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ABSTRACT

A formulation for the free wake analysis of helicopter rotors in incompressible potential flows is presented here. The formulation encompasses both the theory and its numerical implementation. For the case of a single-blade rotor in hover, the formulation is validated by numerical results which are in good agreement with the generalized wake of Landgrebe and computational results of Rao and Schatzle. These results indicate that the formulation does not require any empirical assumption (such as the rate of contraction of the radius of the wake) in order to avoid numerical instabilities. To our knowledge, the results presented here are the first ones ever obtained not requiring any ad-hoc assumptions to avoid such problems. Extension of the theory to compressible flows is also outlined.

LIST OF SYMBOLS

$E(P)$	see Eq. 2.9
$\bar{n}$	normal to surfaces σ_b and σ_w
P	point having coordinates x,y,z
r	$\lvert P-P_*\rvert$
t	time
$\bar{v}$	velocity
Δ	potential discontinuity across wake
σ_b, σ_w	surfaces of body (blade) and wake
φ	velocity potential
ψ	normal wash

1. Introduction

A new methodology for the generation of the wake geometry for the computational aerodynamic analysis of a helicopter rotor is presented in this paper (details of this work are given in Ref. 1). The availability of such a methodology would enhance considerably the present computational capability in this area, which is needed for instance for: (1) performance and structural analysis, (2) evaluation of generalized forces for flutter analysis, and (3) evaluation of the outer potential velocity field for the boundary-layer and separated-flow analysis.

In the classical rotor-wake formulation, the wake is described as a spiral (helicoidal) surface which is obtained from the assumption of uniform vertical flow. This method is not sufficiently accurate for the aerodynamic analysis of helicopter rotors. This yields the need for the development of a methodology for fully-automatic or semi-automatic wake generation. The fully automated wake generation (commonly referred to as 'free wake' analysis) is obtained step-by-step by calculating from the location of a vortex point at a given time step the location at the next timestep: the drawback with this approach is that the free-wake analysis is quite expensive in terms of computer time. On the other hand, a semi-automatic wake generation (commonly referred to as 'generalized wake') may be obtained by expressing the analytical description of the wake geometry in terms of few parameters which are evaluated by fitting experimental results. The generalized-wake analysis is accurate and not more expensive than the classical-wake analysis, but currently requires the use of expensive wind tunnel experiments for the generation of the generalize-wake model. The objective of work presented here is the development of an accurate and general method for free-wake potential aerodynamic analysis which can be used (instead of the more expensive experimental approach) to generate the generalized-wake model for use in a prescribed-wake analysis.

An excellent review on aerodynamic technology for advanced rotorcraft was presented by Landgrebe, Moffitt, and Clark[2]. Additional reviews are presented in Refs. 3-7. Therefore only works which are particularly relevant to the objective and the motivation of the proposed work are included in this brief review, which is not to be considered, by any means, complete. Three items which are relevant to this report and which need a discussion deeper than the ones presented in Refs. 2 to 7 are advanced computational methods (lifting-surface and panel methods), wake roll-up and compressibility. These items are briefly examined in the following.

Consider first advanced computational methods. Lifting-surface theories are presented in Refs. 6 to 7. A third lifting-surface method was developed by Suciu, Preuss and Morino[8] for windmill rotors and yields results which are in excellent agreement with those of Rao and Schatzle[7]. Next consider panel methods, a new methodology recently introduced in aircraft aerodynamics. This methodology (also called boundary-element method) consists of the finite-element solution (over the actual surface of the body) of integral equations for potential aerodynamics. Typically, the surface of the aircraft is covered with source-panels (doublet-, vortex-, and pressure-panels are also used on the surface of the body and of its wake). The intensity of the source distribution is obtained by satisfying the boundary conditions on the surface of the body. An early work on the flow field around three-dimensional bodies by Hess and Smith[9] uses constant strength source-elements to solve the problem of steady subsonic flow around nonlifting bodies. This method has been extended to lifting bodies by including doublet, vortex, and lifting-surface panels (see, e.g., Ref. 10). Work in panel method for unsteady flow around complex configurations include extensions of the doublet-lattice method (Ref. 11) and the work of Morino and his collaborators (Refs. 12-14). This methodology has been extended to helicopter aerodynamics. For instance, the work by Dvorak, Maskew and Woodward[15] presents a method for calculating the complete pressure distribution on a helicopter fuselage. Applications of panel methods to helicopter aerodynamics are also presented by Soohoo, Morino, Noll, and Ham (Refs. 16 and 17). The above remarks indicate that panel-aerodynamics methods are becoming available for the analysis of the complete configuration. The availability of such methods (and corresponding computer programs) enhances considerably the present computational capability for an accurate evaluation of pressure and flow fields.

Next consider the issue of wake dynamics. An excellent review of the problem of the wake roll-up is given in Ref. 2 (where additional works not included here are extensively reviewed). The essence of the state of the art in this area is briefly summarized here. The various aerodynamic analysis of the rotor fall into one of the three following types :

A. Classical wake, i.e., a wake geometry described by a helicoidal spiral with pass obtained from uniform flow assumption.
B. Generalized wake, i.e., a wake geometry obtained by interpolating experimental data in terms of few parameters.
C. Free wake, i.e., a wake geometry obtained computationally as an integral part of the solution.

Analytical models for predicting the geometry of the rotor wake were developed from experimental data by Landgrebe[18], Crews, Hohenemser and Ormiston[19] and Kocurek[20]. Landgrebe's model was used by Rao and Schatzle[7] in their lifting-surface theory, and shows that a considerable improvement in the comparison with experimental results of Ref. 21 can be made simply by using a generalized wake geometry instead of the classical wake geometry. Automatic generation of the wake is considered for instance by Scully[22], Summa[6] and Pouradier and Horowitz[23]. All these works indicate that the algorithms used are unstable unless special constraints (such as specified contraction ratio) are introduced. Another important issue is the one of simplified algorithms which can be used for instance for modeling the far wake: several models are available for the hover case (see e.g., Ref. 24-26). However none of these models is applicable to the case of arbitrary motion considered in this work.

Next consider the issue of compressibility. The importance of compressibility was clearly demonstrated by Friedman and Yuan[3] for the problem of aeroelastic stability (i.e., flutter and divergence) of rotor blades. As mentioned above, compressibility effects are included in the lifting-line theory by Johansson[5] in the lifting-surface method by Rao and Schatzle[7] and in the work by Morino and Soohoo[14]. A general approach is the finite-difference solution of the differential equation used by Caradonna and Phillippe[27].

The work presented here includes the development of a formulation for the time-dependent free-wake aerodynamic analysis of helicopters in hover and forward flight and the validation of the formulation. The formulation is very general: the main restriction is the assumption of potential aerodynamics. This implies in particular that viscous (attached and separated) flows are not included here. For the sake of clarity, compressible flows are dealt with in Appendix A. Additional theoretical results dealing with the issues of wake generation, uniqueness of solution, Kutta condition, Joukowski hypothesis and trailing edge condition are available in Ref. 1. The validation of the formulation includes time-domain free-wake analysis and is limited to a single-bladed rotor in hover. However, the formulation and the numerical algorithm used in the computer program are time accurate (i.e., they yield a steady state solution via an accurate time-domain analysis) and therefore are in theory applicable to time dependent flows, in particular forward flight (of course, validation for this application would be required). The computer algorithm is general in that only the geometry and the motion of the surface of the rotor are needed as input.

2. Wake Dynamics in Incompressible Potential Flows

In this paper we assume that the frame of reference is connected with the undisturbed air. We assume the fluid to be inviscid and incompressible. Hence the motion is governed by the Euler equations and continuity equation for incompressible fluid. These equations form a system of four partial differential equations for four unknowns v_x, v_y, v_z, and p.

Since the frame of reference has been assumed to be connected with the undisturbed air, the boundary condition at infinity may be written as $p = p_\infty$ and $\bar{v} = 0$ for P at infinity. On the body (rotor in our case) it is assumed that the surface of the body is impermeable. This implies

$$(\bar{v} - \bar{v}_b)\cdot\bar{n} = 0 \quad (\text{for P on } \sigma_b) \tag{2.1}$$

where $\bar{n}$ is the normal to σ_b at P. The boundary condition on the wake are discussed later in this section after introducing the concept of potential wake.

The basis of the discussion on the wake dynamics is the well known Kelvin's theorem which states that the circulation Γ over a material contour (i.e., a contour which is made up of material particles) remains constant in time. This theorem is an immediate consequence of the definition of Γ, of Euler equations and of the fact that the density is constant (or, in general, that the fluid is barotropic). Next assume that the flow field is irrotational at time 0. Then according to Stokes theorem Γ is initially equal to zero for any path connected with a surface σ fully inside the fluid volume. Hence, for all these paths, Γ remains identically equal to zero. This implies that curl $\bar{v} = 0$ for almost all the fluid points at all times: the only points to be excluded are those material points which come in contact with the solid boundaries (since for these points, Kelvin's theorem does not apply). In order to simplify the discussion of this issue, let us focus on the case of an isolated blade with a sharp trailing edge and consider only those flows such that the fluid leaves the surface of the blade at the trailing edge. We call these flows <u>attached</u> <u>flows</u>. Hence the points which come in contact with the rotor are only those emanating from the trailing edge and therefore form a surface: such a surface is called <u>wake</u>.

Next consider a well known theorem from vector field theory which states that if a vector field $\bar{v}$ is irrotational then there exists a function, φ , called velocity potential such that $\bar{v} =$ grad φ. Hence our results may be restated as follows: for an inviscid incompressible fluid, a flow field which is attached and initially irrotational is potential at all times and at all points except those of the wake. If the flow is potential, the Euler equations may be integrated to yield Bernoulli's theorem

$$\frac{\partial\varphi}{\partial t} + \frac{1}{2}|\text{grad}\,\varphi|^2 + \frac{p}{\rho} = \frac{1}{\rho}p_\infty \tag{2.2}$$

Using, Eq. 2.2 the continuity equation may be rewritten as

$$\nabla^2\varphi = 0 \tag{2.3}$$

where ∇^2 is the Laplacian operator. Similarly, the boundary condition at infinity is $\varphi = 0$, whereas that on the body becomes

$$\psi = \partial\varphi/\partial n = \bar{v}_b\cdot\bar{n} \tag{2.4}$$

In order to complete the problem, we need a boundary condition on the wake. This condition may be obtained from the principles of conservation of mass and momentum across a surface of discontinuity, which, for incompressible flows, yield $\Delta p = 0$

and $v_n = v_s$ where p is the pressure and $v_n = \bar{v}\cdot\bar{n}$ is the normal component of the velocity $\bar{v}$, whereas v_s is the velocity of the surface (by definition, in direction of the normal $\bar{n}$). Next combine Bernoulli's theorem Eq. 2.4 with the wake condition, $\Delta p = 0$. This yields (see Ref. 1 for details) $D_w \Delta\varphi / Dt = 0$ where

$$\frac{D_w}{Dt} = \frac{\partial}{\partial t} + \bar{v}_w \cdot \text{grad}_t = \frac{\partial}{\partial t} + v_{w_1} \frac{\partial}{\partial x_1} + v_{w_2} \frac{\partial}{\partial x_2} \qquad 2.5$$

with

$$\bar{v}_w = \frac{1}{2}(\bar{v}_1 + \bar{v}_2) \qquad 2.6$$

is the material time derivative for a function defined only over the wake surface. The wake condition may be integrated to yield

$$\Delta\varphi = \text{constant} \qquad 2.7$$

following a point P_w which has velocity $\bar{v}_w$ given by Eq. 2.6. The above equations, with the addition of Joukowski's hypothesis (see Section 4 of Ref. 1), may be used to obtain the solution for φ. Once φ is known, the perturbation velocity may be evaluated using Eq. 2.3 and 2.4. Then the pressure may be evaluated using Bernoulli's theorem, Eq. 2.2.

The integral equation used in this work is a particular case of that introduced by Morino[12] for the general case of potential compressible flows for bodies having arbitrary shapes and motions. The integral equation is based on the classical Green's function method: using Eq. 2.3, one obtains

$$4\pi E(P_*)\varphi(P_*) = - \oint\!\!\oint_{\sigma_b} [\psi \frac{1}{r} - \varphi \frac{\partial}{\partial n}(\frac{1}{r})]\, d\sigma + \oint\!\!\oint_{\sigma_w} \Delta\varphi \frac{\partial}{\partial n}(\frac{1}{r})\, d\sigma \qquad 2.8$$

where $r = |P - P_*|$, σ_b is the (closed) surface of the rotor blade and σ_w is the (open) surface of the wake of the rotor blade. Furthermore, $\Delta\varphi = \varphi_1 - \varphi_2$, whereas $\bar{n}$ is the normal on the side 1 of the wake. Note that $\Delta\varphi$ on σ_w is evaluated from Eq. 2.7. (Note also that the vortex-layer wake of the rotor is represented as an equivalent doublet layer. The proof of the equivalence of doublet layers and vortex layers is given, for instance, in Reference 28.) Finally

$$\begin{aligned} E_* = 1 - \Omega_*/2\pi &= 1 && P_* \text{ outside } \sigma_b \\ &= 1/2 && P_* \text{ on } \sigma_b \text{ (regular point)} \\ &= 0 && P_* \text{ inside } \sigma_b \end{aligned} \qquad 2.9$$

For P_* on σ_b, Eq. 2.8 is an integral equation relating the unknown values of the velocity potential on the surface of the rotor, to the values of ψ (prescribed by the boundary condition on the surface of the blade) and the values of the potential discontinuity on the wake (known from the preceding time history).

Once φ_* is known, Eq. 2.8 may be used to calculate the velocity at any point in the fields as $\bar{v}_* = \text{grad}_* \varphi_*$ (where grad_* indicates differentiation with respect to P_*)

$$\bar{v}_* = -\oint\!\!\oint_{\sigma_b} [\psi \frac{\bar{r}}{4\pi r^3} - \varphi \frac{\partial}{\partial n}(\frac{\bar{r}}{4\pi r^3})] d\sigma + \iint_{\sigma_w} \Delta\varphi \frac{\partial}{\partial n}(\frac{\bar{r}}{4\pi r^3}) d\sigma \qquad 2.10$$

3. Numerical Algorithm

Equation 2.8 may be discretized by dividing the surface of the rotor into N_b surface panels σ_j and assuming that φ and ψ are constant within each panel. Similarly, the wake is divided *into elements* σ_n' *with* $\Delta\varphi$ *constant within each panel.* By imposing the condition that the equation be satisfied at the centroids P_k of the elements σ_k, one obtains (note that according to Eq. 2.9 $E(P_k) = 1/2$, since P_k is a regular point of σ_b)

$$\sum_{j=1}^{N_b} (\delta_{kj} - C_{kj}) \varphi_j = \sum_{j=1}^{N_b} B_{kj} \psi_j + \sum_{n=1}^{N_w} F_{kn} \Delta\varphi_n \qquad 3.1$$

where φ_j and ψ_j are the values of φ and ψ on the j-th panel at time t, whereas

$$B_{kj}(t) = \iint_{\sigma_j} \frac{-1}{2\pi r} d\sigma \Big|_{P=P_*}$$

$$C_{kj}(t) = \iint_{\sigma_j} \frac{\partial}{\partial n}(\frac{1}{2\pi r}) d\sigma \Big|_{P=P_*}$$

$$F_{kn}(t) = \iint_{\sigma_n'} \frac{\partial}{\partial n}(\frac{1}{2\pi r}) d\sigma \Big|_{P_*=P_k} \qquad 3.2$$

In Eq. 3.1 the wake geometry and the values for $\Delta\varphi$ are known from preceeding time steps: in particular they are assumed to be prescribed at t=0. Therefore, Eq. 3.1 is a system of N_b algebraic equations with N_b unknowns φ_j :the values of ψ_j are known from the boundary conditions. (It may be emphasized that if the rotor moves with rigid body motion, the coefficients B_{kj} and C_{kj} are time independent. An analytic expression for the coefficients is given in Ref. 13.)

As mentioned above, the frame of reference is assumed to be connected with the air. This is particularly convenient to discuss the wake dynamics. (The computer program is actually written in a frame of reference connected with the rotor.) Between time t and time t+Δt, the fluid points which were at the *trailing edge at time t move into a new position given by*

$$P_w(t+\Delta t) = P_{te}(t) + \int_t^{t+\Delta t} \bar{v}(P_w(t),t)\ dt \qquad 3.3$$

The location of these points may be is obtained by approximating the above equation as

$$P_w(t+\Delta t) = P_{te}(t) + \bar{v}_{te}(t)\ \Delta t \qquad 3.4$$

(The details for the calculation of $\bar{v}$ at the trailing edge are given in Ref. 1.) It should be noted that as mentioned above, the new locations of the wake points are within a frame of reference connected with the undisturbed air. Therefore while the wake points move, the blade also moves into its new position. Hence, at time t+Δt, we have a new row of wake elements: the value of $\Delta\varphi_n$ assigned to the elements σ_n' is the difference of the values (evaluated at time t) of the potential φ at the centers of the upper and lower blade elements that are in contact with σ_n'. In addition, the new location of the existing wake of elements is obtained as follows: evaluate the velocity at the wake points which are not on the trailing edge using Eq. 2.10 which is discretized as

$$\bar{v}_q = \sum_{j=1}^{N_b} \bar{b}_{qj}\psi_j + \sum_{j=1}^{N_b} \bar{c}_{qj}\,\varphi_j + \sum_{n=1}^{N_w} \bar{f}_{qn}\ \Delta\varphi_n \qquad 3.5$$

where q spans over all the nodes of the wake surface which are not on the trailing edge (the definitions of $\bar{b}_{qj}$, $\bar{c}_{qj}$ and $\bar{f}_{qn}$ are similar to those of B_{kj}, C_{kj} and F_{kn}). Then calculate the new locations as

$$P_q(t + \Delta t) = P_q(t) + \bar{v}_q(t)\Delta t \qquad 3.6$$

Now all the nodes of the wake surface are known. Note that if the numbering of the wake elements is not changed from time t to time t+Δt then, according to Eq. 2.7, $\Delta\varphi_n$ is constant in time. Hence, the new wake geometry and the corresponding values for $\Delta\varphi_n$ are known at time t+Δt and the process may be repeated.

The last issue to be discussed is that of the wake truncation: as the number of time steps grows, the length of the wake also grows. This implies that the computer time per time step also grows. In order to keep computer time within reasonable bounds, it is necessary to obtain a simplified model for the remote element of the wake. While sophisticated intermediate-and far-wake models have been introduced for the hover case (Refs. 24 and 25), these models require ad-hoc assumptions based on empirical data. Since the objective of the present work is to develop a method which may be used to study problems for which such data does not exist (such as maneuvering), it would have been inappropriate to introduce any of the above far-wake models or, for that matter, any model based on experimental data. For this reason in the results presented here, the wake is simply truncated after a certain number of spirals. The implications of this procedure is that the last few spirals are to be considered as modelling for the far wake effects. As indicated by the results presented in Section 4, this is an expensive approach to the problem (the case presented in Section 4 requires approximately eight hours of CPU time on an IBM 370/168). There is need to develop a less expensive approach to the far wake modelling. However, such a model should be based on first principle rather than empirical data, if the methodology proposed here is to be used independently of the experimental analysis.

4. Numerical Results

In order to validate the theory presented above, the numerical algorithm was implemented in a computer program. The results obtained with this program and the comparison with existing data are presented in this section. It is a generally accepted opinion that the wake roll-up problem is harder to solve for hover than for forward flight, because the wake spirals are closer to each other in the hover case. Also the hover case seems to be the only one for which satisfactory results exist. Therefore, in order to test our formulation we have started by studying a hover case. However, in order to validate the time-domain algorithm, the hover case was studied through a time-accurate transient response analysis. These steady-state results are the only ones presented here. We believe that the validation of the formulation will be satisfactory only if more extensive results will confirm the results presented here.

In particular, we chose the case studied by Rao and Schatzle[7] for several reasons, the most important of which is that their formulation (lifting surface with prescribed wake) is based on first principles (no ad-hoc assumption is used except for the wake geometry and zero-thickness blade) and yields results which are in excellent agreement with the experimental ones of Bartsch[21]. The problem considered by Rao and Schatzle[7] consists in a single blade, with tip radius R = 17.5', cut-out radius r_{CO} = 2.33', chord c = 1.083', collective pitch angle Θ_r = 10.61° and twist angle Θ_1 = -5°. The angular speed is Ω = 355 r.p.m. For all the results, the initial wake geometry is a classical wake with $k = \Delta z/2\pi R = \sqrt{C_T/2}$ where Δz is the pitch and C_T = 0.00186 (this is the value obtained by Rao and Schatzle). All the results were obtained using three elements in the chord directions and seven in the span directions for a total of twenty-one elements on each side of the blade. A convergence analysis presented in Ref. 17 indicates that this is sufficient to obtain relatively converged results. The time step is Δt = T/12 where T is the period: this yields twelve elements in the circumferential direction per each wake spiral (there are seven elements in the radial directions because that is the number of elements on the blade in the spanwise direction).

Before discussing the numerical results, it should be noted that as shown in details in Ref. 1, the analysis yields an anomalous behaviour for the last few spirals. This is caused by the fact that the wake is truncated (the following spirals would have a restraining effect on the last spiral: in their absence the last spiral tends to move outward). This behaviour is presented in Figure 1 which shows the vertical displacement of the last vortex line as a function of the azimuth angle at the last time step considered here (that is, at time step 50): the three lines correspond to the three-, five-, and seven-spiral models respectively. It may be seen that the first two spirals are very close for all three cases (whereas the first three spirals are in good agreement for the five- and seven-spiral models). (As mentioned in Section 3, the reason no far-wake model was introduced is because such a model does not exist for arbitrary motion although the far-wake model introduced in Ref. 25 and 26 for the hover case could have been used here in order to improve these specific results). An analysis of the convergence of the iteration for the seven-spiral case is shown in Figure 2 which indicates that the lift distribution appears to be converged: no appreciable changes occur between time steps 40 and 50. The converged five- and seven-spiral results are compared in Figure 3. We believe that the fact that the five- and seven-spiral models are in good agreement on the section-lift distribution implies that only the first one or two spirals have a strong impact on the section lift distribution.

Next, the results obtained with the seven-spiral wake are compared against existing data in Figures 4, 5 and 6. Figure 4 shows a cross section of the wake (first two spirals only) at 90° behind the trailing edge. Also shown in Figure 4 are the location of the tip vortex and of the vortex sheet as predicted by Landgrebe's generalized wake model. Note that Landgrebe's model comes from the experimental data and therefore the tip vortex is not necessarily the location of the last vortex, but rather the 'center of mass' of the rolled-up portion of the vortex sheet. Taking this into account, we do consider this comparison to be very satisfactory especially if one considers the low number of elements used to describe the blade and its wake: much stronger roll-up is expected if a higher number of elements is used. (It may be worth noting that Landgrebe model is only an approximate interpolation of the experimental data.) The results shown in Figure 5 (in which the radial location of the last vortex as a function of the azimuth θ is compared to the radial location of the tip vortex in Landgrebe's model) show similar good agreement. Finally, Figure 6 shows a comparison of our results with those of Rao and Schatzle[7], whose results for a four-bladed rotor are in excellent agreement with the experimental results of Bartsch[21]. Again, we consider that the agreement is satisfactory if one considers the low number of elements used in the analysis and that Rao and Schatzle[7] results are obtained with a prescribed wake. (It may be worth noting that our results are in excellent agreement with their results for classical-wake analysis, see Ref. 17.)

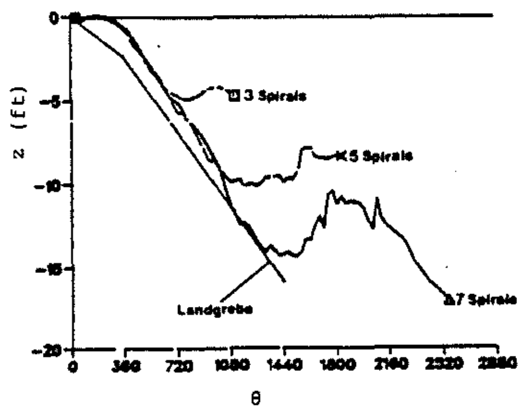

Figure 1

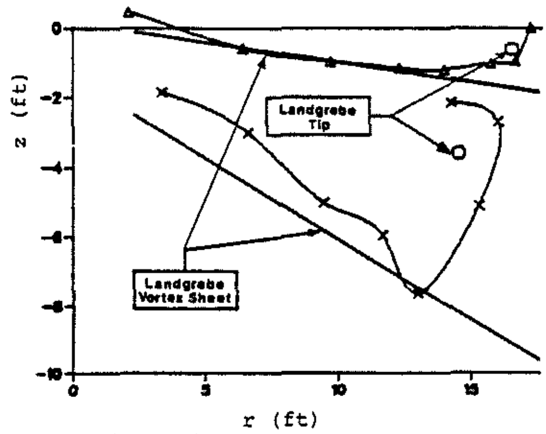

Figure 4

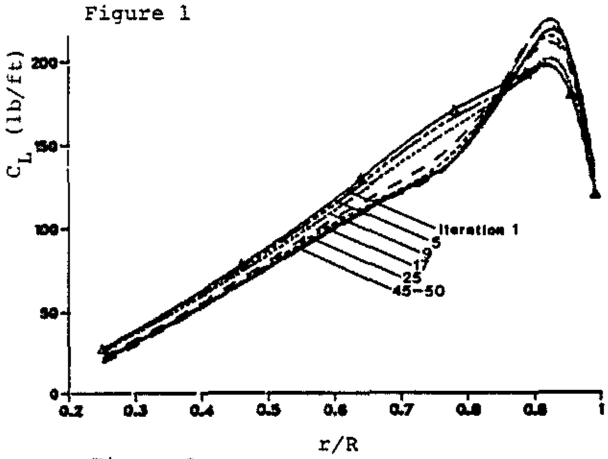

Figure 2

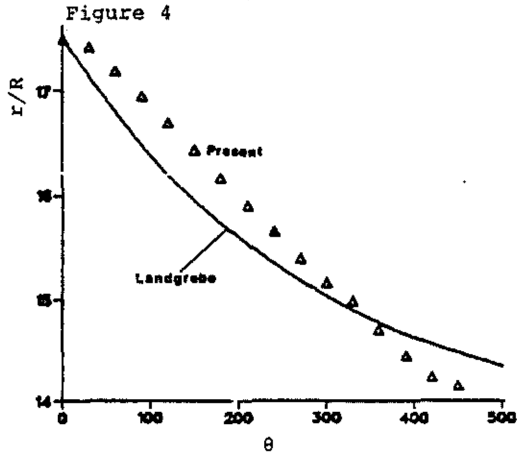

Figure 5

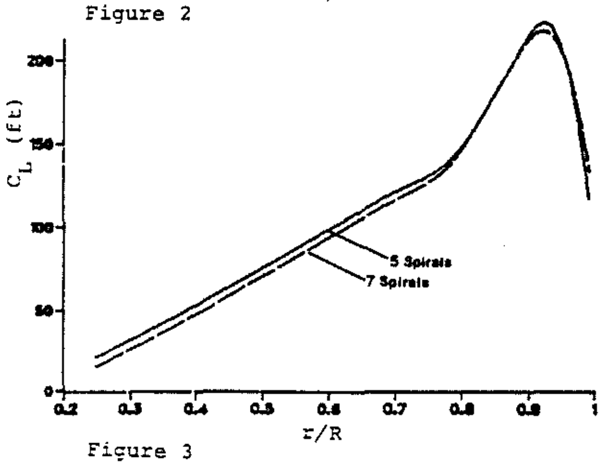

Figure 3

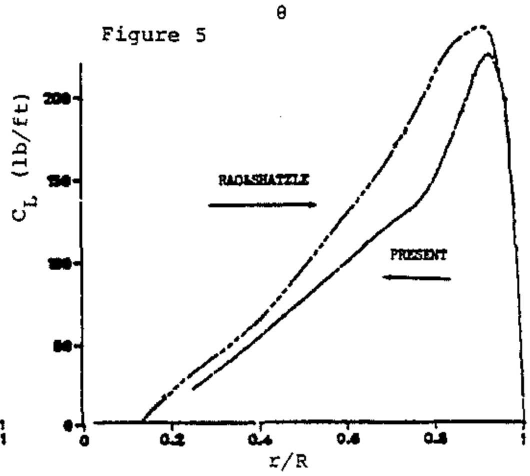

Figure 6

5. Concluding Remarks

From the numerical results presented in Section 4, one may conclude that the algorithm is capable of reproducing the correct trend in wake rollup and pressure distribution. The discrepancy between our results and the existing ones may be due to either the physical approximation (i.e., inviscid flow) or numerical approximation. An analysis of convergence is needed in order to discriminate between the two.

The main accomplishment however, is that the numerical results indicate that the algorithm appears to be free of numerical instabilities, even though no ad-hoc assumption (such as prescribed radial contraction) has been used. More precisely *we believe that the behaviour of the last few spirals of the wake* is due to the truncation of the wake and should not be thought of as a numerical instability in the classical sense: in such a case the vortex line would depart from a smooth-behavior spiral with a disturbance that oscillates and grows in space and time such as that reported by Summa[6]. All of our results are very smooth: an illustrative example of such smooth behavior, is presented in Ref. 1. We believe that this is the first time that such an accomplishment has been reported.

Although the validation has been obtained only for a rotor in hover, the formulation is quite general (the main limitations being irrotationability and incompressibility) and applicable, in particular, to a rotor in forward flight.

Additional work is recommended in the following areas:

1. Convergence analysis: it is expected that a stronger roll-up would be obtained by using a larger number of elements in the radial direction (this in turn would affect the section-lift distribution).
2. Wake truncation: it is recommended that some intermediate- and far-wake model be introduced for the purpose of reducing the number of wake spirals (and hence the CPU time). However, as mentioned above, such models should be based on first principles rather than on empirical data, if the objective is the use of the methodology for calculating generalized wakes.
3. Validation: continue the validation of the formulation by applying it to additional hover cases and then to forward flight cases.

6. References

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A. Compressible Flows

The integral formulation of Section 2 is extended here to the case of compressible flows. The frame of reference is assumed to have arbitrary motion. The surface is assumed to be moving with respect to the frame of reference in order to accommodate structural deformations as well as wake roll-up. However, for the sake of simplicity, such motion is assumed to be small. The general case is considered in Refs. 29 and 30. The formulation is an extension of that introduced in Ref. 31 for acoustics.

The equation for the velocity potential in a frame of reference $\bar{\xi}, t$ connected with the undisturbed air is given by

$$\nabla^2 \varphi - \frac{1}{a_\infty^2} \frac{\partial^2 \varphi}{\partial \tau^2} = \chi \qquad \text{A.1}$$

where χ contains all the nonlinear terms. The boundary conditions are the same as those for incompressible flows. In order to simplify the derivation of the Green's theorem for a frame of reference having arbitrary motion, it is convenient to extend the problem to the whole space by introducing the function $\hat{\varphi} = E\varphi$ where E is given by Eq. 2.9 so that

$$\begin{aligned} \hat{\varphi} &= \varphi \quad \text{inside } \sigma \\ &= 0 \quad \text{outside } \sigma \end{aligned} \qquad \text{A.2}$$

Note that $\hat{\varphi}$ is defined in the whole space and satisfies the equation

$$\nabla^2 \hat{\varphi} - \frac{1}{a_\infty^2} \frac{\partial^2 \hat{\varphi}}{\partial \tau^2} = \hat{\chi} =$$

$$E\chi + \text{grad}\varphi \cdot \text{grad } E + \text{div}(\varphi \text{grad } E) - \frac{1}{a_\infty^2}\left(\frac{\partial \varphi}{\partial \tau} \frac{\partial E}{\partial \tau} + \frac{\partial}{\partial \tau}\left(\varphi \frac{\partial E}{\partial \tau}\right)\right) \qquad \text{A.3}$$

The presence of the grad E and $\partial E/\partial \tau$ terms introduce source layers which act only on the surface σ (which is not to be considered as a boundary of the domain of validity of the equation which is the infinite space). The only boundary is at infinity where we specify $\hat{\varphi} = 0$. Equation A.1 subject to boundary condition on σ_B is equivalent to Equation A.3 in the sense that if a function satisfies Eq. A.3, it also satisfies Eq. A.1 with its boundary conditions. The solution to Eq. A.3 is

$$\hat{\varphi}(\bar{\xi}_*, \tau_*) = \iiiint G dV d\tau \qquad \text{A.4}$$

where $G(\xi, \tau) = -\delta(\tau_* - \tau - \theta)/4\pi\rho$ (with $\rho = |\bar{\xi} - \bar{\xi}_*|$ and $\theta = \rho/a_\infty$) is the well known Green's Function for the wave operator.

For the sake of clarity, consider first the case of a non-lifting rotor in rigid body motion. Introducing a coordinate system $\bar{x}, t$ rigidly connected with the rotor, one obtains (see Ref.

1 for details)

$$-4\pi E_* \varphi_* = + \oiint [\frac{1}{r_M} \frac{\partial \varphi}{\partial n}]_T \, d\sigma$$
$$+ \nabla_* \cdot \oiint [\frac{1}{r_M} \varphi \bar{n}]_T \, d\sigma + \frac{1}{a_\infty^2} \oiint [\frac{1}{r_M} \frac{\partial \varphi}{\partial \tau} V_s]_T \, d\sigma$$
$$+ \frac{1}{a_\infty^2} \frac{\partial}{\partial \tau} \oiint [\frac{1}{r_M} \varphi V_s]_T \, d\sigma + \iiint [E \chi \frac{1}{r_M}]_T \, dV \qquad A.5$$

In Eq. A.5, $[\ ]_T$ indicates evaluation at retarded time t=T, with T such that $T - t_* + |\bar{\xi}(\bar{x},T) - \xi(\bar{x}_*,t_*)|/a_\infty = 0$. Equation A.5 is the desired integral representation. In the limit, as P_* approaches the surface σ, one obtains an integral equation for φ. The numerical solution of such an equation is similar to that given in Section 3.

Next consider the case of rotor/fuselage configuration in which both the rotor and the fuselage move in arbitrary but rigid-body motions. Also for simplicity, assume that the wake remains where it is generated: this is a reasonable assumption when (in a frame of reference connected with the undisturbed air) the velocity of the fluid is small compared to that of the rotor fuselage configurations (this assumption is removed in the analysis of Ref. 30). Hence the surface σ can be broken into three surfaces: the surface of the rotor, σ_r, the surface of the fuselage σ_f, and the surface of the wake, σ_w. For each of these surfaces there exists a frame of reference which is rigidly connected with the surface. In this case (see Ref. 1), one obtains an expression similar to Eq. A.25 with each integral replaced with the sum of three integrals over σ_r, σ_f and σ_w respectively.

If the motion of the surfaces with respect to 'their frame of reference' is small, Eq. A.5 is still valid but such motion 'shows up' in the boundary conditions for $\partial\varphi/\partial n$. The case of completely arbitrary motion is discussed in Ref. 30: the derivation of the equations is quite complex but the final results are slightly more complex than the ones presented here.

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