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IMPROVED HELICOPTER AIRFRAME RESPONSE THROUGH STRUCTURAL CHANGE

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SUMMARY

The single-point response analysis embodied in the Vincent circle theorem, which has been so prominent in the discussion of helicopter airframe vibration, is generalised to the case of multiple responses. Its geometric optimisation is replaced by a straightforward algebraic one. The extension to multiple spring changes extends the study to deal with the important design case of multiple forcing, response and spring change, and this opens up the possibility of refinement of helicopter airframe design for reduced vibratory response.

1 INTRODUCTION

The problem of helicopter vibration reduction may be approached from a number of distinct points of view. We may, for example, seek to reduce the vibratory loads at source by active or passive control of blade pitch, or to reduce coupling between the rotor/gear-box/engine system and the airframe by active or passive isolation systems. Equally we may seek to ensure that residual vibratory forces reaching the airframe produce only minimal response. Indeed, so important is the problem of producing an airframe with the lowest possible transmissibility that it is no understatement to say that all the advantages of good rotor and isolator design could be squandered by inadequate attention to the airframe. Whilst careful airframe design will not, of itself, solve the vibration problem, failure to achieve the best possible airframe configuration will put at risk all other attempts to abate vibration.

The starting point in our investigations is a valid dynamic model of the airframe. For example we may have a NASTRAN model in which we have confidence, or alternatively have measured a set of experimental transfer functions or modes. In either case, we shall assume that it has been possible to construct a matrix of receptances for the appropriate forcing frequency and that these will be representative of the behaviour of the helicopter in flight.

In a number of investigations over the last decade, the use of such a matrix to study the effects of structural change has focussed upon the circle property first observed by Vincent (1973). The discovery that the response locus to a given forcing - when a single structural member is altered - is circular, has been exploited considerably as discussed earlier (Sobey, 1980).

In this paper, we extend the earlier work in which a single response point is surveyed to include as many others as are of interest. A mean square measure of weighted response takes the place of the single response, and in place of the geometrically attractive circle proposition, we find a corresponding algebraic development of the appropriate optimum stiffness for a single spring change. We deal only with stiffness change because that is the most effective and innocuous of the possible structural changes that we may consider, but the ideas are easily extensible to the case where mass is altered. Having solved the problem of choosing the stiffness of a single spring for minimizing the measure function, we extend the ideas to examine the choice of several springs.

This optimum study is dependent on the input loading, which is assumed known. In practice, the load varies with flight condition, and so the value of a design improvement has to be checked against flight condition. In the studies made so far, where the choice of spring values is limited by some practical considerations, it has been found that much the same set of springs is selected as optimal over a considerable flight speed variation. Moreover, the techniques developed in this report are easily extensible to include responses to the loads appropriate to several different flight speeds, simultaneously analysed. Thus we are led to infer that the procedures developed in this paper should lead to a valuable capability for the refinement of airframe design.

2 ANALYTICAL DEVELOPMENT

We use the notation $G_{rf}(= G_{fr})$ to denote the receptance of the structure, giving the complex response along freedom x_r when the structure is forced by a unit generalized force along x_f.

Where several forces co-exist, say F in number, along freedoms x_f $(f=1,2,\ldots F)$, then we write

$$G_{rF} = G_{Fr} = \sum_{f=1}^{F} F_f \exp(i\theta_f) G_{rf}$$

where the generalized force along x_f has magnitude F_f and phase θ_f with respect to the fundamental reference direction of G_{rf}.

The response of the structure is examined when one or more of a set of S springs is adjusted. Typically $s(=1,2,\ldots S)$ denotes a spring given a change of stiffness of magnitude k acting along the identical directions x_p and x_q joining the ends p, q of the spring. We write s, or $\widehat{pq}$, as identifier of this spring and, in general, we study the consequences of varying k to the response of the structure. A second spring say t th in the series, joins freedoms x_u and x_v, or $\widehat{uv}$, and on occasions we refer to a third, the w th joining x_m and x_n, or $\widehat{mn}$.

The fundament relationship between the response z_r along x_r due to a change k in $\widehat{pq}$ with unit force acting is

$$z_r = G_1 - kG_2/(1 + kG_3) \tag{1}$$

where

$$\left.\begin{aligned} G_1 &= G_{rf}, \\ G_2 &= (G_{rp} - G_{rq})(G_{fp} - G_{fq}) \\ \text{and} \quad G_3 &= G_{pp} - 2G_{pq} + G_{qq} . \end{aligned}\right\} \tag{2}$$

Where a series of F generalized forces is applied,

$$G_1 = G_{rF}$$

and

$$G_2 = (G_{rp} - G_{rq})(G_{Fp} - G_{Fq}) \ . \qquad (3)$$

The bilinear nature of the relationship between z_r and k underlies the well-known phenomenon that, as k varies, z_r traces out a circular locus in the response plane (Vincent, 1973). Much has stemmed from this observation.

The comparative simplicity of this relationship has been, at one and the same time, an asset and a liability. For, in identifying the minimum response along x_r, say z_{min}, and its associated spring value, k_{min}, by purely geometric means, one has avoided recourse to more formal minimization techniques. To arrive at such a simple solution may be considered a great boon. However, no attention is paid to the way in which responses within the structure are organized in order to achieve a local optimum, and it is frequently the case that this local minimum is associated with gross deteriorations in response elsewhere in the structure.

This unacceptable situation can only be avoided by making the selection of one or more springs in a way that takes due account of response at a number of locations. We generalize equation (1) to R different locations x_r, $r=1,2,\ldots,R$.

We choose a weighted mean response, M say, where

$$M = \sum_{r=1}^{R} w_r z_r \bar{z}_r \ , \qquad (4)$$

where w_r is a weight associated with the freedom x_r and $\bar{z}_r$ is the conjugate complex of z_r .

Substituting from equation (1)

$$M = \sum_{r=1}^{R} \frac{w_r(G_1 + kG_4)(\bar{G}_1 + k\bar{G}_4)}{(1 + kG_3)(1 + k\bar{G}_3)} \ . \qquad (5)$$

where
$$G_4 = G_1G_3 - G_2 \ . \qquad (6)$$

G_1, G_2, G_4 vary with r, G_3 is a spring combinant independent of r .

M can be written in the form

$$M = \frac{f(k)}{g(k)} = \frac{A + Bk + Ck^2}{1 + Dk + Ek^2} \tag{7}$$

where

$$\left.\begin{aligned} A &= \sum_{r=1}^{R} w_r G_1 \bar{G}_1 \\ B &= \sum_{r=1}^{R} w_r (G_1\bar{G}_4 + \bar{G}_1 G_4) \\ C &= \sum_{r=1}^{R} w_r G_4 \bar{G}_4 \\ D &= G_3 + \bar{G}_3 \\ \text{and} \qquad E &= G_3\bar{G}_3 \;. \end{aligned}\right\} \tag{8}$$

Note that A, B, C, D and E are all real.

Although the analysis is developed for a single flight speed, it is clear from the form of equation (7) that a superposition of the corresponding results for several flight speeds remains of the same form. Hence, in our future development, there is no real distinction between analysis for one forward speed, or for several treated simultaneously.

The stationary values of M are given by $dM/dk = 0$, or

$$fg' - f'g = 0 \tag{9}$$

where primes denote differentiation with respect of k . Substituting for f and g leads to the equation governing the values of k for extreme response:

$$(CD - BE)k^2 + 2(C - AE)k + B - AD = 0 \;. \tag{10}$$

One of these values corresponds to the minimum value of M which we seek, the other to a maximum. We resolve the question of identity in the following way. For the stationary values of M , say V , the relationship between V and k is

$$V = f/g = f'/g' = (B + 2Ck)/(D + 2Ek)$$

or

$$k = (B - DV)/2(EV - C) \;. \tag{11}$$

Writing $v = V/A$, we recover finally the following quadratic equation for the reduced response v:

$$F(v) = v^2 + 2d_1 v + d_2 = 0 , \qquad (12)$$

where $d_1 = (2AE + 2C - BD)/A(D^2 - 4E)$

and

$$d_2 = (B^2 - 4AC)/A^2(D^2 - 4E) . \qquad (13)$$

The exceptional case where $D^2 = 4E$ is discussed later.

Equation (12) has two roots, v_1 and v_2, which we show satisfy

$$0 < v_1 \leqslant 1$$

and

$$1 < v_2 .$$

Now $F(0) = (B^2 - 4AC)/(D^2 - 4E)A^2$.

On introducing the notation that

$$G_p = g_{pr} + ih_{pr} , \qquad r = 1,2,\ldots,R; \; p = 1,2 \text{ or } 4$$

and

$$G_3 = g_3 + ih_3 ,$$

we can write $AC - B^2/4$ in the form

$$\sum_{r=1}^{R}\left(g_{1r}^2 + h_{1r}^2\right)\sum_{p=1}^{R}\left(g_{4r}^2 + h_{4r}^2\right) - \left\{\sum_{r=1}^{R}(g_{1r}g_{4r} + h_{1r}h_{4r})\right\}^2$$

$$= \sum_{r=1}^{R}\sum_{p=r}^{R}\Big\{(g_{1r}g_{4p} - g_{1p}g_{4r})^2 + (g_{1r}h_{4p} - g_{4r}h_{1p})^2$$

$$+ (g_{1p}h_{4r} - g_{4p}h_{1r})^2 + (h_{1r}h_{4p} - h_{1p}h_{4r})^2\Big\}$$

$$+ \sum_{r=1}^{R}(g_{1r}h_{4r} - g_{4r}h_{1r})^2 \geqslant 0 .$$

But $D^2 - 4E = -4h_3^2 < 0$, so $F(0)$ is positive.

$$F(1) = (B - AD)^2/(D^2 - 4E)A^2 \lesssim 0 ,$$

and $F(\infty)$ is positive.

Thus, in general, there is one root for v between 0 and 1 , and one greater than 1 . The minimum is identified as

$$v_{min} = -d_1 - \sqrt{d_1^2 - d_2} . \tag{14}$$

If the structure is optimal with respect to the spring considered,

$$B = AD . \tag{15}$$

Since, in general, v tends to be closer to 1 than to zero, an alternative $u = 1 - v$ is a more suitable base for computing, where u satisfies

$$(D^2 - 4E)A^2u^2 - 2\left[(D^2 - 2E)A^2 + (2C - BD)A\right]u + (B - AD)^2 = 0 . \tag{16}$$

u then, may be sought as a root of equation (16), and a fast algorithm developed to yield u , then v, V and k (from equation (11)).

In the exceptional case where $D^2 = 4E$, or $h_3 = 0$, we have the situation where the effective impedance between the spring ends is a pure stiffnes. In this case

$$u = (B - AD)^2/2\left[(D^2 - 2E)A^2 + (2C - BD)A\right]. \tag{17}$$

If there are no restrictions on the choice of k and all values are acceptable, the criterion for suitability of the s th spring for change is measured by v_{min} and, accordingly, a set of springs may be ranked in order of effectiveness. If k is restricted, to the range (kl,ku) say, it is necessary to check if the minimum is in the range, and if not to pick as achievable v , the value which is the lesser of the values at the limits kl and ku .

In all cases, then, we can examine the suitability of changing a particular spring value for the R response points and the given loading. Suppose we are led to believe that S particular springs hold - individually - promise of a worthwhile reduction in response, how do we organize the calculation so as to realise the best mutual result?

3 THE MULTIPLE SPRING CASE, $S > 1$

Suppose that two springs, the s th and the t th , of stiffnesses k_1 and k_2 respectively, join $\widehat{pq}$ and $\widehat{uv}$. The equation which governs response in this case takes the form:

$$z_r = G_1 + P_2(k_1,k_2)/Q_2(k_1,k_2) \tag{18}$$

where $G_1 = G_{rF}$ as before,

$$P_2(k_1,k_2) = P_{12}k_1k_2 + P_{10}k_1 + P_{02}k_2 \tag{19}$$

$$Q_2(k_1,k_2) = Q_{12}k_1k_2 + Q_{10}k_1 + Q_{02}k_2 + 1 \ . \tag{20}$$

The coefficients P_{12}, P_{10}, P_{02}, Q_{12}, Q_{10}, Q_{02} in equations (19) and (20) are given by

$$\left.\begin{aligned}
P_{12} &= \left[H_5(H_3H_6 - H_1H_7) + H_4(H_3H_7 - H_2H_6)\right] \ , \\
P_{10} &= - H_4H_6 \ , \\
P_{02} &= - H_5H_7 \ , \\
Q_{12} &= H_1H_2 - H_3^2 \ , \\
Q_{10} &= H_1 \ , \\
\text{and} \quad Q_{02} &= H_2 \ .
\end{aligned}\right\} \tag{21}$$

H_1, H_2, H_3, H_4, H_5, H_6, H_7 have the values

$$\left.\begin{aligned}
H_1 &= G_{pp} - 2G_{pq} + G_{qq} \ , \\
H_2 &= G_{uu} - 2G_{uv} + G_{vv} \ , \\
H_3 &= G_{pu} - G_{pv} - G_{qu} + G_{qv} \ , \\
H_4 &= G_{pF} - G_{qF} \ , \\
H_5 &= G_{uF} - G_{vF} \ , \\
H_6 &= G_{rp} - G_{rq} \ , \\
\text{and} \quad H_7 &= G_{ru} - G_{rv} \ .
\end{aligned}\right\} \tag{22}$$

It is easily verified that equation (18) is recovered in the three equivalent circumstances (a) add k_1, then k_2, (b) add k_2, then k_1 and (c) add k_1 and k_2 simultaneously to the basic structure.

Equation (18) takes the place of equation (1) in evaluating response along x_r.

Following procedures similar to those used for the single spring case, we obtain two equations governing the stationary values of M, namely

$$\partial M/\partial k_1 = \partial M/\partial k_2 = 0$$

and these have the form

$$\left.\begin{aligned} F_1(k_1,k_2) &= 0 \\ F_2(k_1,k_2) &= 0 , \end{aligned}\right\} \qquad (23)$$

where F_1 is quadratic in k_1, biquadratic in k_2

and F_2 is biquadratic in k_1, quadratic in k_2.

The coupled equations (23) have, in general 16 roots, amongst which is the minimum sought (and possibly other minima besides).

With S springs fitted, the corresponding equations are of the form:

$$F_s(k_1,k_2,\ldots,k_S) = 0 \qquad s = 1,2,\ldots,S. \qquad (24)$$

where F_s is biquadratic in all the stiffnesses k_1 except k_s in which it is quadratic. The stationary values of this set contain, in general, 2^{2S} solutions. Even in the simplest case of multiple springs, $S = 2$, the computation of the roots of equations (23) is difficult, and for $S > 2$, any procedure is unworkable.

A preferred procedure is to examine each spring in turn, in an order indicated by ranking according to its effect on M when acting alone. For the moment we will ignore any restrictions on the allowable spring changes and consider only open variation of the k's.

The procedure runs as follows:

(1) Establish a basic set of receptance combinations which suffice to describe the behaviour of the structure (see below)

Do (2) and (3) below for $s = 1,2,\ldots,S$.

(2) Select optimum value of k_s.

(3) Update (1) for chosen value of k_s.

(4) Increase s and repeat previous steps.

For each spring, v is either 1 , in which case the system is optimal for that spring, or $v < 1$ and an improvement is possible. Thus, in a cycle of changes to all S springs in order, either $v = 1$ for all springs and no further improvement is possible, or v is reduced. But since $v > 0$, the process must tend to a limit and an optimal system can be found. This minimum may be a local minimum or a global one. For example, if $S = 2R$ it is reasonable to look for solutions in which the response at the R points is vanishingly small. In the case of two springs and a single response freedom, zero response may be achievable - in which case there are two sets of spring values that suffice - or it may not in which case non-zero minima exist. In a more practical situation - at least as far as the helicopter is concerned - R will tend to be greater than $S/2$ and one seeks a non-zero minimum.

A particular example will illuminate this point, but before we examine it, some computational notes are appropriate. For R response points and S springs each cycle outlined above requires the update of a set $\sum^*$ consisting of

(1) R values of G_{rF}

(2) S values of $G_{pF} - G_{qF}$

(3) RS values of $G_{rp} - G_{rq}$

(4) $S(S + 1)/2$ values of $H(s,t) = H(\widehat{pq},\widehat{uv})$

$$= G_{pu} - G_{pv} - G_{qu} + G_{qv} \quad .$$

These $S(S + 3)/2 + R(S + 1)$ combinations of receptances are the only ones which we need to consider in a particular study. The result of introducing a stiffness change k in $\widehat{pq}$ is to change any G_{ij} say, to

$$G_{ij} + UF(G_{ip} - G_{iq})(G_{jp} - G_{jq}) \quad ,$$

where UF is the update factor

$$UF \quad = \quad - k\left(1 + k\left[G_{pp} - 2G_{pq} + G_{qq}\right]\right)^{-1} \quad . \tag{25}$$

Then

$$G_{rF} \rightarrow G_{rF} + UF\ G_2(r) \tag{26}$$

$$G_{uF} - G_{vF} \rightarrow G_{uF} - G_{vF} + UF\ H(\widehat{pq},\widehat{uv})(G_{pF} - G_{qF}) \tag{27}$$

$$G_{ru} - G_{rv} \rightarrow G_{ru} - G_{rv} + UF\ H(\widehat{pq},\widehat{uv})(G_{rp} - G_{rq}) \tag{28}$$

$$H(\widehat{uv},\widehat{mn}) \rightarrow H(\widehat{uv},\widehat{mn}) + UF\ H(\widehat{pq},\widehat{uv})H(\widehat{pq},\widehat{mn}) \quad . \tag{29}$$

In the course of evaluating k for $\widehat{pq}$, $G_2(r)$ given by equation (3) has been evaluated. All other items in equations (26) to (29) are found in the set $\sum^*$ so that the updating process is straightforward.

4 ILLUSTRATIVE CASE, S = 2

For discussion purposes, a case with two springs k_1, k_2 only, is sufficiently general to illustrate a number of practical points. The database used is that of the Lynx stick model (Fig 1), similar to that used by

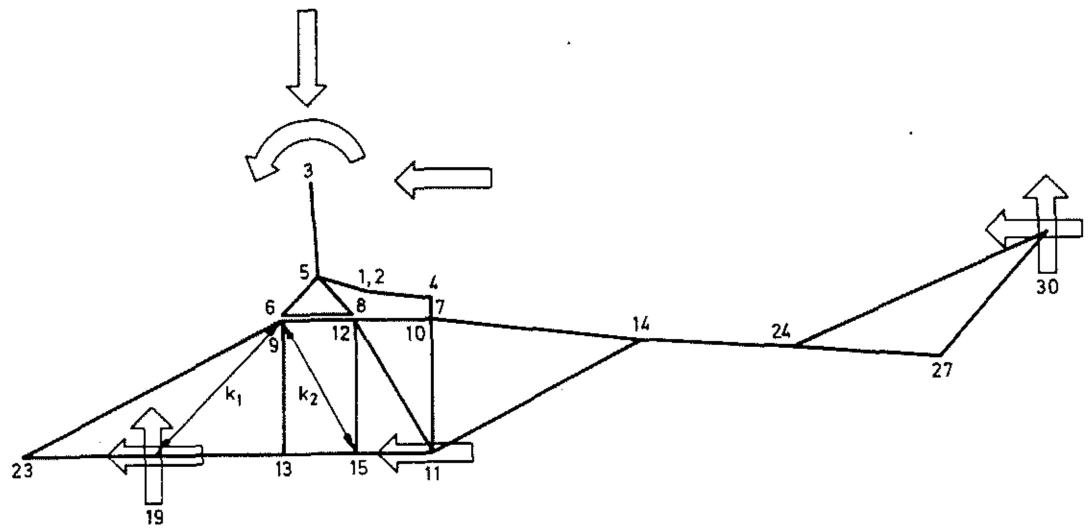

Fig 1 Helicopter stick model

Done (1981), but with an assumed damping. It is used for discussion purposes only and no inferences on the real behaviour of the Lynx helicopter, or of putative improvements in its ride quality are to be made. Indeed, a two-dimensional model is quite unsuitable for that purpose as the nature of the forcing is essentially three-dimensional and induces a structural response asymmetry from side to side within the helicopter.

The case discussed has:

Forcing at node 3 (rotor head)	1 unit F/A at zero phase	
	2 units vertically at 42 degrees	
	25 units pitching at 116 degrees.	
Response	Node 19 F/A	weight 0.8
	Node 19 vertical	weight 1.2
	Node 11 F/A	weight 1.0
	Node 30 F/A	weight 2.0
and	Node 30 vertical	weight 1.5.

The Bode diagram giving the variation of M as a function of frequency is shown in Fig 2 and it is apparent that the response function is strongly influenced by the bending mode some way below nominal exciting frequency. It is thus clear that some improvement is to be expected by an appropriate unstiffening of the structure, and this is verified in Fig 3 which shows a contour map of the measure function M of equation (7) as a function of k_1, k_2, where k_1 is the stiffness change in the member 9-19 and k_2 that of 9-15.

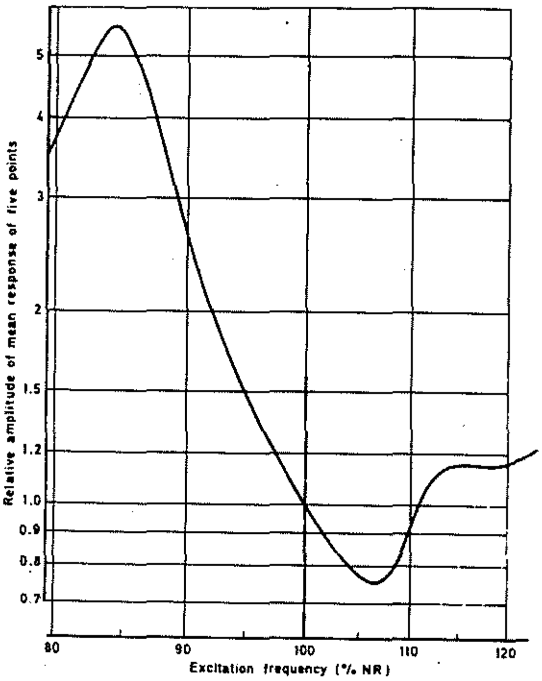

Fig 2 Response of unmodified helicopter as a function of exciting frequency

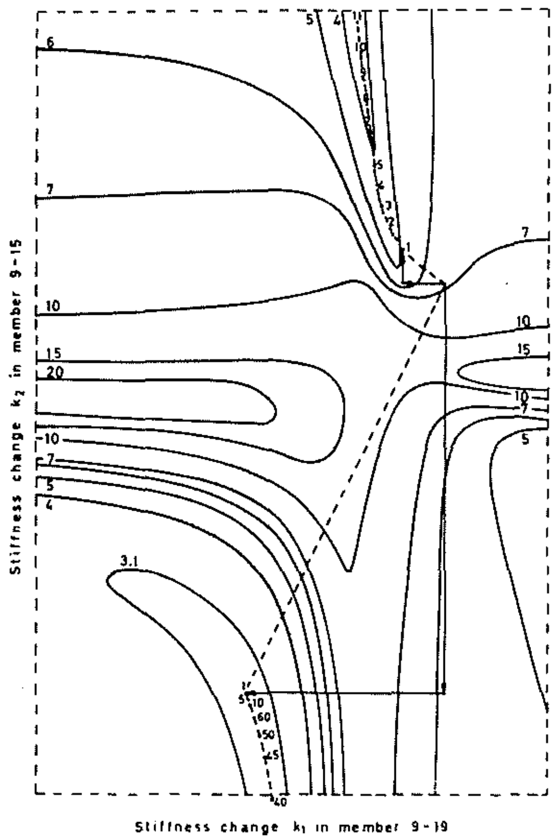

Fig 3 Contour map of measure function for varying k_1, k_2

The sections of this map for $k_1 = 0$ and for $k_2 = 0$ are shown in Fig 4. We note that (1) each has, as expected, a single maximum and minimum

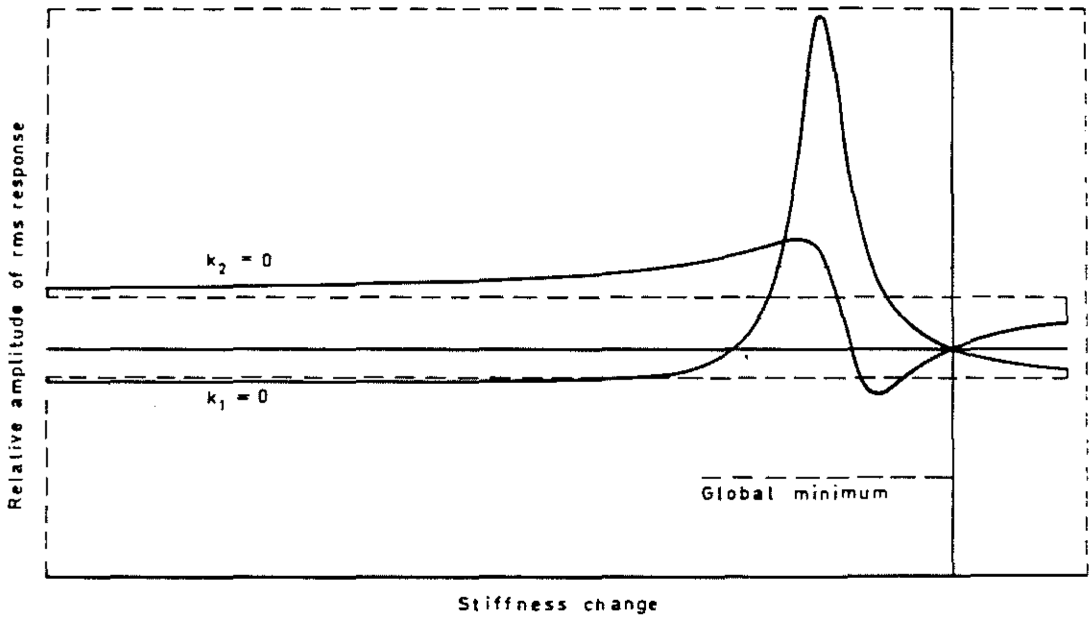

Fig 4 Sections through contour map for $k_1 = 0$ and $k_2 = 0$

(2) the minimum for k_1 varying is sharply defined, but for k_2 varying is not so, and differs but little from the asymptotic value for infinitely large values of k_2 .

Suppose that k_1 is first chosen with $k_2 = 0$; then, with k_1 fixed, k_2 is varied. The resultant change in the double step is a small one to the north-west, heading into a valley which eventually runs north-south. Progress along this course is indicated by the numbering 1,2,... etc. Varying first k_2 and then k_1 achieves two surprising effects. Firstly, the high ground to the south is crossed, and a vicinity of the minimum is reached in only one step. After ten steps the minimum is, for all practical purposes, attained.

To reach this minimum by variation of k_1 , then k_2 , it is necessary to leave the northern valley, go round the world, and re-enter its continuation to the south of the true minimum so that in effect there is a vertical asymptote in the locus of steps. Thus there is a certain value of k_1 , say k_1^* for which k_2 is in effect infinite, when the spring ceases to function in the normal way and imposes a kinematic constraint equalizing displacements at the two ends. For values of k_1 to the east of k_1^* we are in the northern valley; for k_1 to the west of k_1^* we are in the southern one. How do we manage to cross this line?

It is only by rounding error within the computer that such a change is possible, for if our analysis were free from approximation, then the k_1 - then - k_2 process would stay firmly to the east of k_1^* in the northern valley. Progress would become ever decreasing (in the k_1 change) and k_2 would increase indefinitely. Similarly we would follow that the valley with a normal minimization routine, heading for the falling ground to the north-west of the starting point.

In a situation where it is clear that we are heading into a valley and substantially moving in a fixed direction, we may either be in the vicinity of the minimum and converging slowly, or heading up an asymptote. In either case, advantage may be taken of this recognized behaviour and the search procedure may be speeded up by shooting.

5 SHOOTING

Suppose that, in the neighbourhood of a minimum, the contours are simple closed curves, all similar, and similarly situated. To be specific, we take them to be elliptical, although the subsequent argument is general.

A series of steps corresponding to searches along two directions in turn is illustrated in Fig 5.

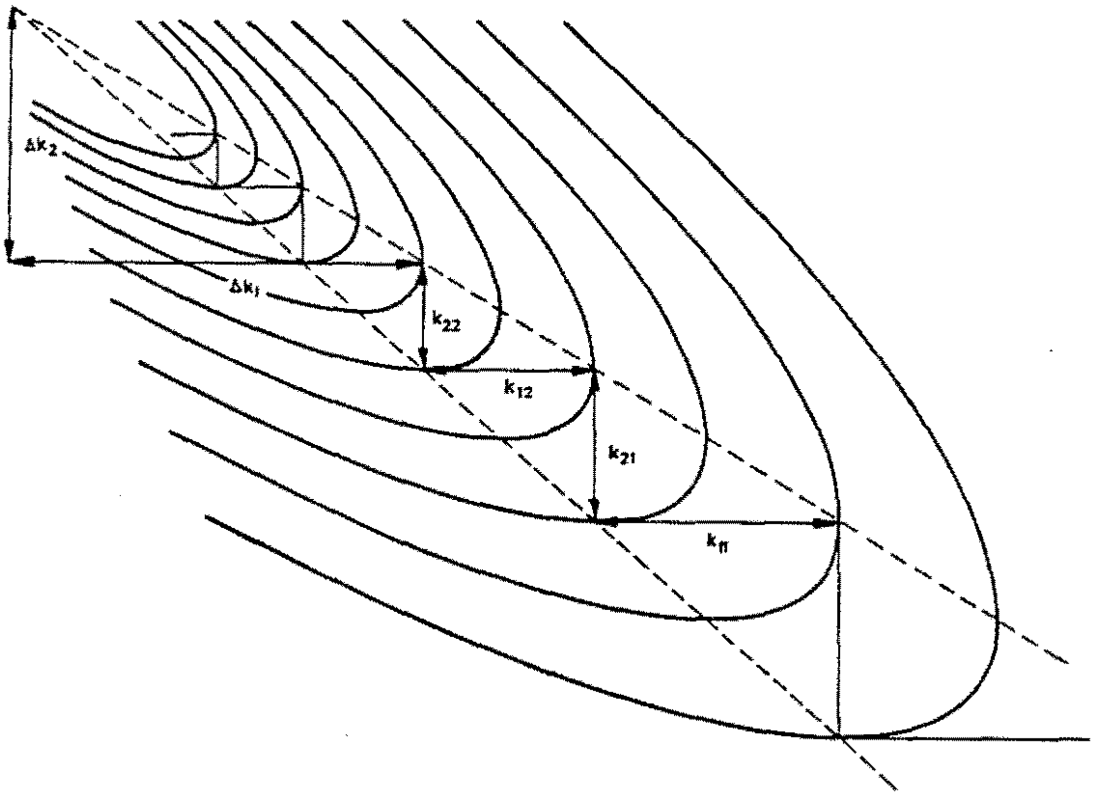

Fig 5 Convergence process near global minimum

Here the law of diminishing returns applies: as we approach the minimum the changes in each k get smaller, and although we are heading towards a minimum, this is never attained. (Had we searched along directions parallel to the axes of the ellipses, convergence in two steps is assured.) To avoid the consequences of a diminishing return, we may, at any stage, recognize the pattern of behaviour and extrapolate forward, or 'shoot'.

In the example of Fig 5, suppose that two complete cycles of iteration are associated with changes:

k_{11}, in k_1 ; k_{21}, in k_2 , in the first cycle

and

k_{12}, in k_1 ; k_{22}, in k_2 , in the second cycle.

Then further increments

$$\Delta k_1 = (k_{12})^2/(k_{11} - k_{12}) \text{ to } k_1$$

and

$$\Delta k_2 = (k_{22})^2/(k_{21} - k_{22}) \text{ to } k_2 ,$$

take the current point to the minimum.

Where successive steps suggest a definite direction of search, the shooting procedure is usually effective, if not in finding a minimum, at least in moving the current point significantly towards the minimum. In an open-ended valley, as occurs in the discussion example, this accelerating procedure is very useful. Its value, however, depends on recognizing when a sufficiently well-defined direction of progress is established. The procedure extends readily to any number of dimensions.

6 LIMITS

In the examples which have been studied, the number of response points R has generally exceeded the number of springs S and only one minimum has been found. However speed of convergence is strongly affected by the order in which the springs are enumerated, as has been seen in the simple example of two springs described above.

Where limits are imposed on the allowed range of springs, several minima may exist and the order of search may now determine which minimum is reached. Taking the example of Fig 3 and imposing arbitrary limits as in Fig 6a, we find that searching first k_1, then k_2, leads to a minimum in the north west valley on the boundary. Taking k_2 then k_1, leads to a minimum in the south-east corner.

Suppose, now, that the limits are all doubled, as in Fig 6b. Taking k_1 then k_2, leads as before to a minimum in the north-west valley, but taking k_2 then k_1 leads to a minimum in the south-west corner. (In fact, quite close to the global minimum.)

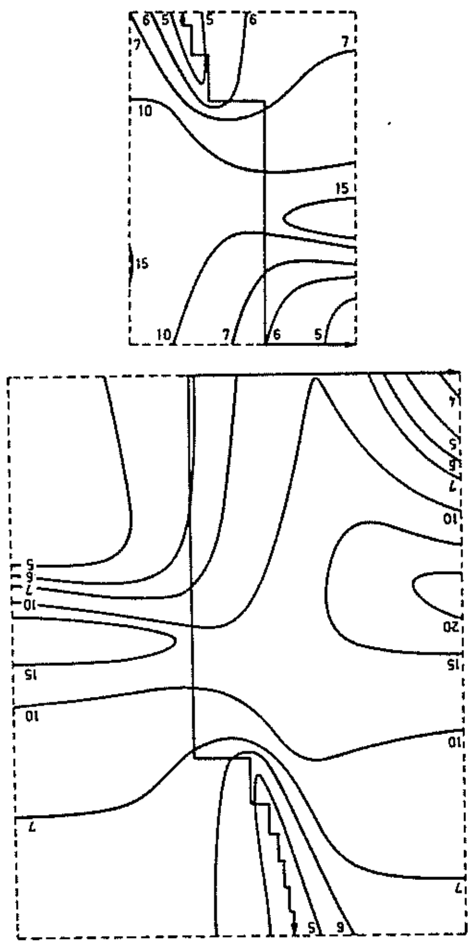

Fig 6a&b Contour maps with limited variation of k_1, k_2

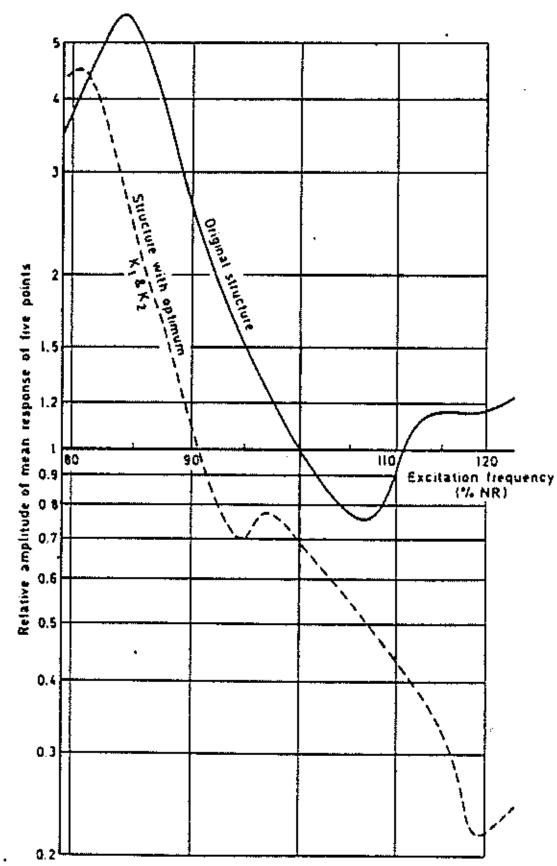

Fig 7 Response of original and of modified structure as a function of excitation frequency

These simple, if artificial, examples show the importance not only of order of search but of the limits themselves. In the first example, approximately equal minima are indicated, each with one increase, one decrease in stiffness. The second example produces significantly different values of the minimum.

This experience has been borne out in investigations of a more comprehensive character with several springs and confirms that both search direction enumeration and the value of the limits are important. The former difficulty can be avoided by repeating the calculation with a different search order, but the problems that arise with the choice of limits can only be resolved with attention to detail.

In particular, it is necessary to look more widely at responses in the helicopter than those included in the measure function. Where there are two apparently equal possible configurations, it will be found that there is a marked difference in the process by which a minimum has been achieved and that consequences away from monitoring points cannot be overlooked.

In Fig 7, the Bode diagram of Fig 2 is recalculated for the structure when modified by the fitting of the two springs k_1 and k_2, with their values adjusted for minimum response at the rotor speed NR . It is clear that a reduction in response of some 30% is possible at NR . However, the response continues to fall with increase in rotor speed to a minimum of about 25% of the original value. This confirms that the choice of springs (9-15 and 9-19) is quite inappropriate for reducing the measure function substantially. It is clear that other members (like 24-30, for example) would be far more effective to change, but the two springs chosen were selected for their usefulness in illustrating points of numerical procedure, and not for their effectiveness in securing a substantial reduction in the response of the helicopter.

To develop these ideas further would take us too far from the main theme and will not be discussed here.

7 VERIFICATION

Since the claims of response improvement lead to a set of spring values which change the basic structure, the modified structure can be analysed *ab initio* using, for example, NASTRAN, and the responses of the revised structure can be examined independently of any analysis used in the selection of the springs.

The analytical value of such a check is, however, only partial, in as much as the theory presupposes a valid dynamic model to start with. Much more valuable would be a programme of work which begins with a piece of real structure, whose modes are identified, and which is analysed as above. The promised improvement in response can then be verified experimentally, thus consolidating the design value of the procedure described. Such a programme of work is in hand in the Helicopter Division of Materials and Structures Department at the Royal Aircraft Establishment.

8 CONCLUSION

In this note, a procedure is presented for the choice of a set of springs in order to achieve a minimal weighted mean square response at a number of locations throughout the airframe over a range of forward speed. An application of a computer program embodying these ideas is presented, and some computational aspects are discussed.

9 REFERENCES

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