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SIMPLIFIED ALGORITHMS FOR ROTORCRAFT MISSION ANALYSIS

by

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SIMPLIFIED ALGORITHMS FOR ROTORCRAFT MISSION ANALYSIS

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1 INTRODUCTION

"Operations Analysis" means a technique of increasing importance (Industries and aircraft Operators), by which the characteristics and usage of a product are evaluated and/or optimized without resorting to experiments; this last feature permits Operations Analysis to examine also products which have not yet been physically accomplished, or products from competitors not available for examination.

An analogy evidently exists between Operations Analysis and Experimentation (fig.1), since both attempt to verify the operational possibilities of the product; in these precincts Operations Analysis has the advantage of being less costly, preventive (applicable to projects not yet accomplished), it allows to compare project alternatives or products that represent the opposition, and to examine the performance not only as a single product but as being part of a fleet, obviously where this may be of interest.

There is a close tie between Operations Analysis and Operations Research due to the spreading use of computer simulations; however, while Operations Research assumes the characteristics of a product as data, Operations Analysis is often called to supply a prior estimate (fig.2).

In this paper we present the application of Operations Analysis techniques to the "helicopter" product, particularly describing the evaluation of unknown technical characteristics with which to carry out a Operations Analysis whose results will be compared to those obtained from the flight manual ("exact" method); the helicopters will be imagined in mission situations set in real places and in relation to past, present, or hypothetical events.

Please note that the bond to situations referring to reality and set in real places serves to avoid unconscious and inevitable simplifying assumptions or, in the case of comparisons with different technical-operational hypotheses, situations that may favour one hypothesis with respect to the others.

The previous characteristics make Operations Analysis interesting, besides than for mission study purposes, also for didactical aims, and this, together with the more scientific aspects, explains the interest provoked in academic environments.

2 MISSION ANALYSIS: APPLICATION TO HELICOPTERS

In this chapter we present the method for calculating results of a helicopter mission using the relative flight manual, supplied by the manufacturer, results which may be considered "exact".

To be more concise, we will refer to a very simple mission, shown in fig.3, composed of an initial warm-up phase, followed by takeoff, an ascent to cruising altitude, a cruise, and a subsequent descent and landing.

To calculate performances using the flight manual it is necessary to know the helicopter's weight in each phase of the mission, since the fuel consumption is affected, besides by environmental factors such as altitude and external temperature, also by the helicopter's gross weight.

The gross weight of the helicopter is obtained by adding the operational empty weight (O.E.W.) to the fuel needed for the mission and that of the payload (obviously variable to the type of mission), and must be inferior, or at the most equal, to the takeoff gross weight (T.O.G.W.); note that since the amount of fuel consumed during the entire mission is unknown, we must assume an initial amount of fuel, for example based on the maximum fuel tank capacity (for a long-range mission). If, at the end of the calculation, the amount of fuel left over is relevant, the procedure will be repeated with a smaller fuel load.

The procedure forecasts, for each phase of the mission, the calculation of the elapsed time and the relative fuel consumption, on the basis of characteristic values (altitude, speed, power or torque, etc.); referring to fig.4 we can follow, step by step, the calculation of the simple mission depicted in fig.3, solving respectively the following phases.

2.1 WARM-UP

In this phase, that we can assume lasts about $t_{wu} = 10$ minutes, the engines idle and we therefore consider the corresponding minimum fuel flow. We refer to the flight manual's diagrams relative to the warm-up phase that relate the fuel flow to altitude z and temperature T; the value f_{wu} of the fuel flow is taken, the fuel consumed is calculated and subtracted from the initial helicopter weight, giving the gross weight at the start of the next phase.

2.2 TAKEOFF

This phase is assumed to last a certain amount of time, for example $t_{to} = 1$ minute, during which we assume that the engine(s) always deliver the maximum available torque, the value obtained from the flight manual's diagrams that relate it to altitude z and external temperature T; with this torque value we then obtain the amount of fuel flow, f_{to} on the diagram "fuel flow - engine torque". Having calculated the takeoff fuel consumption, we subtract its weight from the gross weight at takeoff, giving the weight at the end of takeoff; the time is naturally incremented by the corresponding value t_{to} .

2.3 ASCENT

Before determining duration and consumption for the phase of ascent, we must determine, with a recurring procedure, the altitude that must be reached: this altitude, if not previously defined, is obtained on the basis of the maximum specific cruising range. A tentative altitude value z is chosen with the temperature T estimated at that altitude, according to whether the mission is to take place in hot, standard, or cold climates. From the diagram relative to cruising at altitude z and with temperature T, which gives the indicated speed and the fuel flow related to the helicopter's weight W and the necessary torque (per engine), in correspondence of the curve covering the conditions of maximum range, we obtain the speed V^* and the fuel flow f^* ; therefore

$$Specific\ range = \frac{V^*}{f^*}$$

We then choose another altitude value for maximum range and the above procedure is repeated, obtaining other specific range value. The altitude that offers the maximum value of the specific range is chosen for the cruise and then as ascent end; if not, the procedure is repeated until the values coincide.

Having determined the altitude z_{cr} at the end of the ascent (in order to maximize the cruising efficiency), we can calculate the values of interest for the actual ascent, acquiring an average of the values for fuel flow f_{mrc} , horizontal speed V_{mrc} , necessary torque C_{mrc} and available torque C_d , obtained from the starting and ending altitudes of the ascent (referring to the maximum rate of climb of the previous diagrams) as well as the values of ascending/descending speed, obtained from the corresponding diagram related to the helicopter's weight and the difference in percentage between the previously obtained available torque C_d and necessary torque C_{mrc} ; from the average values of f_{mrc} and V_z we derive the calculation of the fuel consumption and the duration of the climb.

For a precise determination of consumption and time necessary for the ascent, it is necessary, in order to assess the characteristics at the altitude z_{cr} , to hypothesize a reduction Δ in weight corresponding to the consumption during the ascent (as indicated in fig.4): having calculated the consumption for the ascent as explained above, this consumption is compared to the hypothetical reduction in weight Δ , repeating the procedure until the values coincide.

2.4 CRUISING

Having determined the gross weight at the start of the cruise as the difference between the helicopter's weight at the end of takeoff and the fuel consumption during the ascent, and having updated the time, we can then

proceed to calculate the characteristic values of the cruise, once more using a recurring procedure.
Relative to the altitude of maximum range determined in the previous procedure and to the helicopter's weight at the start of the cruise, from the diagram previously used for the calculations at the end of the ascent we can read the values for actual speed V_{cr} and fuel flow f_{cr} in function of weight and for conditions of maximum range. We then calculate for time t_{cr} and for consumption Δ_{cr} for the cruise, calculation which, at this point, will have to be repeated, since the previously considered weight was that of the start of the cruise and not the average. Therefore the procedure is repeated with a more precise value, subtracting half of the calculated consumption from the weight at the start of the cruise and the calculation is repeated until the difference of consumption calculated in two successive steps narrows down to acceptable limits.

2.5 DESCENT / LANDING

For the phase of descent we could operate considering a minimum engine torque and estimating the rate of dive from the diagram previously used for the ascent; however for the majority of helicopter applications it is perfectly acceptable not to consider a value for consumption or duration (or to consider a conventional value "t" for the latter).
Having thus terminated the examination of the (simplified) mission, having obtained the elapsed time, both phase by phase and globally, and the fuel consumption, also phase by phase and globally, from the latter we derive the helicopter's weight at the end of the mission: at this point it is possible to verify if the fuel load at the start of the mission was adequate or not, taking into account the necessary fuel reserve. If the fuel load was not adequate, the recurrence of the procedure is obvious, to be repeated until the values coincide, substituting the supposed fuel load with the estimated fuel consumption of the mission (plus reserve).
Thus determining the fuel consumption for the entire mission and the total duration, it is useful to diagram the derived results, obtaining graphics that show the evolution of cumulative time and fuel consumption related to distance covered (fig.5).
It is appropriate to emphasize that the method here explained refers, as previously mentioned, to a very simple mission comprehending all the fundamental phases, with which we could obtain, with opportune combinations, more complex missions. It is evident that if the mission to be simulated includes, as shown in the example in fig.6, more phases of ascent and descent, a flight altitude cruise, a penetration, and a loitering phase (waiting for optimum landing conditions), etc., for each of these we can repeat the relative procedure of the most suitable phase as explained previously.
Another consideration to take into account when calculating "real" missions is the possibility that the payload could vary during the course of the mission (evacuation, unloading, loading, rescue, fire-fighting, etc.), variations in weight that have to be introduced, during the calculation procedures, at the relevant change of phase.

3 SIMPLIFIED MODEL FOR THE ESTIMATE OF HELICOPTER TECHNICAL SPECIFICATIONS

In the previous paragraph we saw how to determine in an "exact" way, using the flight manual, the characteristic values of a helicopter relative to the calculation of a certain mission. However, the flight manuals are not always readily available (for example, during the analysis of a competing product or a new project); in these cases it is most important to obtain an estimate of the required values using simple, generally applicable tools.
A starting point can be given by the diagram in fig.7 which qualitatively shows the behaviour, at a certain altitude z, of the available P_d and necessary P_n power of a helicopter related to its airspeed V. While P_d can be considered approximately constant to V, its dependence on altitude can be calculated as following:

$$P_d = P_{do}\,\psi(z)$$

with the coefficient $\psi(z)$ having, in relation to z, the behaviour shown in fig.8.
As far as necessary power P_n is concerned, it can be expressed as:

$$P_n = P_i + P_a + P_{pea}$$

where:

- P_i = induced power, that is the necessary power to sustain the helicopter in flight. Fig.9 shows the behaviour of P_i with variations of V expressible as:

  $$P_i = \frac{W^2}{2\,\rho\,S\,V}$$

  This theoretic behaviour may be related in reality with close approximation, except for speeds tending to 0; in particular for $V = 0$ the induced power (power induced at "fixed point") assumes the value:

  $$P_{if} = W\sqrt{\frac{W}{2\,\rho\,S}}$$

  as shown in fig.9; in the same figure the simplified model is shown, adapted to the current work, model that may be expressed as:

  $$P_i = \frac{W^2}{2\,\rho\,S\,V} \quad if\ P_i \leq P_f$$

  $$P_i = W\sqrt{\frac{W}{2\,\rho\,S}} = P_{if} \quad if\ P_i \geq P_f$$

- P_a = necessary power for advancing in horizontal flight, expressed as:

  $$P_a = \frac{1}{2} C_f\,\rho\,V^3\,S_f$$

  where C_f is the helicopter's coefficient of aerodynamic drag referred to the surface S_f (usually the frontal section). The behaviour is visible in fig.10.

- P_{pea} = necessary power for the rotation of the rotor blades and the functioning of auxiliary systems (tail rotor and mechanical accessories included). The behaviour can be, with a first approximation, considered constant with speed V, as shown in fig.10, and is expressible as:

  $$P_{pea} = K_{pea}\,P_{if}$$

The simple model described which, by confronting necessary power to available power (as shown in figs.7 and 10), allows a simple estimate of a helicopter's performances, can be handled perfectly with the knowledge of a few data such as P_{do}, S , S_f, W, certainly available from brochures, as well as the values C_f and K_{pea}, generally not readily available.

An estimate of the two latter values may be attained from other typical data contained in the brochures, particularly typical performances such as hovering ceiling (above ground effect) z_h, the maximum rate of climb V_z at a given altitude (e.g. sea level) and range R.
The first value allows an estimate of the coefficient K_{pea} ; in fact, in a hovering state, therefore on the $V = 0$ axis on the diagram in fig.10, we can state:

$$P_{do}\,\psi(z_h) = P_{if}\,(1 + K_{pea})$$

thus:

$$K_{pea} = \frac{P_{do}\,\psi(z_h)}{W\sqrt{\frac{W}{2\,\rho\,S}}} - 1$$

Supposing that we therefore know the value of K_{pea}, the knowledge of the maximum rate of climb V_z allows the estimate of C_f; referring to figs.7 and 10, we find that the condition of the maximum rate of climb occurs at the airspeed V_{mrc} for which the specific excess power output is highest, and therefore the necessary power is minimal; in particular we find that:

$$V_{zmax} = \frac{P_d - P_{nmin}}{W}$$

where:

$$P_{nmin} = K_{pea}\, P_{if} + \frac{1}{2}\, C_f\, \rho\, V_{mrc}^3\, S_f + \frac{W^2}{2\, \rho\, S\, V_{mrc}}$$

Starting with a tentative C_f value and calculating the maximum value of V_z using the two above equations, in case of coincidence with the known value, the tentative value confirms itself as the value of C_f , otherwise the tentative value is varied until values coincide.

In the flow chart in fig.11 the computer procedure for the above described estimates of K_{pea} and C_f is showed; in the same figure we can see how, in case the estimate of one or both values is impossible (due to the unavailability of z_h or V_z) we can proceed by carrying out a simulation of a mission with range R.

As we can see from the flow chart, we choose an optimal value of the cruising altitude (using the logic of fig.12); the optimized mission obtained (takeoff, ascent, cruise) is simulated comparing the calculated global consumption G_0 with the (supposedly known) real value of G; in case of difference between values, we proceed by repeating the procedure with variations of the values K_{pea} and C_f (as previously mentioned this is best done for only one of the two values, since the other value is thus already estimated to an acceptable degree).

The above procedure, in its simplicity, has proven to be extremely efficient, as shown in the results of the next chapter.

4 APPLICATION EXAMPLE: RESCUE MISSION

Observing that the calculation procedure in the second part of fig.11 essentially simulates a mission, we have given it the possibility, once the values of K_{pea} and C_f are estimated, to vary the payload and eventually the fuel load (refueling simulation) between the various phases of the mission; we have thus obtained an instrument capable of analyzing even complex missions with data taken only from the helicopter's brochures.

In this chapter, we exemplify the application to a rescue mission that aims to evacuate 360 people from a city in hostile territory, following a situation of anarchy due to the collapse of the local regime by external conflict. The mission is described as following:

- 2 hr. pre-alert
- 10 min. taxi (warm-up)
- 1 min. for takeoff (airfield altitude)
- cruise (260 NM) at low altitude and at maximum speed
- 15 min. loitering above the objective
- landing at the gathering point
- 15 min. for jettisoning auxiliary fuel tanks (engines switched off)
- 30 min. for passenger loading (engines idle)
- takeoff from the gathering point
- cruise at low altitude and at maximum speed as in the outward journey

For the mission profile we can refer to the previously illustrated fig.6. It is evident that, due to the large amount of people to be evacuated, several helicopters will undergo the same mission, while the comparison of the results will be relative to only one aircraft.

For comparison purposes, the analysis has been first conducted using the flight manual; the values used are those of the weight of the chosen helicopter, the estimated fuel load needed, the initial altitude and temperature (at the moment of takeoff); the calculation results obtained are, obviously, the values of consumption, duration, and fuel reserve (per helicopter, per mission).

Referring instead to the simplified model, described in the previous chapter, the input data to the relative program are the weight of the helicopter and other simple data obtained from the brochures (S, S_f, z_h, V_{mrc}, P_{do}) from which, with the calculation routines previously described, we estimated the drag coefficient C_f , the coefficient of power necessary for the rotors and the tail rotor and accessories K_{pea}, thus proceeding to the analysis of the desired mission; also in this case we obtain the values of consumption, duration and reserve per helicopter per mission.

The data obtained from the two calculation procedures are compared in the diagrams of figs. 13 and 14, respectively for consumption and duration. We can note a good coincidence between the obtained values, with an error margin within 3-5% for duration and 10-13% for consumption.

The accordance of the results obtained with the two calculation methods and the low error margin, verified also in other simulations of real missions, revels the validity of the techniques adopted by Operations Analysis, especially for the estimation of unknown technical specifications in preliminary studies; it is evident that, due to the remarkable simplifying hypotheses on which such techniques are based, the results obtained can only be interpreted as indicative of the values of the characteristics analyzed, but for this reason they form a good basis for deeper studies.

References

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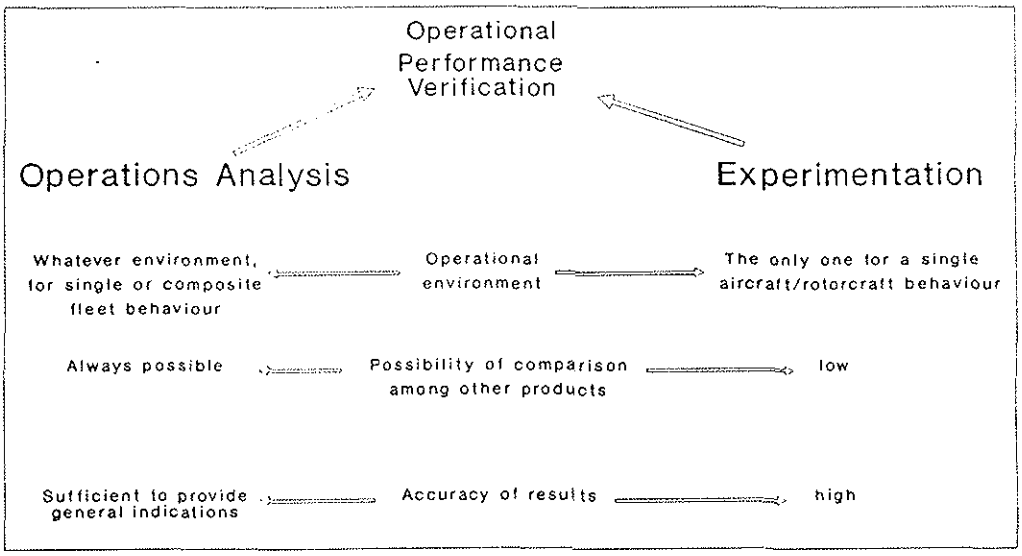


Fig.1

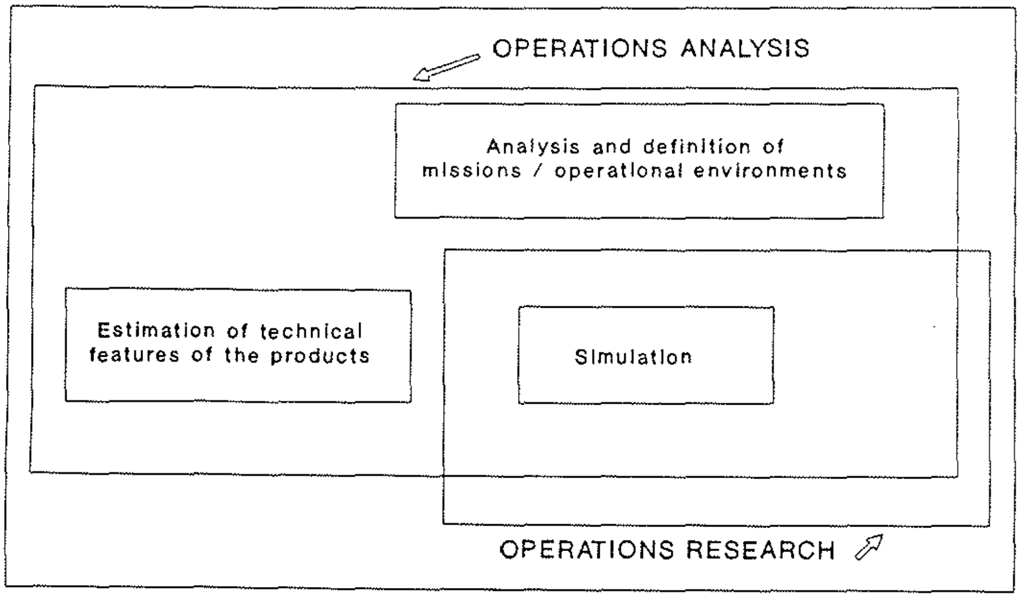

OPERATIONS ANALYSIS
Analysis and definition of missions / operational environments
Estimation of technical features of the products
Simulation
OPERATIONS RESEARCH

Fig.2

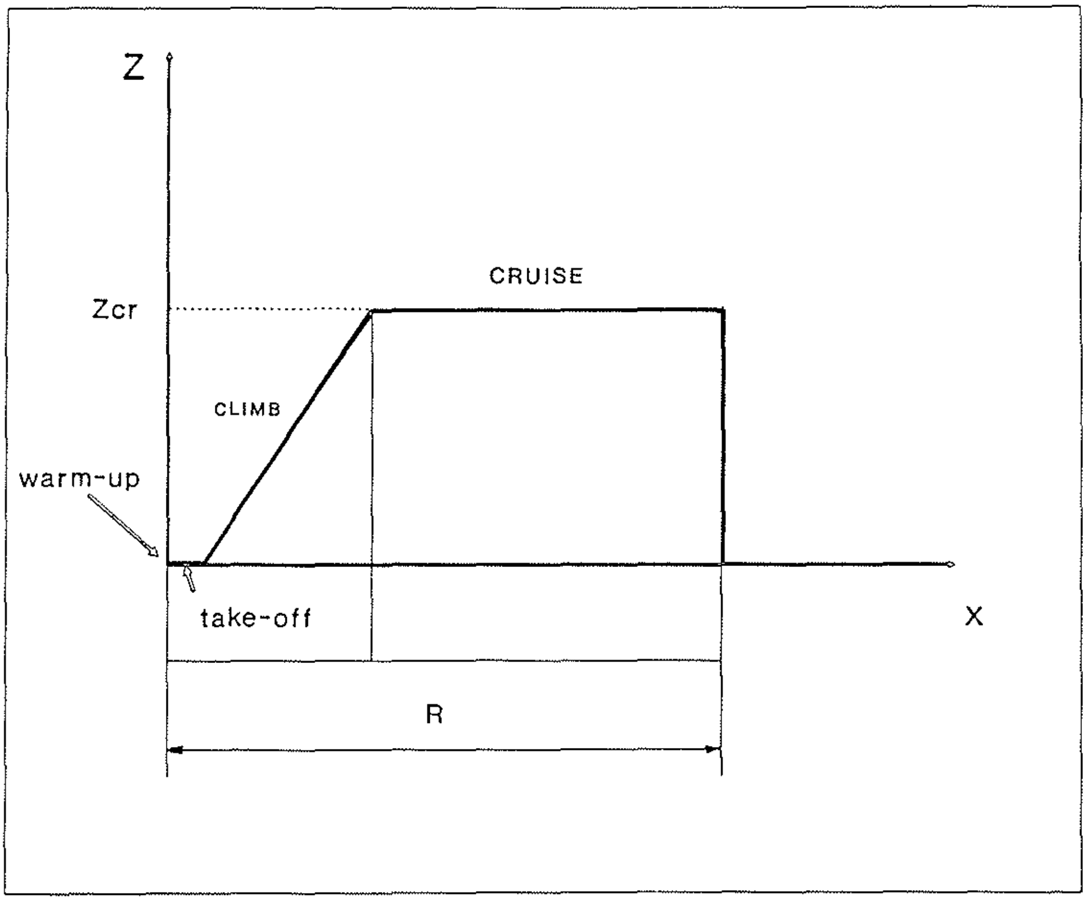

Z
Zcr
CRUISE
CLIMB
warm-up
take-off
X
R

Fig.3

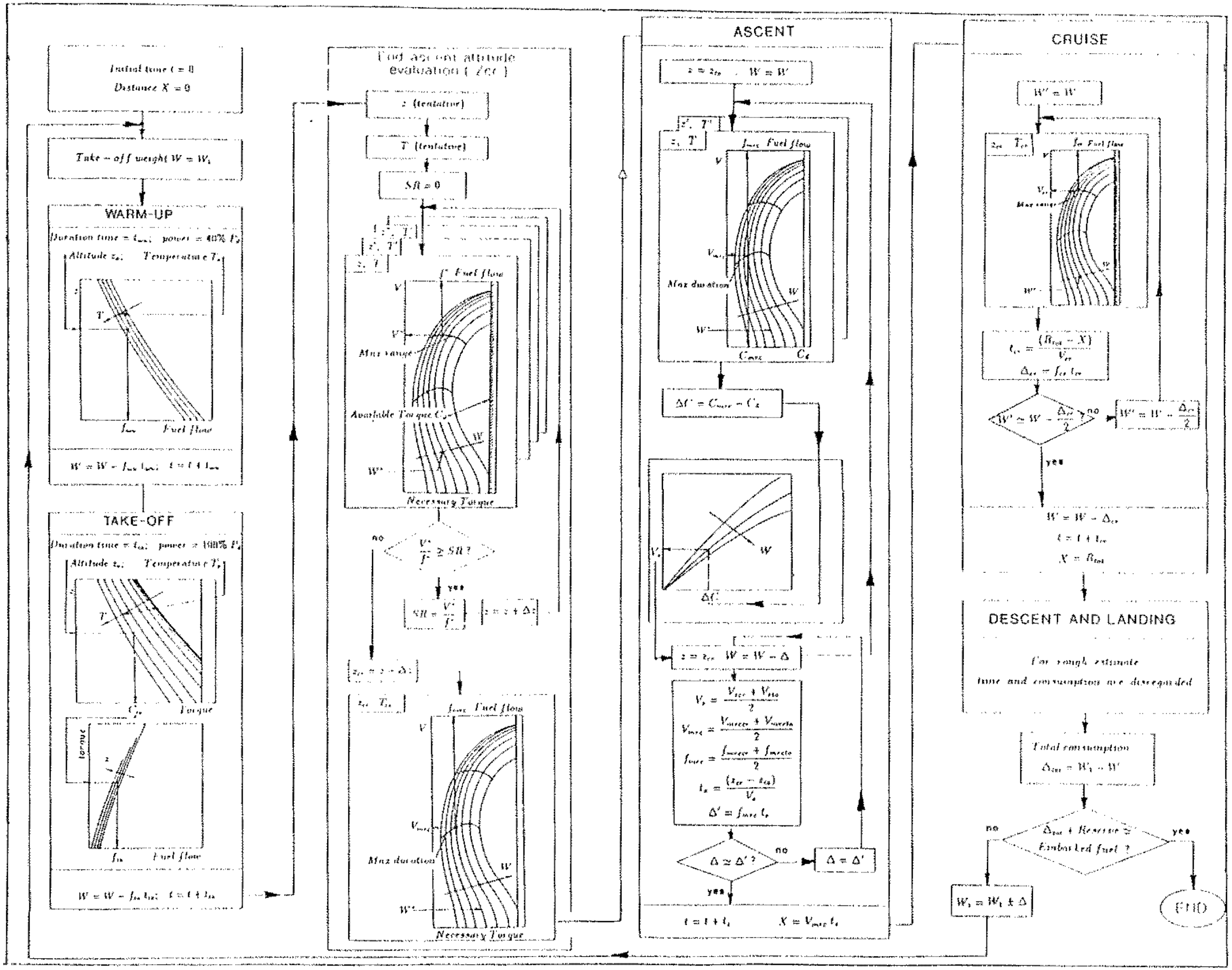
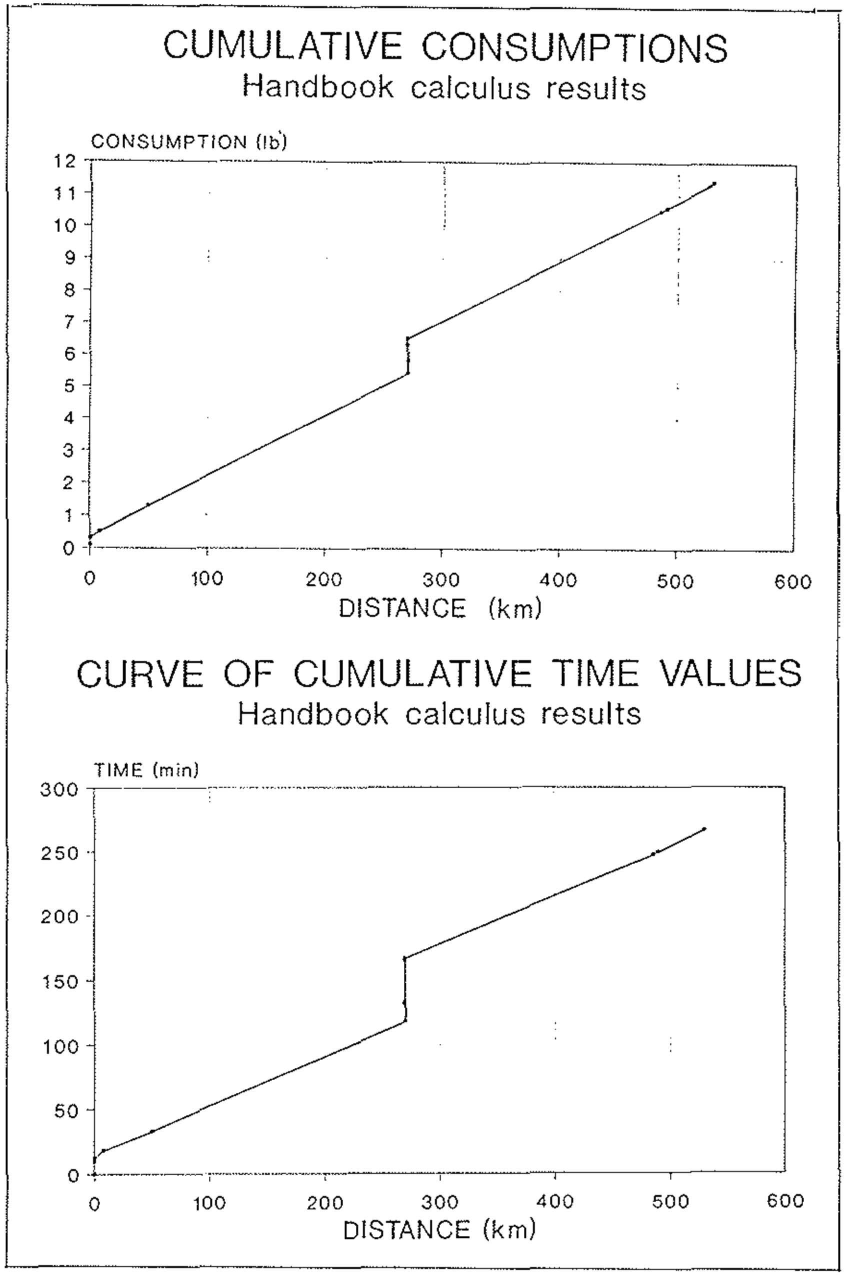

Initial time t = 0
Distance X = 0
Take - off weight W = W_t
WARM-UP
Duration time = t_wu; power = 40% P_t
Altitude z_t; Temperature T_t
T_t
f_wu
Fuel flow
W = W - f_wu t_wu; t = t + t_wu
TAKE-OFF
Duration time = t_to; power = 100% P_t
Altitude z_t; Temperature T_t
T
C_to
Torque
torque
z
f_to
Fuel flow
W = W - f_to t_to; t = t + t_to
End ascent altitude evaluation (Zcr)
z (tentative)
T (tentative)
SR = 0
z, T
f* Fuel flow
V
V*
Max range
Available Torque C_z
W
W*
Necessary Torque
no
V*/f* ≥ SR ?
yes
SR = V*/f*
z = z + Δz
z_cr = z - Δz
z_cr T_cr
f_mcr Fuel flow
V_mcr
Max duration
W
W*
Necessary Torque
ASCENT
z = z_to ; W = W
z, T
f_max Fuel flow
V
V_as
Max duration
W
W*
C_max
C_z
ΔC = C_max - C_z
V_s
W
ΔC
z = z_cr W = W - Δ
V_s = (V_zcr + V_zto)/2
V_mcz = (V_mcrcr + V_mcrto)/2
f_mcz = (f_mcrcr + f_mcrto)/2
t_s = (z_cr - z_to)/V_s
Δ' = f_mcz t_s
Δ ≥ Δ' ?
no
yes
Δ = Δ'
t = t + t_s
X = V_mcz t_s
CRUISE
W' = W
z_cr T_cr
f_cr Fuel flow
V
V_cr
Max range
W'
W
t_cr = (R_tot - X)/V_cr
Δ_cr = f_cr t_cr
W' ≃ W - Δ_cr/2 ?
no
W' = W - Δ_cr/2
yes
W = W - Δ_cr
t = t + t_cr
X = R_tot
DESCENT AND LANDING
For rough estimate
time and consumption are disregarded
Total consumption
Δ_tot = W_t - W
Δ_tot + Reserve ≥ Embarked fuel ?
no
yes
W_t = W_t ± Δ
END

Fig.4


CUMULATIVE CONSUMPTIONS
Handbook calculus results

CURVE OF CUMULATIVE TIME VALUES
Handbook calculus results

Fig.5

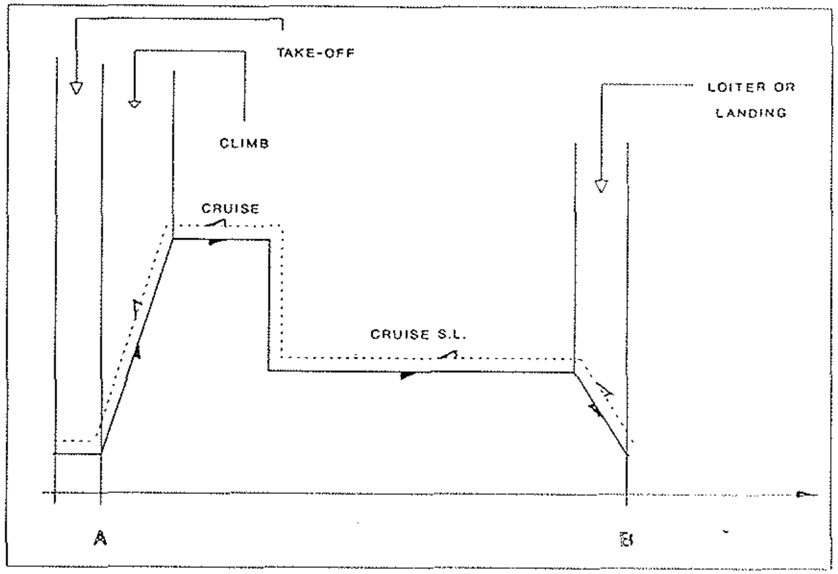


Fig.6

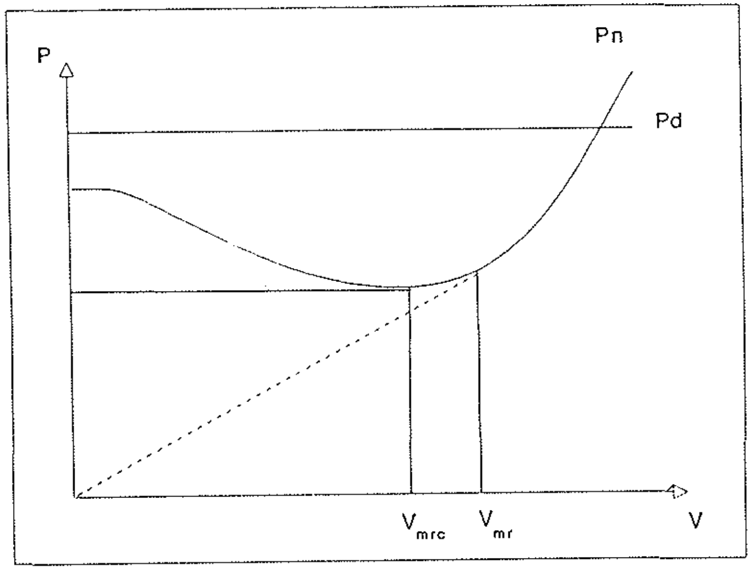


Fig.7

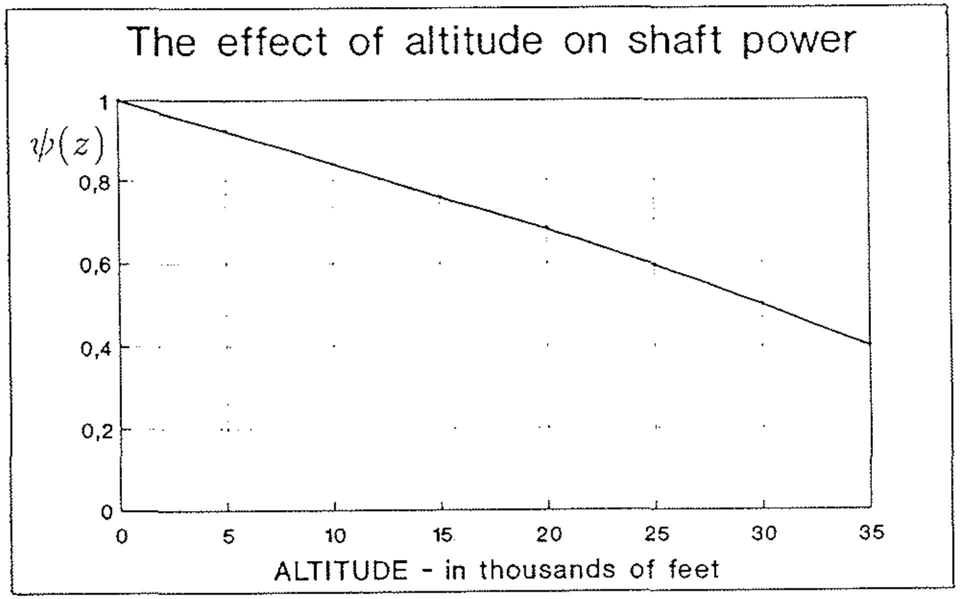


Fig.8

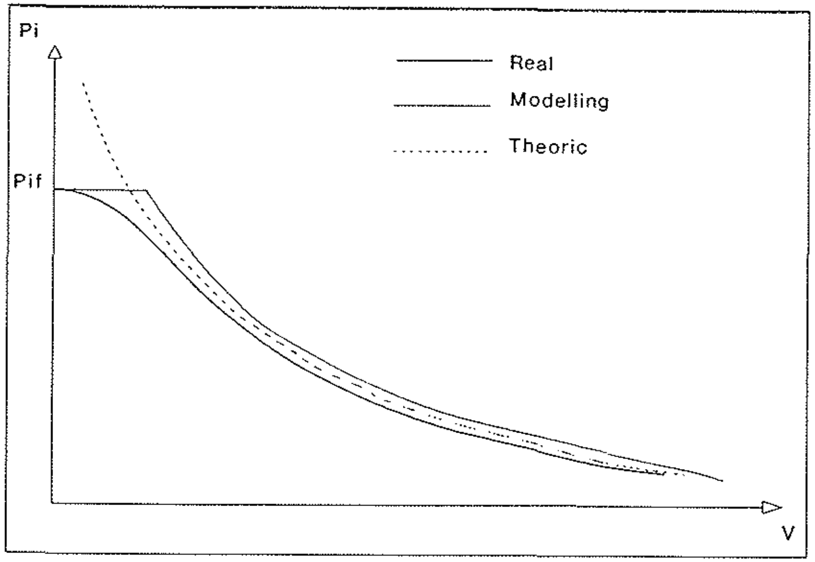


Fig.9

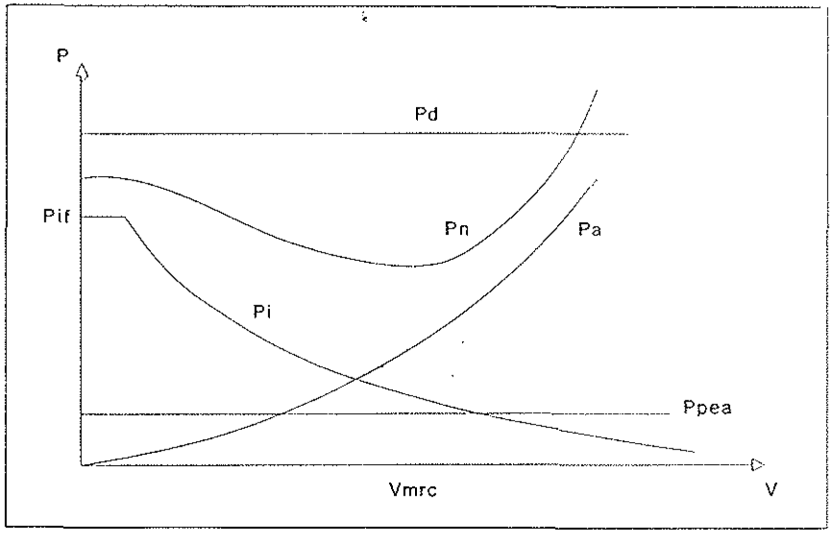


Fig.10

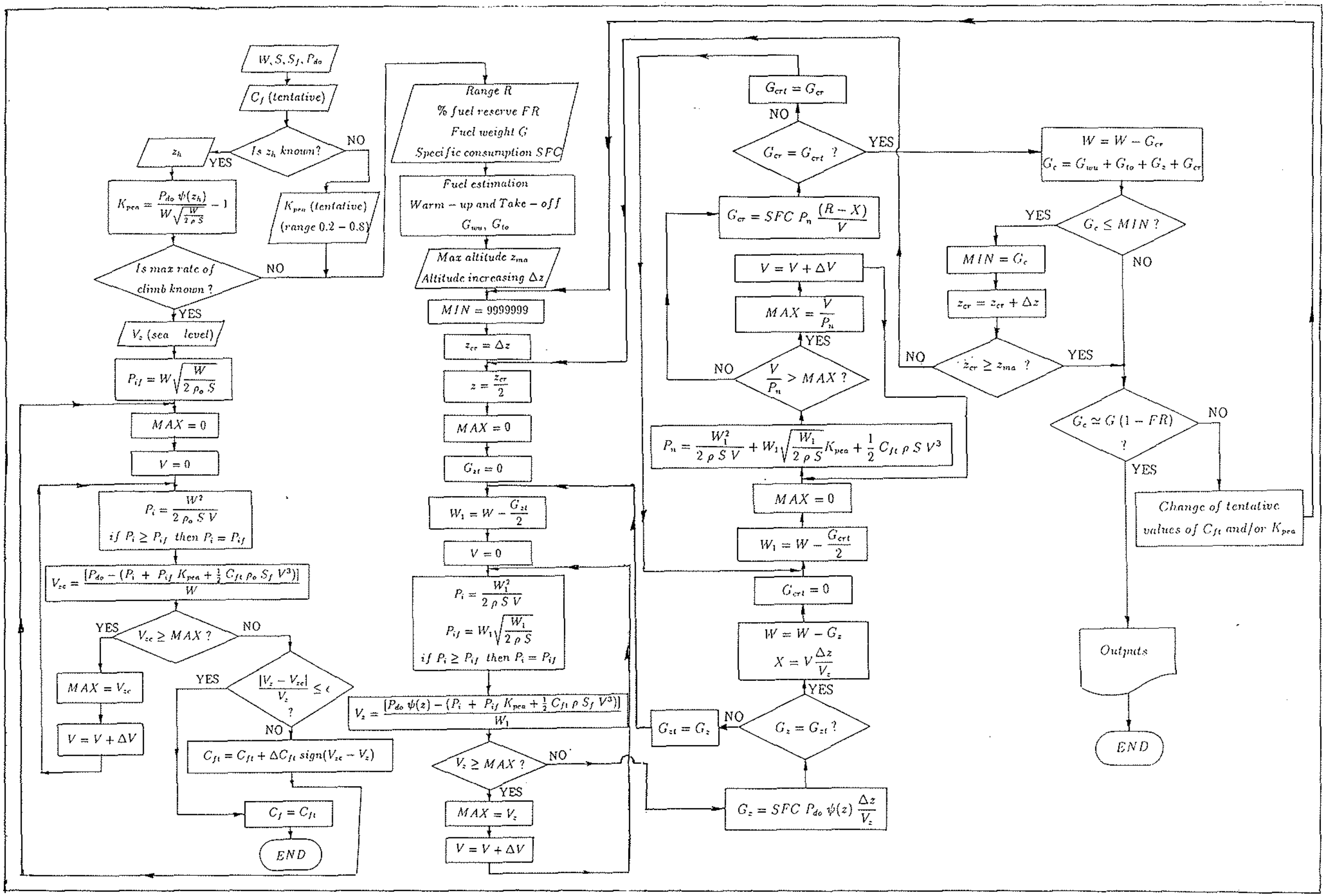

W, S, S_f, P_{do}
C_f (tentative)
Is z_h known?
NO
YES
z_h
$K_{pea} = \frac{P_{do}\,\psi(z_h)}{W\sqrt{\frac{W}{2\,\rho\,S}}} - 1$
K_{pea} (tentative) (range 0.2 – 0.8)
Is max rate of climb known ?
NO
YES
V_z (sea level)
$P_{if} = W\sqrt{\frac{W}{2\,\rho_o\,S}}$
$MAX = 0$
$V = 0$
$P_i = \frac{W^2}{2\,\rho_o\,S\,V}$
if $P_i \geq P_{if}$ then $P_i = P_{if}$
$V_{zc} = \frac{[P_{do} - (P_i + P_{if}\,K_{pea} + \frac{1}{2}\,C_{ft}\,\rho_o\,S_f\,V^3)]}{W}$
YES
$V_{zc} \geq MAX$?
NO
$MAX = V_{zc}$
$V = V + \Delta V$
YES
$\frac{|V_z - V_{zc}|}{V_z} \leq \epsilon$?
NO
$C_{ft} = C_{ft} + \Delta C_{ft}\,sign(V_{zc} - V_z)$
$C_f = C_{ft}$
END
Range R
% fuel reserve FR
Fuel weight G
Specific consumption SFC
Fuel estimation
Warm – up and Take – off
G_{wu}, G_{to}
Max altitude z_{ma}
Altitude increasing Δz
$MIN = 9999999$
$z_{cr} = \Delta z$
$z = \frac{z_{cr}}{2}$
$MAX = 0$
$G_{zt} = 0$
$W_1 = W - \frac{G_{zt}}{2}$
$V = 0$
$P_i = \frac{W_1^2}{2\,\rho\,S\,V}$
$P_{if} = W_1\sqrt{\frac{W_1}{2\,\rho\,S}}$
if $P_i \geq P_{if}$ then $P_i = P_{if}$
$V_z = \frac{[P_{do}\,\psi(z) - (P_i + P_{if}\,K_{pea} + \frac{1}{2}\,C_{ft}\,\rho\,S_f\,V^3)]}{W_1}$
$V_z \geq MAX$?
NO
YES
$MAX = V_z$
$V = V + \Delta V$
$G_{crt} = G_{cr}$
NO
$G_{cr} = G_{crt}$?
YES
$G_{cr} = SFC\;P_n\,\frac{(R - X)}{V}$
$V = V + \Delta V$
$MAX = \frac{V}{P_n}$
YES
NO
$\frac{V}{P_n} > MAX$?
$P_n = \frac{W_1^2}{2\,\rho\,S\,V} + W_1\sqrt{\frac{W_1}{2\,\rho\,S}}K_{pea} + \frac{1}{2}\,C_{ft}\,\rho\,S\,V^3$
$MAX = 0$
$W_1 = W - \frac{G_{crt}}{2}$
$G_{crt} = 0$
$W = W - G_z$
$X = V\,\frac{\Delta z}{V_z}$
YES
$G_{zt} = G_z$
NO
$G_z = G_{zt}$?
$G_z = SFC\;P_{do}\,\psi(z)\,\frac{\Delta z}{V_z}$
$W = W - G_{cr}$
$G_c = G_{wu} + G_{to} + G_z + G_{cr}$
YES
$G_c \leq MIN$?
NO
$MIN = G_c$
$z_{cr} = z_{cr} + \Delta z$
NO
$z_{cr} \geq z_{ma}$?
YES
$G_c \simeq G\,(1 - FR)$?
NO
YES
Change of tentative values of C_{ft} and/or K_{pea}
Outputs
END

Fig.11

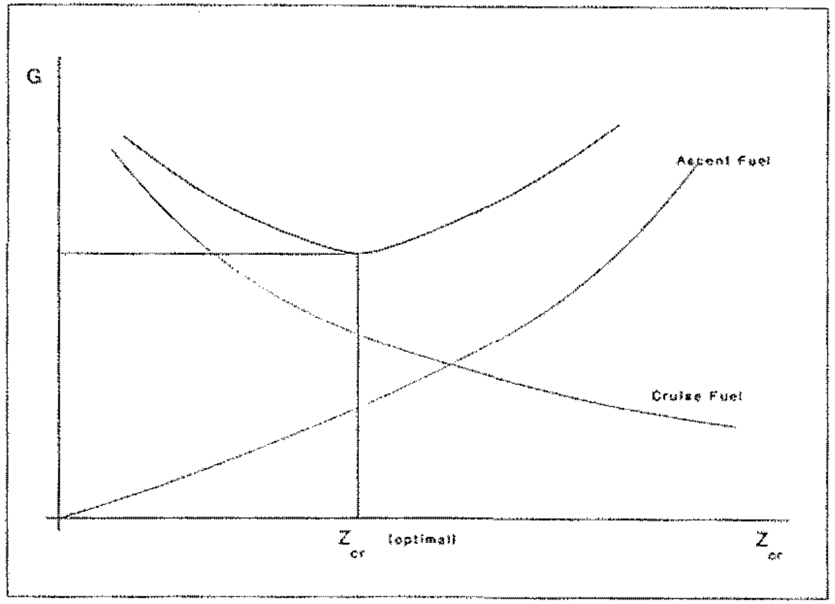


Fig.12

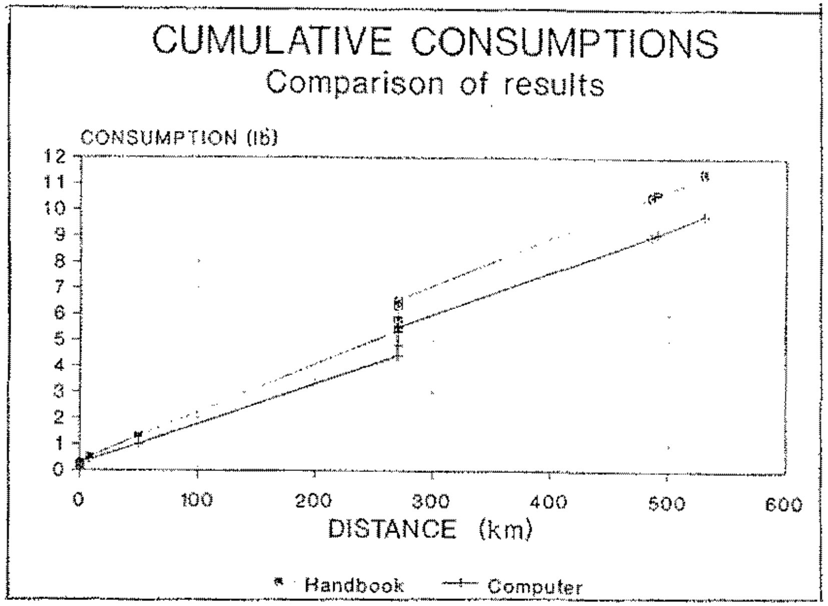


Fig.13

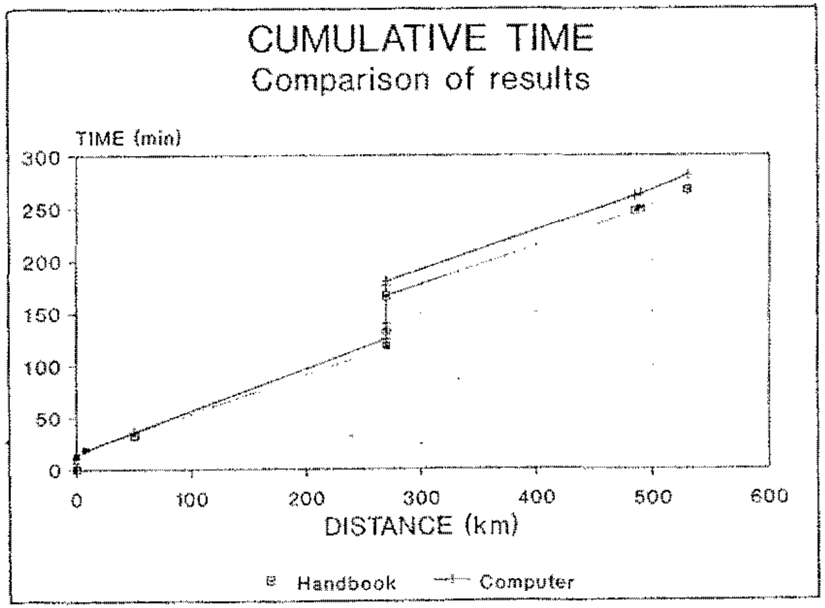


Fig.14