

LOAD LIMITING CONTROL DESIGN FOR ROTATING BLADE ROOT PITCH LINK LOAD USING HIGHER HARMONIC LTI MODELS

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Abstract

This paper discusses the synthesis of a load limiting controller (LLC) for critical helicopter components that are subjected to significant fatigue loading. The development of a (structural) load limit violation detection and limit protection algorithm using a linear time invariant (LTI) model of helicopter coupled body/rotor/inflow dynamics is described. The developed load limiting controller is evaluated in its ability to limit harmonic pitch link loads and its impact on maneuver performance for a typical longitudinal doublet input.

1. NOMENCLATURE

A	LTI state matrix	x	State vector
B	LTI input matrix	x_R	State vector of residualized model
C	LTI output matrix	x_B	Rigid body state vector
D	LTI direct transmission matrix	Y	Augmented output vector
$F(\psi)$	LTP state matrix	y	Output vector
$G(\psi)$	LTP input matrix	\hat{y}	Dynamic trim estimate output from residualized model
G	Constant gain	y_R	Output vector of residualized model
$P(\psi)$	LTP output matrix	ψ	Non-dimensional time
$R(\psi)$	LTP direct transmission matrix	O_0	Average or 0th harmonic term
S	Normalized local sensitivity	O_{nc}	n^{th} cosine harmonic term
U	Augmented control vector	O_{ns}	n^{th} sine harmonic term
u	Control vector	O_k	k^{th} iteration
X	Augmented state vector	LLC	Load limiting control
		LAC	Load alleviation control

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2. INTRODUCTION

A 2012 survey of the past 30 years, carried out within Augusta Westland Limited (AWL) Materials Technology Laboratory, concluded that fatigue failures account for approximately 55% of all premature failures in helicopter components¹. The causes of low cycle fatigue are largely due to aircraft maneuvers, gust loading and through takeoff and landing. Critical helicopter components, classified as Grade-A Vital components by regulatory authorities, are subject to significant fatigue loading in which the failure would result in a catastrophic event. A list of fatigue critical components² on the AH-64A Apache shows that many of the Grade-A Vital components are located in the rotor system, creating challenges for real time load monitoring of those components but also for the development of load alleviation/limiting control schemes.

Current methods for structural health and usage monitoring and load alleviation control rely on distributed sensing and operational monitoring to infer usage and estimate fatigue in critical components. Such inference process is affected by significant uncertainty given that sensors' type and locations are often removed from hot spot areas characterized by maximum stresses. For example, past work³ for limiting pitch link loads has used proxy models of the vibratory loading. A classic example is the Equivalent Retreating Indicated Tip Speed (ERITS) parameter, which has been correlated as a function of airspeed and normal load factor with vibratory pitch link loads from retreating blade stall onset, can be limited to indirectly constrain the pitch link loads.

Recent work^{4,5} at Georgia Tech has developed methods to approximate coupled body/rotor/inflow dynamics using high order Linear Time Invariant (LTI) models. These methods use harmonic decomposition to represent higher frequency harmonics as states in an LTI state space model, and they have been proven to offer the potential for real-time estimation of the effect of control inputs on component dynamic loads which in turn can be used in combination with reduced order structural models to estimate primary damage variables associated with fatigue of critical components. Such real-time estimation of component level dynamic loads, stresses and strains, etc., provides the opportunity for real-time monitoring of component damage variables, and more importantly, the development of control schemes designed to alleviate/limit component fatigue damage.

Recent studies^{6,7} at Penn State have used higher order LTI models for the development of life extending control schemes in the form of load alleviation control (LAC) strategies. The LAC strategies for component life extension aim at reducing component dynamic (e.g., peak-to-peak) loads, leading to reduced peak-to-peak stresses, and hence potentially leading to reduced fatigue life usage. While LAC offers a computationally simpler scheme, it can lead to a conservative design in a specific application at the expense of reduced maneuver performance, as in reducing peak-to-peak dynamic loads, no distinction is made between different harmonic load effects on accumulated component fatigue. A more effective control strategy for component life extension, albeit at a significant computational complexity, is to limit directly the fatigue life usage associated with harmonic loads considering that higher harmonics represent greater number of cycles over time and harmonics that are close in frequency to the natural modes of a component result in a greater modal response.

The present study is aimed at developing a feedback controller for limiting a selected harmonic load component(s) of a rotating blade root pitch link. It makes use of LTI model approximation of coupled body/rotor/inflow dynamics of a helicopter for the real-time estimation of component dynamic loads, which in turn is used for limiting or altering the pilot control inputs in order to achieve component load limiting during aggressive maneuvers.

3. LTI MODEL

A detailed description of the extraction of a higher order LTI model from a high-fidelity nonlinear model of a helicopter is presented in this section. Using the method described in Lopez and Prasad⁵, an LTI model using harmonic decomposition of LTP states with a first order representation (i.e., separate displacement and velocity states) is developed from a full vehicle nonlinear (NL) FLIGHTLAB^{®8} model of a generic helicopter with elastic blade mode shapes and a 33-state Peters-He dynamic inflow model. The LTI model has previously been validated against a nonlinear rotorcraft model and found to be of sufficient fidelity⁵.

Considering an LTP model of the form given in Eqs. (1) and (2), harmonic decomposition for an extraction of LTI model assumes the approximation for the state vector, x , in Eq. (3)

$$(1) \quad \dot{x} = F(\psi)x + G(\psi)u$$

$$(2) \quad y = P(\psi)x + R(\psi)u$$

$$(3) \quad x = x_0 + \sum_{n=1}^N x_{nc} \cos n\psi + x_{ns} \sin n\psi$$

where x_0 is the average component, and x_{nc} and x_{ns} are, respectively, the n /rev cosine and sine harmonic components of x . Likewise, the control u is expanded in terms of harmonic components as

$$(4) \quad u = u_0 + \sum_{m=1}^M u_{mc} \cos m\psi + u_{ms} \sin m\psi$$

and the output y is expanded in terms of harmonic components as

$$(5) \quad y = y_0 + \sum_{l=1}^L y_{lc} \cos l\psi + y_{ls} \sin l\psi$$

where y_0 is the average component and y_{lc} and y_{ls} are, respectively, the l^{th} harmonic cosine and sine components of y .

The LTI approximation of the LTP model given by Eqs. (1) and (2) can be obtained by substituting for harmonic expansions^{4,5} of x , u and y , i.e., Eqs. (3), (4), and (5) into Eqs. (1) and (2). The resulting equations can be represented in state-space matrix form by defining an augmented state vector as:

$$(6) \quad X = [x_0^T \dots x_{ic}^T \ x_{is}^T \dots x_{jc}^T \ x_{js}^T \dots]^T$$

and the augmented control vector as

$$(7) \quad U = [u_0^T \dots u_{mc}^T \ u_{ms}^T \dots]^T$$

where x_0 is the zeroth harmonic component, x_{ic} , x_{is} are the i^{th} harmonic cosine and sine components of x , and u_0 is the zeroth harmonic and u_{mc} , u_{ms} are the m^{th} harmonic cosine and sine components of u , respectively. The state equation of the resulting LTI model is

$$(8) \quad \dot{X} = [A]X + [B]U$$

Likewise, the augmented output vector of the LTI model is defined as

$$(9) \quad Y = [y_0^T \dots y_{lc}^T \ y_{ls}^T \dots]^T$$

Then the output equation of the LTI model can be written as

$$(10) \quad Y = [C]X + [D]U$$

Detailed expressions for the LTI model matrices A, B, C and D have been previously documented⁵.

The LTP model extracted through linearization from the NL model includes 8 body states, 33 inflow states (Peters-He Finite state inflow with 4 harmonics and a maximum radial variation power of 8), and 48 multi-blade coordinate (MBC) rotor states that include rigid flap, rigid lead-lag and coupled elastic modes. Thus, the total number of LTP states is 89. Each of these LTP states is then decomposed into 0-8/rev harmonic components, resulting in 1513 total LTI model states. It should be noted that all 0-8 harmonics may not be required to achieve acceptable fidelity in the LTI model⁵. The nonlinear model is trimmed at 120 knots.

4. DYNAMIC TRIM ESTIMATION ALGORITHM

The dynamic trim estimation algorithm aims at calculating future steady state value of the limited parameter. This ability to estimate future steady state value of the limited parameter is essential in the early detection of limit violation. A detailed description of the methodology used in the development of the dynamic trim estimation of the limited parameter is explained in this section. Dynamic trim is a quasi-steady state condition where the fast dynamics of the aircraft have reached an equilibrium (steady state) while the slow dynamics are still slowly changing. This paper considers a notion of dynamic trim where a certain number of judiciously selected LTI states are considered as slow states while the rest of the LTI state vector represents the fast states. In order to obtain the dynamic trim prediction of the limited parameter at any given time, the process of residualization is used. Residualization is a process based on singular perturbation theory in which a reduced order model is obtained from the LTI model. Through residualization, the LTI model low frequency and steady state are accurately captured but high frequency dynamics are neglected⁹. The residualized LTI model is derived from a quasi-steady representation of the fast dynamics of the full order LTI model. It is assumed that the fast states reach their equilibrium instantaneously with respect to the slow states. In what follows is a derivation of the new reduced order dynamical system and functional relationship that maps the controls and slow states to the limit parameters via the use of residualization. For this study, the limited parameter is chosen to be

harmonic pitch link load but any other helicopter component load could have been selected. The LTI state vector is divided as follows

$$(11) \quad X = \begin{bmatrix} X_s \\ X_f \end{bmatrix}$$

where

X_s = slow states and X_f = fast states

We therefore have the following dynamical system:

$$(12) \quad \begin{bmatrix} \dot{X}_s \\ \dot{X}_f \end{bmatrix} = \begin{bmatrix} A_s & A_{sf} \\ A_{fs} & A_f \end{bmatrix} \begin{bmatrix} X_s \\ X_f \end{bmatrix} + \begin{bmatrix} B_s \\ B_f \end{bmatrix} U$$

As per the assumption that the fast states reach steady state very quickly, we can set $\dot{X}_f=0$ and solve for X_f .

$$(13) \quad A_{fs}X_s + A_f X_f + B_f U = 0$$

$$(14) \quad X_f = A_f^{-1} [-A_{fs}X_s - B_f U]$$

By substituting for X_f from Eq. (14) into Eq. (12), the dynamic equation for the residualized system becomes

$$(15) \quad \dot{X}_s = [\hat{A}]X_s + [\hat{B}]U$$

Where

$$(16) \quad \hat{A} = A_s - A_{sf}A_f^{-1}A_{fs}$$

$$(17) \quad \hat{B} = B_s - A_{sf}A_f^{-1}B_f$$

The output equation is also residualized in terms of the slow states and control as

$$(18) \quad Y = [C_s \quad C_f] \begin{bmatrix} X_s \\ X_f \end{bmatrix} + [D]U$$

$$(19) \quad Y = [\hat{C}]X_s + [\hat{D}]U$$

where

$$(20) \quad \hat{C} = C_s - C_f A_f^{-1} A_{fs}$$

$$(21) \quad \hat{D} = D - C_f A_f^{-1} B_f$$

$$(22) \quad Y = [y_0^T \dots y_{lc}^T \ y_{ls}^T \dots]^T$$

Using the residualization procedure described above, an initial study was conducted to assess the fidelity of different reduced order LTI models for

prediction of blade root pitch link loads. In this regard, three different reduced order LTI models were considered. The first model was an 8th order LTI model (or 8th order model) derived with slow states consisting of 0th harmonic components of body velocities (U, V, W), body angular velocities (P, Q, R) and body pitch and roll attitudes (θ, ϕ). The resultant slow state vector is defined as

$$(23) \quad X_s = [x_{B_0}]$$

The second reduced order model was a 10 states LTI model (or 10th order model). For this model, in addition of the slow states included in the 8th order model, the 0th harmonic of the longitudinal (β_{1c_0}) and lateral (β_{1s_0}) flapping were also retained as slow states, thus capturing the low-frequency cyclic flap mode in addition to the body modes as part of the slow dynamics. The resulting slow states vector is defined as

$$(24) \quad X_s = [x_{B_0}^T \ x_{B_0} \ \beta_{1c_0} \ \beta_{1s_0}]^T$$

Finally, the third model was a 14th order LTI model (or 14th order model). In addition to the slow states retained for the 10th order model, the 1st harmonic cosine and sine components of the coning ($\beta_{0_{1c}}, \beta_{0_{1s}}$) and differential coning ($\beta_{d_{1c}}, \beta_{d_{1s}}$) were retained as slow states for the construction of this model, basing it on a recent study in the literature⁷. The slow state vector of the 14th order model can be defined as

$$(25) \quad X_s = [x_{B_0}^T \ \beta_{1c_0} \ \beta_{1s_0} \ \beta_{0_{1c}} \ \beta_{0_{1s}} \ \beta_{d_{1c}} \ \beta_{d_{1s}}]^T$$

It is important to note that the 1st harmonic components of coning and differential coning included in the 14th order model contribute to coning and differential coning modes, which theoretically are faster than the low frequency cyclic flap mode. However, they are similar in form to the 0th harmonic components of longitudinal and lateral flapping in arriving at their contributions to rotating blade pitch link loads, and hence, may play a dynamic role in the estimation of the rotating pitch link loads. This aspect was investigated as part of the initial study.

The three different reduced order LTI models were compared in their ability to predict the dynamic trim value of the pitch link load arising from pilot control input, body motion and rotor states retained as part of the slow states. Towards this, a comparison is made between the body states responses from all three reduced order LTI models and the full-order LTI model to a longitudinal doublet input. Figure 1

is a plot of the percentage change in longitudinal cyclic control variation applied to all four models (8th, 10th, 14th and full order LTI models). All other controls are held fixed at their trim values. The resulting vehicle angular rate response (P, Q, R) and body velocity component response (U, V, W) predictions from the models are shown in Figs 2 and 3, respectively. It is seen from Figs. 2 and 3 that all reduced order LTI models prediction of body velocity and angular rate responses are close to the full-order LTI model response.

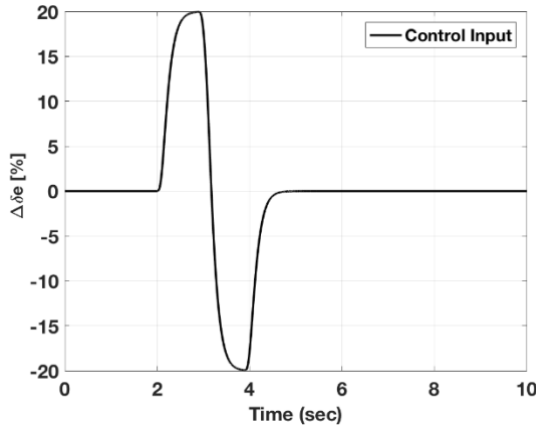


Figure 1. Percentage change from trim of longitudinal cyclic control input.

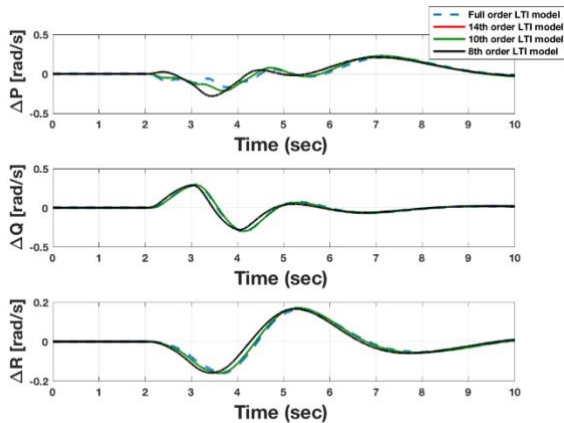


Figure 2. Body angular rate response from full and reduced order LTI models for the selected longitudinal control input (see Fig.1).

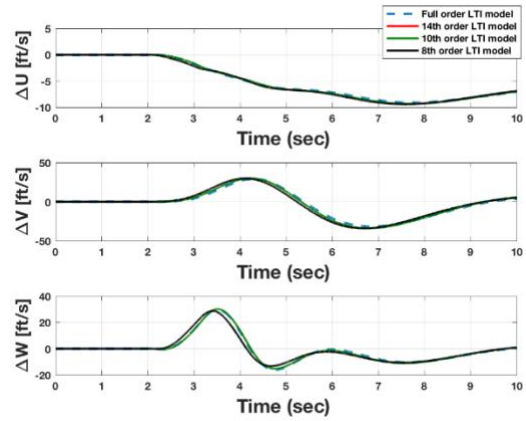


Figure 3. Body velocity response from full and reduced order LTI models for the selected longitudinal control input (see Fig.1).

Figure 4 shows the variation of reference blade harmonic pitch link load (magnitude of 4/rev) output predicted by the full-order LTI model and all the different reduced order LTI models. It is seen in Fig.4 that the harmonic pitch link load output from all the reduced order LTI models lead in time to that from the full-order LTI model. In a sense, with the reduced order LTI models, an estimate of the future value of the pitch link load is obtained before it actually happens, thus providing lead time for altering pilot control inputs for an effective load limiting control strategy. Further, the 10th and 14th order LTI models predictions of the 4/rev pitch link load are almost identical, suggesting that the 10th order LTI model retains similar fidelity of the 14th order LTI model in its prediction of the 4/rev harmonic pitch link loads. This aspect is also clear from the eigenvalue plots of different order LTI models shown in Fig. 5. It is seen from Fig. 5 that the low frequency cyclic flap mode eigenvalues for the 10th and 14th order LTI models are nearly identical. Hence, only the 10th order LTI model in place of the 14th order model was considered in the subsequent load limiting control study. The 10th order LTI model predictions of the 4/rev harmonic pitch link load when compared to that of the 8th order LTI model is better, especially in capturing of the peak magnitude predictions of the full order LTI model. In order to assess the impact of the loss of fidelity of the 8th order LTI model in capturing of the peak magnitude of 4/rev harmonic pitch link load on the load limiting controller performance, both the 8th and 10th order LTI models were considered in the load limiting controller synthesis.

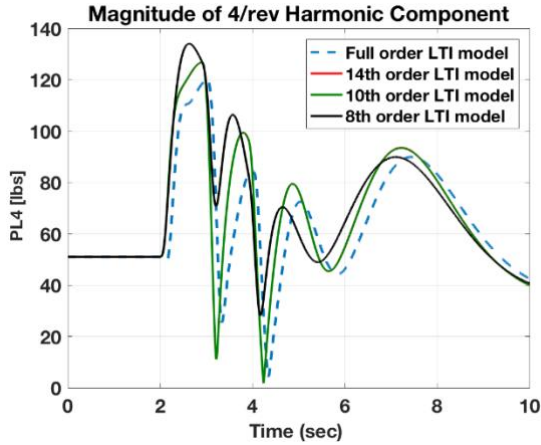


Figure 4. Variation of 4/rev harmonic component of reference blade pitch link loads for the selected longitudinal control input (see Fig.1).

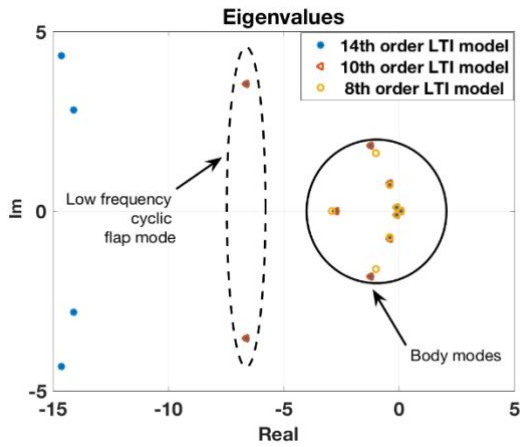


Figure 5. Eigenvalues of reduced order LTI models.

5. LOAD LIMITING CONTROL SYNTHESIS

A detailed description of the proposed load limiting control algorithm is presented in this section. It makes use of real time estimation of the limit parameter in dynamic trim to predict future limit violation and uses that information to alter pilot control input via a feedback controller in order to avoid limit violations. With known values of the slow states and control at the current time instant t , the dynamic trim value of the limit parameter at $t+\Delta t$ due to a step change in control input is estimated using the residualized model (reduced order LTI model). The magnitude of the step input used is equal to the difference between the input from the pilot at time $t+\Delta t$ and t . This represents a one-step prediction where we estimate the value of the limit parameter in dynamic trim at $t+\Delta t$ while the aircraft is still at t . If the estimated value of the limit

parameter is below the set limit, then the pilot control input is allowed as is without any modification. If a limit violation is predicted by the residualized model, the pilot control input is limited (reduced) through a feedback loop to avoid limit violations. The process is repeated over a pre-selected value of Δt . It is important to note that when a pilot applies any desired control input, no extra effort is needed to make sure that the input does not result in such an aggressive maneuver that would cause a limit violation. The proposed load limiting controller takes action without the pilot's awareness to help in reducing excessive control action.

When limit violation is predicted by the residualized model, the load limiting controller reduces or limits the control input through a feedback loop. In order to come up with the appropriate feedback control law, this study makes use of the local sensitivity approach¹⁰. The local sensitivity method is employed to establish the needed reduction in control deflection for limit avoidance. At any time $t+k\Delta t (k \in \mathbb{N})$, if a limit violation is detected using limit parameter estimate \hat{y}_k from the residualized model, the pilot control input is modified by $G^* \Delta v$ where Δv is computed from

$$(26) \quad \Delta v = S(\hat{y}_k - y_{lim})$$

where S is the normalized value of the local sensitivity and is calculated using

$$(27) \quad S = \frac{\left(\frac{\partial y}{\partial u}\right)^{-1}}{\text{norm}\left[\left(\frac{\partial y}{\partial u}\right)^{-1}\right]}$$

where

$$(28) \quad \left(\frac{\partial y}{\partial u}\right) = \frac{\hat{y}_k - y_{R(k-1)}}{u_k - u_{k-1}}$$

A block diagram representation of the proposed load limiting control (LLC) algorithm is shown in Fig. 6. The value of G in Fig. 6 is set to 1 in this study.

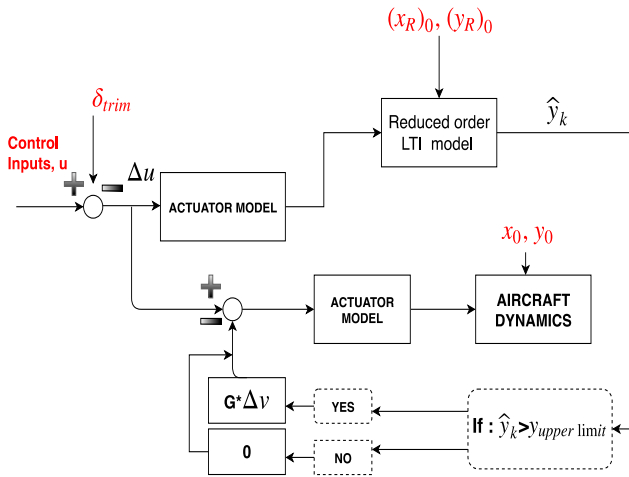


Figure 6. Load limiting control (LLC) algorithm.

In the diagram above, $(x_R)_0$ and $(y_R)_0$ represent the initial state and output vectors of the reduced order LTI model while x_0 and y_0 are those associated to the vehicle model.

6. RESULTS

The proposed load limiting control algorithm was evaluated in simulation for the case of harmonic axial blade pitch link loads arising from the longitudinal doublet input shown in Fig.1. Specifically, limiting of the magnitude of 4/rev pitch link load was considered. Furthermore, a study of the impact of selecting different reduced order LTI models on the closed loop system was also performed. Only the 8th order and 10th order models were used in this study as the 14th order and 10th order models predictions of the magnitude component of the 4/rev pitch link load were seen to be very similar (see Fig.4).

The full-order LTI model⁴ extracted from the nonlinear model of the generic helicopter in FLIGHTLAB[®] was used as the truth model in this initial proof-of-concept study of the proposed load limiting controller.

The upper limit for the pitch link 4/rev load magnitude was arbitrarily set at 100 lbs for the load limiting control law. The magnitude component of the 4/rev pitch link load is obtained using

$$(29) \quad y_{4/rev} = \sqrt{y_{4c}^2 + y_{4s}^2}$$

Simulated variations of the reference blade root pitch link 4/rev load magnitude without (labeled 'No

LLC') and with the proposed load limiting control law (labeled 'With LLC') are shown in Fig. 7 for the case of a doublet longitudinal cyclic input of Fig. 1. It can be observed from Fig.7 that with LLC, the 4/rev magnitude of the pitch link load stays within the selected limit using either of the 8th and 10th order LTI models. However, it is important to note that during the time period where limit exceedance is detected, the 10th order model allows for a more efficient load limiting as the magnitude of the 4/rev harmonic pitch link load rides the limit boundaries whereas the 8th order model allows for some slight exceedance. Furthermore, Fig.8, wherein the longitudinal cyclic control input with and without LLC are compared, shows that the pilot control input is modified whenever the 4/rev load exceeds the selected limit. As the 4/rev load magnitude increases with increasing control input, the load limiting control law alters the input so as to keep the load within the selected limit. From Fig.8, it can be observed that using the 10th order model does not lead to premature control action from the LLC in order to avoid limit exceedance. In a sense, using the 10th order model over the 8th order model in the load limiting control design allows for less sacrifice in maneuverability. In a more general sense, it is seen from Figs. 7 and 8 that the proposed LLC scheme takes corrective action in altering the pilot control input only when necessary.

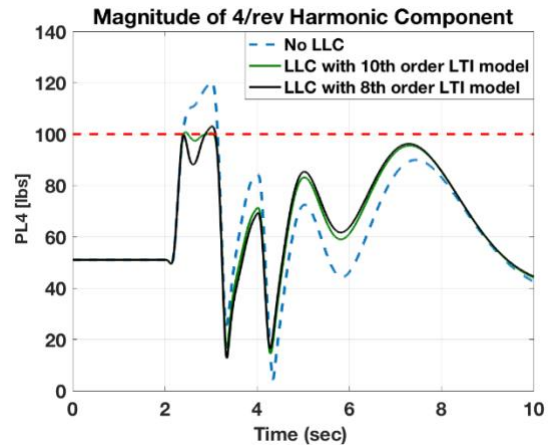


Figure 7. Variation of 4/rev harmonic component of reference blade pitch link loads with and without LLC.

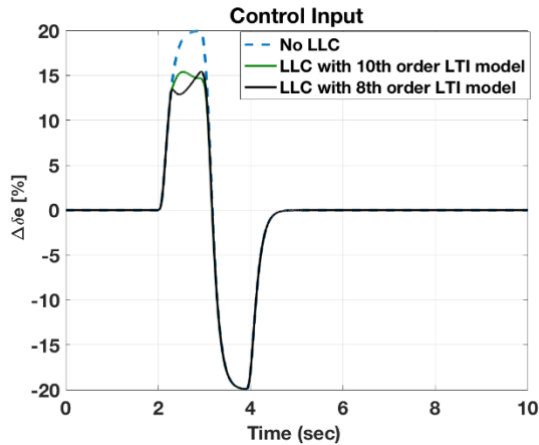


Figure 8. Percentage change from trim of longitudinal cyclic control input with and without LLC.

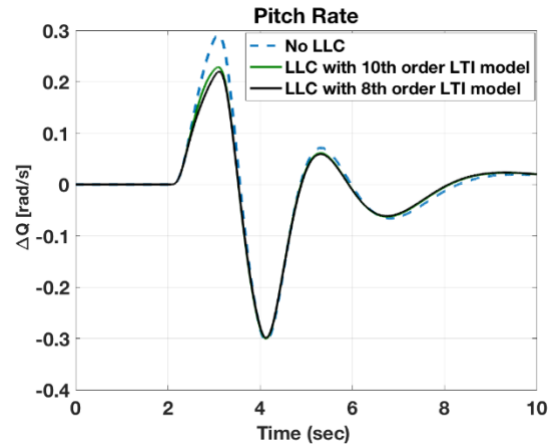


Figure 9. Body pitch rate response with and without LLC.

Figure 9 shows the variation of body pitch rate response with and without LLC. It is seen from Fig. 9 that, for the selected doublet maneuver, as the body pitch rate increases, the magnitude of the 4/rev harmonic pitch link load increases as well. With LLC, it is seen that the achievable maximum pitch rate for the selected control input is reduced in order to keep the pitch link load within the selected limit. Moreover, when the load limit is not exceeded, the pitch rate response is somewhat similar to the case without LLC. This again shows that the proposed load limiting control law does not lead to a conservative design, i.e., pilot control is modified only when necessary. Figure 9 can also serve to corroborate the previously mentioned fact that the 10th order model leads to less sacrifice in maneuverability compared to the 8th order model. The pitch rate profile of the vehicle with a LLC using the 10th order model is not reduced as much as the one with a LLC using the 8th order model when limit exceedance is detected.

While the proposed load limiting control law is synthesized to limit the magnitude of the 4/rev load, its effect on the 4/rev sine and cosine components are shown in Figs. 10 and 11. As expected, both cosine and sine parts of the 4/rev pitch link load get reduced to allow for the magnitude of the 4/rev load to be within the prescribed limit.

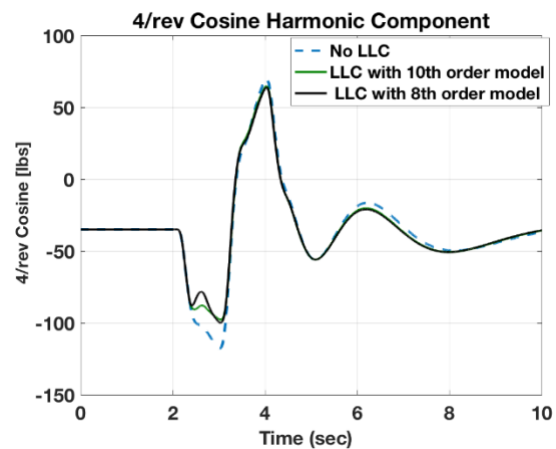


Figure 10. Variation of 4/rev cosine harmonic component of reference blade pitch link loads with and without LLC.

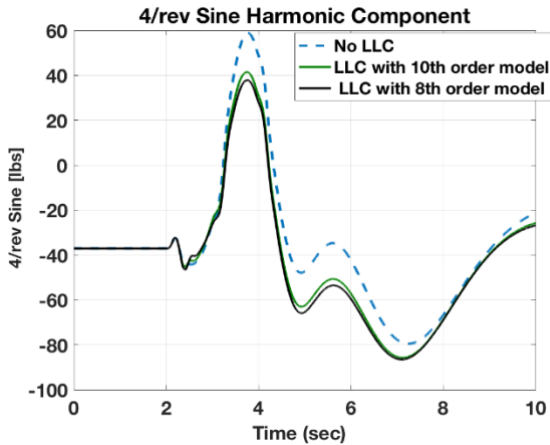


Figure 11. Variation of 4/rev sine harmonic component of reference blade pitch link loads with and without LLC.

It is also of interest to analyze how limiting one harmonic component impacts other harmonic components of the pitch link load. Figure 12 shows the magnitude of 1/rev pitch link load variation with and without LLC. It is clear from Fig. 12 that the peak magnitude of 1/rev pitch link load is also reduced using the proposed LLC for limiting the 4/rev load. This reduction in the magnitude of 1/rev is noticed irrespective of the reduced order model selected. Though not shown, similar reductions in peak magnitudes of other harmonics of pitch link loads were observed from the simulation results.

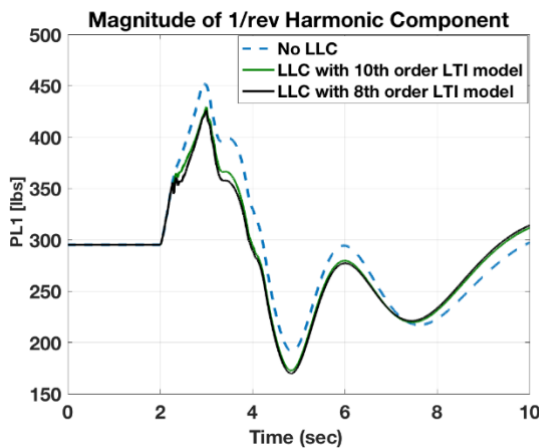


Figure 12. Variation of 1/rev harmonic component of reference blade pitch link loads with and without LLC.

7. CONCLUDING REMARKS

An approach for real time load limiting control law for limiting helicopter component loads during aggressive maneuvers is presented in which a linear time invariant (LTI) model of a helicopter coupled body/rotor/inflow dynamics and the notion of residualization are used. The load limiting control law developed in this paper uses a two-step process to achieve the desired task, viz., limit violation detection and limit avoidance. The limit violation detection part of the algorithm uses a reduced order model representation to perform a one-step prediction in order to calculate future steady state value of the component load to detect future limit violation due to pilot control inputs. The limit avoidance makes use of the notion of local sensitivity to calculate the required reduction of the pilot control needed in order to avoid load limit exceedance.

The proposed load limiting control scheme is evaluated in simulation for limiting an individual harmonic component of blade root pitch link loads arising from a longitudinal doublet maneuver. In the proof-of-concept results presented, a linear model extracted at 120 knots from a nonlinear model of a generic helicopter in FLIGHTLAB[®] was used as the truth model. The presented results show promise in the ability of the proposed load limiting control (LLC) law to limit harmonic components of the pitch link loads through a required reduction in the aggressiveness of the maneuver. The proposed load limiting control law is also seen to be somewhat robust in the choice of the reduced order model used in the dynamic trim estimation algorithm.

While the proposed load limiting control law shows promise from the results for an example control doublet, future work is needed in establishing its performance when one chooses to limit different harmonic components of loads. Further evaluations need to address the robustness of the proposed scheme in the presence of significant model uncertainty using a nonlinear model.

8. ACKNOWLEDGMENTS

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