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ROTORCRAFT WAKE MODELING: PAST, PRESENT AND FUTURE

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Abstract

Rotorcraft wake modeling is still a major concern in rotorcraft design and analysis as it influences the aerodynamic, aeroacoustic, and aeroelastic behavior of the vehicle. This paper presents a comprehensive overview of rotorcraft wake modeling, in particular in the last decade, from the experimental, theoretical and computational viewpoints. Present capabilities are discussed and trends for future development are explored based on a recent Army Research Office (ARO)-sponsored Workshop in Wake Modeling held in March 2009 at the Georgia Institute of Technology, in which experts from all over the globe discussed current status and limitations, as well as future needs and trends in wake modeling. This paper summarizes this workshop and juxtaposes it with current research on-going in the area of wake experimentation and numerical modeling. Advances in computational fluid dynamics (CFD) have improved near- and mid-wake modeling capabilities, but hybrid methods that employ vortex element (VE), vorticity transport (VT) or vorticity confinement (VC) methods will be necessary for long-age (far-wake) modeling in the near-term. A new hover experiment with high-accuracy measurements that tie the blade loading, rotor performance and wake characteristics is needed for numerical model correlation and development.

1. BACKGROUND AND MOTIVATION

The wake of a rotor is at once an extremely complex flowfield, but also one where the dominant phenomena are amenable to simple description. While the Landgrebe[1,2] and Gray[3,4] characterization of the wake structure into tip vortices and helical vortex sheets during the 1960's and 1970's still holds true to a large extent, details of the wake have been refined since that time. The transition from the near wake to far wake is known to occur through deterministic vortex-pairing rather than through chaotic processes, even at high Reynolds number. Similarly, mysterious "jitter" phenomena have mostly been shown to be adequately explained through predictable vortex interactions.

Many of the issues facing rotorcraft researchers today intricately involve the rotor wake, and interactions with the environment are particularly prominent for both military and civil applications. There is a national objective to extend the ability of rotary-wing vehicles to operate to a greater extent within urban environments. This requires full understanding of and the ability to control the wake to minimize noise due to blade-vortex interaction (BVI) and rotor wake interaction with nearby infrastructure. The need to understand and resolve brownout, in which the rotor wake entrains sand or other particulates and convects them into the pilot's visual field, has risen to prominence as a result of recent military engagements where safety and survivability are key factors in the deployment of rotary-wing vehicles. Similarly, sling load dynamics and shipboard operations require further understanding of the interaction of the wake with its environment. While the study of wakes from rotating blades has had its roots in the helicopter area, an important extension of this research is in the field of sustainable energy, particularly wind turbine design and analysis. The knowledge and accurate convection and dissipation of individual wakes in wind farms are critical to exploit and propagate this green technology.

The rotor wake can be deconstructed into three major "fields": near, mid and far, as illustrated in Fig. 1. The near field lies closest to the rotor, and it includes the region where the wake initially leaves the rotor blade and forms the classic character of the tip vortices. The accurate solution of this region is important to the prediction of blade airloads, blade-vortex interactions, rotor vibration and aeroacoustic signatures. The mid field encompasses the helicopter fuselage, so that wake resolution is important for characterization of rotor-fuselage interactions, empennage buffet, and interior noise. The far field includes the region where wake-environment issues such as ground effect, sling loads, brownout, shipboard operations and formation flying are important.

Fig. 2 illustrates the state of helicopter performance prediction from 1985 to the present. The rotor figure of merit rises considerably at larger weight (thrust) requirements, and along with it, the need for accurate prediction has increased. In 1985, prediction of blade aerodynamics was primarily accomplished via panel and blade element-based potential flow methods. Navier-Stokes calculation of a rotor in hover was just being demonstrated, but it was still far too slow and expensive to be used as industry tool. As the capability of computing power has exploded in the past two decades, the drop in computing cost and advances in computational algorithms have brought the ability of Reynolds-averaged Navier-Stokes (RANS) calculations well within reach of users even at the design stage of rotorcraft. The issues in blade computational fidelity have shifted to turbulence modeling for the wake and separated rotational flows.

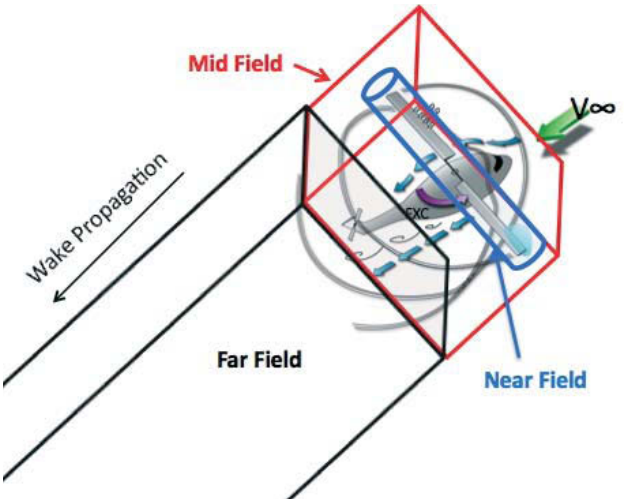


Figure 1: Wake regions of interest. The near field encloses the rotor blades, while the mid field encompasses the fuselage and area immediately surrounding the rotor. The far field extends beyond the vehicle to a distance defined by the investigation. Permission granted for use of the helicopter cartoon by F.X. Caradonna, AFDD.

Meanwhile, new strides are being made to characterize the wake flow field experimentally and with analytically-based computational tools. Application of accurate non-intrusive measurement techniques in facilities where vortex-wall interactions are minimized have enabled clean measurements of vortex core structure in axial and in edgewise flight to large wake ages. These analytical tools have permitted flight simulations to address operational problems in the laboratory and have guided research down productive paths to address solutions in a cost-effective manner.

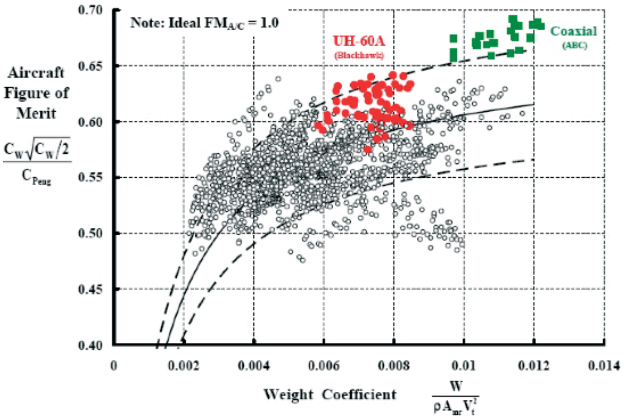


Figure 2: Improving rotor figure of merit at higher thrust coefficients requires improved rotor wake prediction capability. Originally from Harris;[5] this modified version from Tung.[6]

While these significant strides have been made in rotorcraft aeromechanics, the level of advancement in modeling and understanding of rotor wakes follows at a slower pace. There still remain first-order uncertainties in modeling rotor wakes from physical laws. Therefore, in March of 2009, an international workshop[7] was held on the state of prediction technology for rotorcraft wakes. Experts in rotorcraft and wind turbine wake research presented recent advances and discussed future directions of research in wake modeling. The questions asked at the recent ARO rotor wake workshop included:

1. What is the structure of the tip vortex at its origin, and what factors influence it?
2. How much of the blade bound vorticity actually ends up in the tip vortex?
3. How fast does the tip vortex diffuse/ dissipate/decay?
4. What is the role of turbulence in these processes?
5. What are the physical phenomena responsible for observed "vortex jitter"?
6. What is the state of computational capability for the tip vortex as a function of age in hover and forward flight?

7. How well are long-age phenomena such as ground vortex rollup, and tail rotor/main rotor interaction captured?

8. What is the status of turbulence modeling for helicopter rotor wakes?

This paper summarizes the state of knowledge and advances with respect to rotor wakes from both experimental and computational perspectives. This paper archives the technical discussions from this workshop and juxtaposes it with experimental and predictive milestones over the past quarter century, in particular during the past decade. To help summarize this vast field, this paper relies heavily on three other archival studies in addition to the recent ARO Workshop. The first is the 1985 review on helicopter aerodynamics by Phillippe *et. al*,[8] which was accomplished as part of an AGARD volume on Aeromechanics. The second source is a review on helicopter rotor aerodynamics in 1997 by Conlisk,[9] the same year that the American Helicopter Society conducted a Technical Specialists Meeting on rotorcraft aeromechanics and acoustics. Finally, a history of computational developments for rotorcraft by Strawn[10] provides a background for a more complete historical perspective.

In each of the perspectives of rotor wakes, a discussion of the state-of-the-art and recent advancements is first presented, followed by a discussion of the pertinent workshop questions with regard to each perspective. Finally, a list of conclusions and recommendations extracted from the workshop are provided.

2. EXPERIMENTAL ADVANCEMENTS

The wake of a rotor is at once an extremely complex flowfield, and one where the dominant phenomena are amenable to simple description. Table 1 classifies available experimental databases, and Table 2 lists the issues that were addressed. While they are by no means exhaustive, they do capture what are believed to be the most relevant and accessible references. The Gray deconstruction[3, 11] of the hover wake vorticity structure into tip vortices and helical vortex sheets of the 1950's, and the Landgrebe extension[1] into the complex interactions of forward flight done in the early 1960's, still hold true to a very large extent. Several other details and implications have been proven since then. The transition from the near wake to the far wake is known to occur through deterministic vortex-pairing rather than through chaotic processes, even at high Reynolds number. Similarly, mysterious "jitter" phenomena have mostly been shown to be adequately explained through predictable vortex interactions, whether in the near or far field of the rotor.

Table 1: Organization of experimental test cases

Experiment Type	Reference
Reviews	Desopper[12]
Hover	Gray,[3, 13] Castles,[14] Landgrebe,[2] Caradonna and Tung,[15] Tung *et. al*,[16] Piziali and Felker,[17] Lorber,[18] Norman and Light[19]
Axial	Castles,[14] Caradonna*et. al*[20]
Edgewise	Gray,[11] Wilson and Mineck[21] Landgrebe *et. al*,[22] Lorber[23]
Ground Effect	Empey,[24] Curtiss[25] Light,[26] Cimbala,[27] Brand[28]
Brownout	Johnson and Leishman[29]

Meanwhile, the unexplained disconnect between the correct wake geometry calculated from blade loading, and the correct thrust calculated using the wake geometry appears to have survived to the present. Recent results, detailed later in the paper, suggest that this can be attributed to a basic misconception in the application of potential flow-vortex element analysis to rotor wakes. Patient application of non-intrusive measurement techniques in facilities where vortex-wall interactions are minimized, have enabled clean measurements of vortex core structure in axial and in edgewise flight to large wake ages. This has spurred a retrospective evaluation of data existing in the literature. The traditional assumption that all the vorticity outboard of the bound circulation peak will roll into the tip vortex has been discounted by specific experiments via correlation of data across numerous independent experiments performed by several organizations. A large part of this vorticity is actually convected into the edge of the inboard vortex sheet. The extent to which this occurs depends on the details of the blade tip shape, demanding high-resolution numerical techniques to capture in predictive analysis.

These experimental findings enable a new look at prediction techniques, potentially offering great simplification. Meanwhile, issues such as turbulence continue to be extremely important in predicting the initial rollup of shear layers into tip vortices, and in predicting the size evolution of the tip vortex and the spreading of the inboard blade wake. These, in turn, are essential for future efforts to design blades for low Blade Vortex Interaction (BVI) loads, noise, and better performance.

The evolution of the near wake in fact conforms quite well to the schematic descriptions developed by Gray for the near wake in hover and Landgrebe for the wake in forward flight, except that the mutual vortex interactions included by Landgrebe also occur in the hover case as the wake ages. The tip vortices roll around each other, in behavior that is similar in appearance at any given cross-section to the interaction of two co-rotating vortices in a two-dimensional shear layer. These are

Table 2: Issues explored in experiments

Type of Analysis	References
Model Scale, Reynolds Number, Mach Number	Hein and Chopra[30]
2D vs, 3D non-rotating vs. rotating	McAlister[31]
Inflow	Elliott and Althoff,[32] Peters[33]
Blade-Vortex Interaction	Hubbard and Harris,[34] Lorber,[35] Caradonna,[20] HART-II Team[36]
Stall	McCroskey and Fischer,[37] Bousman,[38] Carr,[39] Chandrasekhara and Carr[40] , Yu[41]
Near Wake	McAlister,[42] Thompson,[43] Wadcock,[44] Light,[45] Felker[46] , Adams[47]
Transition/Pairing	Caradonna *et. al*[48]
Circulation/Vortex Strength	Thompson,[43] Ghee,[49] Kim[50]
Turbulence issues	Ramasamy *et. al*,[51,52] Mahalingam and Komerath[53]

strongly helical vortices, however, and thus the stretching associated with this rollup must have strong influences. The rollup proceeds until the cores come close together and then lose definition in the merger. It is logical to expect turbulent processes to become significant at this stage, and that the strong axial flow slows down, with an accompanying rapid growth in core size. The merged vortices may then weaken much quicker than their components did before merger, and this then delineates the start of the "far wake". The discrete structures in the far wake may then break down as occurs in shear layer growth processes. While these processes look very complex as the number of blades increases, they still appear to be obey descriptions that need not resort to any non-deterministic (or chaotic) phenomena until the vortex mergers occur. Meanwhile the inboard blade wake, which is basically a thin, flat shear layer developed from the turbulent boundary layers over the surfaces of the blade, may develop a strong counter-rotating feature at its edge, and convect down faster than the tip vortex. Interactions with older tip vortex segments may occur.

2.1. Wake Features Still to be Resolved

While many features of the wake have been resolved, and their physics understood, there are a number of significant observations in experiments that have not been resolved. Some of the more important questions or observations that remain to be further explored are:

2.1.1. The transition to the far wake occurs through deterministic, periodic, repeatable pairing events, for a 2-bladed rotor.

Conducting clean hover experiments in ground facilities has always posed difficult problems. Fig. 3 shows an experimental setup used at Ames Research Center by Frank Caradonna and colleagues to study the performance of a rotor in axial flow. The rotor is mounted along the axis of the $7' \times 10'$ wind tunnel in the $30' \times 30'$ settling chamber rather than in the test section. In 1996, they explored the idea of obtaining hover performance as the asymptotic limit of zero climb rate. As part of this effort, a set of white light sheets and intensified cameras captured the behavior of the wake of this rotor. The tip Mach number of the rotor was near 0.7, and the Reynolds number was in the typical operating regime of a tail rotor. Fig. 4 shows a sequence of video images at $9°$ collective and 3.5 fps "climb speed", clearly showing how the tip vortices remain as strong entities beyond $720°$ of vortex age, and then interact and merge during the transition to the far wake. Until this merger occurred, the vortex trajectories remained cleanly periodic and repeatable. However, when the tunnel flow velocity was reduced below a certain point (where the settling chamber velocity fell below 1m/s), the entire wake became unstable. This was attributable to the fact that vortices encountered downstream obstructions and meandered back upstream, causing violent transient interactions. This hypothesis was reinforced when obstructions were placed downstream, it was observed that the behavior of the wake could be forced to become unsteady even at higher velocities. The nature of the test configuration with the long return path and large settling chamber minimized the possibility of vortices interacting with obstructions downstream and returning to the inflow plane in a way that is usually difficult to achieve with ground-based "hover chambers". Thus, this evidence indicates that vortices persist as distinct entities far beyond the traditional limits where the vorticies are considered to enter the far-field region and dissipate.

It is not known if this deterministic, periodic behavior can be repeated as the number of blades is increased above the 2 blades in the experiment. However, the visible proof of this behavior for a 2-bladed rotor encouraged Jain and Conlisk[54] to undertake calculations of wake vortex behavior to long wake age to study mutual interactions of the vortices. They were able to capture the mutual interaction of the vortices observed in experiments and the resulting vortex trajectory. This capability allowed them to obtain much better agreement with other published experimental wake trajectories than was possible from prior lifting line/ free wake calculations. As noted later, this also allowed computation of vortex rollup and re-ingestion phenomena in ground effect. The point made here is that a simple model of deterministic vortex interactions resulted in both efficient and accurate prediction of rotor wakes to large ages.

Figure 3: Facility used to conduct interference-free axial flow experiment[48] for a 2-bladed rotor in the Ames $30{\times}30$ settling chamber of the 7×10 tunnel.

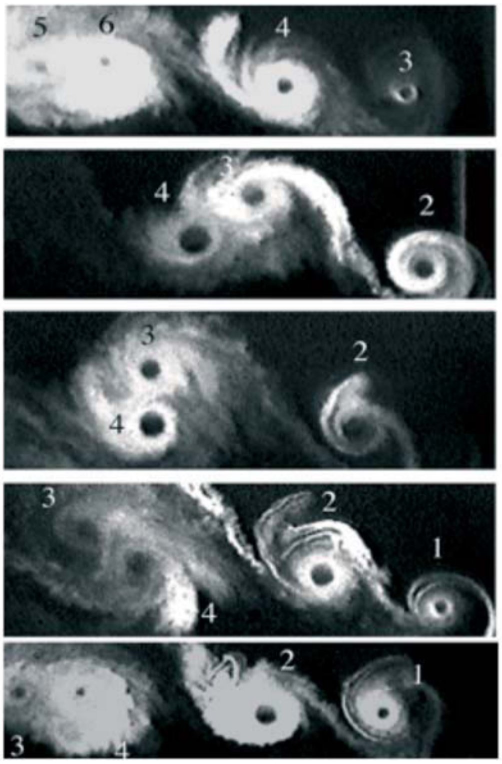


Figure 4: Vortex Pairing Sequence in the wake of the 2–bladed rotor at 9° collective, 3.5 fps axial climb speed, in the facility shown in Fig. 3. Smoke/light sheet cross-section images show $< 50\%$ core growth at 720° of wake age.[48]

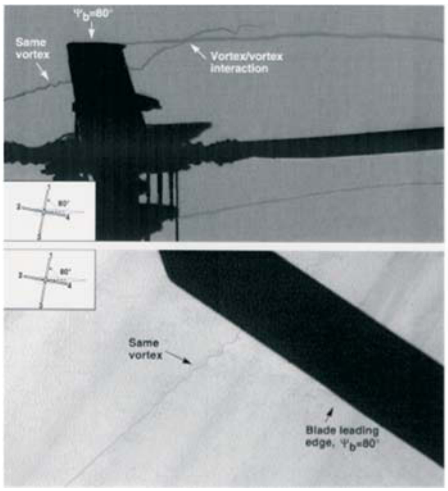


Figure 5: Shadowgraphy visualizes the tip vortices from an S-76 Rotor, at a Tip Mach number of 0.605. Two simultaneous projections (top and side views) of the rotor wake interactions. From Shinoda and Johnson,[55] courtesy NASA.

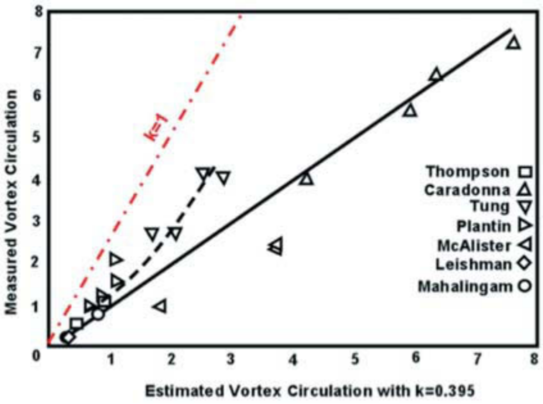


Figure 6: Measured tip vortex strength compared with the estimated value of peak bound circulation near the tip of the corresponding rotor blades, from experiments in various facilities.[56] The factor k is what is used to multiply the peak bound circulation to match the vortex strength data.

2.1.2. Vortex core size remains small for large vortex ages (several revolutions).

Further proof of the long persistence of tip vortices, comes from visualization of the density gradient across tip vortex cores. Fig. 5 is a NASA Ames shadowgraphy experiment[55] on a Sikorsky S-76 rotor at a tip Mach number of 0.605. Thus the highest Mach number difference across the core can only be 0.6, even if the tip vortex starts out at a full strength value computed from the peak bound circulation at the tip (more on this later). It is well-known that shadowgraphy and schlieren techniques do not work in room temperature flows for differences below Mach 0.25. In other words, even after some 360 degrees of rotor age, the vortex strength remains close to its original strength.

Coalescing the above two observations, the tip vortices in hover or low-speed climb should stay well organized in deterministic rather than chaotic behavior, and their strength should dissipate very slowly with increasing vortex age. Fig. 6, which summarizes data from several independent experiments clearly illustrates this behavior. This appears to be in contradiction to some existing formulations where a large turbulent diffusion or other such process is necessary to explain why the measured tip vortex strength is much lower than the peak bound circulation on the blades.

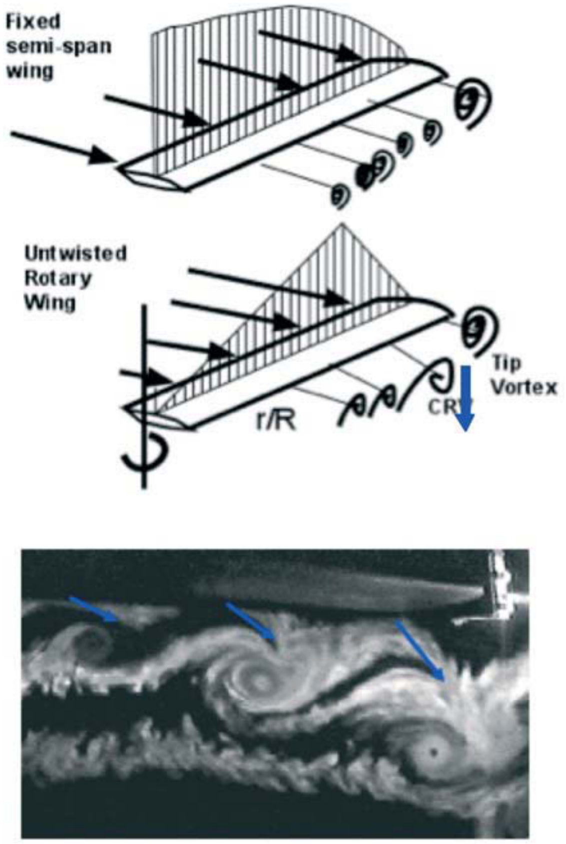


Figure 7: a) Schematic illustration of the difference between fixed-wing and rotary wing vortex system development. The rolled-up edge of the rotor blade's inboard vortex sheet, marked "CRV" has a sense of rotation opposite to tip vortex (TV). A strong downflow develops between them. From Komerath[56] based on the work of Kim.[50] b) laser sheet image of the vortex pair in the wake of a 2-bladed rotor in forward flight. From Kim.[50]

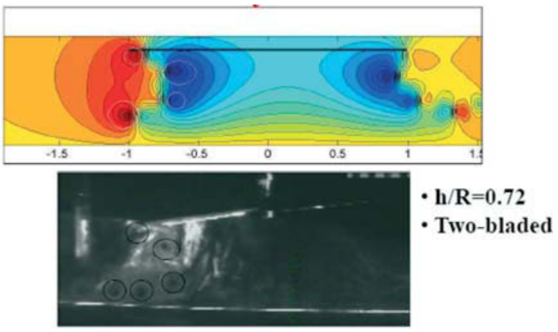


Figure 8: Comparison[57] between computed (above) and experimentally captured tip vortex recirculation ahead of a 2-bladed rotor in strong ground effect, at low advance ratio.

The Fig. 5 image also illustrates other phenomena that may be described as "jitter" of the vortices. These may be due to instabilities triggered for example by blade-vortex or vortex-vortex interaction, and it may lead under some circumstances to an obstruction of the axial flow in the vortex core accompanied by swift growth and destruction of the vortex core. Wadcock[44] has reported a sharp contraction in tip vortex core size that he attributes a strain on the vortex due to interaction with the first blade passage. He notes that earlier reports of rapid core expansion and diffusion can be traced to the use of only a single-point sensor across a vortex that will exhibit cycle to cycle "jitter".

2.1.3. The Characterization of the Rolled-up Structure At the Edge of the Inboard Vortex Sheet.

Kim *el. al*[50] demonstrated that in the case of a highly-loaded, unwisted blade, the edge of the inboard vortex sheet of the rotor blade rolls up into a counter-rotating vortex (CRV), whose strength approached 50% of that of the tip vortex. Experiments by Ghee and Elliott[49] at Langley employing a substantially larger (but still model scale) rotor confirmed this finding. While this has not been directly shown in numerical computations, Egolf[58] cites calculations that attribute up to a 20% reduction in tip vortex strength to the interaction with the inboard vortex sheet. He also demonstrated by smoke visualization images from UTRC wind tunnel tests and computations that a "dual vortex" or counter-rotating vortex pair can be seen above the rotor blade in some flight conditions. If this presence of a strong rolled-up CRV can be observed for full-scale main rotors at high thrust coefficients, it should have substantial effects on the structure of the wake and hence on inducted velocities and the thrust computation. The experimental cases of Kim and Ghee appear to have established that such a structure must exist in most model scale experiments at moderate or higher thrust coefficients. This finding will have a profound influence on the relation between the peak bound circulation on the blade and the strength of the tip vortex. However, there is no evidence of similar counter-rotating vorticity in the computations reported by Narducci[59] for a large fixed blade twisted to approximate the circulation distribution of a typical rotor in hover, with correlations to experiments at the Boeing Vertol Wind Tunnel. The circulation distribution shown in this case has a small hump near the tip, but this does not appear to generate a visible counter-rotating vortex structure at the edge of the inboard sheet. One may postulate that the counter-rotating pair phenomenon could be due to aspects of the rotation (such as radial flow) that are not present in fixed-wing experiments.

Fig. 6 indicates that up to 55% of the peak bound circulation may be lost from the tip vortex shortly downstream of the blade tip. The outlier datum in Fig. 6, showing over 90% capture of the tip bound circulation peak is from an experiment conducted by Tung and Caradonna,[16] an experiment that is often cited as a basic test case for CFD. As illustrated in Fig. 7, the near-saw-tooth shape of the bound circulation on a rotor blade, as opposed to the monotonic shape on a corresponding fixed wing, implies that the sense of rotation of the inboard vortex sheet is indeed counter to that of the tip vortex (TV). Where the inboard sheet rolls into a CRV, the counter-rotating vortex pair induces a strong downward flow. A measurement technique that detects only the induced velocity magnitude cannot distinguish the effect of the TV from that of the CRV. Thus if the induced velocity is used as a measure of tip vortex strength, and the measurement point is in between the TV and CRV, then a spuriously high estimate of the TV strength will result, albeit one that may capture the effect of a vortex of strength equal to that given by the peak bound circulation. This appears to have occurred in the case of the outlier point. Further investigation of the research that resulted in this point finds that the researchers were very concerned that their previous experiment (also using hot-wires, the best available technique at the time) gave TV strength that was only about 50% of bound circulation, and hence they modified their curve fit procedure and extended the region used to estimate TV strength well beyond the region used in the previous experiment. Thus it appears that the low estimate of the initial paper by Caradonna and Tung[15] was the correct one.

The role of turbulence to explain this behavior has generated diverging hypotheses. Jain and Conlisk[54] have shown through order of magnitude estimates that this loss cannot be due to turbulence, and indeed if there were any turbulent phenomena that caused such a sharp drop in vortex strength, there is no way to explain the long persistence of the vortex, shown for instance in Figs. 4 and 5. Instead, the presence of turbulence should diffuse or dissipate the vortex swiftly, within 90° degrees of vortex age. This conundrum led to much confusion in the 1990's, when combined with the phenomena of a strong, persistent tip vortex interacting with facility walls. In several experiments, researchers used single-point laser velocimeter probes to scan across vortex cores, using azimuth-resolved sampling synchronized to the rotor. When the vortex position "jittered" due to the transient vortex interactions from cycle to cycle discussed previously, the effect was that the apparent vortex core was "smeared" over several degrees of rotor azimuth, losing definition, and leading to erroneous agreement with CFD codes that suffered from large numerical dissipation due to poor grid resolution and low order spatial algorithms. Adding to the uncertainty were theoretical analyses, coming at a time when "chaos theory" was seen as the avenue to predict turbulence, suggesting that the wake of a rotor must be fundamentally "unstable", swiftly degenerating into chaos.

With the introduction of particle image velocimetry (PIV), this error should be avoidable, although the resolution of the core velocity profile from PIV is difficult because the core rarely has sufficiently detectable seed particles in it, and those that are seen inside may not follow the local flow direction due to centrifugal effects. This problem is exacerbated as the rotor scale and Reynolds number increase, the vortex gets stronger, and measurement distances increase. Recently a series of papers[51,52,60] reports on experiments using stereoscopic PIV that indicate that the turbulence in the shear layers rolls up into the vortex core. Based on these experiments, a Reynolds number-based vortex model has been proposed to empirically simulate this behavior.[61] These tests note "Significant turbulence activity up to two core radii from the tip vortex axis." Asymmetric flow characteristics in the tip vortices were associated with "a pronounced anisotropic distribution of eddy viscosity, a typical characteristic of flows with high streamline curvature."

2.1.4. The Behavior of the Wake in Ground Effect.

The issue of calculating and measuring the rotor wake in ground effect situation has been studied for a long time, since ground resonance has destroyed many vehicles, ground effect is crucial to survival in autorotation landings due to the destructive effects of unsteady outflow induced by a rotor downflow at the ground, and more recently because of the emergence of brownout (or whiteout) as a key survivability issue in military operations. This field is too large to do justice in this paper, and there was no review or summary of these issues at the recent Workshop. The references mentioned in Table 1 deal with the basic issues of ground effect. More recently, Saijo[62] and Ganesh[57,63,64] dealt with the reported problem of sharp transient loads experienced in low-speed flight close to the ground where the test condition was believed to be steady. They showed how to deconstruct this problem by first establishing that the wake was very steady (vortex positions repeatable to a high tolerance from cycle to cycle) at their OGE condition (Rotor disc located 2.7R above the ground) at advance ratios above 0.03. As the ground height was decreased, unsteadiness clearly set in. They then showed that at low advance ratios (below 0.06 at a ground clearance of 0.77R) tip vortices would interact with the ground and were then entrained ahead of the rotor disc, sometimes entering the inflow causing sharp transients. The disparate variety of interaction geometries and locations possible caused these events to occur at widely separated intervals, translating in the case of a full-scale rotorcraft to the order of several seconds or even minutes between events. On the other hand, as the advance ratio increased, the region where the tip vortices met the ground and moved under the vehicle. This meant that the recirculation into the inflow could no longer occur. In this regime, the wake structure was much more stable. Note that if a tail rotor were present, vortex ingestion into that rotor would continue to cause transients; however, the Saijo and Ganesh configurations did not include a tail rotor. They then placed fuselages of different generic geometries below the rotor and again confirmed steady vortex trajectories in the space above the fuselage OGE. Measured loads on the fuselage were periodic and contained no aperiodic transients. This was expected because any random vortex effects would be cancelled along the length of the fuselage. As the advance ratio was varied, however, they showed that there were measurable sudden excursions in the values of the side, lift and drag forces on the fuselage (which was not connected physically to the rotor). Much more interestingly, as the ground vortex position crossed the fuselage center of mass, there were large excursions and reversals in yaw and pitch moments. To a pilot flying slowly near the ground, these would certainly occur as sharp transients and control reversals, occurring with very small changes in advance ratio that could be caused by mild breezes. Analytically, these two sources of unsteadiness (vortex ingestion versus ground vortex position change) could be separated, the former requiring long-age computations of vortex trajectories and interactions, while the latter requires only a quasi-steady computation of ground vortex strength and position over a range of advance ratios. Jain and Conlisk have shown[54] the ability to compute the wake to large ages, with interactions, as mentioned above. Pulla and Conlisk[57] extended this to the above problem, and showed the ability capture the re-ingestion situation, as shown in Fig. 8. Ganesh also captured the strength of the ground vortex and found it to be 4 to 5 times that of individual tip vortices, corresponding to the number of tip vortex turns that merge into the ground vortex. He found no evidence of discrete tip vortex structures within the rolled-up ground vortex. In contrast, at low Reynolds number, a wall jet at the ground, comprised of discrete, clearly-identifiable vortex cores, has been captured in experiments by Lee *et. al* in 2008, reported by Quackenbush.[65]

2.1.5. The Sudden Descent Problem.

Brand[28, 66] has analyzed the flight experiences of the sudden descent problem encountered by the tiltrotor aircraft. He shows that the catastrophic loss of lift suspected in this problem is actually a case of the rotor blade experiencing sharp downflow, resulting in reduction of angle of attack rather than any possibility of rotor stall. The downflow is a result of the accumulation of tip vorticity into the “vortex ring state”. There is some similarity in the fluid mechanics regarding the accumulation of tip vortices into a strong ground vortex in ground effect. The established vortex ring was shown to be a self-propelling structure that can lead or follow the descending rotor. An increase in advance ratio is seen to be the way to escape this condition.

2.1.6. Wake interactions in the far-field.

A recent source of knowledge on the behavior of rotor wakes is the surge in research on wind turbines. The Reynolds number range of interest here is even larger than that involved in full-scale helicopter aerodynamics, but the tip Mach numbers are generally low. Two samples of current work in this field were reported at the Workshop. Massouh *et. al*[67] report detailed PIV data in the wake of wind turbine models in a low-speed wind tunnel, albeit at relatively small model scale and Reynolds number.

Another valuable aspect of wind turbine research to those interested in rotorcraft aerodynamics is that the interaction between the wakes of different, closely spaced turbines is of strong interest, and large scale visualizations are available on such interactions in wind farms from natural condensation or smoke studies. Dobrev and Massouh[68] also showed analyses of the interactions between wakes of turbines. For these analyses, simpler singularity-based methods must be used, because of the numerical demands posed by the multiple interactions.

3. COMPUTATIONAL ADVANCEMENTS

Trends in the development of computer hardware and its associated cost continue to follow the 1965 trend predicted by Gordon Moore[69] now known as Moore’s Law. Microprocessor capacity has increased exponentially, while cost has decreased similarly, so that coupled with the development in parallel computing capability, significant strides in the solution of the rotor in hover (Fig. 9).

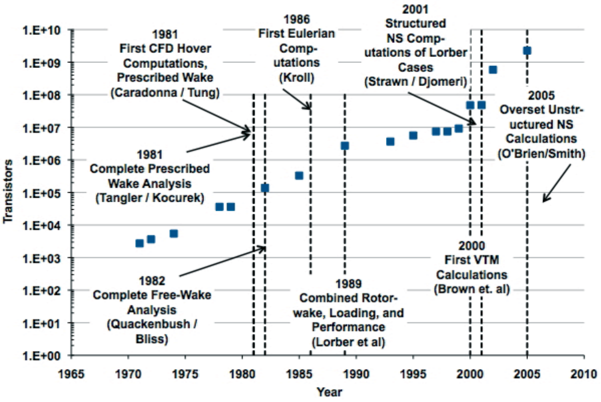


Figure 9: Microprocessor development predicted by Gordon Moore[69] compared with numerical simulations for hovering rotors.

3.1. The Development of Computational Methods

Since the advent of the computer, rotorcraft researchers have looked to computational means as a mechanism to improve the knowledge of the physics of the complex rotor flow field and to improve engineering predictions of rotorcraft behavior for design, simulation and analysis. A review of the development of primitive-variable Computational Fluid Dynamics (CFD) for rotorcraft published by Strawn[10] is recommended as a general review. A synopsis of the history of these methods, specifically with regards to rotorcraft wake predictions is provided here.

3.1.1. The Development of Vortex Element Methods

The earliest line of numerical research in wake simulations has resulted in a set of vortex element methodologies that are typically implemented in a class of codes known as comprehensive codes. A review of the most popular of these methods has been detailed in Ref. 70 and 71. Here, the rotor wake is modeled as one or several vortex elements (VE) trailing from each blade using the potential or “singularity-based” simplification of the fluid mechanics equations of motions. Earliest models assumed the wake structure to be composed of a tip vortex plus a vortex sheet, based on the Gray[11] 1956 model. The helical vortex assumption of the steady wake was later modified to include the capability to model “free” wake motion that the prescribed helical formulation did not allow. Multiple trailers of vortex elements, also known as filaments, were introduced to more physically represent the wake distribution of vorticity, including vortices shed from the root. Egolf[58] gives an excellent synopsis of VE methods specific to rotorcraft applications.

There are several methods of implementation of these VE methods currently in use. Lagrangian calculations of incompressible vortex element dynamics using the Biot-Savart law[70] or combinations of Eulerian and Lagrangian methods[72] are popular VE methods for rotor wake codes. A very efficient implementation is the constant vorticity contour (CVC) method where the ordering of the wake structure is a function of the wake vorticity distribution.[65,73] Vortex core models are used to overcome solution instabilities in the near and far wake in these methods. Instability observed in vortex models for hover in the near wake in early studies using time marching schemes appears to have been overcome in more recent work using relaxation schemes.

An alternative implementation of VE methods developed by French researchers[74] involves the use of vortex points or blobs to avoid solution instabilities. These address issues such as vortex merging, which can be difficult if a specific ordering, such as vortex lattice ordering, is implemented. Conversely, many more points are needed to model the wake, so that the overall cost may increase substantially.

This concept has been shown to accurately predict the loading and wakes on the rotor in hover and forward flight[73,75–78] for many flight conditions These methods offer fast turnaround times on a single PC (some even real-time), but require an estimate of the wake strength and location at the rotor blade. These methods were first coupled to lifting-line and panel methods, but they have also been coupled to Euler or RANS methods since the latters’ advent as production codes in the 1990’s. Because VE methods are free from the numerical diffusion problems afflicting CFD schemes (see following section), they are ideally suited for propagating vortices over long distances and times, as illustrated in Fig. 10. Moreover, vortex methods avoid the complexities of mesh generation and thus are very well suited when frames in relative

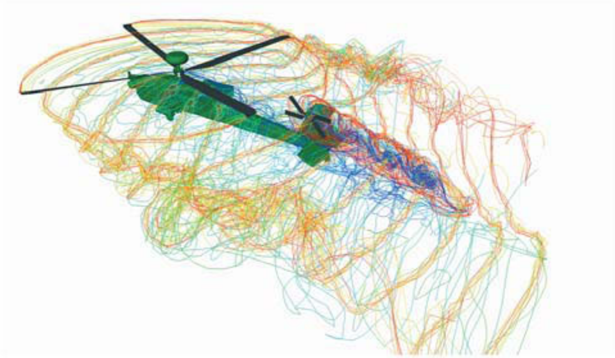


Figure 10: Simulation of an AH-64 full multi-rotor configuration using the inviscid Lagrangian wake model, CHARM.[79] Used with permission.

The cost of the Lagrangian wake scales with the number of elements squared, so that additional vortex elements added to the model to refine the wake representation can result in dramatic computational cost increases. Multiple wake trailers and point vortices can be distributed across a number of processors with varying success. The most successful methods to reduce cost are multipole methods, such as the parallelized formulation in CHARM,[65] which reduce the $\bigcirc(n^2)$ cost to $\bigcirc(nln(n))$ cost. The focus of the computational cost reduction efforts of today are focused on CFD/CSD coupling trim using VE methods using massively parallel computers.

3.1.2. Potential-Based Formulations

During early computational efforts, the focus was on hybrid methods where method to solve the simplified forms of the Navier-Stokes equations for a rotor blade were combined with an analytical wake representation to reduce the size of the problem so that it could be resolved on the limited memory computers that were the norm at that time. Initial simulations of a rotor in hover were performed by Caradonna and Tung[15] for a limited computational domain modified by surface transpiration and outer boundary variables determined from an integral solution of the wake outside of the computational boundaries. Comparisons with the experimental hover data showed good correlation and spurred further computational endeavors in the field. By the 1980's full potential codes, such as FPR[80] were applied to hover and, applying the transpiration concept from earlier successes, were able to simulate lifting forward flight configurations[81] using a finite-difference transonic small disturbance method.

Efforts to develop potential-based formulations of the simplified Navier-Stokes equations slowed during the late 1980's and in the 1990's as computer power increased and Euler/Navier-Stokes methods were developing. One method that continued development during this time was the HELIX hover code,[82,83] which is a combined Eulerian/Lagrangian method where the velocity field from the wake vorticity is inserted into the near-blade computational code. Recent upgrades to this code replace the potential near-body solver with a Navier-Stokes solver.[78,84]

Recently, D'Andrea has revisited the use of panel methods and shown[85,86] that for some flight conditions a full-span unstructured-hybrid panel method is capable of predicting rotor wakes very accurately when coupled to a CVC formulation to capture the wake. He notes that even with a parallel implementation, the use of vortex lattice methods in the wake is impractical for problems such as brownout where 100 rotor revolutions may be necessary[87] to capture the in ground effect phenomena.

3.1.3. Euler/Navier-Stokes Formulations

As noted by Strawn,[10] one of the major goals of developing Euler/Navier-Stokes methods was the ability to capture rotor wakes without resorting to the hybrid methods necessary with potential-based solvers. Contemporary use of the acronym "CFD" has become synonymous with computational methods that resolve the primitive variable (ρ, u, v, w, e) formulation of the Reynolds-averaged (RANS) equations of motion. These methods typically employ finite-difference or finite-volume techniques to resolve the flow field (but not always), and the turbulence closure occurs via an eddy viscosity parameter that is computed from a statistical model of the scales of turbulence. Much of the development of these solvers has focused on the ability to resolve the vortical flow around the blades, so that the tip vortex and vortex sheet can be computed via first principles, ostensibly resulting in a more physically correct simulation.

Conventional rotor configurations are well suited to resolution by overset structural RANS CFD methods, given that their shapes can be readily described by quadrilateral shapes. For hover cases, a structured mesh comprised of a single zone is an efficient way to model the rotor blade. However, CFD methods introduce a numerical dissipation into the solution, so that high density grids are required to capture rapidly changing flow field characteristics without losing important characteristics. This pertains of course to the topic of interest here, the capture and propagation of the rotor wake. Strawn and Djomehri[88] in 2002 computed the wake of a UH-60A rotor in hover using embedded momentum sources to mimic the rotational flow that is generated by the blades of the rotor. They noted a persistent grid dependence in the simulation, and even with a 64 million node structured grid, the numerical dissipation of vorticity still overwhelmed the physical behavior of their system.

Extending CFD to forward flight meant that two frames of motion representing the rotating blades and the stationary background must be computed during the same simulation. Lee[89] and Steijl[90] have both demonstrated variations of sliding boundary conditions between rotor and background grids, but the most versatile method of modeling different frames of reference is the overset method.[91] This method provides the ability to model arbitrary motions, including elastic motion, without the problem of overly complex boundary condition development. Overset methods have been successfully implemented and applied in a number of rotorcraft applications.[92–95]

Computational simulation of mid field wakes, such as for rotor-fuselage interaction (RFI), strains even further the resources required by structured methods. Actuator disks will provide good time-averaged loading on fuselages,[96–98] but the rotor wakes are lost as individual rotor blades are not modeled within the sources. At minimum, time-accurate, blade-referenced source modeling of the rotor is required to obtain an estimate of the instantaneous fuselage surface pressures[94] due to the individual blade wake passage. The exact representation of the moving rotor blades is required to evaluate accurately the complete detail of the unsteady wake and its interaction with the fuselage or empennage. Potsdam and Strawn[93] studied a full-span V-22 configuration with moving rotor blades. Their structured overset RANS solver required numerous overset grids to model the complex geometry. WIth a total of 47.6 million nodes, they were able to correlate the unsteady wake-fuselage interaction to adverse flight control and aerodynamic phenomena observed in flight.

Given the limitations of structured RANS methods, unstructured and Cartesian grid topologies have been considered as options to structured grid topologies to reduce the number of computational nodes and the complexity of the grid generation process that is required as a prerequisite for the simulation of the aerodynamics of modern helicopter configurations. Unstructured RANS methods use either a fully tetrahedral grid topology or a mix of prisms, tetrahedral and hexahedral cells to comprise the grid geometry. This approach permits complex geometries to be modeled much more rapidly than is the case with their structured method counterparts, and a single grid surrounding the fuselage can be extended efficiently to form the background or far-field grid. Rapid changes in configurations are also possible with overset methods, permitting hubs, rotors, and struts to be added (or removed) from the model with little difficulty. Unstructured methods are not without their problems, as current solvers cannot efficiently obtain high-order spatial resolution (e.g., 4^{th} or 6^{th} order), and their computational overhead and cost per iteration is significantly higher than with structured methods. Cartesian methods[99] do not require body-fitted grids, minimizing the time-consuming grid generation. Problems still exist in modeling rapid geometry changes and viscous simulations about complex geometries due to the nature of the boundary conditions. A comparison[99] of hybrid, Cartesian and unstructured methods with mixed methods of rotor modeling demonstrated the advantages and shortcomings of these various methods for RFI, while Renaud *et al.*[100] compared the performance of two overset structured and one unstructured method with rotor disk models. An alternative that is currently being explored is to use overset near-field grids (structured or unstructured) in conjunction with Cartesian background grids solved by a separate Cartesian solver,[101] as illustrated in Fig. 11.

3.1.4. Alternative Navier-Stokes Formulations

The problems encountered in propagating the wake without significant dissipation with traditional (or, as they are sometimes referred to, primitive-variable-based) RANS methods have resulted in research into alternate formulations to alleviate this problem. An alternative approach which has shown great promise to date in addressing long-age wake propagation is the class of methods based on the solution of the RANS equations in vorticity-velocity form. By casting the vorticity as the primary conserved variable, the numerical diffusion of vorticity in the wake is avoided by suitable choice of the numerical algorithm that is used to advect the vorticity through the computational domain. One of the most successful of such approaches is the Vorticity Transport Model (VTM), developed by Brown and his students at Glasgow University (and earlier at Imperial College London)[102,103] that employs a grid-based CFD solver for the wake. This model has been applied to a number of different rotor wake problems with exceptional success to date. These applications include near-field wake modeling for airloads and aeroacoustics (including BVI),[104–106] mid-field rotor-fuselage interaction[107,108] and rotor-rotor interaction,[109] and far-field helicopter-aircraft wake interaction,[110] vortex ring state,[111] ground effect[112] and brownout.[87] The CFD solver from VTM has been enhanced and implemented in modular form in another code known as VorTran-M with similar successes, as reported for example on wake analyses.[110,113]

There has been some effort to solve the vorticity transport equations by revisiting particle-based methods[114] rather than the Eulerian formulation discussed previously. Recent versions of this is known as the Particle Vortex Transport Model (PVTM), although the published record to date[115] indicates only partial success. Another implementation[116] applies the method, now referred to as the Particle Vortex Method (PVM), to augment a dynamic inflow model within a comprehensive code. While inflow angles are improved, there was no discussion of airloads or on the wake modeling abilities of the PVM method. Major hurdles still remain when applying particle methods to high Reynolds number three-dimensional flows. If viscous effects are included, then periodic remeshing is needed to maintain accurate representation of the diffusion operators.[114] For inviscid flows (or the advection step of viscous methods), particle methods suffer from instabilities and loss of accuracy associated with failing to enforce a divergence-free vorticity field and particle disorder.[114] The apparent flow modeling flexibility offered by the unstructured and non-connected particle representation is generally offset by the need in all such methods, to maintain adequate particle overlap for accuracy. Thus when stretching occurs, particles must be continually added to maintain accuracy and limit the non-physical growth of $\mathbf{V} \cdot \mathbf{\Omega}$.

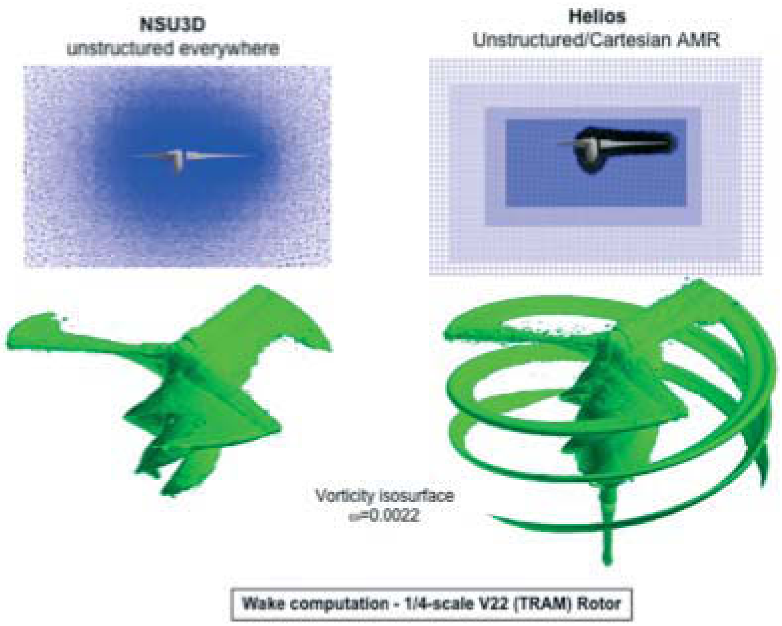


Figure 11: Comparison of wake resolution for TRAM rotor in M=0.1 climb conditions using NSU3D and Helios; (left) unstructured NSU3D everywhere, (right) unstructured near-body with NSU3D, off-body with high-order Cartesian AMR using SAMARC

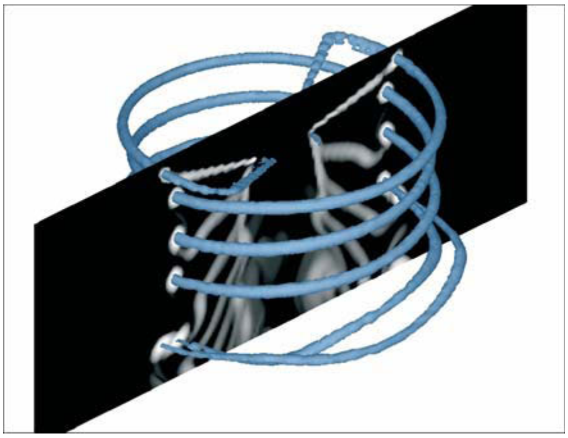
A second alternative Navier-Stokes formulation that has shown some success in capturing rotor wakes is the vorticity confinement (VC) method. Developed by Steinhoff,[117,118] it has been applied to rotorcraft wakes,[119] and in particular to brownout.[120,121] This method solves the discretized Navier-Stokes equations with the addition of an extra term that "confines" vorticity to only a few grid cells. As a result, vortices are convected without numerical spreading over long wake ages. The computational savings result from smaller grids, which can be simplified further using Cartesian grids. Qualitatively, the methodology appears to work very well, although correlations to experimental wake information, similar to VTM correlations (Fig 12), has not, to the authors' knowledge, been published.

All of these methods typically rely on some other computational mechanism to calculate the vorticity arising on surfaces in the flow, i.e. the vorticity in the near-field of the rotor. While most of these have relied on lifting-line or panel-based methods to rapidly generate these data, the most accurate near-blade computational methodology remains conventional CFD methods. Thus, coupling of these methods with CFD to take advantage of the best characteristics of both methods is a logical step. These methods are collectively known as hybrid methods and are discussed in the following section.

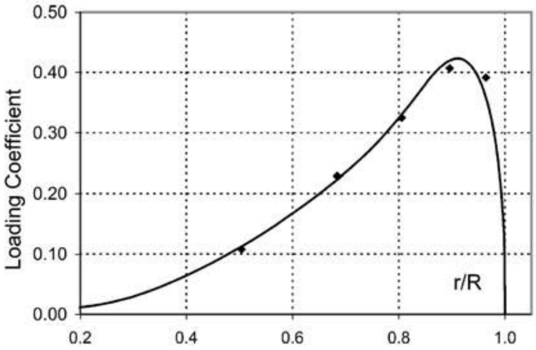
3.1.5. Hybrid Formulations

As noted in the prior sections, each of the computational methods applied to the wake problem has advantages and disadvantages. It was recognized early on in the 1970's and 1980's that a combination of methods that merge the best characteristics of each method, while mitigating the disadvantages, showed promise in tackling the wake modeling issue. This class of simulations is known collectively as "hybrid" methods. Early efforts attempted to address the cost of the CFD simulation, along with the problematic wake dissipation, by creating a mixed Eulerian-Lagrangian formulation, although this is no longer the case as hybrid methods that couple CFD with Eulerian vorticity transport methods are being developed.

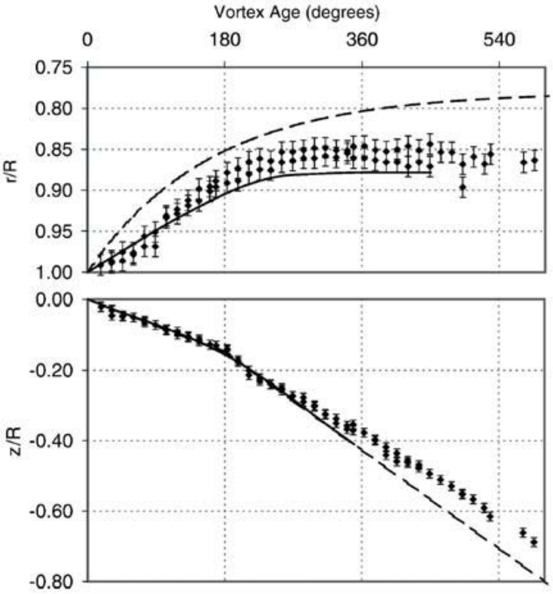
Initial hybrid methods typically utilized the more costly computational method to capture the blade near-field physics, either with full-potential methods in the 1970's and 1980's[80,122–124] or more recently, RANS methods.[125,126] These hybrid methods will be referred to as CFD-Lagrangian wake methods. Most of these hybrid methods apply free- or prescribed-wake methods to resolve the far-wake, reducing the computational grid requirements, but at the expense of the physics, as the limitations in the free-wakes still remain. Additional simplifications, such as modeling one blade on the rotor further reduces the cost of quasi-steady simulations, such as hover and steady level flight.



(a) Wake visualization of a two bladed hovering rotor (800,000 grid cells, 50 cells/radius, 6 cells/chord)



(b) Blade Loading



(c) Comparison of vortex trajectory with experiment.[103] (Symbols are VTM, Solid line is Caradonna & Tung, Dashed Line is Kocurek &Tangler)

Figure 12: Hovering wake predictions using the Vortex Transport Method.

TURNS-UMARC has shown very good correlation of airloads for forward flight,[127] and HELIX versions have been shown to predict figure of merit at or within experimental error bounds.[78] Issues that are still being investigated for these methods tend to center on boundary condition issues associated with the interface when strong levels of vorticity are exchanged between the methods.

More recent hybrid methods that are under development combine the RANS CFD at the blade near field with vorticity transport or vorticity confinement methods in the far field. This hybrid methodology is more consistent than CFD-Lagrangian wake hybrid methods as the Navier-Stokes equations are being solved in both the near-field and wake modules. VorTran-M has been coupled with an unstructured solver, RSA3D[113] and coupling efforts are underway with OVERFLOW and FUN3D, while VC has been coupled with TURNS,[128] and is being coupled with UM-TURNS. Here, the vorticity (or velocity vector) from the wake method can be directly inserted into the CFD grid to alleviate many of the interface issues CFD-Lagrangian wake hybrid methods must contend with to conserve the RANS equations. The primary formulations of these vorticity methods still tend to be incompressible, so that density changes must be handled with care to preserve continuity. The current primary drawback with these methods is that their implementation is designed for serial processors, which means that their full effectiveness has not yet been evaluated.

3.2. Wake Modeling Problems Still to be Resolved

While many advancements have been made with computational codes, there are still major wake modeling issues that need to be resolved. Some of the more important questions or observations necessary for wake modeling and the current efforts in the area include:

3.2.1 Core modeling

Egolf[58] points out that despite large advances in computational methods and computer speed, analytical modeling of rotor wakes continues to be essential due to the high cost of CFD-based methods. Within the VE methods, the core is modeling via analytical algorithms based on empirical observations. Bhagwat and Leishman[129] summarize a number of vortex core models and how the model can impact hover simulations. However, core modeling remains a fertile area for research, as experimental techniques to measure the behavior of the vortex core are improving with technology advancements. These core models were developed from small scale experiments, so that there are a number of questions relating the model assumptions to the vortex core behavior. Egolf[58] specifically cites the assumption of the initial core sizes and decay rates. Both Egolf and Quackenbush[65] also list entrainment or agglomeration behavior in the wake for interactional aerodynamics and brownout as an area for additional research.

Expounding on the interactional questions, further understanding of "distinct" vortices (e.g., tip vortex, vortices from abrupt sweep changes) with the inboard sheet, or with other vortical features of complex blade tip shapes is necessary so that dual vortices inboard of the rotor and the roll-up of the root vortices can be correctly modeled.

Near-wake core modeling issues can be alleviated by coupling the VE method with an Euler/RANS CFD code that captures the blade vorticity. Here, the analytical modeling is replaced with first-principles methods, which, if applied correctly, should provide a more physical representation of the near field wake. However, these methods have their own limitations and are not a panacea for all near-wake issues, as will be discussed shortly.

3.2.2. The role of turbulence modeling in defining the near-field (and mid-/far-field) wake

Transition and turbulence modeling are the issues that tend to be most often cited when explaining differences between CFD and experimental results. When resolving the flow field about the rotor blade, the shed wake is directly related to the solver's ability to predict the blade loading and rotor performance. It is this aspect of RANS CFD that has received the bulk of the attention during the CFD development and is summarized very aptly in Strawn *et. al.*[10] The ability of RANS methods to predict aeroelastic effects will also impact the blade loading and subsequent strength and location of the prominent wake vortices. Recent CFD/CSD coupling efforts have significantly improved the ability of CFD to predict blade loading (e.g., 127, 130, 131), and has recently been applied to BVI[132]

For rotor applications that include transonic flows and stalled flows, it has been shown that algebraic models do not correctly simulate the behavior of rotor airfoils under these conditions.[133] One- and two-equation models appear to capture the overall behavior of the rotor, including the tip vortex, well for many applications,[59,132] but they miss dynamic stall rotor characteristics.[130,134] Advanced turbulence models are gaining attention after they showed promise during the DARPA Quiet Helicopter program.[130] Nichols *et. al*,[135] Szydlowski and Costes,[134] and Lynch and Smith[136] have recently demonstrated dynamic stall or improvements in capturing complex wake physics using various advances in turbulence simulation techniques.

Transition remains a key issue in some simulations, in particularly hover, as noted by Narducci.[59] Many legacy codes assume fully turbulent flows or have some user-defined transition location, but only a few include transition modeling as part of the turbulent closure methodology. Transition modeling appears to have mixed influence on the prediction of loading for dynamic applications,[137,138] improving static or low-frequency integrated loads, but having less effect as the reduced frequency increases.

Transition and turbulence modeling issues become even more important when advanced concepts are studied. Active flaps include unsteady effects and regions with differing characteristic Reynolds numbers, so that even for small flap excursions, differences in turbulence models are observed.[126,139] Applications involving small scales, such as micro air vehicles, exhibit dominant viscous effects with thick boundary layers and separation bubbles, which are not well-captured by current methods designed for larger scale and higher Reynolds-number helicopter problems.[140]

3.2.3. Capturing the vortex core characteristics

It is necessary to capture the characteristics of the vortex core (strength, velocity profiles) as well as its location with respect to the rotor blade to ensure that the vortex is propagated correctly in the wake. These characteristics are intricately linked to the blade loading, orientation and tip geometry, which were discussed in the prior section. As part of the workshop, the question was asked, "How many grid points does it take to model the vortex core?" There is not a single number, as it is tied to the grid topology and to the spatial scheme of the methodology. A more appropriate question might be cast as, "How can the vortex core be modeled accurately and efficiently in RANS-based solvers?" This question has been addressed in RANS-based research for a number of years, and the most recent advances are discussed here.

Original RANS solvers relied on 2^{nd}-order solvers and single zones to model the vortices shed from the rotor blade, in particular the tip vortex. Dissipation and dispersion were immediately identified as a problem, and higher-order spatial stencils were applied to the problem as early as the 1990's,[141] and have continued to be evaluated for vortex capturing.[135,142,143] While the increase of the spatial order will improve the ability of the RANS methodology to capture wake features on the same grid, as expected, they come with the penalty of higher cost, and more importantly, difficulties with the higher-stencil requirements at the grid boundaries and overlapping grid fringing. Compact stencils, such as Padé schemes, will somewhat alleviate the latter problem; however, many codes use explicit boundary conditions and these compact methods typically require implicit boundary schemes and significant code restructuring to implement. Recently, Péron and colleagues at ONERA[144] were able to obtain a seven-fold decrease in the computational time to attain comparable results for a 44 million node grid using a 2^{nd}-order spatial stencil with a 6 million node grid and a 3^{rd}-order stencil on Cartesian grids. Initial grid densities were comparable to those reported by Potsdam *et. al*[95] necessary to capture details of the shed vortices in the mid and far field wake. Wissink *et. al*[145] attained comparable vortex wake resolution using a 5^{th}-order spatial scheme over a 2^{nd}-order scheme with a three-dimensional grid 27 times smaller than the original.

Recent studies with automatic mesh refinement (AMR) combined with overset meshes and/or moderately high-order methods combine the characteristics of each to address vortex core modeling with fewer high-order stencil issues encountered in prior studies. Narducci,[59] Péron[144] and Wissink[145] presented results specific to capture the vortex core characteristics at the March 2009 workshop. These researchers reported similar results in their independent studies. Vortex peak-to-peak magnitudes suffered 50% or more error compared to experiment when captured via 2^{nd}-order stencils, with typically 10-15% error or less using 5^{th}- or 6^{th}-order schemes, as illustrated in Fig. 13. Comparable results appeared to be observed for vortex core velocity distributions and location. Narducci noted that using an Euler-based solver in the background to propagate the vortex to long wake ages, the same level of error at $2c$ behind the blade was observed at $20c$ behind the blade. The addition of AMR to the vortex grid resulted in improvements in the core predictions roughly equivalent to a two-order increase in the spatial scheme.

These studies to date focus primarily on the spatial aspect of the algorithms. The temporal integration of the RANS equations of motion may also be modified to increase the local time step to minimize computer costs. To date, this remains in need of more study for rotor wake applications.

3.2.4. Simulation of long-age wake phenomena

Once the simulation details necessary to accurately capture cross-sectional properties of the vortex have been determined, as discussed in the prior section, the propagation of the wake to long ages within the context of a rotor grid is addressed. RANS will convect the wake for approximately one revolution on grid sizes typically applied in production settings,[146] but Potsdam *et. al*[95] have demonstrated that no matter what type of grid is applied (structured, unstructured, or Cartesian), the grid cell should have a side no greater than 10% chord in the wake area if the detailed features are to be captured. Wissink reports[145] that grid cell sides of about 1% chord are necessary to propagate vortex strength and advection over long wake ages, although it has minimal influence on the computed blade loads. This indicates that a more coarse wake grid can be applied for situations where blade loading is necessary, resolving the far wake only when interactional results are required.

Researchers have attempted to improve wake propagation characteristics by increasing the grid resolution and using higher-order algorithms (as discussed in the prior section). Depending on the configuration, the vortex-dominated wake region occupies typically significantly small portion (10-40%) of the computational domain. The cost of applying these concepts everywhere in the computational domain is prohibitively expensive, and very high order algorithms ($> 5-6$) may not be conducive to massive parallelization primarily due to the issue of computational stencil growth. This concept has been recognized, and alternate concepts that take advantage of this wake-to-domain fraction are being studied.

There are two areas of research underway, not mutually exclusive, that may resolve the problem within the next five years: automatic mesh refinement and mixed topology overset grids. Automatic mesh refinement (AMR),[147] or h-refinement allows the grid to be clustered in the area of the vorticity, and potentially adding new cells in the area as well. The impact of feature-based mesh refinement to improve vortex capturing was demonstrated via static updates,[59,94,144,146] as illustrated in Fig. 14. Researchers have also investigated the concept of embedding refined overset grids that follow the vortex path in the wake has also been investigated with successful tracking of a single shed vortex.[59,148] The key to making automatic mesh refinement routine will be the ability to resolve the flows during a time-accurate simulation without user intervention. This has already been demonstrated by Ruffin[99] and Wissink[101,145] for Cartesian grids, and research is being funded to accomplish this for unstructured meshes.[149] As most CFD runs are now parallel, the ability to automatically load-balance the changing grid so that the grids are partitioned to minimize overlap and communications between processors will be crucial to maintaining a cost-effective process.

Robert Narducci[59] points out that the use of AMR may be problematic for the near-rotor wake in hover and descending flight where the multiple vortices remain very close to the rotor. Feature-based AMR assumes that large errors are associated with large gradients, so that AMR not be able to distinguish wake features appropriately in this region. An alternative to feature-based adaption may be the use of adjoint-based adaption, where the

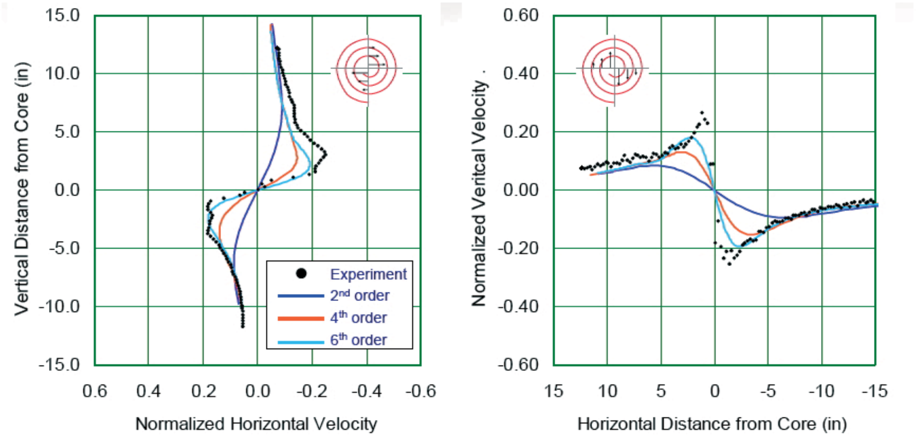


Figure 13: Comparison of vortex resolution with spatial stencil accuracy for a vortex at $x = 17.8c_{tip}$ from the quarter-chord location. The vortex is shed from a non-rotating generic tiltrotor blade at $M_\infty = 0.15$ and $\alpha = 6.5°$. Experimental results were obtained in the Boeing-Vertol Wind Tunnel. From Narducci,[59] used with permission of the author.

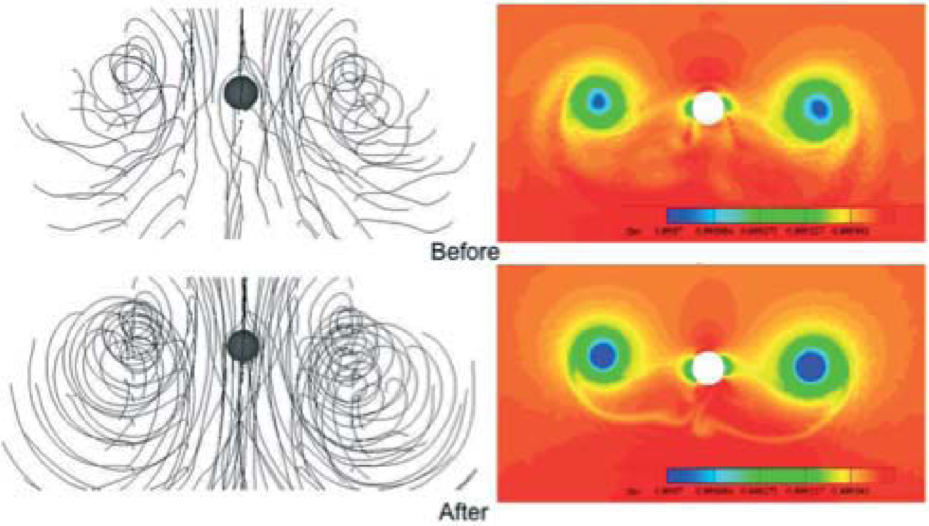


Figure 14: Comparison of wake resolution for the Georgia Tech rotor-fuselage using FUN3D with and without mesh refinement.

estimated error of the function is related to the residual errors of the solution (primal and adjoint) to refine the local mesh. Methodologies have been developed[150,151] for compressible flow equations with good success. Extension of this to the application of the rotorcraft hover wake problem is warranted.

Another concept to improve wake modeling for CFD in the near-term is the use of mixed topology grids that are overset, and in some cases use AMR to further refine the grid. This concept is not new, as a Cartesian grid forms the background grid in OVERFLOW.[152,153] However, this does not fully exploit the Cartesian grid as the solver still must resolve the Cartesian grid using a body-fitted algorithm. The Helios project[145] plans to resolve complex near-body configurations with an unstructured code, while computing the Cartesian grid using a different solver designed for Cartesian grids (with AMR). As illustrated in Fig. 11, the hover wake is more accurately captured using a combination of high-order algorithms and background mesh in a Cartesian solver. He also noted that each node in the 5^{th}-order Cartesian solver can be computed 7 times faster (CPU time/node) than its counterpart in the unstructured 2^{nd}-order solver. It should be noted that Cartesian overset grids require more points, so that the CPU time is offset by the requirement for more grid nodes. Péron *et. al*[144] have performed similar studies using the HART-II[36] baseline case with BVI for an inviscid structured near-body grid with an adaptive Cartesian solver in the background. Their focus was on the impact of the higher-order spatial stencils, and did not provide timing studies with reference grids.

A longer-term solution that may also significantly improve the near-field simulations is the development of unstructured high-order algorithms. Using the discontinuous Galerkin (DG) formulation, unstructured topologies can be resolved so that each cell is essentially independent of the other grids, rendering the boundary condition issues seen in traditional structured high-order methods moot. This is not a new concept, however, prior applications[154,155] of DG formulations to rotorcraft have primarily applied a high-order DG stencil at every node in the grid, with the result that the methodology is burdened with a high CPU cost. Recognizing that the wake-to-domain factor and applying high-order stencils only where needed (p-refinement) will alleviate the high cost. Shelton[156,157] has developed a DG-based unstructured method that successfully employs hp-refinement based on flow field features using an automatic refinement algorithm to determine the levels of refinement needed to resolve the feature (Fig. 15), and Shelton and Smith are further extending this method to rotor wake applications.

For the near-term, hybrid methods appear to be the best method to apply to applications that require long-age wake propagation. Phillips and Brown[87] report that 100 revolutions are needed to capture the brown-out problem, and that 25 revolutions may be necessary to the model transients. Based on current research, it is apparent that classic CFD techniques will require more development to be able to solve these wake-dominated problems.

If classic (primitive-variable) CFD methods can be used to accurately capture the near field characteristics of the wake close to the blade (and fuselage), then the vortex element-based, vorticity transport or vorticity confinement approaches to resolve the fine wake may be appropriate. Even using these methods, efficiency is still a paramount issue. Parallelization efforts for these techniques are needed to improve the turn-around time of the simulations and are underway.

4. DISCUSSION

A discussion between the participants of the workshop was held at the end of the presentations by researchers. This section provides a short synopsis of the discussion in relation to the workshop questions.

4.1 What is the structure of the tip vortex at its origin, and what factors influence it?

One prominent research scientist noted that it was surprising that this question is still not resolved. He believes that there should be a database or characterizations of the tip vortex, including strength and location, and how it varies with blade tip, advance ratio, etc.

The remainder of the discussion focused on the influencing factors of the tip vortex at its origin. Several researchers noted that turbulence has little to do with the formation of the tip vortex, but a well-known CFD researcher noted that while must of the vortex generation is inviscid in nature, the CFD simulations are required to be viscous for other reasons. He has noted that the viscous-inviscid interaction rearranges the vorticity; it does not destroy it. Another noted that twenty to twenty-five years ago, dual vortices were observed for negatively loaded advancing blades, but CFD has yet to duplicate these results.

4.2 How much of the blade bound vorticity actually ends up in the tip vortex?

Experimental correlations such as Fig. 6 indicate this range to be 45% - 50% based on several published data sets from a wide range of independent experiments, and speakers cited additional experiences where ranges of

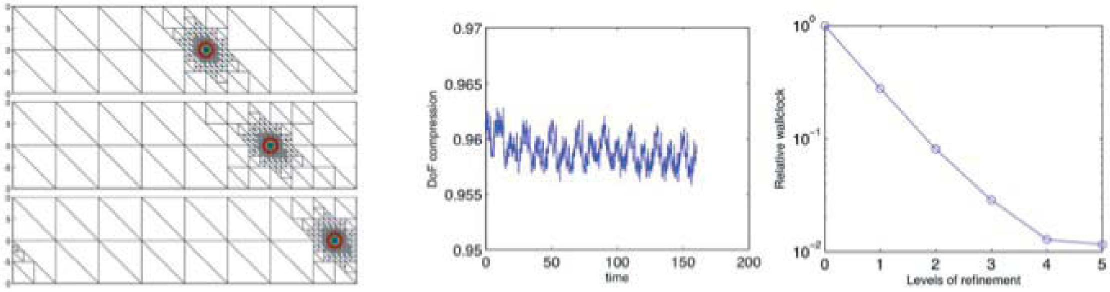


Figure 15: Vortex capture using hp-refinement in a multi-resolution discontinuous Galerkin method. Left: Illustration of vortex convection; Middle: Degrees of freedom describing the compression of the grid; Right: Wallclock time versus refinement. From Shelton[156]

80%-100% were observed. The remainder of the bound vorticity was said to move inboard and verified by others who noted that one blade that was studied showed a second vortex formed inboard at high advance ratios, which rolls up slowly. The Tram tip vortex was noted to roll up very slowly, due to its high twist, while the UH-60 blade was noted to have 100% of its bound vorticity in the tip vortex.

This question immediately brought forth the rebuttal question, “Do we need to model the inboard bound vorticity?” The answer was given, that if indeed the characteristics in bound vorticity roll-up vary with blade configuration, then it will have to be modeled to capture the tip vortex characteristics and hence the correct blade loading.

4.3 How fast does the tip vortex diffuse/ dissipate/decay? What is the role of turbulence in these processes?

Most of the participants believed that the inviscid or time-averaged characteristics of the tip vortex are more important than the details of the turbulence within the vortex. That is, the time-averaged behavior of the vortex is on the engineering scale, which is important, while small-scale turbulence is not of primary issue, and, at the present time, require too many assumptions to be able to model computationally. It was believed by some that the turbulent structure of the vortex may be important to acoustics and blade interactions, as outside of the core there are turbulence effects with some diffusion, while the core may have relaminarized.

4.4 What are the physical phenomena responsible for observed “vortex jitter”?

As vortices are, by definition, not steady, unsteady effects observed in experiments should be expected. Certainly in hover there is pairing of the vortices, so that unsteadiness exists and grows with time. It was noted that care are should be taken in defining vortex unsteadiness as tunnel unsteadiness has caused some incorrect conclusions to be drawn. For example, in some tunnel tests of fixed wing configurations, unsteadiness of the tip vortex was observed, but found later to be due to the interaction of the wake with the unsteadiness of the flow in the wind tunnel.

4.5 What is the state of computational capability for the tip vortex as a function of age in hover and forward flight?

This topic does not have the high visibility that topics modeling complex configurations and interactions have recently had. Correlations in the past show that CFD does not compare well with experiment, in particular as the wake ages. The state of the art in the computational capability for this topic needs to be assessed so that the conclusions can characterize the problems that exist in CFD. To do this experimental data that include good wake information for correlation, as well as information on how the experimental data were obtained is needed. A rotor test that includes different blade planforms with the same solidity and thrust was suggested. The characterization of the turbulence should be systematically studied as part of this test. Any conclusions from CFD should be based on a full numerical modeling of the test (including tunnel walls, sting mounts, etc.).

4.6 How well are long-age phenomena such as ground vortex rollup, and tail rotor/main rotor interaction captured?

From presentations at the workshop and other avenues, it appears that hybrid methods involving vortex elements, vorticity confinement and vorticity transport methods can model the rotor wakes over long ages, while traditional CFD cannot yet accomplish this. It was noted that there is a need for good data sets with which these methods can be correlated so that further conclusions regarding their ability to accurately follow the wake path and maintain vortex characteristics can be assessed.

5. CONCLUSIONS

A wake modeling workshop was held in March 2009 on the topic of rotor wake modeling, attended by experts from the United States, Europe and Asia. There is one overarching recommendation from the workshop for a new isolated rotor hover test, using state-of-the-art measurement techniques, to provide a high-quality database for correlation with computational methods, including vortex-element-based (comprehensive) codes. The scatter on prior tests is large, and there are not the data necessary to provide full correlation with numerical methods, linking the wake, blade loading and overall performance of the rotor. This hover test should include a systematic study of the influence of blade planforms and tips, keeping other parameters constant so that conclusions can be concretely drawn from both the experiment and computational correlations. Conclusions of the current state of the art and recommendations for future research paths pertaining to the experimental and computational modeling of rotor wakes are enumerated below.

Some simplifying findings from experiments are listed below:

1. Experimental evidence proves that rotorcraft tip vortices persist with small core size for several revolutions of wake age.
2. When facility wall interactions are reduced sufficiently, the wake of a 2-bladed rotor does indeed evolve in a deterministic manner. Long time scales and sharp transients appear IGE, or when other obstacles are present in the wake path
3. The transition to the far wake occurs through deterministic processes of interaction between vortices, leading to vortex mergers.
4. Tip vortex strength is substantially lower than the peak bound circulation. The difference is attributable to ingestion of the tip shear layer into the counter-rotating inboard vortex sheet before the tip vortex is formed. Hence, tip shape details are very important. This finding is surprising, coming after so many decades of research, but so far stands.
5. Both in ground effect and in the “vortex ring state” problems, rolled-up vortex structures develop, with strength equivalent to that of several turns of the tip vortex. Lift loss in sudden descent, and sharp transient loads with small advance ratio changes IGE, are attributed to the altered flow direction due to the strong velocities induced by these rolled-up structures.

Computational methodologies :

1. Wake structures in the blade near field are well-captured using CFD for most flight conditions, except for regions with dynamic stall. Further development of advanced turbulence simulation techniques that are appropriate to RANS production size grids and time requirements are warranted.
2. The influence of transition continues to be an unknown quantity for the prediction of the rotor airloads and wake. It appears from studies thus far that transition is important for steady and low-frequency flight conditions that involve rotor blade stall, but is less important for high-frequency phenomena.
3. Using combinations of higher-order schemes, grid adaptation, and/or a mix of grid/solver topologies, CFD can capture 4-5 revolutions of the rotor wake without excessive grid requirements. Wakes for longer time periods must be modeled with hybrid methods involving vortex elements, vorticity transport or vorticity confinement in the wake.
4. Further correlations with experimental data are needed to fully document the accuracy of the wake methods for long wake-ages, indicating the need not only for a hover test case, but additional high-quality experiments, particularly for in-ground-effect flight and small scale rotary wing designs.

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