presented for the selection process. In this work, the normalized geometric ranking method given in [20] is used for the selection process.

4.4 Genetic Operators

Genetic operators used in genetic algorithms are analogous to those which occur in the natural world: reproduction (crossover, or recombination), and mutation. The probability of these operators will affect the efficacy of the GA. The genetic operators for RCGA are described below.

The crossover operator is a primary operator in GA. The role of crossover is to recombine genetic information from the two selected solutions into even better solutions. The crossover operator improves the diversity of the population. The form of the crossover operator depends on the string representation. Now, we describe four different crossover operators used in this problem. Let A and B be the two parents selected for the crossover operation from the population and given as

$$A = \{a_1, a_2, ...a_i, a_{i+1}, ...a_j, a_{j+1}, ..a_{n/2}\}$$
$$B = \{b_1, b_2, ...b_i, b_{i+1}, ...b_j, b_{j+1}, ..b_{n/2}\} \quad (5)$$

where a_i and b_i are integers belonging to the possible ply angle set.

a) Two Point Crossover: In this operator, two crossover points i and j are selected randomly in the parents, where $(i < j, i, j \leq n/2)$. The offspring produced by swapping the selected ply angles between the crossover points are

$$A' = \{a_1, a_2, \cdots, b_i, b_{i+1}, ...b_j, a_{j+1}, ..a_{n/2}\}$$
$$B' = \{b_1, b_2, ...a_i, a_{i+1}, ...a_j, b_{j+1}, ..b_{n/2}\} \quad (6)$$

b) Uniform Crossover: In this operator, the cross over points are selected randomly. The ply angles in the selected crossover points are swapped between the parents. Let the randomly selected crossover points be 2, i and j. The offspring produced are given below.

$$A' = \{a_1, b_2, .., b_i, a_{i+1}, .., b_j, a_{j+1}, .., a_{n/2}\}$$
$$B' = \{b_1, a_2, .., a_i, b_{i+1}, .., a_j, b_{j+1}, .., b_{n/2}\} \quad (7)$$

The mutation operators are used to avoid the local minima and premature convergence of the algorithm by introducing diversity in the population.The mutation operators used in this study are explained below. Let A be the parent selected for the mutation operation.

$$A = \{a_1, a_2, .., a_i, .., a_j, .., a_{n/2}\}$$

a) Swap mutation: In swap mutation, two mutation points are randomly selected. The selected ply angles in the mutation points are swapped to generate the new offspring. Let i and j be the mutation points selected. The offspring $A^{'}$ produced is

$$A^{'} = \{a_1, a_2, \cdots, a_j, \cdots, a_i, \cdots, a_{n/2}\}$$

b) Heuristic mutation: In this operator, a single mutation point is randomly selected. The ply angle in the selected mutation point is replaced with a randomly selected value from the possible set of values. For example, let us consider the following string for mutation operation and the mutation point is highlighted in boldface.

$$A = \{0, 15, \mathbf{45}, 0, 30, 90\} \quad (8)$$

Now the mutation point is replaced with the randomly selected ply angle value, say 60. The offspring produced by this operator is

$$A^{'} = \{0, 15, \mathbf{60}, 0, 30, 90\} \quad (9)$$

4.5 Fitness Function

Fitness is the driving force in GA. In RCGA, the solutions represent the possible angles of the plies in the laminate. Based on these angles, the stiffness values are calculated using the VABS code. Using these stiffness and the desired values, the fitness of the solution is calculated. The GA will try to maximize the fitness.

4.5.1 Termination Criterion

The maximum number of generations is commonly used as the termination criteria. Hence, in RCGA, the maximum number of generations is used to terminate the algorithm.

5 Numerical Results

The rotor blade considered in this study is a uniform blade equivalent of the BO-105 rotor blade. The NACA0015 airfoil section with 304.8 mm (12 inch) chord is selected for this study. The details of the airfoil section are given in Fig. 1. The skin and the D-spar are considered to be made of graphite/epoxy composite material. The stacking sequence of skin and D-spar is selected to provide frequencies typical of a stiff-in-plane rotor [22]. The macrolevel effective material properties (E_1, E_2, G_{12}, G_{23}, ν_{12}, ν_{23}) of graphite/epoxy are considered as random variables with normal distribution. The COV for material properties are taken from a micromechanics study in which the fiber and matrix properties are considered as random variables [2] and given in Table 1. We see that there is considerable variability in the

Table 1: Material properties of graphite/epoxy

Property	Mean	COV(%)
E_1 (GPa)	141.96	7.0
E_2, E_3 (GPa)	9.79	4.0
G_{12}, G_{13} (GPa)	6.00	12.0
G_{23} (GPa)	4.80	3.0
ν_{12}, ν_{13}	0.30	3.5
ν_{23}	0.34	3.0

COV with the Poisson's ratios ranging around 3% and some of the shear modulus going up to 12%.

In robust design, the first step is to identify the sensitive random variables. A COV of 1 percent and a normal distribution is considered for all the six material properties. The impact of randomness in each material property on the rotor blade cross-sectional stiffness is studied. That is, each material property is considered as a random variable with 1 percent COV while other properties are kept at its deterministic or baseline value. The statistics of the blade cross-sectional stiffness are calculated using 500 MCS. The cross-sectional analysis of the rotor blade is carried out with the FEM based VABS code. The sensitivity results are shown in Fig. 11. The results show that the randomness in longitudinal Young's modulus E_1 has a higher impact on the cross-sectional stiffness than other material properties for the baseline blade. Further, the uncertainty in E_1 has different impacts on the flap, lag and torsion stiffness with the impact on GJ being lowest. Other than E_1, the variations in shear modulus G_{12} leads to considerable scatter in the blade cross-sectional stiffness. In particular, the GJ shows a strong effect of G_{12}. However, these uncertainty results are for the specific stacking sequence and thickness of the skin and spar of the baseline rotor blade.

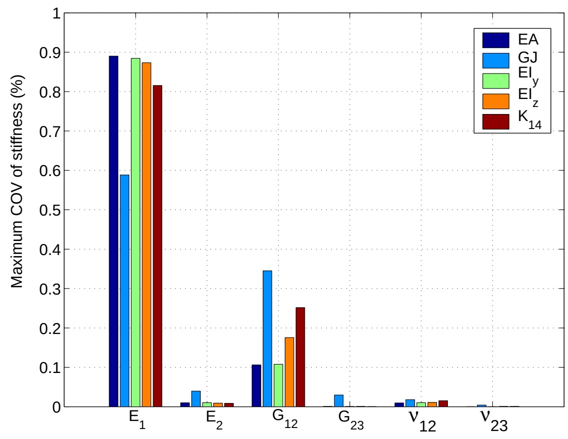

Figure 2: Sensitivity of baseline cross-sectional stiffness to material properties

5.1 Stacking Sequence

For a given airfoil cross section with n number of plies, the stiffness varies with the ply angle values of laminate. Therefore, the uncertainty impact also can vary with the ply angle values. To study the variation of uncertainty with stacking sequence, the ply angles of skin ($[\pm\theta_3]_s$) and D-spar ($[\pm\theta_6]_s$) are varied from 0 to 90 degree. The standard deviation of stiffnesses with respect to the variation in material properties are calculated with the FORM. For example, the standard deviation of torsional stiffness can be given as

$$\sigma_{GJ}^2 = \left(\frac{\partial GJ}{\partial E_1}\sigma_{E1}\right)^2 + \left(\frac{\partial GJ}{\partial E_2}\sigma_{E2}\right)^2 + \left(\frac{\partial GJ}{\partial G_{12}}\sigma_{G12}\right)^2 + \left(\frac{\partial GJ}{\partial G_{23}}\sigma_{G23}\right)^2 + \left(\frac{\partial GJ}{\partial \nu_{12}}\sigma_{\nu 12}\right)^2 + \left(\frac{\partial GJ}{\partial \nu_{23}}\sigma_{\nu 23}\right)^2 \tag{10}$$

In the above equation, the first derivative of cross sectional stiffness with respect to each material property is calculated with the central finite difference scheme. The COV of each material property is considered as the step size in central finite difference scheme for calculating the first order derivatives. For example, the sensitivity of GJ with respect to the E_1 is calculated as

$$\begin{aligned} \triangle GJ &= \frac{GJ^a - GJ^b}{E_1^a - E_2^b} \\ \mathrm{E}_1^a &= 1.07E_1 \\ \mathrm{E}_1^a &= 0.93E_1 \end{aligned} \tag{11}$$

where E_1 is the mean value of longitudinal Young's modulus. The sensitivity of the stiffnesses with respect to material property for different ply angles are shown in Figs. 3 to 6. Note, the sensitivity of stiffnesses with respect to elastic moduli are dimensional whereas with respect to Poisson's ratio are nondimensional and therefore, shown in separate figures. The variation in sensitivity of flap and lag stiffness are similar and therefore, only lag stiffness is shown.

It is observed that the sensitivity of flap and lag stiffness to G_{12} is higher than the sensitivity to longitudinal and lateral modulus when the ply angle values are close to 30 degree as shown in Fig 4. The torsional stiffness is highly sensitive to the shear modulus G_{12} and G_{23} than the Young's modulus. The torsional stiffnesses is highly sensitive to the Poisson's ratio ν_{12} near 45 degree whereas the sensitivity of lag and flap stiffness are maximum near 20 degree.

For calculating uncertainty impact, the sensitivity factors are multiplied with the standard deviation of corresponding material properties as given in Eq. 4. The variation of these multiplied factors are shown in Figs. 7 and 8. It is observed that the randomness in longitudinal Young's modulus and shear modulus G_{12} have higher impact on the cross-sectional stiffness than other material properties. Therefore, the randomness in E_1 and G_{12} values can be considered as a major factors in the uncertainty analysis.

The standard deviation of torsional stiffness reaches its maximum value when the ply angle values are at 45 degree. The standard deviation of flap and lag stiffness are higher when the ply angle values approach zero and decrease when the ply angle value is greater than 30 degrees.

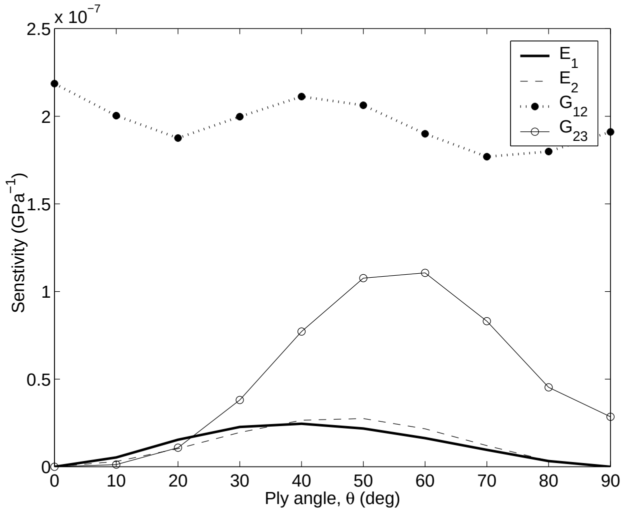

Figure 3: Sensitivity of torsional stiffness to elastic moduli

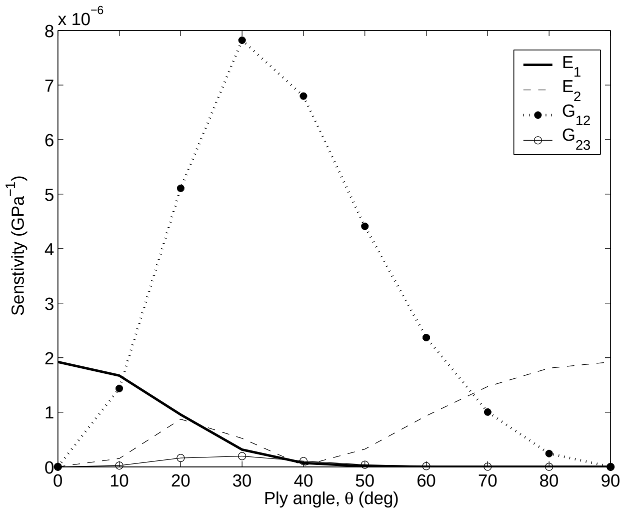

Figure 4: Sensitivity of lag stiffness to elastic moduli

5.2 Robust Optimization

In robust design and optimization, the structure is optimized for a specified requirement and its varia-

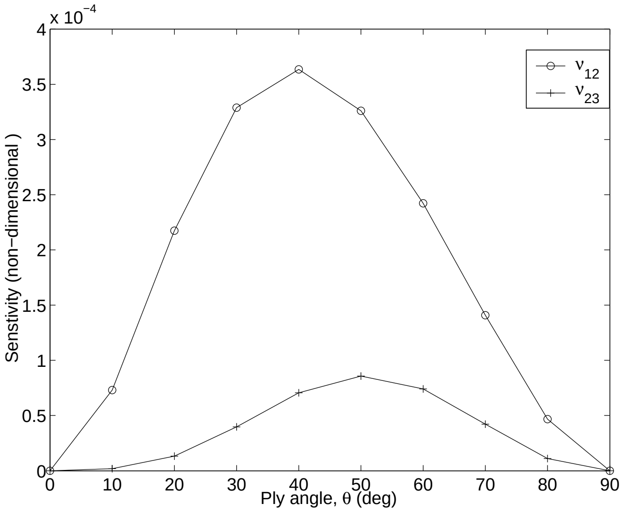

Figure 5: Sensitivity of torsional stiffness to Poisson's ratio

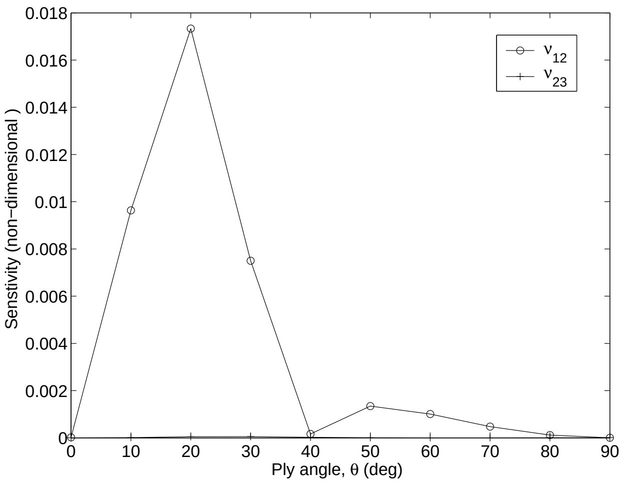

Figure 6: Sensitivity of lag stiffness to Poisson's ratio

tion with respect to the randomness in design variables is also minimized. The baseline rotor blade shown in Fig. 1 is designed to match the stiff-in-plane rotor properties. The SD of cross-sectional stiffnesses of the baseline rotor blade are given in Table 2.

Table 2: Statistics of cross-sectional stiffness

Stiffness	Baseline	SD
$GJ/m_o\Omega^2R^4$	0.0037	2.157e-004
$EI_y/m_o\Omega^2R^4$	0.0057	3.595e-004
$EI_z/m_o\Omega^2R^4$	0.1249	0.00759

Figures 7 and 8 show that the scatter in torsional and bending stiffness (lag and flap) reaches a maximum at different ply angle values. The minimization of variation in torsional and bending stiffness simultaneously may result in a non-robust solution. Therefore, only the scatter in bending stiffness is considered to be minimized. As the flap and lag stiffness follow simi-

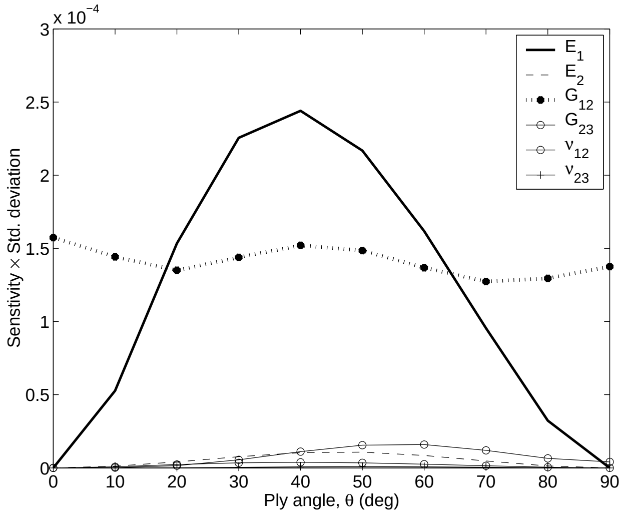

Figure 7: Effect of stacking sequence on SD of torsional stiffness

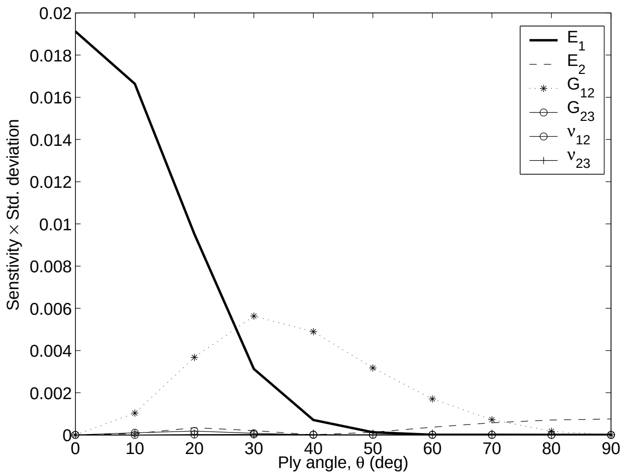

Figure 8: Effect of stacking sequence on SD of lag stiffness

lar scattering, the minimization of SD of lag stiffness leads to reduction in flap stiffness also. For the robust optimal design of rotor blade, the optimization problem can be written as

$$Min, \sigma(EI_z) \tag{12}$$

The stacking sequence of skin and D-spar of rotor blade are considered as design variables. The design variable vector θ can be given as

$$\begin{aligned}
\theta &= [\theta_{spar},\ \theta_{skin}] \\
\theta_{spar} &= [\pm\theta_1, \pm\theta_2, \pm\theta_3, \pm\theta_4, \pm\theta_5, \pm\theta_6]_s \\
\theta_{skin} &= [\pm\theta_7, \pm\theta_8, \pm\theta_9]_s
\end{aligned} \tag{13}$$

The above stacking sequence represent symmetric balanced laminate for skin and D-spar and consists of six design variables of D-spar and three design variables of skin of the rotor blade. The allowable ply angle design values are given as

$\theta \in \{0^o, 5^o, 10^o, ..., 90^o\}$

While the rotor blade is optimized for minimizing the uncertainty effects, the dynamic properties of the rotor blade such as rotating natural frequencies will deviate from the baseline rotor properties. The rotating natural frequencies depends on the cross-sectional stiffness of the blade. Therefore, the constraints are placed on the deviation of candidate stiffness value from the baseline stiffness value. The constraints can be given as

$$Max\ (g) \quad \leq 0.20 \tag{14}$$

where the inequality constraint g is defined as

$$g(\theta) = abs\left[\frac{EI_y^t - EI_y}{EI_y^t}, \frac{EI_z^t - EI_z}{EI_z^t}, \frac{GJ^t - GJ}{GJ^t}\right] \tag{15}$$

Here, the superscript t corresponds to the baseline values of stiffness. The above formulation tries to design the rotor blade to match the baseline stiffness value with minimum impact of material uncertainty. The material properties E_1, E_2 and G_{12} are considered as random variables in the robust design. The standard deviation can be calculated by FORM as shown below.

$$\sigma_{EI_z}^2 = \left(\frac{\partial EI_z}{\partial E_1}\sigma_{E1}\right)^2 + \left(\frac{\partial EI_z}{\partial E_2}\sigma_{E2}\right)^2 + \left(\frac{\partial EI_z}{\partial G_{12}}\sigma_{G12}\right)^2 \tag{16}$$

The real coded genetic algorithm is used as the optimization tool. The optimal ply angles from the best five runs of robust optimization and their percentage reduction in the SD from the baseline design are given in Table. 3 and 4, respectively. From the robust design perspective, the case 4 and 5 results show a 11 to 13 percent reduction in SD of torsional stiffness and 12 to 23 percent reduction in SD of flap and lag stiffness. However, the rotating natural frequencies play a major role in rotor blade design. Therefore, the rotating natural frequencies of the five robust optimal results are given in Table 5. In helicopter blades, the integer multiples of rotor speed should not coincide with rotating natural frequencies. The torsion rotating frequencies of case 3, 4 and 5 are closer to the 4/rev frequency. The torsion frequencies of case 1 and 2 are better than the remaining three optimal results. Therefore, the case 1 robust optimal design is considered as the best design.

Table 3: Robust optimal ply angles

Case	D-Spar	Skin
1	[25,30,80,0,65,10]	[30,25,65]
2	[55,15,25,5,65,20]	[35,25,85]
3	[5,25,65,5,25,65]	[60,60,10]
4	[80,0,0,5,75,15]	[25,70,35]
5	[0,25,50,50,15,20]	[65,70,10]

Table 4: Reduction in standard deviation

Case	GJ(%)	EI_y(%)	EI_z(%)
1	1.38	21.98	19.74
2	6.23	21.99	23.70
3	8.74	21.28	19.55
4	10.72	12.32	23.26
5	12.86	21.16	20.52

Table 5: Rotating frequencies of robust designs (/rev)

Case	Flap	Lag	Torsion
Base	1.1242	1.4383	4.1690
1	1.1137	1.3278	4.2231
2	1.1137	1.3106	4.1097
3	1.1137	1.3212	4.0189
4	1.1176	1.3073	3.9765
5	1.1137	1.3162	3.8879

The SD of cross-sectional stiffnesses of case 1 robust design normalized with the SD of baseline is shown in Fig. 9. The histograms of flap, lag and torsional stiffness of case 1 robust design are generated by MCS and shown in Figs. 10 to 12. The histograms clearly show a contraction in the scatter of flap and lag stiffness when compared to the baseline histograms.

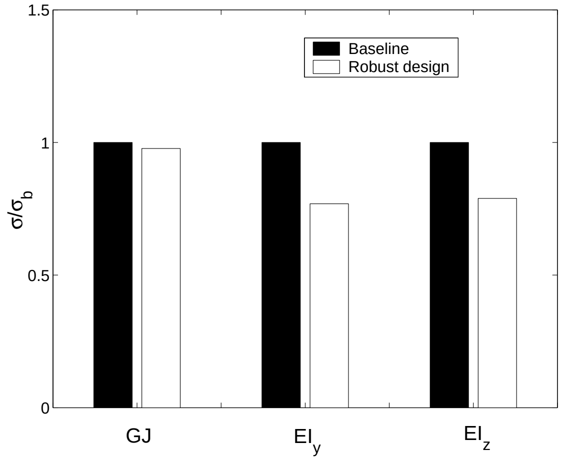

Figure 9: SD of baseline and robust designs, normalized with baseline values

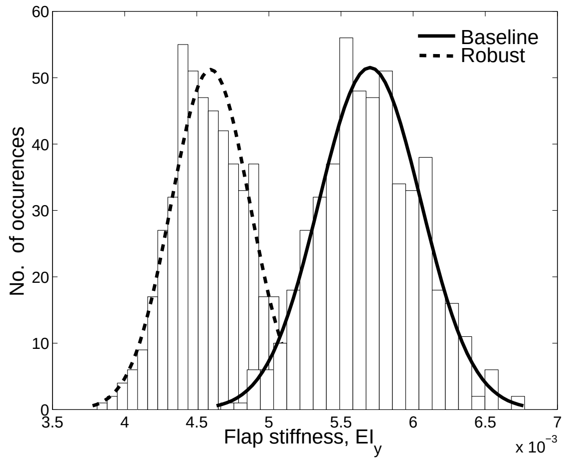

Figure 10: Histograms of flap stiffness

6 Conclusion

In this paper, the effect of material uncertainty on the design optimization of a composite rotor blade is studied. A statistical First-Order Reliability Method (FORM) is used to propagate the input uncertainties through the finite element based cross-sectional analysis code to evaluate the variations of output response. The statistical properties are then used in a robust optimization process. The first order derivatives required for the FORM are obtained from central finite difference scheme. The FORM based statistical analysis is computationally efficient compared to MCS for the robust design of rotor blades. The sensitivity analysis results show the uncertainty in longitudinal Young's modulus, E_1 and shear modulus, G_{12} have major impact on the cross-sectional stiffness. The numerical results of robust optimization show designs which are more robust than the baseline design. The robust design shows 12-23 percent reduction in uncertainty in lag and flap stiffnesses while the mean values match with the required stiffness properties.

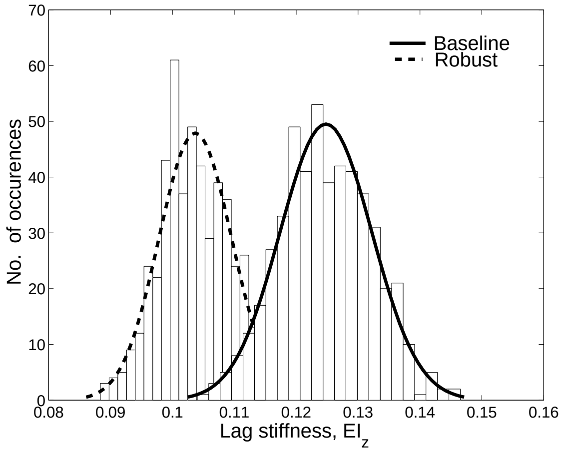

Figure 11: Histograms of lag stiffness

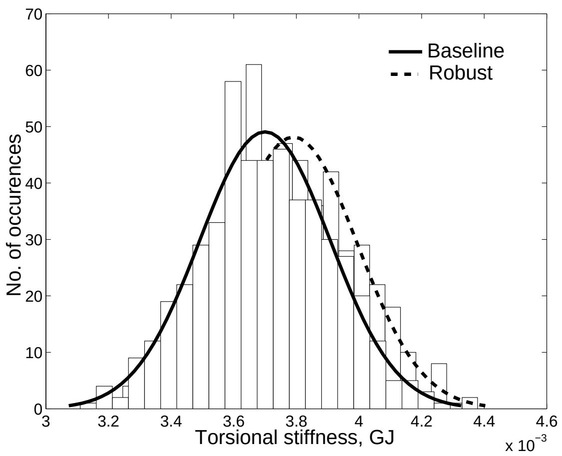

Figure 12: Histograms of torsional stiffness

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