COMPLEX MATHEMATICAL MODEL OF THE FLOW'S VISCOUS SEPARATING FROM THE SMOOTH SURFACE OF THE BODY.

M. I. Nisht,

Military Aviation Technical University.

The common problem of a viscous flow over an arbitrary body is described by the full system of Navier-Stokes equations. The numerical solution of this problem in the wide range of the alterations of the body's movement parameters, in particular, by its separated flow, however, runs across significant mathematical and computing difficulties at big Reynolds numbers. Even utilizing the simpler Reynolds equations while solving such problems requires the considerable rise of the computer fast acting.

That's why nowadays numerical solutions of the problems of a flow's viscous separating by using of the full Navier-Stokes equations system or Reynolds equations are limited by just some particular cases.

As you know, the flow's viscous separating from the smooth surface of the body takes place because of interaction of the positive pressure gradient and the boundary layer, which thickness increases towards the stern of the body. The main problem of its mathematical modeling is determination of the place where the boundary layer separates from the smooth surface of the body. This requires attraction of the flow-surface viscous interaction mechanism. The complex approach to the mathematical modeling of the two-dimensional separated flows is set out in Reference[1]. It is based on the mathematical synthesis of inviscid incompressible fluid models and a viscous boundary layer. In this work, the approach is integrated on the three-dimensional aspect of a viscous separated flow over a finite-span wing. As in the two-dimensional case, for example, at the airfoil separated flow modeling[1], mathematical modeling of the three-dimensional viscous flow over a wing is executed by the unification of the non-linear unsteady inviscid flows method of calculating and the boundary layer methods of calculating. With this, places of the flow separation from the body's surface are defined according to the theory of the boundary layer. When calculating a laminar or turbulent boundary layer both integral and differential methods are used which allows to take flow unsteadiness into account in details.

The problem of calculating of the wing aerohydrodynamical characteristics is solved by the method of discrete vortices basing on the perfect medium scheme and N.E.Zhoukowsky's vortex theory.

The vortex model of the viscous incompressible separated flow over a wing is shown in Figure1. According to the Prandtl hypothesis, the spatial flow is conditionally divided into two regions: the field of the viscous flow in the three-dimensional boundary layer near the wing surface and the field of the inviscid flow everywhere outside the wing, boundary layer and its continuation behind the wing.

In the common case of the separated flow over the wing, the flow can go down from its training and lateral edges, as well as, depending on the angle of incidence α, from its upper or lower surface along the line R_x; with that the surfaces of the tangential gap or the velocity- free vortex sheets σ_i are formed.

The second separation, i.e. formation of the secondary boundary layer and its separating from the wind surface, is possible in the reflexive flow field beyond the boundary layer separation line R_*. However, experimental and theoretical investigations have shown that this phenomenon is seldom observed (in case of the flow around a wing, not a cylinder), and practically doesn't influence considerably the flow picture and aerodynamical characteristics, so the secondary separations are not taken into account in this work.

When mathematical stating of the problem it is considered that the flow is vortice-free everywhere outside the wing, boundary layer and vortex wake (vortex sheets σ_i), and the examined unsteady non-linear problem of the external inviscid flow parameters definition is described by the Laplace equation for a velocity potential. When solving it the following conditions are used:

- boundary condition of no-penetration of a wing surface S

- condition of the disturbance attenuation on the infinite distance from a wing and Vortex Sheets σ_i

- Chaplygin-Zhoukowsky hypothesis of finiteness velocities on the trailing and lateral wing edges free vortex sheets go down from

- elementary condition of circulations immutability in time along any closed curve enveloping a wing and its vortex wake

Besides, calculating a flow field in its outer potential region demands knowledge of the position of a boundary layer separation line R_* on the wind surface, as well as parameters and disposition of all vortex sheets σ_i.

Calculating of an outer inviscid flow is executed by the method of discrete vortices[2]. A design vortex scheme of the wing is shown in Figure 2. A persistent vortex layer modeling a wing surface and its free vortex wake is changed by systems of discrete rectilinear vortex cuts (cross and longitudinal) with constant in length circulations. A boundary condition of the wing surface no-penetration is fulfilled in control points situated in the middle between neighbouring cross and longitudinal vortex cuts. According to the Chaplygin-Zhoukowsky hypothesis, the control points, not discrete vortices, are the nearest to the trailing and lateral wing edges. To fulfil the conditions on the vortex sheets σ_i the latter are considered free, i.e. moving along the paths of the particles. Systems of the discrete vortex cuts modeling a wing surface and its wake answer the boundary condition in the infinity.

A persistent process of boundary conditions, flow parameters, circulations and an aerodynamical load change in time is replaced by staged one. It is considered that they change spasmodically in some estimated time moments, but remain unchangeable in periods of time between them.

When examining flow around a wing without slip you can limit yourself to modeling only its right half, and take into account the influence of the left half basing on symmetry.

To receive a design vortex scheme the surface of the right half of the wing is divided by cross and longitudinal sections into design panels, each containing one cross discrete vortice and one control point. Unknown quantities in each design time moment are circulations of these cross vortices on the wing surface, as well as of free vortices formed in a design period of time on the trailing edge and on the separation line R_* in every design strip. Circulations of the longitudinal discrete vortices on the wing surface and in systems σ_i owing to the vortex systems closing can be expressed through circulations of the corresponding cross vortex cuts.

When executing the boundary condition of the wing surface no-penetration in control points and immutability of circulation along the loops every one of which envelops one of the design strips and a vortex wake behind it in each design moment, we have a system of linear algebraic equations for definition of the unknown circulations on the wing surface and behind the trailing edge. This system of equations solving is carried out in every design moment of time, starting with the first one when a flow is considered completely inviscid, and a boundary layer not yet formed. Circulations of the longitudinal vortices are calculated basing on the circulations of the cross vortices got from the system of equations, and velocity and pressure fields are received from the known circulations of all the vortices. Therefore, Cauchy-Lagrange integral is used to determine unsteady pressures on a wing surface.

Thus, a solution of a problem of an outer (inviscid) flow around a wing in every design moment of time provides with distribution of the velocities and pressure on a wing surface, which allows passing to a three-dimensional boundary layer parameters design. This problem is solved in a quasistationary approximation. An outer flow (in the inviscid flow region) is calculated as unsteady, and a three-dimensional boundary layer is considered quasistationary and is calculated using an integral method according to a momentary distribution of velocities and pressures at every design step.

A boundary layer parameters design is carried out along current lines using a three-dimensional boundary layer system of equations. To determine a place of a boundary layer separation different criteria are used: a surface friction coefficient striving for a zero, a sharp increase of a thickness of an impulse or a parameter forms loss, a boundary layer curve along a wing span, etc. It is supposed that all the boundary layer turbulence joins the outer flow from the boundary layer on a separation line R_* at each design step in time, and circulations of discrete vortices modeling a boundary layer separation are determined providing this condition. Further (after separation) movement of these vortices, as well as free vortices formed on a trailing and lateral edges, is carried out along the paths of fluid particles. It permits to design a vortex structure of a wing at every design moment of time, i.e. to define a form and a position of vortex sheets σ_i in space. Using the proposed complex model, numerical researches of particularities of the separated viscous flow around finite-span wings, including mechanization, wide range of angles of incidence and Reynolds numbers are carried out.

Figure 3 shows a designed disposition of a boundary layer separation line on the upper surface of a rectangular wing with a NASA airfoil section (aspect ratio λ=5, thickness ratio $\bar{c}$=0.12) at two Reynolds numbers and different angles of incidence α. It is obvious that an angle of incidence increase causes a removal of the separation line to the leading edge, and vice versa, Reynolds numbers increase causes a removal to the training line, which corresponds to the known experimental data.

Figures 4 and 5 present velocity fields in rectangular wing sections near the central ($\bar{z}$=0.25) and the end ($\bar{z}$=2.25) sections at a subcritical (α=18°) and supercritical (α=20°) angle of incidence. It is obvious that a separation point is near a trailing edge in the end sections, and it is removed to the leading edge in the root sections.

Figures 6…8 show a comparison of summary aerodynamical characteristics of a rectangular wing designed according to a scheme of a non-separated inviscid flow, a separated viscous flow, and results of an experiment at $Re=1\cdot10^6$. While appropriate results at non-separated flow regimes are given by the non-separated flow scheme it is necessary to use a viscous separated flow model at big angles of incidence. It provides both critical and most advantageous angles of incidence and a maximum lift coefficient, as well as modeling of a non-linear character of dependences of aerodynamical coefficients on angles of incidence.

Influence of a flap deflection on the aerodynamical characteristics of a wing at the separated viscous flow around it is illustrated in Figure 9.

The following conclusions based on the realized explorations can be drawn.

1. Calculation of the influence of the viscosity at the method of discrete vortices with the help of the complex mathematical model can get with high reliability aerodynamical characteristics of real wings at a wide range of angles of incidence at interesting in practice Reynolds numbers.

2. A role of viscosity at designing of a separated flow around smooth bodies comes only to the definition of forces of friction and places of flow separation.

3. Use of different criteria of separation leads to approximately similar results, i.e. a kind of criterion doesn't influence the separated flow deeply.

4. A viscosity and a boundary layer influence vortex movement and normal forces poorly, acting on a viscous flow around a body. Interaction of vortices and their influence on the flow around the body's surface is more essential.

5. Dissipation of vortices caused by molecular and turbulent viscosity is a matter of principle only when modeling of a distant vortex wake, but it may not be taken into account in case of a near wake and a non-streamlined surface.

6. The main features and macroeffects of a turbulent flow in a wake after a streamlined body at big Reynolds numbers do not depend on a medium viscosity and are effectively modeled by vortex systems and described by unsteady equations of inviscid fluid.

7. A formed separated flow around a body and its aerodynamical characteristics depend on prehistory of a movement, so these problems must be solved in an unsteady statement.

8. Navier-Stokes equations can be considered at an applied point of view as determining at modeling of separated and turbulent flows.

References

1. Belotserkovsky S.M., Kotovsky V.N., Nisht M.I.,Fedorov R. M., Two-dimensional Separated Flows – CRC Press. Boca Ration, Ann. Arbor, London, Tokyo, 1992.

2. Belotserkovsky S.M., Lifanov I.K. Method of Discrete Vortices. CRC Press, USA, 1993.

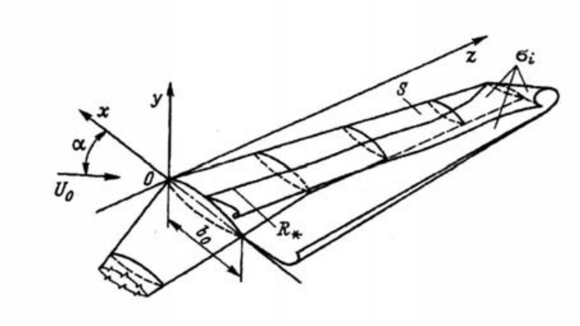


Figure 1. A vortex model of a separated viscous flow around a wing at $\alpha>0$.

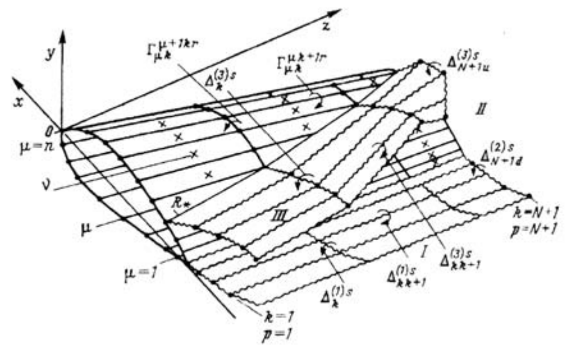
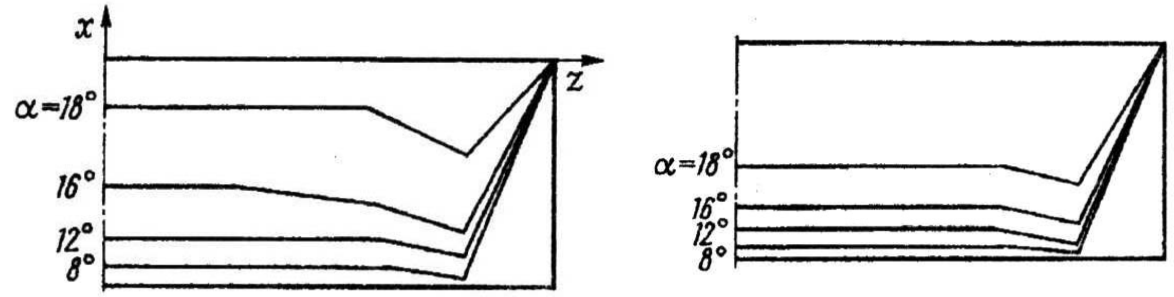


Figure 2. A vortex design scheme of a wing.



a) b)

Figure 3. Influence of a Reynolds number Re and an angle of incidence α on a separation line disposition on a rectangular wing ($\lambda=4$, $\bar{c}=0.12$): a) $Re=1.28\cdot10^6$; b) $Re=6\cdot10^6$.

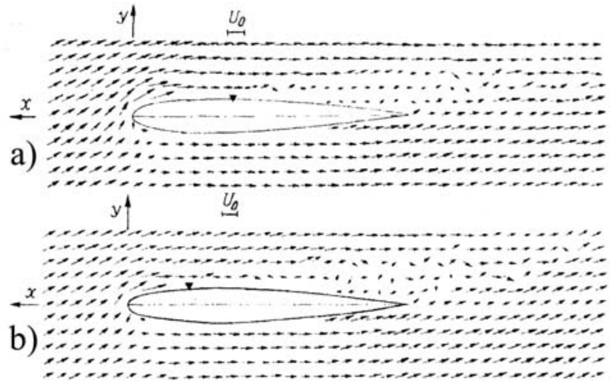


Figure 4. A velocity field in a near to central section ($\bar{z}=0.25$) of a rectangular wing ($\lambda=5$; $\bar{c}=0.12$): a) $\alpha=18°$; b) $\alpha=20°$; ∇- a point of separation.

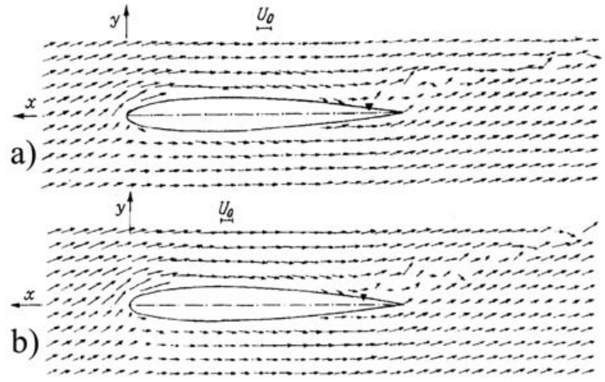


Figure 5. A velocity field in a near to end section ($\bar{z}=0.25$) of a rectangular wing ($\lambda=5$; $\bar{c}=0.12$): a) $\alpha=18°$; b) $\alpha=20°$; ∇- a point of separation.

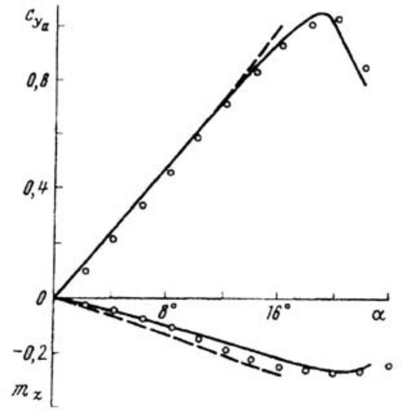


Figure 6. Coefficients $C_{y\alpha}$ and m_z of a rectangular wing ($\lambda=4$, $\bar{c}=0.15$) dependences on an angle of incidence α at $Re=1\cdot10^6$: dashed line, non-separated flow; solid line, separated flow; open circles, experiment.

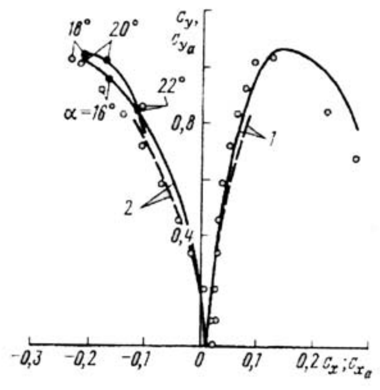


Figure 7. Polars of the first(1) and second(2) kinds of a rectangular wing ($\lambda=4$, $\bar{c}=0.15$) at $Re=1\cdot10^6$: dashed line, non-separated flow; solid and solid with points lines, separated flow; open circles, experiment.

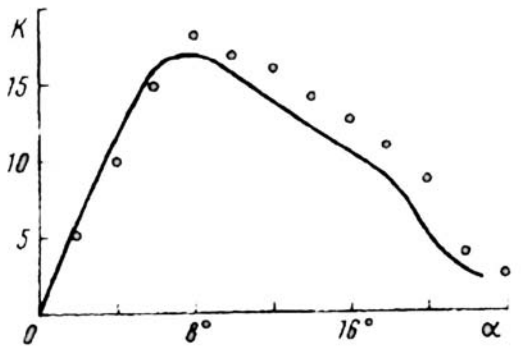


Figure 8. A lift-to-drag ratio K of a rectangular wing (λ=4, $\bar{c}$=0.15) dependence on an angle of incidence α at Re=$1\cdot10^6$: solid line, calculation; open circles, experiment.

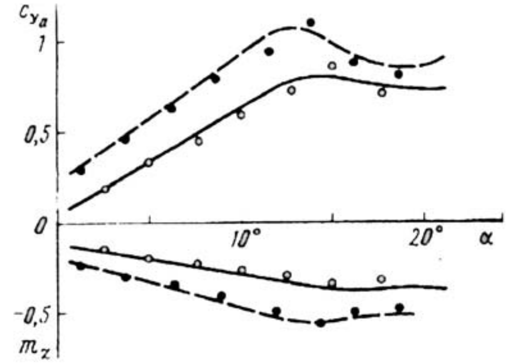


Figure 9. Coefficients $C_{y\alpha}$ and m_z of a wing (λ=4.7, χ=1.75°, η=1.5, $\bar{c}$=0.11) with a flap ($\bar{b}$=0.3, $\bar{l}$=0.33) dependence on an angle of incidence α at Re=$0,7\cdot10^6$: open circles, δ=0 (calculation); points, $\delta=40^0$ (calculation); solid line, δ=0 (experiment); dashed line, $\delta=40^0$ (experiment).