

THE SOLUTION OF THE HELICOPTER FLIGHT DYNAMICS TASKS
BY THE METHODS OF OPTIMAL CONTROL THEORY

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A B S T R A C T

The expansion of the coaxial helicopter manoeuver capabilities demanded the study of flight dynamics at the modes with limited values of flight parameter and application for this purpose of the optimal control theory modern methods.

The solution of most unclassic tasks, i.e. those with constraints is based on the Pontrjagin's maximum principle. To solve the helicopter optimization control task, as an unlinear object, the Krotov's iteration method - the method of improvement is selected.

Based on the developed approaches and on the system of mathematic models proposed by the author a package of applied programs is created to solve a number of practical tasks for definition of the helicopter manoeuver capabilities at the modes with the flight parameters limiting value, to examine complex flight modes and to provide for the helicopter flight modes automation.

From a number of solved tasks the following are examined: determination the effect of the desing constrains upon the helicopter heading turn angle in hover and the Ka -32 helicopter autorotation landing task at different flight weights.

I. INTRODUCTION

Exploration by traditional methods of mathematic and half full scale modelling of flight dynamics does not permit to solve the full range of problems conserving the helicopter maneuverability especially when using the maximal capabilities and the limit values of flight parameters. Such situation dictated the necessity to use new methods and particularly the methods of the optimal control theory.

When applied to the helicopter flight dynamics the optimal control methods permit to solve a whole range of problems concerning :

- determination of the helicopter maneuvering capabilities in such modes where the flight parameters (speed, overload etc.) reach their limit values which permits to evaluate the helicopter capabilities depending upon various design limitations at the initial design stage;

- investigation of complex flight models in order to reduce the volume of flight testing especially the testing connected with a certain amount of risk like landing with engines failed or autorotation landings;

- automation of various helicopter flight modes.

II. FORMULATION OF THE HELICOPTER CONTROL OPTIMIZATION TASK.

1. The method selection

The modern optimal control theory is based on Pontrjagin maximum and Bellman relativity principles.

The new prospects of solving existing nonlinear problems appeared upon the development at the 1970-ies by Soviet scientists V.F.Krotov and V.I.Gurman of a theory based on the general adequate conditions of optimality and of iterative methods for solving optimal control problems - the improvement methods.

The advantages of the method are its orientation on the nonlinear models, practical absence of limitations on the form of the right sides of the initial differential equations system, a wide possibility of utilizing the engineering knowledge on the object of control and the modes explored.

The investigations carried out by the author show that it is also possible and suitable to use the improvement method for the purpose of identifying the object mathematic model. On this case one and the same method is used for solving the dual problem of control identification and optimization which will provide for considerable gains at practical application due to the continuity of the programmed implementing of both the problem sides.

2. Mathematic model construction.

Modelling of the helicopter motion dynamics with the full consideration of all its peculiarities calls for very complex object descriptions due to a wide range of motion parameters variations a notably larger number of helicopter degrees of freedom as compared to a fixed wing aircraft.

It is most important to maximally simplify the model describing the object by getting rid of all complex dependencies which are not relevant for solving the problem in question. But on the other hand neglecting the important dependencies may render the recommendations worked out in the process of problem solving useless.

Generalization of its long standing experience permitted the Kamov Design Bureau to create a number of helicopter motion models for solving the problems of helicopter control optimization in various flight modes.

Table 1

models chars	fully linear	parti- ally linear	linearized		nonlinear	
			long. (side) motion	3 D motion	long. (side) motion	3 D motion
The order of the initial equation system	3	3	7	14	7	14
The order of control vector	1	1	2	4	2	4
Right si- des (line- arization degree)	linear with cons- tant coef- fici- ents	linear of 1-2 nonli- near- ity	partially linea- rized; the main nonlinearities are implied		nonlinear airframe characteristics, nonexplicit func- tions.	
Number of cons- traints - current	6	4	up to 10	up to 20	up to 10	up to 20
- final	1	2	4	up to 10	4	up to 10
The right side deri- vation	a n a l y t i c a l				Trasfer to a discrete scheme	
Examples of practi- cal appli- cation	The influ- ence of de- sign para- meters	Auto- mation of verti- cal modes	Maneu- vers in single plane in a limited range of parame- ters variation	3 D ma- neuers	Maneuvers in the single plane full range of mo- tion parameters variation	3 D ma- neuers

The models presented in Table 1 differ in the non-linearity degree of the differential equations systems right sides describing the helicopter motion, in the number of limitations, the dimentional representation of the control vector etc.

3. Determination and implementation of constraints.

The practical solution of a helicopter control optimization problem demands taking account of various constraints on power, overload, travel of controls, speeds, angles etc.

For implementation of constraints a penalty functions method is used which reduces the task to a sequence of problems on minimization of a certain auxiliary function which coincides with the initial function within the limits of the constraints and sharply increases beyond those limits, i.e.:

$$f(z)=0 \quad Z_{\min} \leq Z \leq Z_{\max} ; \quad f(z) = \begin{cases} Z > Z_{\max} \\ Z < Z_{\min} \end{cases}$$

Here Z may denote a control vector component as well as a phase vector component.

The difficulties in using the penalty functions method for practical problem solutions are most often connected with unsuitable scaling. It is necessary to take into account the fact that state vector components may differ by a whole order of magnitude or more (for example, the change of flight speed is measured by values of 10..20 m/s and the change of rotor speed by 0.1 1/s and so on).

Keeping that in mind it is recommended to normalize penalty function exponential indices setting approximately the maximal expected change of parameters, for example:

$$f(z) = \beta e^{\alpha(z-Z_{\max})} = \beta e^{\alpha Z_{\max}(\frac{z}{Z_{\max}} - 1)}$$

A penalty function coefficient control algorithm (fig.1) is developed so that when the solution is far from the optimum at the beginning of the solution process the penalties are "soft", i.e. small α coefficients and large β coefficients and later the penalty gets "harder" by increasing α coefficients and decreasing β coefficients (fig.2).

Full functional : $I = I^x + J$

Useful side of the functional: $I^x = q \Delta X_k$

Penalized side of the functional: $J = F_k(t_k) + \int_{t_n}^{t_k} f^0 dt$

$$F_k(t_k) = (1-q) \sum_{i=1}^N \beta_i (X_i - X_{i, req})^2$$

$$f = (1-q) \sum_{i=1}^M \beta_i \left[e^{\alpha_i (Z_i - Z_{i, max})} + e^{\alpha_i (Z_{i, min} - Z_i)} \right]$$

$X = (V_x, V_y, \varphi, \omega, \dots)$; $Z = (\varphi, \sigma_B, \omega, n_y, N, \dots)$

CONTROL ALGORITHM

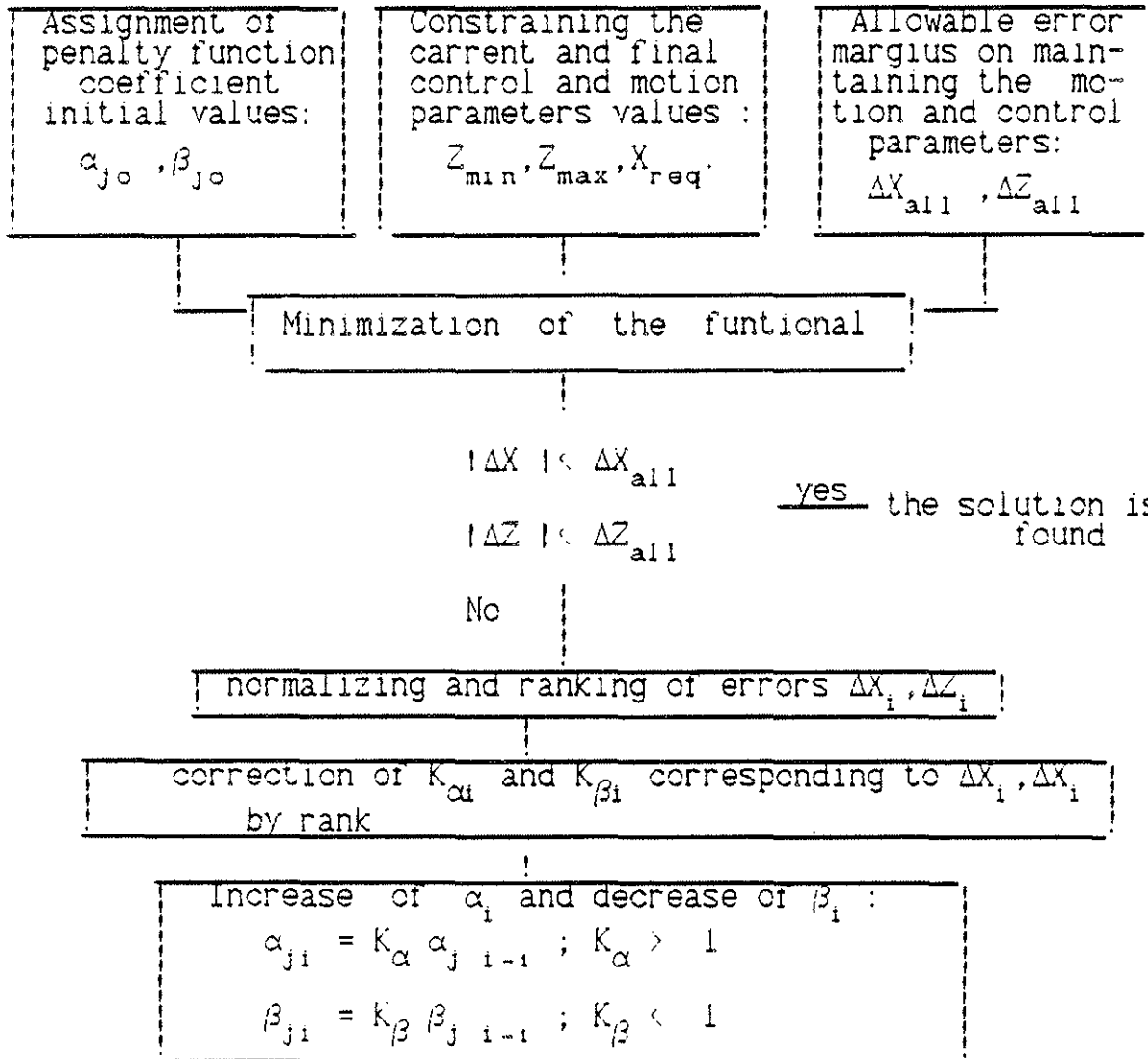


fig.1 Penalty method coefficients control

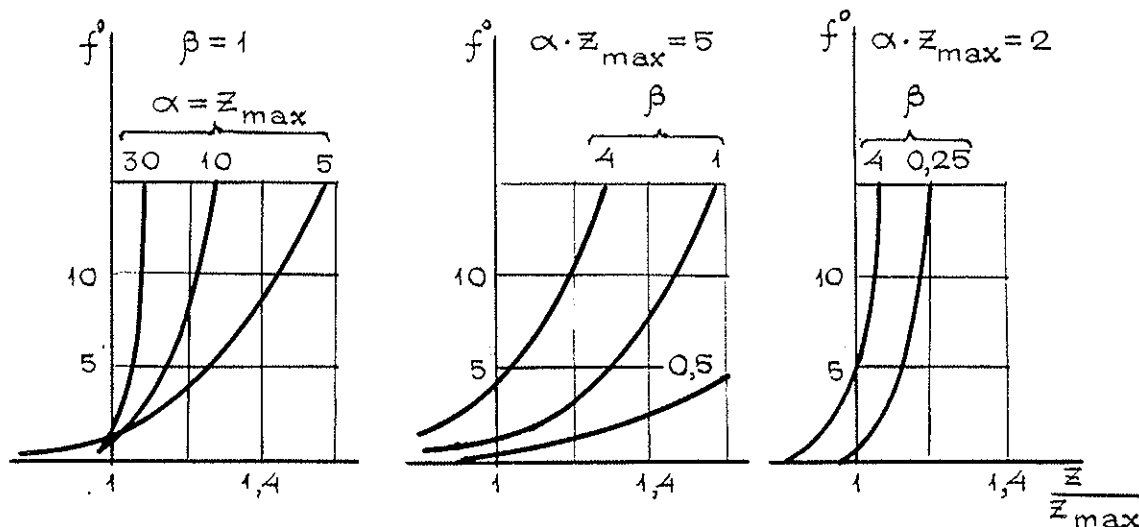


fig.2 The effect of the exponential index α and scale coefficient β value on the penalty function f^0 .

4. Selection of initial approximation.

The improvement method as all iterative methods assumes the selection of initial approximation control which may be obtained from several sources:

1. the control may be determined on the basis of the data on flying a similar mode by the helicopter in question during flight testing;
2. the results of mathematic and scale modelling may be used;
3. in the absence of the above mentioned data analogues data on some other helicopter may be used;
4. in a general case initial approximation control may be set quantitatively under physical considerations on the basis of engineering experience.

All the sources mentioned above may be quite successfully utilized. It should be pointed out that the more complex the task is and the more nonlinearities and constraints are there, the more important the selection of the initial approximation becomes.

III. SEQUENTIAL CONTROL IMPROVEMENT METHOD

1. Method description.

Suppose that there exists a differential equations system describing the system being examined:

$$\dot{X} = f(t, x, u), x \in R^n, u \in R^r \quad (1)$$

with initial conditions: $X(t_n) = X_n$, and the first approximation is set which is an aggregation of phase trajectory $X^I(t)$, control program $U^I(t)$ and process termination time t_n^I :

$$m^I = (X^I(t), U^I(t), t_n^I).$$

The task is as follows: from D solutions of system (1) such a second approximation $m^{II} \in D$ shall be found so that the functional

$$I = F(X(t_n)) + \int_{t_n}^{t_k} f^0 dt$$

diminishes its value: $I(m^{II}) < I(m^I)$.

The improvement procedure according to the method presented may constitute a procedure of second order or of a simpler first order when it coincides with the known procedure of gradient descent in the functional space.

In the process of solving the helicopter control optimization problems some modification of the improvement method were developed with the aim of expanding its capabilities. One of the modifications proposes the state-in-space discretization supplemented by the operations of analysis and search for optimal discretization steps which is relevant for solving the problems with nonlinear right sides of the equations system describing the motion of control object.

To solve the identification problem another modification was developed - a method for dynamic system unknown constants determination upon a run of experiments which permits to correct the mathematical model to make it correspond to the control object.

2. Improvement procedure.

The improvement procedure consists of the following steps:

1. By integrating the initial system(1) in the interval from t_n to t_k at $U=U^I(t)$ we get the aggregation of first approximation phase trajectories - a "support" m^I :
$$m^I = (X^I(t), U^I(t), t_k^I).$$

At this "support" we calculate and memorize the value of the functional:
$$I^I = I^I(m^I).$$

2. Now we determine $t_k^{II} = t_k + \delta t_k$, where

$$\delta t_k = - K_t \text{Sign} [F_{t_k}^I - H^I(t_k)];$$

by this for the first order methods:

$$H = \psi' f - f^0 \alpha, \quad (\psi' f - \text{vector product of } \psi \text{ and } f).$$

3. We determine an auxiliary vector function $\psi(t)$ as a solution of differential equations system $\dot{\psi} = - \left(\frac{\partial H}{\partial X} \right)^I$, which is integrated from right to left at the initial conditions: $\psi(t_k^I) = - \frac{\partial F}{\partial X_k} (t_k^I, X^I(t_k^I))$.

In the process of integration we calculate sequentially the vector derivatives (on each of the control vector components):

$$H_u = \left(\frac{\partial H}{\partial U} \right)^I = \psi' f_u - f_u^0, \quad \alpha = \psi' \left(\frac{\partial f}{\partial U} \right) - \left(\frac{\partial f^0}{\partial U} \right) \alpha$$

and determine the control correction δU , which may be uniform all over the "support": $\delta U = -K_u \frac{H_u}{|H_u|}$, as well as proportional

to the derivative value H_u : $\delta U = -K_u \frac{H_u}{|H_u|_{\max}}$;

4. We determine the control parameters vector of second approximation $U^{II} = U^I + \delta U$ and replace the initial approximation on control by this vector. By integrating the initial system at the new second approximation control we obtain the second "support": $m^{II} = (X^{II}(t), U^{II}(t), t_k^{II})$.

At each "support" the functional is calculated and compared with the preceeding one. At the decrease of the functional the calculation is continued with the vector components of the control gain coefficient K_j preserved or increased. At the increase of the functional which certifies to passing the minimum point, the proximity condition of neighbouring approximations control is checked. The solution is considered reached when the neighbouring approximations functional values as well as the control values coincide with the required degree of accuracy.

IV PRACTICAL APPLICATIONS

1. Peculiarities of the method application.

On the process of the method practical application some additional developments appeared to be necessary and the most important of them were developing a series of motion models and the penalty method coefficients control algorithm as described above. Two more problems also required settling - the necessity to check the found extremum for the absolutness and to find the optimal relationship of the control vector components.

Solving practical problems is shown for two models describing the helicopter motion. Both of them belong to the series of models presented in Table I. One of them is the simplest linear model and another is one of the most complicated nonlinear models.

2. Simple model problem solution.

The task is set to determine the effect of the design constraints upon the helicopter heading turn angle in hover with fixing the heading angle at the end of the turn. The influence of the design parameters manifest itself in limiting the maximum turn rate and the control power available (control travel available).

The motion equations describing the helicopter motion in this mode have the following form:

$$\frac{d\Delta w_y}{dt} = \bar{M}_y^{wy} \Delta w_y + \bar{M}_y \Delta \varphi$$

$$\frac{d\Delta \psi}{dt} = \Delta w_y$$

$$\frac{d\Delta \varphi}{dt} = \frac{1}{T} (U - \Delta \varphi), \quad \text{where}$$

$\Delta \varphi$ - the change of the heading control actuator position;

U - displacement of pedals transformed to the dimensional representation of the actuator;

\bar{M}_y^{wy} - helicopter relative damping;

\bar{M}_y^φ - relative yaw control power;
 T - time constant of the actuator;
 $\bar{M}_y^{W_y}, \bar{M}_y^{\Delta\varphi} T$ - const.

In the process of turn the constraints on turn rate ΔW_y , control travel $\Delta\varphi$ and actuator movement speed $\dot{\Delta\varphi}$ were implied; at the last moment the value of the turn rate W_y is limited:

$$|\Delta\varphi| \leq \Delta\varphi_m; \quad |\dot{\Delta\varphi}| \leq \dot{\Delta\varphi}_m; \quad |W_y| \leq W_{y_m}$$

$$|W_y(t_k)| = W_{y_{k-1m}} = 0.$$

The parameter optimized is the heading angle (maximum heading angle should be available).

The mathematic model for optimization problem solving has the following form:

$$f_1 = \frac{dX_1}{dt} = A_{11} X_1 + A_{12} X_3$$

$$f_2 = \frac{dX_2}{dt} = X_1$$

$$f_3 = \frac{dX_3}{dt} = -A_{33} X_3 + A_{33} U$$

with limiting phase and control components:

$$|X_1| \leq X_{1m}; \quad |X_3| \leq X_{3m}; \quad |\dot{X}_3| \leq \dot{X}_{3m};$$

and limiting the final parameter value:

$$|X_1(t_k)| = X_{1_{k_m}}$$

So according to the task set we get the value of the unpenalized functional equal to :

$$I^* = -X_2(t_k).$$

Implementation of all above listed constraints in the form of penalties brings us to the following form of the penalized functional :

$$I = F_k(t_k) + \int_{t_n}^{t_k} f^0 dt$$

where the terminal side implies both the unpenalized functional and the turn rate constraints at the moment of the process termination:

$$F_k(t_k) = -q X_2(t_n) + (1-q)\beta_4 (X_1(t_k) - X_{1k_m})^2,$$

and the integral side implies all the constraints of the current parameters (both control and phase coordinates):

$$f^0 = (1-q) \left[\beta_1 \left[e^{\alpha_1 (U - U_m)} + e^{\alpha_1 (-U_m - U)} \right] + \right. \\ \left. + \beta_2 \left[e^{\alpha_2 (X_1 - X_{1m})} + e^{\alpha_2 (-X_{1m} - X_1)} \right] + \right. \\ \left. + \beta_3 \left[e^{\alpha_3 (A_3 U - A_3 X_3 - \dot{X}_{3m})} + e^{\alpha_3 (\dot{X}_{3m} - A_3 U + A_3 X_3)} \right] \right].$$

Exponential character of the penalty function is adopted for more stringent constraints and square character as less stringent is adopted for limiting the turn rate at the moment of the process termination.

The dependences show that even a small number of constraints lead to a quite complicated expression of the minimized functional.

The check of the extremum for absolutness is done by one of the simplest engineering methods i.e. solutions obtained for controls with different initial approximation are compared (fig.3), since the problem is relatively linear the convergence is ensured by a rough enough setting of the initial approximation control.

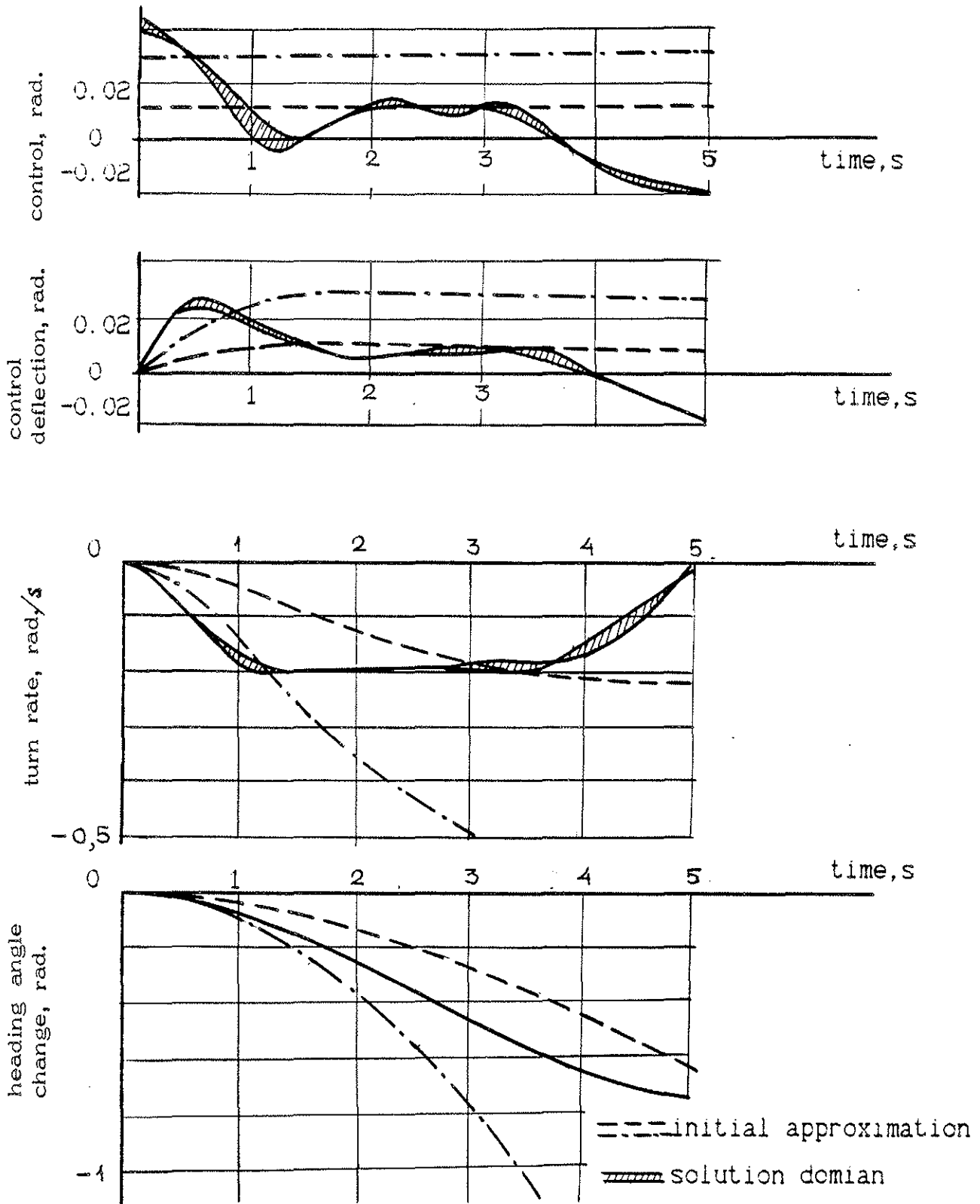


fig.3 The change of motion and control parameters in the process of making a heading turn in hover.

The resulting dependences of the maximum heading turn angle in the function of the time available at different constraints on speed and control power were obtained (fig.4).

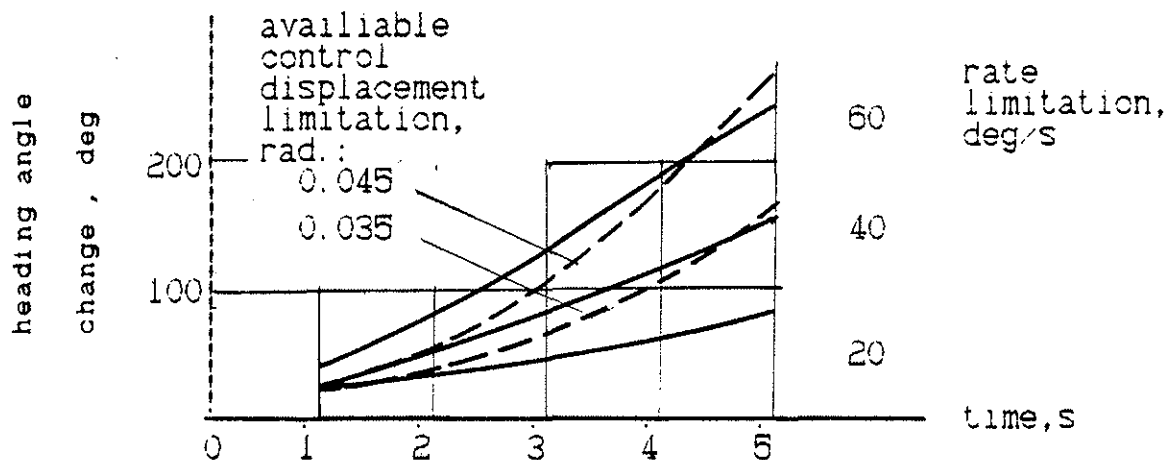


fig.4

3. Definition of a helicopter autorotation landing performanc.

The model used to solve the control optimization problem in case of an autorotation landing should imply all the specific features of a helicopter as object of control, when the motion parameters change in a wide range of flight speed (both horizontal and vertical), pitch angle ect. variation which necessitates the use of a non-linear helicopter motion model.

Due to the aerodynamic symmetry of the coaxial design helicopter the motion of which is examined here, the helicopter side motion at an autorotation landing may be neglected. It simplifies the task setting but the model remains substantially nonlinear.

Motion equations. Designations adopted:

- G - helicopter flying weight ;
- J_z - longitudinal moment of the helicopter inertia;
- J_p - polar moment of rotors inertia ;

- V_x, V_y - speed vector components in a coordinate system connected with the rotors shaft;
 ϑ, W_z - pitch angle and pitch rate;
 $Y_{HB}, X_{HB}, M_{z_{HB}}$ - lift force, longitudinal force and longitudinal moment of the rotors;
 $Y_{\PiЛ}, X_{\PiЛ}, M_{z_{\PiЛ}}$ - lift force, longitudinal force and longitudinal moment of the airframe;
 M_k - rotors torque;
 φ_3 - rotors locking angle;
 φ - rotors collective pitch angle;
 δ_z - resultant force vector deviation at longitudinal control.

Motion equations in a general form:

$$\frac{dV_x}{dt} - V_y W_z = \frac{g}{G} (X_{HB} + X_{\PiЛ}) - g \sin(\vartheta - \varphi_3) = \bar{X};$$

$$\frac{dY}{dt} + V_x W_z = \frac{g}{G} (Y_{HB} + Y_{\PiЛ}) - g \cos(\vartheta - \varphi_3) = \bar{Y};$$

$$\frac{dW_z}{dt} = \frac{1}{J_z} (M_{z_{HB}} + M_{z_{\PiЛ}}) = \bar{M}_z;$$

$$\frac{d\Delta\vartheta}{dt} = W_z;$$

$$\frac{d\Delta H}{dt} = V_x \sin(\vartheta - \varphi_3) + V_y \cos(\vartheta - \varphi_3);$$

$$\frac{dW}{dt} = - \frac{1}{J_p} M_k.$$

Rotors forces and moments coefficients are defined on the basis of Glauert-Lokk theory and the works of I. P. Bratukhin, M. L. Mil, R. P. Pein, B. N. Juriev. The inductive speed value is determined with consideration to its singularities in the area of low horizontal and high

vertical speeds (according to Pein, for example) . The correspondence of the rotors thrust coefficient and the inductive speed value is established by solving a nonlinear equation of the 4th order relating these values.

When determining the torque its profile component is set according to nonlinear dependencies precalculated with the help of a special rotors performance calculation program. The same program is used to define the rotor maximal lift constraints implied by the model.

At practical application of this model its identification was carried out upon the results of a prototype helicopter flight testing run in the modes nearing the mode examined.

The main constraints. The task of control optimization in case of an autorotation landing consists of determining the character of the longitudinal control and the rotors collective pitch control changes which most completely answers the requirements to helicopter motion parameters change at the moment of landing.

The following current parameters should be constrained :

- control travel;
- pitch angle;
- maximum rotors lifting ability ;
- rotors speed changes.

At the final moment the following parameters are constrained:

- forward speed;
- flight altitude ($H=0$ at the moment of landing);
- pitch angle at landing;
- pitch rate.

The parameter to be optimized is the vertical speed at the moment of landing (its absolute value should be minimized).

Presenting the constraints in the form of penalty functions in a way similar to that used in the previous example, we get the following form of a functional to be minimized:

$$I = qX_k(t_k) + (1-q) \sum_{i=1}^N \beta_i (X_i(t_k) - X_{i, \text{all}})^2 +$$

$$+ (1-q) \int_{t_n}^{t_k} \sum_{i=1}^M \gamma_i \left[e^{\alpha_i (Z_i - Z_{i, \text{max}})} + e^{\alpha_i (Z_{i, \text{min}} - Z_i)} \right] dt$$

where: N - number of parameter constraints at the last moment of time t_k , $N=4$

X_i - motion parameters, constrained in $t=t_k$;

M - number of current motion and control parameters constraints; $M=5$;

Z_i - motion and control parameters constrained in the process of the mode realization.

Since the number of constraints and consequently of penalty function coefficients is great the application of the penalty method coefficients control algorithms described above (II.3) is of a particular importance for this task.

At the presented example of solving a problem the dependence of the collective pitch angle change, as an initial approximation control, is simplified only qualitatively reflecting the character of the control defined upon the results of a prototype flight testing and the longitudinal control is retained constant. As shown on fig.5, the optimal control in difference to the initial approximation control enables the helicopter to make a landing with a forward speed not exceeding 50...60 km/h, vertical speed not exceeding 2 m/s and a pitch angle of 10 - 14°.

After performing a set of calculations, the recommendations were developed to carry out the helicopter flight testing in the mode in question and in particular the effect of the helicopter flying weight upon its vertical speed of landing was determined.

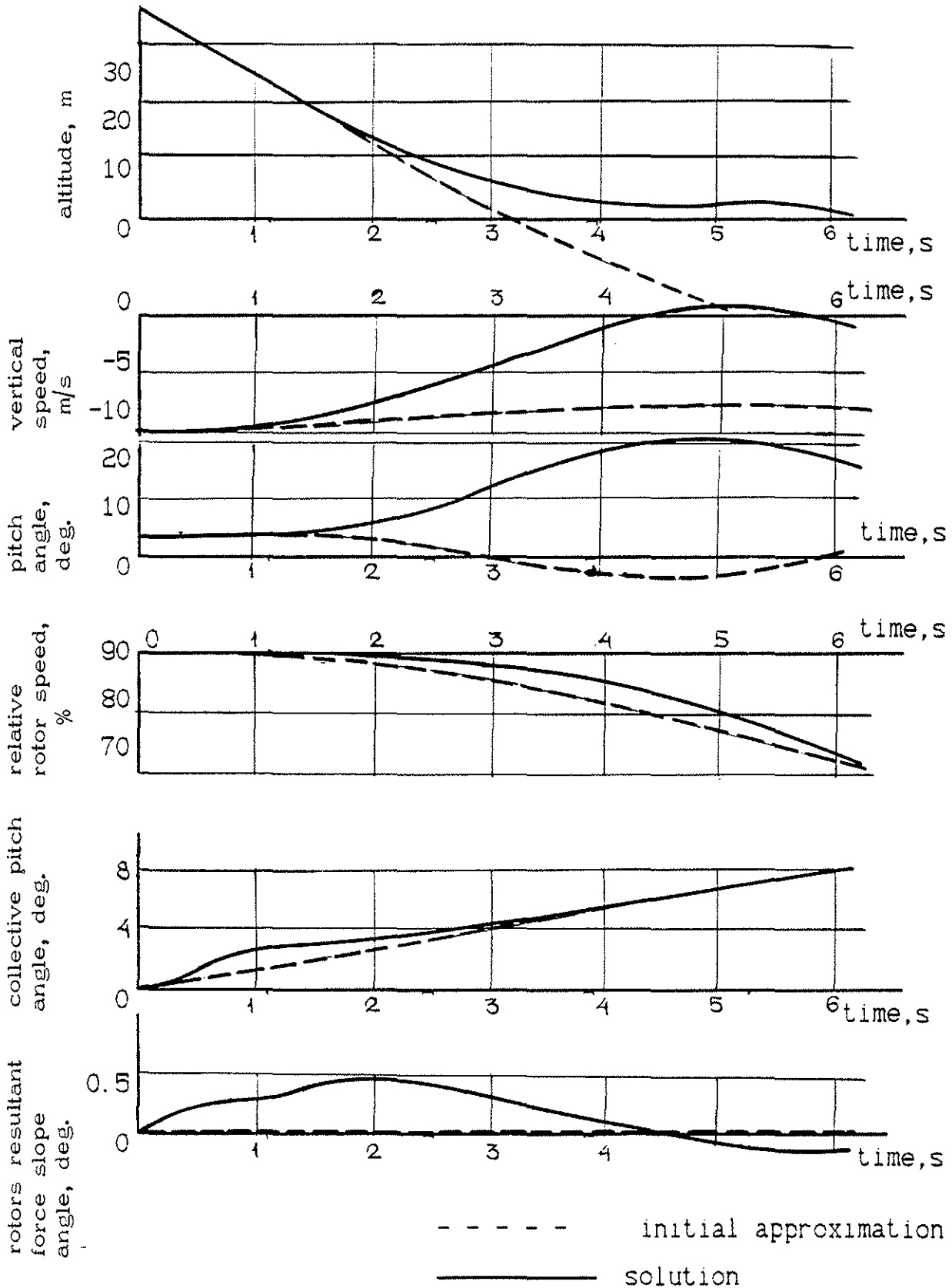


fig.5

CONCLUSION

The experience gained, permits to express a hope that optimal control theory methods will become as traditional as other existing mathematic method for the helicopter flight dynamics study.

In comparison to the traditional mathematical modelling techniques demanding a large scope of intermediate results analysis, the application of the optimal control method permitted to reduce, due to automation, the time of calculation by 1-2 orders of magnitude.

The solution of the helicopter flight dynamics task by the optimal control methods permit to identify and to utilize more effectively the existing reserves and to guarantee that all the existing constraints are observed.