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HELICOPTER AND TILT-ROTOR INVERSE SIMULATION: METHODS, FEATURES, PROBLEMS AND CURES.

R Bradley,
School of Computing and Mathematical Sciences,
Glasgow Caledonian University,
G4 0BA
UK
r.bradley@gcal.ac.uk

D G Thomson,
Department of Aerospace Engineering,
University of Glasgow,
G12 8QQ
UK
douglast@aero.gla.ac.uk

Nomenclature

x,u	*state and control vectors*
y	*output vector*
T	*thrust*
β, θ	*disc tilt and vehicle pitch angles*
q	*pitch rate*
I_{yy}	*pitch moment of inertia*
l	*nominal shaft length*
t	*waypoint interval*
dt	*integration step*
n	*number of steps*
A, B	*system and control matrices*
C	*output matrix*
P, Q	*discrete system and control matrices*
P^*, Q^*	*discrete waypoint system and control matrices*
$P^\dagger$	*waypoint transition matrix*
λ, μ	*continuous and discrete eigenvalues*
Γ	*normalised torque*
ψ, r	*heading angle, yaw rate*
ψ_0, r_0	*initial heading angle, yaw rate*
$()_s$	*two time-scale quantity*
$()_d$	*demanded quantity*
I_{xx}, I_{zz}	*Moments of inertia*
η	*control deflection*
M_η	*control derivative*
T_y	*power plant pitching moment*
Λ	*grouped aerodynamics*
τ	*actuator time constant*
$\overline{D}$	*output control matrix*

Abstract

This paper describes the development of the technique of rotorcraft inverse simulation since the first implementation of it as a practical research tool by Thomson. It presents an explanation of some of the difficulties that have been encountered by researchers during the various extensions of the original 'pure' method to cater for more advanced modelling environments. The important properties of these method can be derived from a consideration of rudimentary helicopter with basic dynamics. The identification of the source of the difficulties can lead to a satisfactory modification of the method to eliminate unwanted effects. The challenge of resolving the trade-off between the elimination of these unwanted effects and the retention of those which are beneficial is discussed.

Introduction

Inverse simulation is a method of predicting the control actions that are needed to pilot a helicopter through a given manoeuvre. It has, therefore, particular value in providing an initial indication of the performance capabilities of a vehicle design and, for example, in evaluating the effect on practical manoeuvres of any anticipated configurational change: such as, for example, the effect of vertical tail size on the performance of the side-step manoeuvre. Many of the standard manoeuvres (MTEs) described in ADS33 [1] have been translated into a form where they can be simulated in order to assess a vehicle's capability and the pilot's general control strategy. It has also been useful in the design of such evaluative manoeuvres [2]. The method has also been used to investigate handling qualities and pilot workload [3] and developments of the basic method and their applications have been widely reported [e.g. 4, 5, 6].

The initial development of rotorcraft inverse simulation as a practical research tool by Thomson [7] demonstrated the benefits of the inverse approach. Its obvious potential encouraged other researchers to modify the basic method in order to overcome some of its implementation difficulties and to avoid what may be perceived as some of its unwanted properties. There is a need, therefore, to understand the problems of 'pure' inverse simulation and to appreciate how alternative techniques deal with them and, further, to recognise the disadvantages they, in turn, introduce. It is then possible for new researchers to decide what method is most appropriate for their investigations.

In the sections to follow, the essential characteristics of pure inverse simulation are discussed first and the origins of those aspects of the method that are regarded as disadvantages are identified. The method of integration due to Hess [4] is discussed next. The integration method solves some of the implementation difficulties of the 'pure' method but at the expense of being prone to an unusual instability - the probable source of which is identified. An analysis of the two time-scale method of Avanzini [5] follows. This method is quite different to the two previous methods in its approach and it sacrifices some important properties of inverse simulation which may be crucial in certain investigations. Finally, the method of Nonlinear Dynamic Inversion (NDI), developed by Smith [8], is discussed in the context of inverse simulation even though its initial application was in aircraft control. It turns out to be directly relevant to the issues discussed in this paper.

The important properties - whether desirable or undesirable - of each of the methods are remarkably robust and can be explained using a rudimentary helicopter model [9]. The impact on more realistic models is demonstrated through the properties of a linearisation of a state-of-the-art flight mechanics simulation model and, where appropriate, responses of the full, non-linear simulation.

Inverse simulation.

The starting point for the 'pure' inverse simulation is a simulation model of the rotorcraft expressed as a system of first order differential equations with a state vector *x* and control vector *u*. The dimension of x depends on the sophistication of the model. For example, it may be simply a 6 DOF model with an actuator disc rotor representation or the blades may be modelled individually with a finite-state dynamic inflow representation. Similarly the dimension of the control vector depends on the configuration of the vehicle. For a conventional helicopter there are four controls but for a tilt-rotor in helicopter mode there may be five. The simulation model is written, therefore,

$$\dot{x} = f(x,u)\,,$$

and it is required to follow a prescribed manoeuvre defined by an output or constraint equation on the state vector of the form

$$y = g(x)\,,$$

where *y* is a time dependent description of the flight path. For a conventional helicopter the dimension of *y* is four so that in a typical application the earth referenced velocities and heading angle (or side slip angle) are specified. For a tilt-rotor an additional constraint, such as the angle of roll, may be applied. (We restrict the discussion in this paper to the case where the dimension of the control vector is the same as that of the output vector.) The inverse problem is to find the control u, as a function of time, which ensures that the flight path constraint is satisfied and the prescribed manoeuvre flown. In order to achieve this it is necessary to differentiate the constraint equation to obtain

$$\dot{y} = \frac{\partial f}{\partial x}\dot{x} = \frac{\partial f}{\partial x}f(x,u)\,,$$

which equation may, it appears, be solved for *u* in terms of *x* and the rate of change of *y*. Usually, this is not the case and further differentiations are required to bring in sufficient knowledge of *u*. What is happening here is that the direct consequences of control activity for the system are being established. For example, vehicle accelerations rather than velocities are typically the direct result of control activity. After the required manipulation the final relationship may be written

$$\bar{y} = h(x,u),$$

where $\bar{y}$ is the differentiated constraint vector made up of the requisite number of differentiations of the components of the original output y. This equation may be solved to give an explicit expression for the control vector

$$u = g(x,\bar{y}),$$

where $\bar{y}$ involves the rate of change of y and higher derivatives. With this control incorporated into the simulation equation the inverse system is described by

$$\dot{x} = f(x, g(x,\bar{y})).$$

The simulation is driven by the manoeuvre as described by the vector $\bar{y}$. Solving for u in this manner is not a trivial exercise for a simulation model of any practical benefit. The technique originally developed by Thomson relied on considerable algebraic manipulation of the helicopter equations of motion in order to recast them in an inverse form, finally incorporating a numerical procedure for finding the pitch and roll attitude angles. The task of recasting, analytically, a simulation into inverse form becomes progressively more difficult as the sophistication of the underlying flight mechanics model increases. This situation is a genuine limiting factor of the application of 'pure' inverse simulation.

Another issue is that the dynamics of the constrained system above are quite different to the 'controls-fixed' properties of the original [10]. Typical responses of an inverse simulation of a generic helicopter model configured to resemble a Westland Lynx [11] carrying out an accel-decel (or longitudinal repositioning manoeuvre), of 10 seconds duration and a maximum horizontal velocity of 35 kt, are shown in Fig. 1.

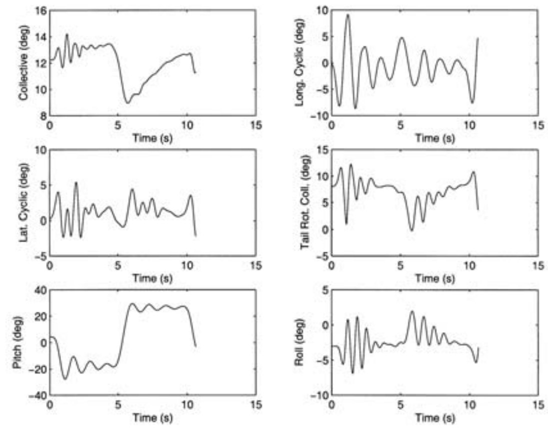

Figure 1. Inverse simulation responses for an accel-decel manoeuvre.

Clearly observed on the control and attitude responses are lightly damped oscillations which are not associated with the free modes of the aircraft. The phenomenon is easy to explain us when the inverse of the VSH model (Fig. 2) is constructed.

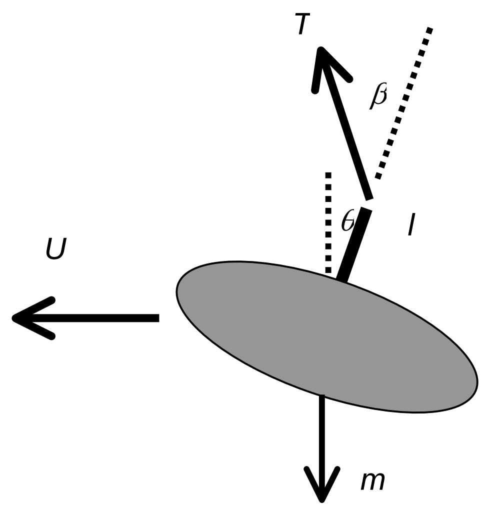

Figure 2. The VSH model

This rudimentary vehicle is controlled by a thrust T, representing the rotor thrust, which is variable in magnitude and direction. The equations for longitudinal motion from Ref [9] with β and θ small may be written

$$\dot{U} = g(\beta - \theta)$$

$$\dot{q} = -\frac{mgl}{I_{yy}}\beta$$

$$\dot{\theta} = q$$

where U is the horizontal velocity, θ is the pitch attitude and q the pitch rate. The angle β is the rotor disc tilt relative to the shaft. The parameters m, I_{yy} and l are the vehicle mass, moment of inertia in the pitch axis and shaft length respectively. With no vertical motion the thrust T is equal to the weight mg. The manoeuvre is defined by specifying the horizontal velocity - as, for example, in an acceleration-deceleration manoeuvre.

$$y = U$$

Following the procedure above this output equation is differentiated to give

$$\dot{y} = \dot{U} = g(\beta - \theta)$$

so that the required control is readily determined as

$$\beta = \theta + \dot{y}/g\,.$$

Given the desired profile for y this input does, indeed, achieve the required manoeuvre. Its effect on the rotational dynamics is expressed by

$$\ddot{\theta} + \frac{mgl}{I_{yy}}\theta = -\frac{mgl}{I_{yy}}\dot{y}/g\,.$$

This equation reveals a persistent oscillatory behaviour driven by the vehicle acceleration. It is clear that this arises because the rotor tilt which is being used to control the acceleration of the vehicle also affects the rotational dynamics. The same analysis applies to the roll axis in lateral manoeuvring. This behaviour is evident in practical flight mechanics models. Table 1 compares the control -fixed modes of a generic helicopter model populated with UH60 data in trimmed flight at 20 kt with those of the system constrained in inertial velocities and heading.

Table 1 Eigenvalues of free and constrained modes (conventional helicopter).

Free (controls fixed)	**Constrained**
-4.3304	-0.3886 ± 5.4897i
-1.5983	-0.1803 ± 2.4075i
-0.2349 ± 0.8195i	0.0000
-0.1443 ± 0.4947i	0.0000
-0.4730	0.0000
-0.1231	0.0000
0.0000	0.0000

The table confirms the significantly modified dynamics of the inverse system. In the constrained case, the non-zero eigenvalues repesent oscillatory modes in pitch and roll of 1.1 sec and 2.61 sec respectively. The additional four zero eigenvalues are associated with the four constraints (There is a zero eigenvalue in the free case associated with the heading angle). The effect on control responses is illustrated in Figure 3 where the simulation model is pressed through a 25 second triple jinking manoeuvre (a slalom path, but with the heading along the centre line). Oscillations, of period approximately one second are clearly visible - and importantly persist well beyond the end of the manoeuvre at 30 seconds. It is this phenomenon which is often felt to be unrealistic control activity.

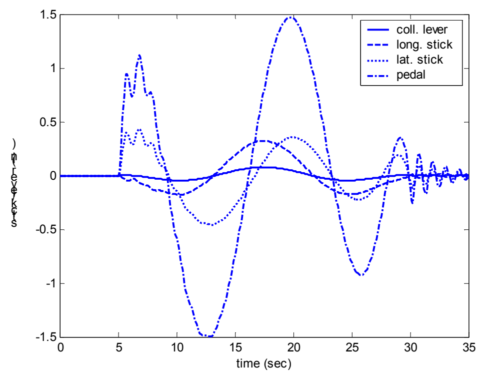

Figure 3. Control responses for triple jink manoeuvre

The constrained modes correspond to lateral and longitudinal oscillations with the vehicle rocking about the centre of mass together with four additional zeros denoting the four constraints. For a tilt rotor configuration where the bank angle can additionally be constrained results solely in a pitching instability as shown in Table 2 and an additional zero eigenvalue.

Table 2. Free and constrained modes (tilt rotor).

Free (controls fixed)	**Constrained**
0	0
-1.2497	-0.1772 ± 2.2590i
-0.7271	0.0000
0.1905 ± 0.3506i	0.0000
0.1291 ± 0.4047i	0.0000
-0.4339	0.0000
-0.1612	0.0000
	0.0000

In a practical application these oscillations which are attributed to the zero dynamics of the constrained system may be minimised by the careful selection of smooth flight path profiles. Nevertheless these dynamics are still present and are a direct result of the inverse formulation. They are a feature (not a problem) of inverse dynamics and can provide useful information about control strategy [12]. If they are unacceptable in a particular application then it is not 'pure' inverse simulation that should be adopted.

Method of Integration.

Because of the need to implement inverse simulation by analytic or numeric differentiation it is often termed the *differentiation* method of inverse simulation. As discussed above, the tractability of this approach reduces as model sophistication increases. In order to overcome this problem and to employ a simulation model in a more convenient forward simulation, the method of integration (or waypoint method), as it is called, integrates the simulation over a chosen interval (here called the waypoint interval) with the controls set at the values which result in the desired flight path being achieved at the end. The values for the controls are not known in advance, of course, and are usually found through an iterative scheme based on Newton-Raphson. Formally the method can be expressed via

$$x_t = x_0 + \int_0^t f(x, u_0)\,dt$$

$$y_t = g(x_t)$$

from which the required control u_0 may be found. It is essential that the waypoint interval is sufficiently large as to allow control activity to be adequately detectable in the output variables. Otherwise changes in u_0 lose significance and the algorithm will fail. What is not often recognised is that the essence of the zero dynamics is retained in this method. The VSH can demonstrate this property quite simply. If the control β_0 is made constant through the interval [0,t] then the pitch rate may be written

$$q = q_0 - \frac{mgl}{I_{yy}}\beta t$$

In turn, the pitch attitude becomes

$$\theta = \theta_0 + q_0 t - \frac{mgl}{I_{yy}}\beta\frac{t^2}{2}$$

and finally, the horizontal velocity at time t may be written.

$$U = U_0 + g(\beta t - (\theta_0 t + q_0\frac{t^2}{2} - \frac{mgl}{I_{yy}}\beta\frac{t^3}{6})).$$

In order to examine the zero dynamics, U and U_0 are set to zero and the transition matrix over the waypoint interval is

$$\begin{bmatrix} q \\ \theta \end{bmatrix} = \begin{bmatrix} 1 & -\frac{mgl}{I_{yy}}t \\ t & 1 \end{bmatrix}\begin{bmatrix} q_0 \\ \theta_0 \end{bmatrix}$$

to first order in t. The eigenvalues of the transition matrix are

$$\lambda = 1 \pm i\sqrt{\frac{mgl}{I_{yy}}}t$$

and recall that $\exp(\pm i\omega t) \approx 1 \pm i\omega t$ to regain the familiar frequency of the zero dynamics.

It is possible that some control of the zero dynamics can be maintained by a judicious choice of the waypoint interval. This aspect may be investigated via a linear, time independent, model with system and control matrices A and B respectively. Consider a constraint step of t starting at time zero with state vector x_0 and ending with x_n at $t_n = t$ after n integration steps. The required output can be set at zero since we are investigating stability so we have:

$$\dot{x} = Ax + Bu_0$$

where u_0 denotes the value of the control vector, held constant over the constraint step, and

$$Cx_n = 0.$$

Indexing quantities by the intermediate integration steps 1 to n, ($nh=t$), the transition of the state vector from time t_r to t_{r+1} may be written:

$$x_{r+1}=Px_r+Qu_0$$

where u_0 is to be determined, and where the particular form of the transition matrices P and Q depend on the numerical method chosen for the integration step.
Over the whole interval t the stepping process may be written

$$x_n=P^*x_0+Q^*u_0$$

where

$$P^*=P^n$$

and

$$Q^*=(I+P+...+P^{n-1})Q.$$

At the end of the constrained step the requirement is that

$$Cx_n= CP^*x_0+CQ^*u_0=0.$$

This condition may be achieved by selecting

$$u_0 = -(CQ^*)^{-1}C * P * x_0$$

provided det$(CQ^*)\neq0$ – which should indeed be the case since that is the whole purpose behind the integration approach. The robustness of the solution and some guidance on the choice of the interval t can be interpreted by inspecting the condition number of CQ^*. Figure 4 shows the variation of the condition number with waypoint interval for the UH60 configured model introduced above. The basic integration (explicit method) step, dt, is approximately 15° of rotor revolution and the waypoint interval is varied up to four rotor revolutions. On the evidence of Figure 4, a waypoint interval greater than one rotor revolution should be employed for this helicopter.

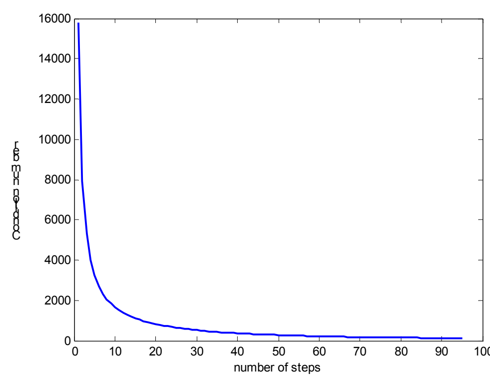

Figure 4. Condition number with waypoint interval

Further, the equivalence of the constrained dynamics should be revealed by the eigenvalues of the transition matrix, $P^\dagger$ for a single constraint step:

$$P^\dagger=P^*+Q^*(CQ)^{-1}C^*P^*.$$

Figure 5 shows the migration of the eigenvalues of the constrained system for the UH60 configuration introduced above as the waypoint interval is varied. The eigenvalues, μ, of the discrete system are converted to an equivalent continuous form, λ, through:

$$\mu = \exp(n\lambda dt),$$

recast as:

$$\lambda = \frac{1}{ndt}\left(\log(abs(\mu) + i\arg(\mu)\right).$$

It is clear that the constrained dynamics persist and retain their oscillatory character well beyond what would be a limit of constant control position, eventually becoming unstable. Certainly, for this model there appears to be no possibility of significantly modifying the oscillatory behaviour.

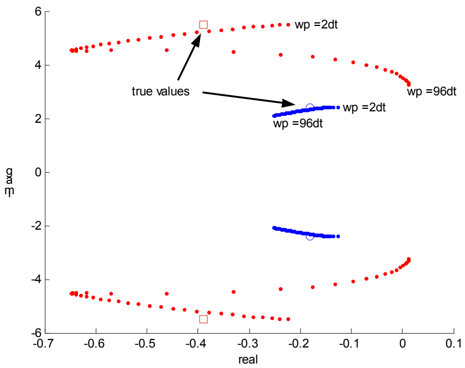

Figure 5 Migration of eigenvalues with waypoint interval (wp).

The method of integration therefore satisfactorily solves the problem of tractability and retains the features of constrained dynamics. It does, unfortunately, introduce a new problem.

Waypoint Instability.

The method can suffer from what appears to be a weak numerical instability [4] with an oscillation of increasing amplitude and period twice the waypoint interval superimposed on the responses. This phenomenon is illustrated in Fig. 6 for the waypoint method applied to the Lynx model of Fig.1 carrying out a 30m pop-up over a distance of 500m at a forward speed of 80kt. The integration time step is 0.04 sec and the waypoint interval is 0.16 sec

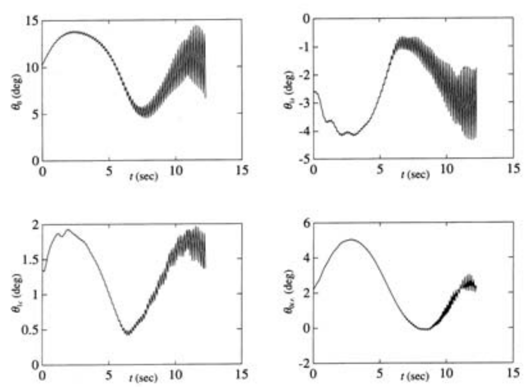

Figure 6. Responses in pop-up manoeuvre using waypoint method.

Again, the origin of this behaviour is simply illustrated. The VSH can be imbued with simple yaw dynamics of the form

$$\dot{r} = \Gamma$$

$$\dot{\psi} = r$$

where Γ is a normalised controlling torque. At the end of the interval [0,t] we have

$$\begin{bmatrix} r \\ \psi \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ t & 1 \end{bmatrix} \begin{bmatrix} r_0 \\ \psi_0 \end{bmatrix} + \begin{bmatrix} t \\ t^2/2 \end{bmatrix} \Gamma$$

Following the earlier procedure, the heading is set to zero at the beginning and end of the waypoint interval the yaw rate satisfies

$$r = -r_0$$

That is, the yaw rate changes sign at each way point resulting in marginal stability. In practice, the aerodynamic and other effects can make a practical simulation either slightly unstable or stable. Table 3 shows the eigenvalues of the transition matrix of the UH60 model for a waypoint step of approximately one rotor revolution. The values indexed 2,3 and 4,5 correspond to the oscillations of the constrained system and feature in Fig. 3 above. The eigenvalue with index 1 corresponds to the waypoint instability. In this case its magnitude is greater than one so it is unstable.

Table 3. Eigenvalues of transition matrix (one rotor rev)

index	eigenvalue (discrete system)
1	-1.0259
2,3	0.8187 ± 0.5064i
4,5	0.3296 ± 0.8426i
6-9	0.0

The responses from a simulation of the waypoint method for the UH60 for the same conditions are shown in Figure 4 and the initial state is proportional to the corresponding eigenvector. The constrained variables are the inertial velocities and the heading angle These do, indeed, come to zero at the waypoints but there are increasing excursions in between. If the responses were only plotted at the way points the instability would not be observed in the constrained variables but the other states would display an alternating sign and increasing magnitude.

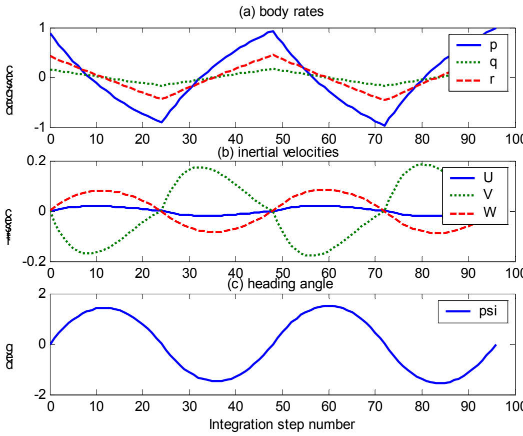

Figure 7. Detailed responses for integration method

This behaviour does not feature in the pure inverse method because of the differentiations present in its formulation so that it is the second derivative of ψ that is prescribed. Similarly Rutherford [6] notes that the integration method does not display the waypoint instability when accelerations are specified at the waypoints. This observation reinforces the analysis above.

In view of the vehicle's independent control of roll angle it is interesting to examine the eigenvalues of a tilt rotor vehicle. Table 4 shows the eigenvalues of a waypoint transition matrix for a 6 DOF linearisation of a generic tilt rotor model configured to resemble a raw XV15. The way point interval corresponds to a single rotor revolution.

Table 4. Eigenvalues of waypoint instability for tilt rotor.

index	**eigenvalue (discrete system)**
1,2	0.9261± 0.3197i
3	-1.0955
4	-1.0764
5-9	0.0

The eigenvalues with indices 1 and 2 are a complex pair corresponding to the constrained oscillations. Since their magnitude exceeds unity the mode is unstable. The eigenvalues with indices 3 and 4 are both waypoint related since they are characteristically close to -1 in value. There are two such eigenvalues because for the tilt rotor configuration the additional constraint on the roll angle acts in the same way as that on the heading angle. Neither has a natural direct aerodynamic restoring force.

Despite its high frequency, the waypoint instability is related to the rigid body dynamics. A full, 30 state, time dependent linearisation displays the same characteristic behaviour, as shown in Fig 8 which shows the responses with an initial state composed of a combination of the two waypoint eigenvectors.

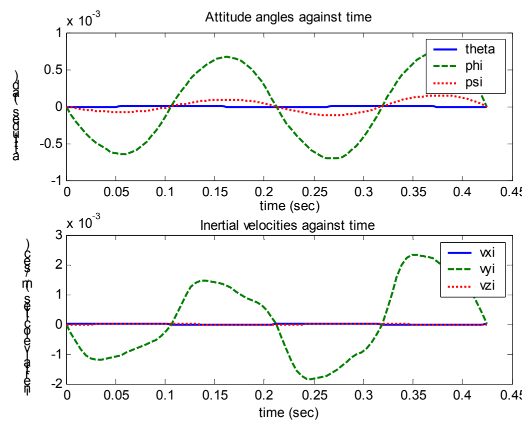

Figure 8. Responses of a tilt rotor vehicle illustrating waypointing instability.

Further analysis reveals that the precise values of the waypoint-related eigenvalues are dependent on the numerical method used to perform the simulation. This is due to the high frequency of this mode which is sufficient to influence significantly the truncation error terms.

This instability then, is a genuine problem of the waypoint method. That is, the waypoint instability is an artefact of the waypoint method. Fortunately, it is easily solved. If two waypoint steps are performed and then an intermediate value calculated via a weighted average then the oscillation is eliminated. The simple yaw dynamics analysed above for the VSH become:

$$\left.\begin{array}{c} r_0 \\ r_1 = -r_0 \\ r_2 = r_0 \end{array}\right\} \to r_1{}^* = \tfrac{1}{4}r_0 + \tfrac{1}{2}r_1 + \tfrac{1}{4}r_2 = 0 .$$

This procedure should be incorporated formally in the normal integration method. It involves two steps of the usual scheme to make a single averaged step so twice as much effort is involved but it completely eliminates the waypoint instability, while retaining the characteristics of the other dynamics. Table 5 shows the discrete eigenvalues for the averaged method and should be compared with Table 4. The eigenvalues with indices 1 and 2 are virtually preserved, those with indices 3 and 4 are virtually eliminated as required since they related to the waypoint mode. The remainder become 0.25, replacing 0.0 which simply represents the fact that departure from reference values are not eliminated in a single step but rapidly decay.

Table 5. Eigenvalues of the averaged system.

index	**eigenvalue (discrete averaged system)**
1,2	0.9019 ± 0.3079i
3	0.0023
4	0.0015
5-9	0.2500

If a fourth order integration scheme is used for the integration then the simple three-point averaging above reduces the order for the combined scheme. Other averaging schemes may be developed to give higher order but naturally the computational effort is increased.

Two time-scale method

The two time-scale method developed by Avanzini [5] exploits the fact that the rotational dynamics of the vehicle are much faster than the translational dynamics so that during a realistic manoeuvre the latter may be considered in equilibrium with the angular velocity zero. The lateral dynamics are solved with the attitude angles acting as pseudo controls and from knowledge of the attitude angles the angular velocity can be determined to provide, finally, the rotational control actions.

The procedure is again easily demonstrated using the VSH which has its lateral dynamics expressed in earth coordinates - an essential prerequisite. Firstly, the pitch rate q is set to zero which, from

$$\dot{q} = -\frac{mgl}{I_{yy}}\beta$$

implies that the control β is also zero. Using the suffix s to denote 'slow' quantities

$$\dot{U} = g(\beta - \theta)$$

becomes

$$\dot{U}_s = -g\theta_s$$

so that knowledge of the longitudinal acceleration provide the angle of pitch θ_s. From

$$\dot{\theta}_s = q_s$$

and

$$\dot{q}_s = -\frac{mgl}{I_{yy}}\beta ,$$

the control is finally calculated as

$$\beta = -\frac{I_{yy}}{mgl}\ddot{\theta}_s = \frac{I_{yy}}{mgl}\frac{\dddot{U}_s}{g}$$

This applied to the simulation model to calculate the final vehicle responses.

$$\dot{U} = \frac{I_{yy}}{mgl}\frac{d}{dt}\left(\ddot{U}_s + \frac{mgl}{I}U_s\right)$$

It is clear that the longitudinal velocity calculated in this manner can vary from the desired U_s . In fact, the right hand side has a zero at the frequency of the zero dynamics, so that the component of the manoeuvre velocity at that frequency is ignored by the two time-scale method. Fig. 9 shows the velocity profile of a triple jink manoeuvre for a UH60 type of vehicle. This is the same manoeuvre as in Fig.3 but here there is an absence of oscillatory behaviour in the control activity.

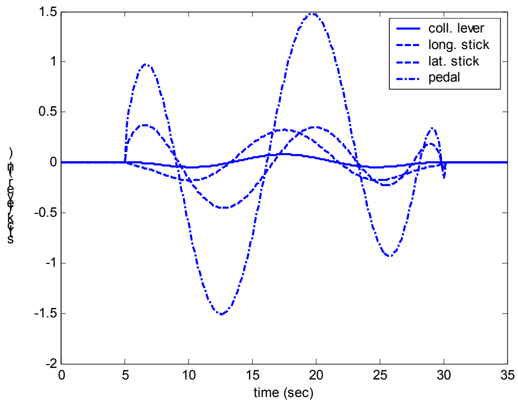

Figure 9. Control actions for triple jink manoeuvre (two time-scale method)

For this relatively gentle manoeuvre the specified trajectory is flown accurately, as shown by the lateral velocities in Fig 10.

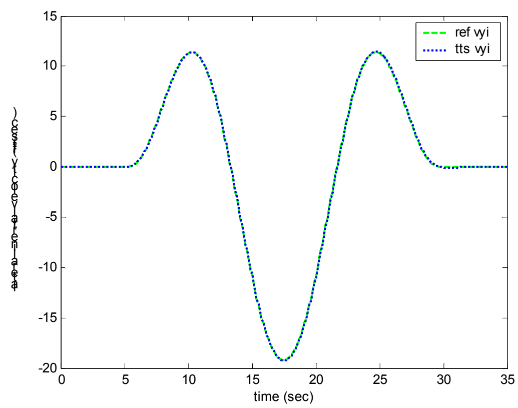

Figure 10. Lateral velocities for triple jink manoeuvre (two time-scale method)

When the time interval of the manoeuvre is reduced form 35 sec to 5 sec the required trajectory is beginning to be followed less well, as shown in Fig. 11, due to the input characteristics discussed above.

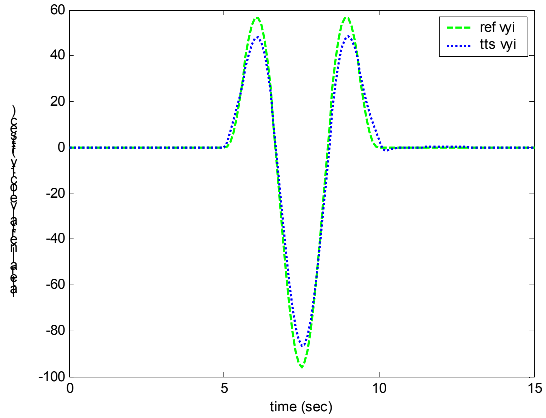

Figure 11. Lateral velocities for fast triple jink manoeuvre (two time-scale method)

The aggressiveness of the manoeuvre has been selected to illustrate the departure form the required flight path that can result from the two time-scale method. This reflects the trade-off in the two time-scale method. The conventional zero dynamics are entirely eliminated by using the attitude angles as pseudo controls but at the cost of poor representation of aggressive manoeuvres. The zero on the input simply ignores that component of the manoeuvre - but ‘pure’ inverse simulation must achieve the manoeuvre precisely and so increases its control activity to counteract the attenuating effect of the input dynamics. This trade-off is for the simulation engineer to assess in each particular situation.

A final comment on the two time-scale approach is that it does require some manipulation of the simulation model in order to implement it. It does not appear to be as complex a requirement as is required for the pure inverse method but it is a factor to be taken into account. For example it is necessary to formulate the translational dynamics in Earth-referenced velocities because the body referenced quantities vary with the speed of the rotational dynamics. Some computational time may be saved by this method because the translational dynamics, being slower, can be solved accurately on a coarser time grid

<u>Nonlinear Dynamic Inversion.</u>
A method of control law design pioneered by Smith [8], initially for fixed wing aircraft, is the technique of Nonlinear Dynamic Inversion (NDI). Its application to rotorcraft has recently been described by Howitt [13] and, following this author, the method is introduced by considering the instantaneous pitching dynamics of a fixed wing aircraft.

$$I_{yy}\dot{q} = (I_{zz} - I_{xx})rp + I_{xz}(r^2 - p^2) + \Lambda + M_\eta \eta + T_y$$

In this equation, $\dot{q}$ is the pitch rate, η is the control surface deflection, T_y is the moment from the power plant and Λ represents grouped aerodynamic terms. The remainder of the quantities are the standard inertial terms. If, at this instant, there is a demanded pitch rate $\dot{q}_d$ which requires a control deflection η_d then, accordingly,

$$I_{yy}\dot{q}_d = (I_{zz} - I_{xx})rp + I_{xz}(r^2 - p^2) + \Lambda + M_\eta \eta_d + T_y \; .$$

Subtraction eliminates the nonlinear terms (the heart of the technique) and leads to a relationship between the error in the current pitch rate and the required correction to the control.

$$(\eta_d - \eta) = (\dot{q}_d - \dot{q})\frac{I_{yy}}{M_\eta} \; .$$

This correction, in the control context , may be applied, for example, through the series actuator of the automatic flight control system. The occurrence of η linearly in the pitch rate equation is important in this development and will need to be considered when the application to the general case is discussed but first NDI is applied to the VSH.

In this case the demanded horizontal acceleration is given by:

$$\dot{U}_d = g(\beta_d - \theta) \, ,$$

where β_d is the required control input. Subtracting the instantaneous acceleration gives the control correction required.

$$(\beta_d - \beta) = (\dot{U}_d - \dot{U})/g \; .$$

In order to implement this control, the VSH needs to be equipped with an actuator, assumed to be of time constant τ.

$$\tau\dot{\beta} = (\beta - \beta_d) \; .$$

When combined with the VSH equations above, the modes have eigenvalues which satisfy

$$\lambda^3 + \frac{1}{\tau}\lambda^2 + \frac{mgl}{I_{yy}\tau} = 0 \; .$$

For fast actuation this gives eigenvalues $-1/\tau$, and $\pm i\sqrt{mgl / I_{yy}}$ approximately which represent the actuation and constrained dynamics respectively.

The output equation for the general case is not linear in the controls and it is necessary to consider the approximate form

$$\bar{y} = h(x, u_0) + \frac{\partial h}{\partial u}(u - u_0)$$

where u_0 is a reference control position - typically for a reference trim state. The required reference output $\bar{y}_{ref}$ is obtained from the control u_{ref} where:

$$\bar{y}_{ref} = h(x, u_0) + \frac{\partial h}{\partial u}(u_{ref} - u_0) \, .$$

Subtracting again gives a relationship between the control vector and the error in the output.

$$(\bar{y}_{ref} - \bar{y}) = \frac{\partial h}{\partial u}(u_{ref} - u) = \overline{D}(u_{ref} - u) \, ,$$

from which the update to the controls may be derived in terms of the error in the output and the inverse of the Jacobian control matrix $\overline{D}$ (in the terminology of Bradley [13]) at the reference state

$$(u_{ref} - u) = \overline{D}^{-1}(\bar{y}_{ref} - \bar{y}) \, .$$

Finally, this update is applied via a pseudo actuator with time constant τ.

$$\tau\dot{u} = \bar{D}^{-1}(\bar{y}_{ref} - \bar{y})$$

Figure 12 shows the responses of an NDI based inverse simulation of a tilt-rotor XV15 type vehicle carrying out a longitudinal accel-decel manoeuvre from trim speed of 30 m/s and an actuation time constant of 0.01 sec. There is clearly adequate tracking of the reference velocities in this case and the characteristic oscillations of the constrained dynamics may be observed on the longitudinal stick response. The eigenvalues of the modes of the combined system at the initial trim state are compared in Table 6. (Those not shown are numerically zero.) The eigenvalues associated with the constrained pitching oscillation are in close agreement and, for the NDI, those associated with the five control actuators are close to 100, corresponding to the selected time constant.

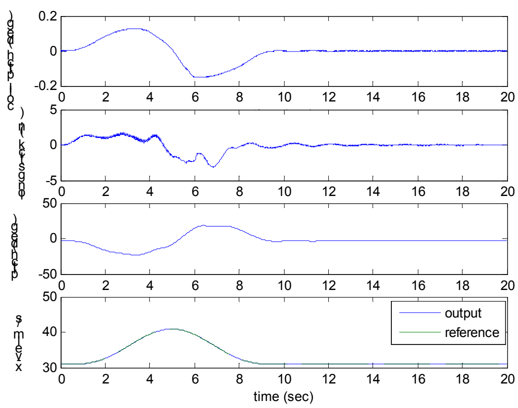

Figure 10. Responses of an NDI-based inverse simulation for a tilt rotor type aircraft.

Table 6. Eigenvalue comparisons for NDI controlled inverse simulation.

Inverse model	**NDI-based inverse model**
Constrained mode eigenvalues	**Constrained mode eigenvalues**
-0.294±3.80i	-0.225 ±3.78i
	Actuator mode eigenvalues
	-100.1±0.1i
	-100.8
	-101.1
	-100.5

The issues with the use of NDI are the validity of the linearisation $\bar{D}$ and the choice of the value for the time constant, τ. The latter can determined by trial and error if necessary using the need to be well separated from the time constants of the body dynamics. If a more precise indication is required then the eigenvalues of the combined system may be inspected along the lines of Table 10. The 'non-ideal' pilot discussed by Thomson [15] is related to this issue. The use of the Jacobian to update the control inputs provides a type of dynamic Newton iteration and, if necessary, it can be updated during the simulation, although in the authors' limited experimentation with this type of inverse simulation this strategy has not been needed.

In summary, the NDI approach is a convenient way to implement inverse simulation particularly when many simulation environments possess options to generate the required linearisation. The method may sacrifice the precise accuracy of the zero dynamics and the flight path. Additionally, it adds to the dimensionality of the problem by incorporating addition control dynamics.

Conclusions

This paper has discussed the features and problems of several approaches to helicopter inverse simulation. Features are properties that are genuine characteristics of the inverse approach and as such may be regarded as not requiring cures. Problems are undesirable artefacts, usually side effects of a particular formulation and do require attention.

- For a state of the art simulation model the pure (differentiation) method is very difficult to implement. The two time-scale method also needs a significant amount of algebraic manipulation. Alternatives such as the waypoint (integration) method or the NDI approach offer a much simpler task.
- The constrained oscillations of the zero dynamics are a genuine feature of all approaches except the two time-scale method. The question to be resolved in a particular investigation is whether or not these dynamics are pertinent to it.
- All of the methods discussed, with the exception of the two time-scale method follow the flight path references accurately, if not with the precision of pure inverse simulation. The method of integration sacrifices accuracy in between waypoints in order to facilitate implementation, and the NDI approach builds in a lag on the correct control action for the same reason. Technically, the parameters of the method can be adjusted to follow the references as closely as is required.
- The waypoint instability of the integration method is simply a nuisance and best eliminated by incorporating averaging in the simulation. It is not sufficient, for a fully nonlinear model, to average after the event.

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