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PREDICTION OF OFF-DESIGN PERFORMANCE OF TURBO-SHAFT ENGINES A SIMPLIFIED METHOD

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SUMMARY

The estimation of the off-design performance of a given gasturbine is generally based on detailed performance characteristics of compressor and turbine(s). During many preliminary studies these characteristics are often not available. In earlier work the author has developed approximate methods for the estimation of the off-design behaviour of turbojet and turbofan engines, using thermodynamic relationships only. This method is based on a matching procedure with generalized mass flow characteristics of the turbine(s) or exhaust nozzle(s).
In the present paper the method is adapted to turbo-shaft engines. Some examples are given for an actual engine type, which show a fair agreement between the calculated off-design performance and the data derived from the manufacturers' engine brochure.

CONTENTS

1. Introduction

In aircraft project work the design team will often avail of manufacturers' performance data for the engine(s) of the aircraft. Sometimes - especially in early stages of a project - the engine performance characteristics have to be predicted by the designers themselves, because detailed engine data are lacking. To match the engine thrust or power to the aircraft requirements sometimes "rubberized" engines are used, for which the data are found from existing engines by scaling. In this case the cycle characteristics of the engine such as compressor pressure ratio, turbine entry temperature etc. are specified beforehand. For arbitrary cycle characteristics the design-point characteristics of gasturbines (jet or shaft-power engines) can easily be estimated, using state-of-the-art values for the component efficiencies. The calculation method is found in most text books on gasturbines and aircraft propulsion, e.g. Ref. 1. The effects of compressor pressure ratio and turbine entry temperature on the specific thrust (power) and specific fuel consumption can easily be derived to optimize the engine for a given design point. But for engines with a given design point, the off-design performance can not be found as easily, because of additional matching conditions between the engine components.
In order to obtain off-design equilibrium conditions of a given engine, methods have been developed to match the engine components, requiring performance maps of the compressor(s) and turbine(s), (Ref. 1). In most cases these detailed component characteristics will not be available to the aircraft designer.
To be able to predict off-design performance of turbojet- and turbofan-engines with given design-point characteristics, the author has developed calculation methods using gasdynamic relationships only, and based on some simplifying assumptions, which fix the operating points of the engine components (Ref. 2).
In the present paper the same method is adapted to turboshaft engines with a free power turbine.

2. List of symbols

A	cross sectional area
c_p	specific heat at constant pressure
c_v	specific heat at constant volume
F	fuel flow
H	heating value of fuel
M	Mach number
m	mass flow
N_t	rotational speed of turbine
p	static pressure
p_t	total pressure
P_{br}	shaft horse power
R	gas constant
T	static temperature
T_t	total temperature
T_g	gross thrust
V	velocity
γ	specific heat ratio ($\gamma = c_p/c_v$)
ε_c	compressor pressure ratio
η_B	combustion efficiency
η_c	polytropic compressor efficiency
η_R	ram pressure recovery (intake)
η_t	polytropic turbine efficiency
η_m	mechanical efficiency

φ turbine entry temperature ratio
π^* total pressure ratio of turbine

Subscripts:

0,1 ... 7	stations for engine flow (Fig. 1)
a	air
g	gas
crit	critical condition
des	design-point condition

3. *General equations for cycle calculations*

Fig. 1 gives the general lay-out of a shaft turbo engine with free power turbine. In this figure the characteristic engine stations are numbered and these numbers are used as subscripts for all flow quantities in the corresponding stations.
In cycle calculations the change of the flow is described for each engine component separately. Within the engine the flow condition is fixed by the total pressure and the total temperature.

- Intake (0 → 2). For given ambient conditions and flight speed (or Mach number) the compressor intake conditions are given by:

$$p_{t_2} = \eta_R\, p_{t_o} = \eta_R\, p_o \left\{1 + \frac{\gamma_a - 1}{2} M_o^2\right\}^{\frac{\gamma_a}{\gamma_a - 1}} \tag{3.1}$$

$$T_{t_2} = T_{t_o} = T_o \left\{1 + \frac{\gamma_a - 1}{2} M_o^2\right\} \tag{3.2}$$

The flow is considered as adiabatic with a pressure loss in the intake, characterized by the intake efficiency: $\eta_R = p_{t_2}/p_{t_o}$.

- Compressor (2 → 3). For the compressor ratio: $\varepsilon_c = p_{t_3}/p_{t_2}$ and polytropic efficiency η_c one finds:

$$\frac{T_{t_3}}{T_{t_2}} = \varepsilon_c^{\frac{\gamma_a - 1}{\eta_c \gamma_a}} \tag{3.3}$$

- Combustion chamber(s) (3 → 4). In most calculations the turbine entry temperature (TET, T_{t_4}) and a pressure drop over the combustion chamber are assumed.
  In the following the ratio p_{t_4}/p_{t_3} will be considered as a known performance parameter of the combustion chamber. Introducing the heating value H of the fuel and the combustion efficiency η_B, the fuel flow F is related to the enthalpy change in the combustion chamber by the equation:

$$\eta_B\, H\, F = m_a\, c_{p_m} (T_{t_4} - T_{t_3}) \tag{3.4}$$,

  where c_{p_m} is a mean value of the specific heat c_p.

- Compressor/turbine matching. The power equation for the compressor and its turbine (compressor-turbine) is:

$$m_a\, c_{p_a} (T_{t_3} - T_{t_2}) = \eta_m\, m_g\, c_{p_g} (T_{t_4} - T_{t_5}) \tag{3.5a}$$

  Due to bleed or cooling air extraction and the fuel mass, the gas mass m_g differs from the air mass m_a; the gas properties after the combustion chamber (index g) are distinguished from those of air (index a); *see Table I of the appendix for standard values used in this study.*
  A modified expression for (3.5a) can be found by introducing eq. (3.3):

$$\varepsilon_c^{\frac{\gamma_a - 1}{\eta_c \gamma_a}} - 1 = \eta_m \frac{m_g\, c_{p_g}}{m_a\, c_{p_a}} \frac{T_{t_4}}{T_{t_2}} \left(1 - \frac{T_{t_5}}{T_{t_4}}\right) \tag{3.5b}$$

- Compressor-turbine (4 → 5). For given flight condition (T_{t_2}), given compressor ratio ε_c and turbine entry temperature T_{t_4} the turbine outlet temperature T_{t_5} follows from (3.5b). Using the polytropic turbine efficiency η_t, the total pressure ratio of the turbine becomes:

$$\frac{p_{t_5}}{p_{t_4}} = \left(\frac{T_{t_5}}{T_{t_4}}\right)^{\frac{\gamma_g}{\eta_t(\gamma_g-1)}} \qquad (3.6)$$

- Power-turbine (5 → 6). The shaft horse power extracted from the gas flow by the power-turbine is:

$$P_{br} = \eta_m' \, m_g \, c_{p_g} \, (T_{t_5} - T_{t_6}) \qquad (3.7)$$

Using the polytropic turbine efficiency η_t', the total pressure ratio for the power turbine follows from:

$$\frac{p_{t_6}}{p_{t_5}} = \left(\frac{T_{t_6}}{T_{t_5}}\right)^{\frac{\gamma_g}{\eta_t'(\gamma_g-1)}} \qquad (3.8)$$

In principle the temperature T_{t_6} is a design variable of the engine, which determines the division of the available power of the gasgenerator (between section 0 and 5) over the shaft and the exhaust jet. For turboprop engines the power division between the propeller shaft and the exhaust jet can be chosen such that the maximum total propulsion power is obtained. In this case the optimum jet velocity is found to be proportional for the flight speed (Ref. 3). Thus for turboshaft engines, designed for operation at static condition or low flight speeds, the condition for maximum power dictates an expansion over the power turbine to almost ambient pressure (low jet velocity).

- Exhaust nozzle (6 → 7). For shaft-power engines with low exit jet velocities a first approximation of T_{t_6} can be obtained from eq. (3.8) with the assumption: $p_{t_6} = p_o$. In this case the total temperature T_{t_6} and the jet temperature T_7 are also about equal; for e.g. $V_7 = 60$ m/sec the difference between these temperatures is: $\Delta T = V_7^2/2c_{p_g} \cong 1{,}6$ K.

Using $T_{t_6} = T_7$ the jet velocity can be found from the condition that the mass flow has to leave the engine through the exit nozzle area A_7 at the pressure p_o:

$$V_7 = \frac{m_g}{\rho_7 \, A_7} = \frac{R_g \, T_7}{p_o} \, \frac{m_g}{A_7} \qquad (3.9)$$

Using Bernouilli's equation for incompressible flow the total pressure ratio for the jet pipe becomes:

$$\frac{p_{t_6}}{p_o} = 1 + \tfrac{1}{2} \, \frac{\rho_7 \, V_7^2}{p_o} = 1 + \tfrac{1}{2} \, R_g \, T_7 \left(\frac{m_g}{p_o \, A_7}\right)^2 \qquad (3.10)$$

From this result a corrected value of $\frac{p_{t_6}}{p_{t_5}} = \frac{p_{t_6}}{p_o} \Big/ \frac{p_{t_5}}{p_o}$ can be found for use in eq. (3.8) to obtain a second estimate for T_{t_6} and the power-turbine output P_{br}.

4. Design-point performance

The design point of the turbo engine of the type considered here is fixed by the following engine characteristics:

- free stream condition: p_o, T_o, M_o
- compressor pressure ratio ε^c
- turbine entry temperature T_{t_4}
- the component efficiencies η_c, η_t, η_t', the mechanical efficiencies η_m and η_m' and the pressure ratio p_{t_4}/p_{t_3} for the combustion chamber(s)
- the mass ratio m_g/m_a and the division between shaft and jet power, characterized by the choice of

T_{t_6} or a related condition (e.g. jet velocity v_7 or $p_{t_6} = p_o$).

The specific power, specific thrust and specific fuel consumption at the design point are fixed by these engine parameters. The shaft horse power, jet thrust and fuel consumption result from the air mass flow, which characterizes the size of the engine.

As an example the design-point performance for a range of compressor pressure ratios and turbine-inlet temperatures is presented in Fig. 2, using the assumption: $p_{t_6} = p_o$.

5. Off-design matching procedures for different engine types

In design-point calculations the compressor pressure ratio ε_c and the turbine entry temperature T_{t_4} may be chosen as independent variables. However, if an engine with fixed design-point characteristics is considered at other flight conditions and/or turbine-entry temperatures (fuel flows), the compressor pressure ratio has become a dependent variable. The methods developed by the author for the estimation of the engine equilibrium condition in off-design cases are based on the use of generalized characteristic curves for the mass-flow of the turbine and/or the exhaust nozzle to match the flow conditions of the engine components.

Most turbojet engine operate with a choked exhaust nozzle. In this case the operating point(s) of the turbine(s) are fixed and full analytical expressions of the engine-off design performance can be derived (Ref. 2). For turbojet and turbofan engines operating with unchoked exhaust nozzle flows, a similar, but somewhat more complicated method is derived in the same Reference, based on the assumption of a choked condition of the (low-pressure) turbine(s). The same approximation is used in Ref. 4 for the estimation of the off-design performance of turboprop engines.

For all methods mentioned above the use of the mass-flow characteristic curve of the exhaust nozzle is essential (Fig. 3). The use of this curve is, however, less suitable for turbo-shaft engines, since due to the low exhaust nozzle pressure ratios of these engines the mass flow is very sensitive with respect to the pressure ratio.

Moreover, over the full operating range of most turbo-shaft engines the pressure ratio of the power turbine at lower engine ratings will drop below the critical pressure ratio for choked turbine flow.

For this reason a calculation method different from Refs. 2 and 4 has been developed in the present paper.

6. Off-design performance calculations for turbo-shaft engines

The characteristic curve for the mass flow of a turbine can generally be expressed in the following relationship, taking the compressor-turbine as example:

$$\frac{m_g \sqrt{T_{t_4}}}{p_{t_4}} = f\left(\frac{p_{t_4}}{p_{t_5}}, \frac{N_t}{\sqrt{T_{t_4}}}\right) \tag{6.1}$$

Particularly, in the unchoked condition the mass flow is dependent on the generalized turbine rotational speed N_t, but to a first approximation the effect of turbine speed on the mass flow characteristic can be neglected and an unique relationship between the generalized mass flow and the pressure ratio can be used. Fig. 4 show these generalized curves for the compressor- and power-turbine.

In order to match the mass flows of these turbines the mass flow of the compressor-turbine can be expressed in terms of the exit conditions (station 5) instead of the entry conditions (station 4):

$$\frac{m_g \sqrt{T_{t_5}}}{p_{t_5}} = \frac{m_g \sqrt{T_{t_4}}}{p_{t_4}} \; \frac{p_{t_4}}{p_{t_5}} \sqrt{\frac{T_{t_5}}{T_{t_4}}} \tag{6.2}$$

Introducing eq. (3.6) this relationship becomes:

$$\frac{m_g \sqrt{T_{t_5}}}{p_{t_5}} = \frac{m_g \sqrt{T_{t_4}}}{p_{t_4}} \left(\frac{p_{t_4}}{p_{t_5}}\right)^{1 - \frac{\eta_t(\gamma_g - 1)}{2\gamma_g}} \tag{6.3}$$

For a given value of the turbine efficiency η_t this equation gives a unique relationship between $m_g \sqrt{T_{t_5}}/p_{t_5}$ and p_{t_4}/p_{t_5}, which is also shown in the left part of Fig. 4. The mass flows of both turbines can be matched by means of this curve.

In the matching procedure two cases can be distinguished:

(a) the power turbine is choked (region A_1), in which case the operating point of the compressor turbine (point A_2) is fixed independent of the pressure ratio p_{t_5}/p_{t_6} of the power turbine,

(b) the power turbine is unchoked; in this case the operating point of the compressor-turbine depends on the pressure ratio p_{t_5}/p_{t_6} of the power turbine, as indicated by the points B_1 and B_2.

To develop an off-design calculation method for case (b) the characteristic curve of the power turbine in the unchoked region has to be available. This curve will here be approximated by a formula derived by Linnecken (Ref. 5):

$$\left(\frac{m_g \sqrt{T_{t_5}}}{p_{t_5}}\right)^2 = \text{const.} \left[\left(1 - \frac{1}{\pi^*_{crit}}\right)^2 - \left(\frac{1}{\pi^*} - \frac{1}{\pi^*_{crit}}\right)^2 \right] \tag{6.4}$$

where: $\pi^* = \dfrac{p_{t_5}}{p_{t_6}}$ and π_{crit}^* denotes the critical pressure ratio. Fig. 5 shows this relationship for $\pi^*_{crit} = 2.5$, which is a typical value for two-stage turbines. In the following the design-point condition will be used as a reference condition. In most cases the compressor-turbine will operate in the choked region over the operating range of the engine, hence: $\dfrac{m_g \sqrt{T_{t_4}}}{p_{t_4}} = \text{const.}$

Eq. (6.3) can then be written as:

$$\frac{m_g \sqrt{T_{t_5}}/p_{t_5}}{(m_g \sqrt{T_{t_5}}/p_{t_5})_{des}} = \left[\frac{p_{t_4}/p_{t_5}}{(p_{t_4}/p_{t_5})_{des}}\right]^{1 - \frac{\eta_t(\gamma_g - 1)}{2\gamma_g}} \tag{6.5}$$

Combining eqs (6.4) and (6.5) gives:

$$\frac{p_{t_4}/p_{t_5}}{(p_{t_4}/p_{t_5})_{des}} = \left[\frac{\left(1 - \frac{1}{\pi^*_{crit}}\right)^2 - \left(\frac{1}{\pi^*} - \frac{1}{\pi^*_{crit}}\right)^2}{\left(1 - \frac{1}{\pi^*_{crit}}\right)^2 - \left(\frac{1}{\pi^*_{des}} - \frac{1}{\pi^*_{crit}}\right)^2}\right]^{\frac{1}{2 - \frac{\eta_t(\gamma_g - 1)}{\gamma_g}}} \tag{6.6}$$

This equations give a unique relationship between the pressure ratios p_{t_4}/p_{t_5} of the compressor-turbine and $\pi^* = p_{t_5}/p_{t_6}$ of the power turbine for unchoked conditions of the latter.

For the total pressure ratio over the engine holds the condition:

$$\frac{p_{t_6}}{p_o} = \frac{p_{t_6}}{p_{t_5}} \frac{p_{t_5}}{p_{t_4}} \frac{p_{t_4}}{p_{t_3}} \frac{p_{t_3}}{p_{t_2}} \frac{p_{t_2}}{p_o} \tag{6.7}$$

Assuming $\dfrac{p_{t_6}}{p_o} = 1$ and $\dfrac{p_{t_4}}{p_{t_3}} = \text{constant}$, the compressor-ratio $\varepsilon_c = \dfrac{p_{t_3}}{p_{t_2}}$ follows from:

$$\frac{\varepsilon_c}{(\varepsilon_c)_{des}} = \frac{p_{t_5}/p_{t_6}}{(p_{t_5}/p_{t_6})_{des}} \; \frac{p_{t_4}/p_{t_5}}{(p_{t_4}/p_{t_5})_{des}} \; \frac{(p_{t_2}/p_o)_{des}}{p_{t_2}/p_o} \tag{6.8}$$

From eq. (2.5b) for the power matching condition of the compressor and its turbine is obtained by assuming $\eta_m \dfrac{m_g \, c_{p_g}}{m_a \, c_{p_a}}$ constant:

$$\frac{\varepsilon_c^{\frac{\gamma_a-1}{\eta_c\gamma_a}}-1}{(\varepsilon_c)_{des}^{\frac{\gamma_a-1}{(\eta_c)_{des}\gamma_a}}-1}=\varphi\,\frac{1-T_{t_5}/T_{t_4}}{1-(T_{t_5}/T_{t_4})_{des}} \tag{6.9}$$

In this equation a generalized turbine-entry temperature ratio is introduced:

$$\varphi=\frac{T_{t_4}/T_{t_2}}{(T_{t_4}/T_{t_2})_{des}} \tag{6.10}$$

For the compressor mass flow we find:

$$\frac{m_a\sqrt{T_{t_2}}}{p_{t_2}}=\frac{m_a}{m_g}\,\frac{m_g\sqrt{T_{t_4}}}{p_{t_4}}\,\frac{p_{t_4}}{p_{t_3}}\sqrt{\frac{T_{t_2}}{T_{t_4}}}\cdot\frac{p_{t_3}}{p_{t_2}} \tag{6.11}$$

For choked condition of the compressor-turbine $(\frac{m_g\sqrt{T_{t_4}}}{p_{t_4}}=\text{const})$ this leads to:

$$\frac{m_a\sqrt{T_{t_2}}/p_{t_2}}{(m_a\sqrt{T_{t_2}}/p_{t_2})_{des}}=\frac{\varepsilon_c}{(\varepsilon_c)_{des}}\,\frac{1}{\sqrt{\varphi}} \tag{6.12}$$

From the equations derived above the off-design performance can be calculated readily.
A range of values of $\pi^*=p_{t_5}/p_{t_4}$ (in the unchoked region of the power turbine) is assumed. For each value of π^* follows p_{t_4}/p_{t_5} from (6.6), ε_c from (6.8), φ from (3.6) and $\frac{m_a\sqrt{T_{t_2}}}{p_{t_2}}$ from (6.12).
Note that in eq. (6.8) the differences in flight conditions between the design and off-design state are taken into account by the term p_{t_2}/p_o and that for each value of φ the corresponding turbine-inlet temperature T_{t_4} depends on T_{t_2}.
From the engine conditions fixed by the calculation above, the shaft-horse power and fuel flow can be estimated from eqs. (3.7) and (3.4). Also the jet velocity and thrust can be found, and a correction for $p_{t_6}/p_o\neq 1$ can be applied as described in section 2.
Some simplified relationships can be derived for engine conditions with choked power-turbine, which fixes the point of the compressor-turbine independent from $\pi^*=p_{t_5}/p_{t_6}$ (point A_2 in Fig. 4). As in this case T_{t_5}/T_{t_4} = constant in eq. (6.9) one finds:

$$\varepsilon_c=\left[1+\varphi\left\{(\varepsilon_c)_{des}^{\frac{\gamma_a-1}{(\eta_c)_{des}\gamma_a}}-1\right\}\right]^{\frac{\eta_c\gamma_a}{\gamma_a-1}} \tag{6.13}$$

This equation combined with eq. (6.4) fixes a unique working line in the generalized performance map of the compressor for all engine conditions:

$$\varepsilon_c=f\left(\frac{m_a\sqrt{T_{t_2}}}{p_{t_2}}\right) \tag{6.14}$$

It may also be useful to express the engine performance in generalized coordinates. E.g. for the shaft-horse power eqs. (3.7) and (3.6) yield:

$$\frac{P_{br}}{p_{t_2}\sqrt{T_{t_2}}} = \eta_m' \, c_{pg} \, \frac{m_g\sqrt{T_{t_4}}}{p_{t_4}} \, \frac{T_{t_5}}{T_{t_4}} \sqrt{\frac{T_{t_4}}{T_{t_2}}} \, \frac{p_{t_4}}{p_{t_3}} \, \frac{p_{t_3}}{p_{t_2}} \left\{1 - \left(\frac{p_{t_6}}{p_{t_5}}\right)^{\frac{\eta_t'(\gamma_g-1)}{\gamma_g}}\right\} \tag{6.15}$$

For a fixed operating point of the compressor-turbine (choked power turbine) the generalized shaft horse power with reference to the design condition becomes:

$$\frac{\dfrac{P_{br}}{p_{t_2}\sqrt{T_{t_2}}}}{\left(\dfrac{P_{br}}{p_{t_2}\sqrt{T_{t_2}}}\right)_{des}} = \frac{\varepsilon_c}{(\varepsilon_c)_{des}} \sqrt{\varphi} \, \frac{1 - \left(\dfrac{p_{t_6}}{p_{t_5}}\right)^{\frac{\eta_t'(\gamma_g-1)}{\gamma_g}}}{1 - \left(\dfrac{p_{t_6}}{p_{t_5}}\right)_{des}^{\frac{\eta_t'(\gamma_g-1)}{\gamma_g}}} \tag{6.16}$$

With the approximation: $p_{t_6} = p_o$ one finds from eq. (6.8):

$$\frac{p_{t_5}/p_{t_6}}{(p_{t_5}/p_{t_6})_{des}} = \frac{\varepsilon_c}{(\varepsilon_c)_{des}} \, \frac{p_{t_2}/p_o}{(p_{t_2}/p_o)_{des}} \tag{6.17}$$

For the conditions with choked power turbine eqs. (6.12), (6.16) and (6.17) determine the main engine characteristics directly as function of the turbine entry temperature ratio φ and the flight condition (speed and altitude).
It should be noted that in eqs. (6.9) and (6.13) one may assume: $\eta_c = (\eta_c)_{des}$; however, the effect of variations of η_c is easily taken into account if a curve $\eta_c = f(\varepsilon_c)$ is available. Over the operating range of the engine any change of the turbine efficiencies is neglected.
All calculations above are based on the assumption of a choked condition of the compressor-turbine over the operating conditions of the engine. If necessary the matching procedure of Fig. 4 can also be applied for unchoked conditions of this turbine. Then the equivalent expression of (6.4) for the compressor-turbine has to be used to obtain modified equations analogeous to eqs. (6.6) and (6.12).

7. Example for an actual engine

To illustrate the validity of the method described in section 6, an example is presented for the turbo-shaftengine Allison Model T 63-A-5 (Ref. 6). For this engine the design point is chosen and from brochure data the flow conditions are estimated at the different engine stations, introducing a limited number of assumptions.
These data for the design point (sea level static, shaft speeds $N_1 = N_2 = 100\%$) are presented in Table II in the Appendix.
From these data the engine off-design performance is calculated and compared with the performance as obtained from the manufacturers' brochure. The off-design performance (sea level static) is estimated for different compressor turbine speeds N_1 at constant power turbine speed ($N_2 = 100\%$).
Both turbines of the engine have two stages and a critical pressure ratio $\pi^*_{crit} = 2.5$ has been assumed. From the data in Table II follows that the power-turbine is unchoked in the design-point condition, but the compressor-turbine is in the choked condition. It is assumed that over the engine rating range of interest, the compressor-turbine remains in the choked condition, which is verified afterwards.
The engine brochure data allow the calculation of the compressor efficiency η_c as a function of the pressure ratio ε_c, as shown in Fig. 6. This curve is used in the off-design performance estimation to avoid any discrepancy due to variations in η_c. The off-design calculation is performed for a range of power-turbine pressure ratios $\pi^* = 1.3 - 2.5$ (up to the choking point).
Table III in the Appendix shows the successive steps of the calculation. For each value of π^* an off-design condition of the engine is fixed.
Since the compressor-turbine speed (N_1) is not a variable in the calculation method, the air mass flow is used as independent variable for the comparison between the estimated data and the data from the engine brochure. Figs. 7 a-e show this comparison. In view of the large range of engine ratings, the agreement of calculated and brochure data appears satisfactory for the compressor pressure ratio ε_c and the engine shaft horse-power P_{br}.

The fuel flow has been estimated by assuming $\dfrac{m_a \, C_{pm}}{\eta_B \, H}$ in eq. (3.4) constant and equal to the value at the design point. The agreement between calculation and brochure data is less satisfactory, as is also the case for the (rather small) gross thrust (Figs. 7 c and d). It is found that the calculated outlet temperature of the compressor-turbine T_{t_5} is too high in comparison with the brochure data (Fig. 7 e).

From Table III in the Appendix it can be seen that at the lowest engine ratings ($\pi^* = 1.3$ and 1.5) the compressor-turbine is no longer choked, but according to the Linnecken equation the change of the mass flow $\frac{m_g \sqrt{T_{t_4}}}{p_{t_4}}$ is negligible for the corresponding turbine pressure ratios p_{t_4}/p_{t_5} (Fig. 5).

Another check of the calculation method has been made for the same engine by estimating the effect of the ambient temperature on the engine performance. This check has been performed for sea-level pressure and compressor-turbine speed $N_1 = 100\%$. From the engine brochure the compressor pressure ratio could be obtained in relation to the ambient temperature for this engine condition (Fig. 8a). Since the matching procedure in the first eight columns of Table III in the Appendix is independent of the ambient condition, the values of ε_c in this table correspond to ambient temperatures to be derived from Fig. 8a. With the appropriate value of the ambient temperature T_o ($= T_{t_2}$) one can easily estimate the engine performance from φ and $\frac{m_a \sqrt{T_{t_2}}}{p_{t_2}}$ in Table II. The result is presented in Fig. 8 b and c (see also Table IV in the Appendix).
A fair agreement between the calculation and the brochure data is obtained.

8. Concluding remarks

Although for the case of an actual engine the agreement shown above is not perfect, the examples show that the method developed in this paper can be used with some confidence to estimate the off-design performance of a shaft-power engine in preliminary design studies, e.g. for parametric variation of the engine characteristics in aircraft project work.
However, the method seems suitable also for other applications to extend the brochure data of a given engine type beyond available data. Two cases can be mentioned especially:
- correction for atmospheric conditions and flight speeds at a given engine rating,
- extrapolation to higher turbine temperature e.g. for emergency ratings during one-engine out flight conditions. This extrapolation is shown in Fig. 7 a - e for conditions beyond the design-point.

Furthermore, the examples given in section 7 are for a constant shaft-power speed only ($N_2 = 100\%$).
All calculations for the described method can easily be performed with a small electronic pocket-calculator at the designers' desk.

9. References

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Appendix: Tables (additional to section 7)

	air	gas	units
$\gamma = \frac{c_p}{c_v}$	1.4	1,333	-
c_p	1005	1147	$m^2 s^{-2} K^{-1}$
$R = c_p - c_v$	287.14	286.53	$m^2 s^{-2} K^{-1}$

Table I: Standard values for air and combustion gases.

Engine brochure:

- Shaft horse power : P_{br} = 305 hp = 227.44 kW
- Fuel flow : m_f = 94,35 kg/h
- Gross thrust : T_g = 142.4 N
- Air flow : m_a = 1.42 kg/s
- Compressor discharge total pressure: $p_t = 6.23\ 10^5\ N/m^2$
- Compressor discharge total temperature: T_{t_3} = 556 K
- Gas producer turbine outlet temperature: T_{t_5} = 1008 K
- Gas producer turbine speed: N_1 = 100%
- Power turbine speed: N_2 = 100%

Derived data:

- Compressor pressure ratio : ε_c = 6,15
- Compressor efficiency : η_c = 0,79
- Compressor turbine entry temperature: T_{t_4} = 1245 K
- Power turbine, outlet temperature : T_{t_6} = 861 K
- Exhaust jet velocity : V_7 = 100.3 m/s
- Exhaust nozzle pressure ratio : $\frac{p_{t6}}{p_o} = 1.022$
- Exhaust nozzle exit area : $A_7 = 344.8\ 10^{-4}\ m^2$
- Total turbine pressure ratio : $\frac{p_{t4}}{p_{t6}} = 5.717$
- Turbine efficiencies : $\eta_t = \eta_t' = 0,847$
- Pressure ratio, compressor turbine : $\frac{p_{t4}}{p_{t5}} = 2.714$
- Pressure ratio, power turbine : $\frac{p_{t5}}{p_{t_6}} = 2.107$

Assumed data:

- No loss of air flow mass : $m_g = m_a$
- Combustion chamber pressure ratio : $\frac{p_{t4}}{p_{t3}} = 0,95$
- Mechanical efficiency, compressor turbine: η_m = 0,99
- Mechanical efficiency, power turbine : η_m = 0,95

Table II: Design-point data of Allison Model T63-A-5 for sea level static conditions ($p_o = 1.01325\ 10^5\ N/m^2$, T_o = 288 K).

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\pi^* = \dfrac{p_{t_5}}{p_{t_6}}$	$\dfrac{p_{t_4}}{p_{t_5}} \Big/ \left(\dfrac{p_{t_4}}{p_{t_5}}\right)_{des}$	$\dfrac{p_{t_4}}{p_{t_5}}$	ε_c	η_c	$\dfrac{T_{t_5}}{T_{t_4}}$	φ	$\dfrac{m_a \sqrt{T_{t_2}}/p_{t_2}}{(m_a \sqrt{T_{t_2}}/p_{t_2})_{des}}$
assumed	eq. (6.6)	-	eq. (6.8)	Fig.6	eq. (2.6)	eq. (6.9)	eq. (6.12)
2.5	1.009	2.738	7.363	0.790	0.808	1.141	1.121
2.3	1.007	2.733	6.760	0.790	0.808	1.074	1.061
2.107*)	1.000	2.714	6.150	0.790	0.809	1.000	1.000
2.0	0.993	2.695	5.797	0.790	0.810	0.968	0.958
1.9	0.983	2.668	5.452	0.790	0.812	0.932	0.918
1.7	0.952	2.584	4.724	0.788	0.818	0.860	0.828
1.5	0.982	2.421	3.905	0.780	0.829	0.773	0.722
1.3	0.733	2.048	2.933	0.744	0.855	0.730	0.558

*) design point

Table III: Off-design performance calculation at different engine ratings.

(9)	(10)	(11)	(12)	(13)	(14)
$\pi^* = \dfrac{T_{t_5}}{T_{t_6}}$	$\dfrac{T_{t_5}}{T_{t_6}}$	T_{t_4} (K)	T_{t_5} (K)	P_{br} (kW)	m_a (kg/s)
-	eq. (2.8)	-	-	eq. (2.7)	-
2.5	0.824	1420.5	1147.8	350	1.59
2.3	0.838	1337.1	1080.4	288	1.51
2.107*)	0.854	1245.0	1007.2	227.5	1.42
1.9	0.873	1160.3	942.2	169.5	1.30
1.7	0.894	1070.7	875.8	119	1.18
1.5	0.918	962.4	797.8	73	1.03
1.3	0.946	908.9	777.1	36	0.79

*) design point

Table III: Continued.

(15)	(16)	(17)	(18)	(19)	(20)	(21)
$\pi^* = \frac{p_{t_5}}{p_{t_6}}$	T_{t_6} (K)	V_7(m/s)	T_g(N)	$\frac{T_{t_3}}{T_{t_2}}$	$\frac{F}{F_{des}}$	F(kg/h)
-	-	eq. (3.9)	-	eq. (2.3)	eq. (2.4)	-
2.5	945.8	123.3	196.0	2.059	1.347	127.0
2.3	905.4	112.1	169.3	1.996	1.174	110.75
2.107*)	860.1	100.2	142.3	1.929	1.000	94.35
1.9	822.5	87.7	114.0	1.847	0.837	79.0
1.7	783.0	75.8	89.4	1.756	0.679	64.1
1.5	735.4	61.9	63.8	1.647	0.511	48.3
1.3	735.1	47.6	37.6	1.512	0.384	36.2

*) design point

Table III: Continued

$\pi^* = \frac{p_{t_5}}{p_{t_6}}$	ε_c	T_o (C)	T_{t_4} (K)	T_{t_5} (K)	P_{br}(kW)	m_a(kg/s)	F(kg/h)
2.5	7.36	-38	1159.1	936.6	316.1	1.76	114.6
2.3	6.76	-13.3	1207.1	975.4	272.1	1.58	105.1
2.107*)	6.15	+15	1245.0	1008.0	227.4	1.42	94.35
2.9	5.10	+34	1285.7	1040.6	204.7	1.32	89.7
1.9	5.45	+54	1316.1	1068.7	180.4	1.22	83.9

*) design point

Table IV: Off-design performance calculation for different ambient temperatures.

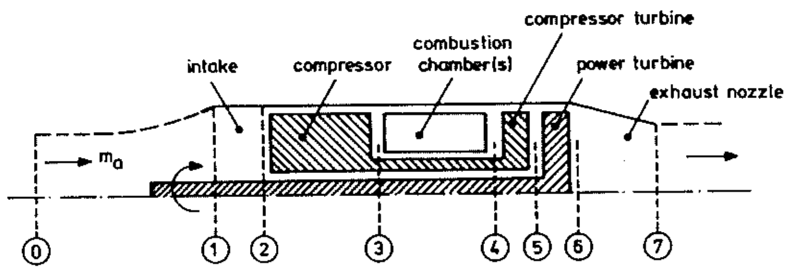

Fig. 1. Shaft-power turbo engine with free turbine.

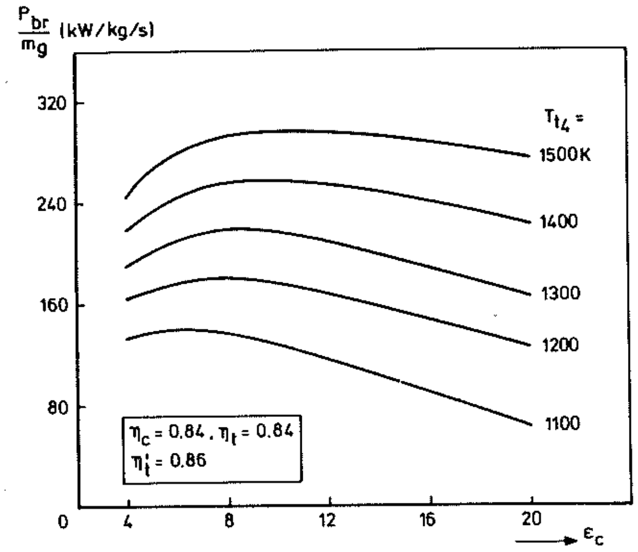

Fig. 2. Design-point performance.

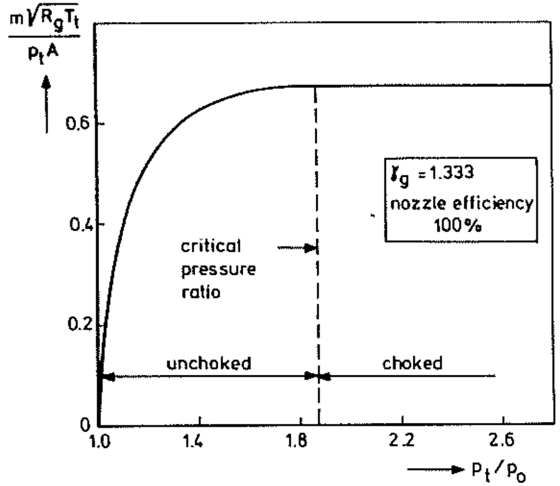

Fig. 3. Characteristic curve for exhaust nozzle.

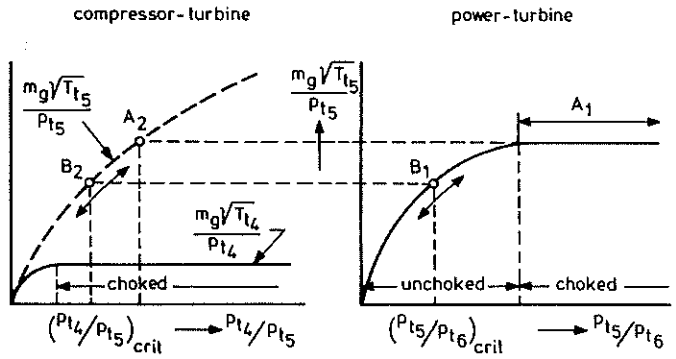

Fig. 4. Matching of the two turbines.

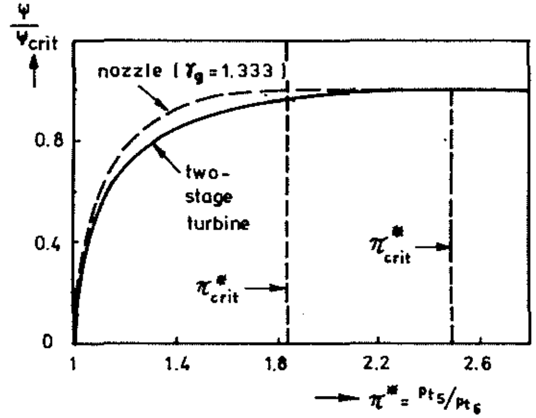

Fig. 5. Turbine mass flow according to Linnecken (π^*_{crit} = 2,5).

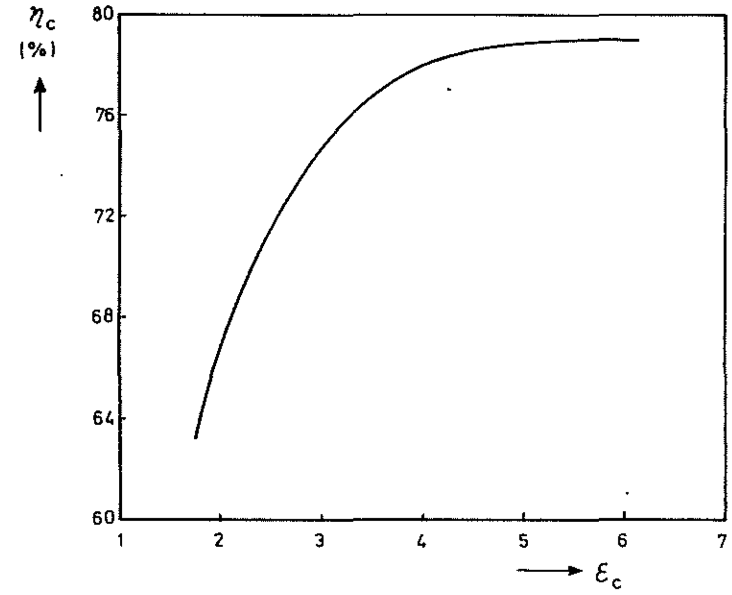

Fig. 6. Compressor efficiency.

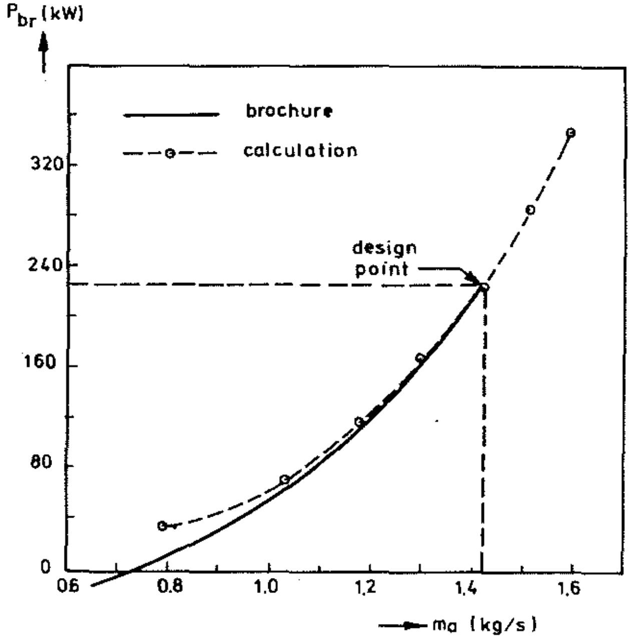

Fig. 7a: Shaft horse power

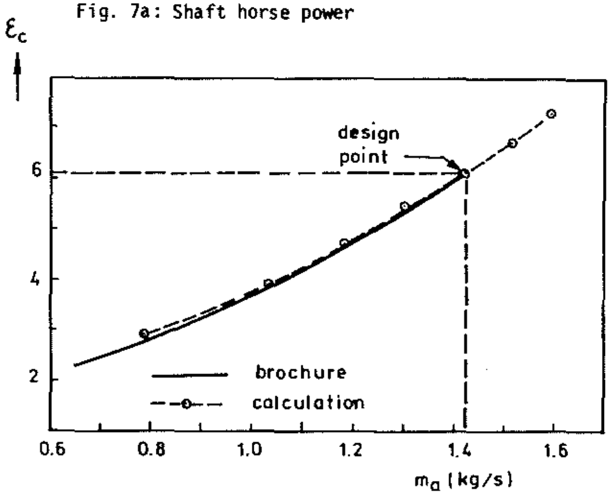

Fig. 7b: Compressor pressure ratio

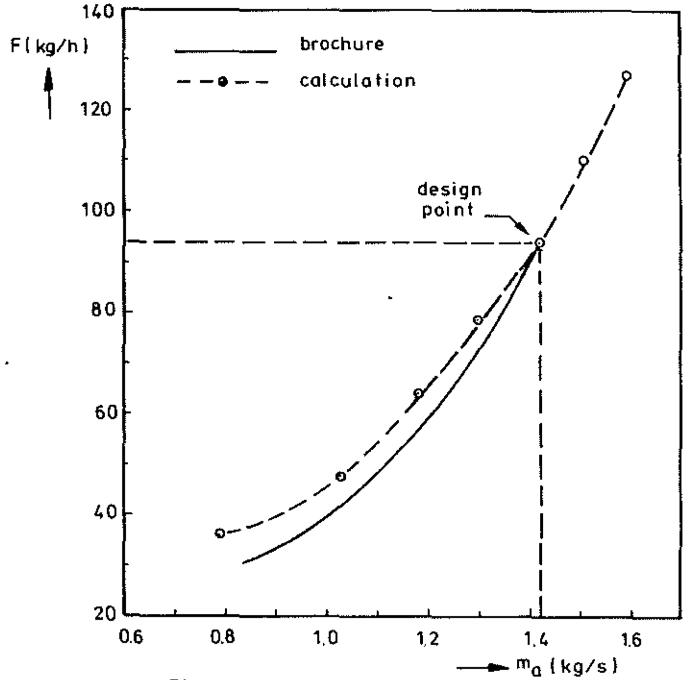

Fig. 7c: Fuel flow

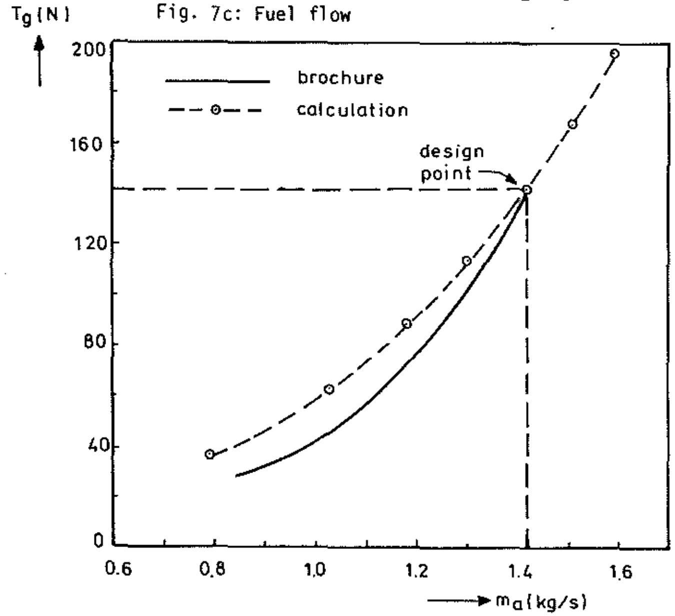

Fig. 7d: Gross thrust

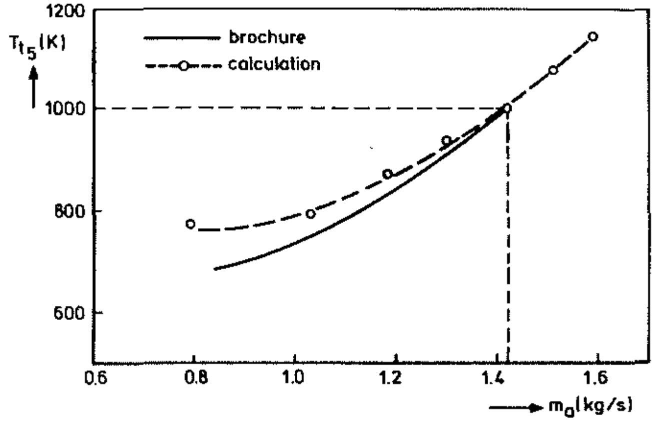

Fig. 7e: Compressor-turbine outlet temperature

Fig. 7. Off-design performance at different engine ratings (sea-level static).

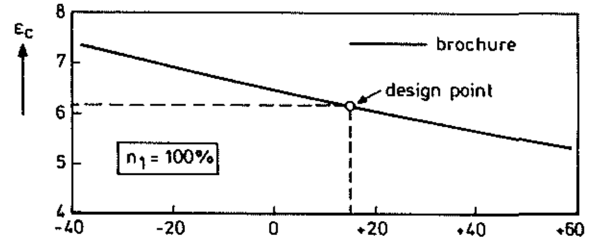

Fig. 8a: Compressor pressure ratio.

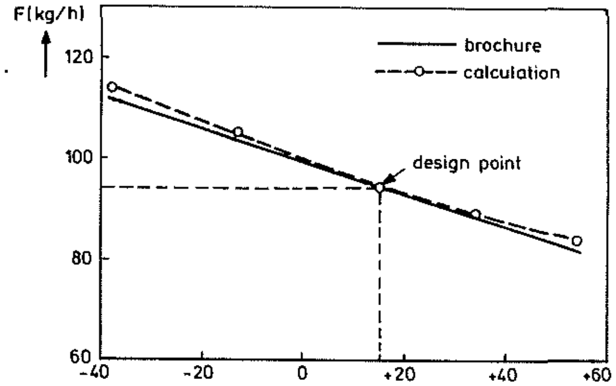

Fig. 8b: Fuel flow

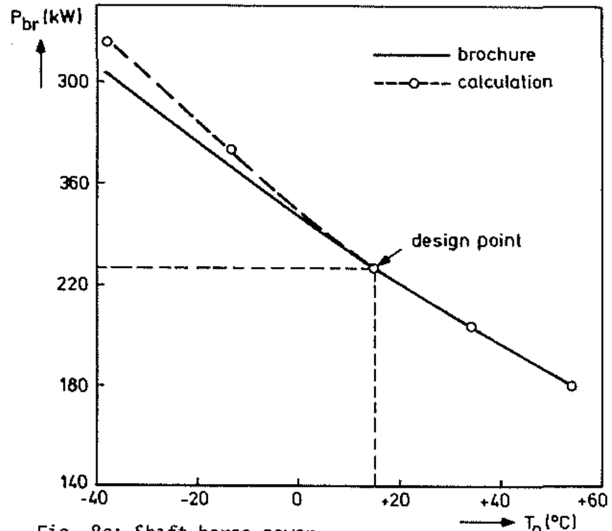

Fig. 8c: Shaft-horse power

Fig. 8. Off-design performance at different ambient temperatures (sea level static).