Buckling Behaviour of Composite Shells under Combined Loading

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ABSTRACT

The research project presented in this paper aims at improving the knowledge of the buckling behaviour of composite shell structures through an extensive experimental analytical and numerical investigation. Comparison of the numerical and analytical results with the experiments will allow validation of the numerical tools that can be used in further wide parametric studies. The main objective is to produce strength design criteria for composite cylinders under combined loading. The current status of the project is presented and the first experimental and numerical results discussed. Furthermore, a theoretical investigation, performed to analyse the effects of the lamination geometries on the buckling and post-buckling behaviour of imperfect composite cylindrical shells is presented.

INTRODUCTION

The use of composite shell structures has been increasing in the last two decades mainly in the aircraft and spacecraft industry but the lack of generally applicable design criteria for composite shells and panels is currently an inhibiting factor in the efficient use of composite materials. Although the use of numerical simulations for the analysis of different types of civil, marine and aerospace engineering structures is commonly accepted as a replacement of expensive experimental investigations, some complex physical problems may only be solved by means of a combined experimental, analytical and numerical programme. A typical example is the buckling behaviour of shells.

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The main objectives of the present research are to improve the knowledge of the behaviour of composite materials in shell structures and to provide scientific background for a better exploitation of the material properties together with control of the influence of processing conditions on product performance [1]. Furthermore, the results will form suitable background material, through numerical and experimental studies, for the development of Eurocodes on composite shell structures and thin-walled components under combined loading.

The research is being developed along the following steps :

- evaluation of statistical properties of geometric imperfections on several series of cylindrical specimens made of composite materials with different lay-up configurations
- development of characteristic imperfection models for cylinders made of composite materials to include in strength prediction tools
- buckling tests of a series of cylindrical specimens with different stacking sequences and load combinations
- recording of the pre and post-buckling response of

these models together with the failure mechanism, in a form suitable for comparison with analytical and numerical models

- assessment of the reliability of buckling tests by comparing the results of nominally identical test components
- comparison of analytical and numerical predictions with the experimental results in order to validate the numerical models that can be used in further wide parametric studies
- development of strength design criteria for composite cylinders under axial compression, torsion and combined loading using both the experimental and numerical results.

EXPERIMENTAL PROGRAMME

Cylinder geometry and material properties

The cylindrical specimens considered in the present work are made with pre-preg fabric lay-up on a cylindrical mandrel and have the following dimensions

radius= 350 mm
total length = 700 mm

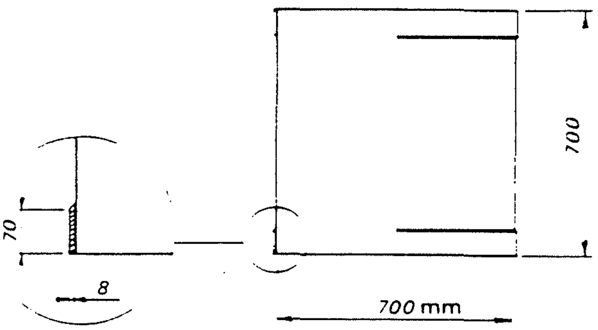


Fig. 1- Cylindrical specimen geometry

It must be noted that the cylinders present two thicker parts at the top and bottom to facilitate the fixing of the specimens into the loading rig (Fig. 1). As a consequence the actual length of the thin cylindrical shell is reduced to 550 mm. The layers are made of a orthogonal Kevlar fabric embedded into an epoxy resin matrix. The elastic properties of each lamina are the following

$E_1 = E_2 = 23450$ MPa
$\nu_{12} = 0.2$
$G_{12} = G_{23} = G_{31} = 1520$ MPa

being x_1 and x_2 the in-plane axis directed the fibers in the lamina, and x_3 is normal to the lamina midplane.

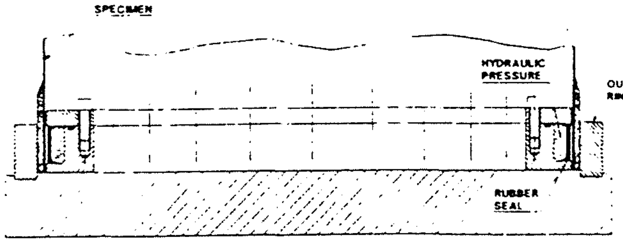
Cylindrical specimens made with the following stacking sequences are available :

a) cross-ply cylinders made with 4 laminae at $0°/90°$ to the cylinder axis (total thickness t=1.04mm)

b) angle-ply cylinders made with 4 laminae at $\pm 45°$ to the cylinder axis (t=1.04mm)

c) quasi-isotropic cylinders made with 8 laminae and the following stacking sequences $(45/-45/0/90)_S$ $(45/0/90/45)_S$ $(0/90/45/-45)$ (t=2.08mm)

Note that because of the particular elastic properties of each lamina the orientation $+/-45°$ and $0°/90°$ become coincident and the cylinder surface is actulally a thicker lamina with the same orthotropic elastic properties. This is not the case for other inclinations.



HYDRAULIC END GRIP

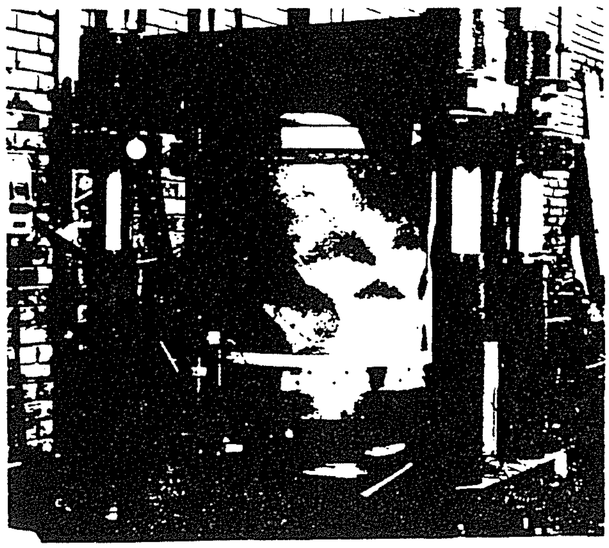


Fig.2- Loading rig

Test apparatus

The loading rig is shown in Fig. 2. Axial force is provided by a hydraulic ram, but the actual load applied to the specimen is controlled by four adjustable screw stops, acting on the four corners of the loading platform. Thus, the loading machine is displacement controlled with a very good accuracy. The lower end-plate is supported by three rollers laying on a horizontal surface (Fig. 3a), when torsion is to be applied by an independent mechanical system, or on sloped surfaces (Fig. 3b), for a fixed ratio of compression to torsion.

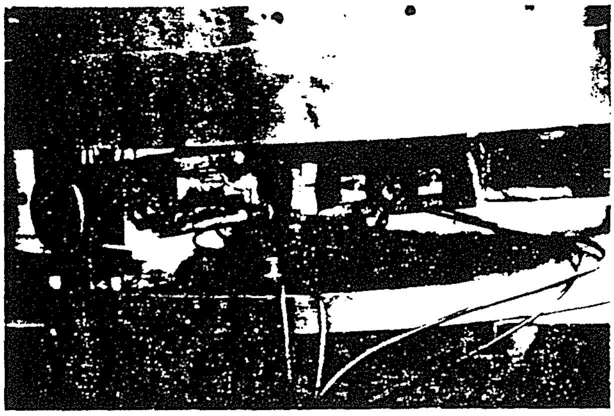


Fig.3a- Flat supports of the rollers for the loading cases of independent torsion and compression.

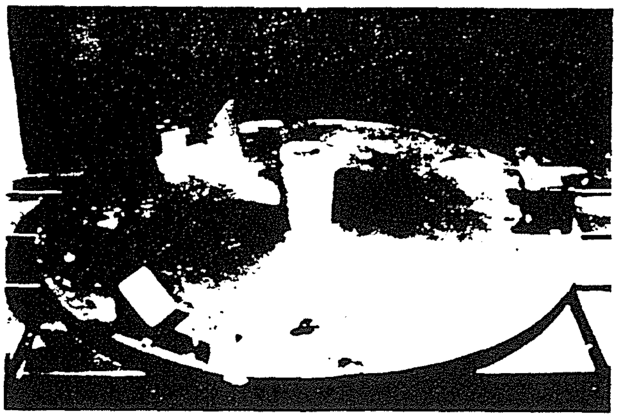


Fig.3b- Sloped supports of the rollers for the loading case of combined torsion and compression.

Compression load and specimen shortening are measured, during the test, on three equally spaced points, corresponding to the three supporting rollers. This gives a measure of the accuracy of the loading process, in terms of load and displacement uniformity.

Mapping of geometric imperfections

Shape imperfections have been detected by an ad-hoc designed apparatus (Fig. 4) where the outer and the inner surface of the shell are scanned by two LVDT transducers. Data acquisition and surface scanning are controlled by a PC, with an extreme flexibility on sampling pitch in both direction. Surface data are stored in a digital form suitable for subsequent computations.

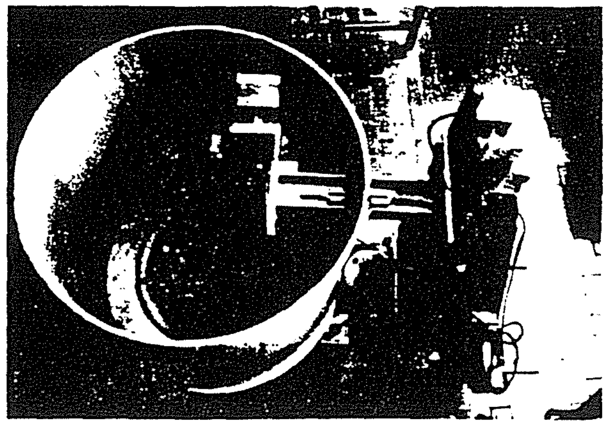
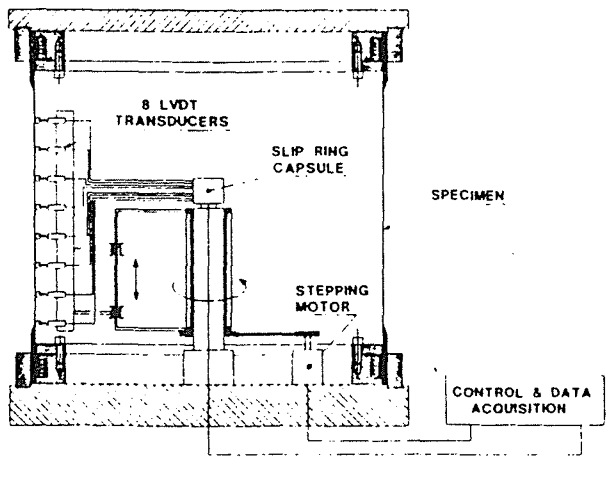


Fig.4- Apparatus for imperfection shape survey.

The imperfection surface of each cylinder has been recorded in a regular mesh with an interval of 1 cm axially and 2 cm circumferentially. This results in 110 measurements along any circumferential line and 46 measurements along any generator. At each point, measurements were taken on both the inside and outside cylindrical surface.



INNER SURFACE MAPPING IN THE LOADING RIG

ANALYSIS OF GEOMETRICAL IMPERFECTION MEASUREMENTS

General methodology

Imperfection sensitivity has long been recognised as the main factor for discrepancies between experimental buckling loads and analytical predictions of shell structures, in general, and of cylindrical shells subject to meridian compression, in particular. In recent years, significant effort has been directed at detailed measurement of imperfections on cylindrical test specimens, as well as some full-scale components [2]. In most of these studies a standard method of data analysis has been adopted, based on the concept of 'best-fit cylinder'[3]. Thus, the 'raw' imperfections obtained from LVDT readings form the input data to a program that calculates a 'best-fit' cylinder through the entire grid of measured points and then re-computes the imperfections from this artificial 'perfect' surface.

This concept has enabled a unified datum to be established for shell imperfections and can be of particular use in comparative studies [2].

Following the 'best-fit' procedure, the resulting imperfections are analysed using two dimensional harmonic analysis to produce a set of Fourier coefficients, i.e.

$$w_0\,(x,\theta) = \sum_{m=1}^{\bar{m}} \sum_{n=0}^{\bar{n}} \xi_{mn} \sin \frac{m\pi x}{L} \sin(\, n\theta + \phi_{mn})$$

where ξ_{mn} and ϕ_{mn} are the Fourier coefficients obtained by a discrete measurements of the imperfection function, $w_0(x,\theta)$, at a number of points on the cylinder surface ($0 \leq x \leq L$ and $0 \leq \theta \leq 2\pi$).

It is worth noting that the above expression represents a half-range sine expansion in the axial direction, thus, imposing zero imperfection values at the two cylinder ends. Although this is not strictly correct, the error introduced is confined to the end regions and, provided the number of terms calculated is not too small, is not significant. In fact, in the current programme both half-range (sine and cosine) as well as full-range expansions were evaluated by considering the following error function

$$e = \frac{1}{N} \sum_{i=1}^{N} (w_i^{BF} - w_i^{F})^2$$

where w_i^{BF} is the imperfection value at point i after 'best-fit' analysis, w_i^F is the imperfection value at point i using Fourier representation and N is the total number of imperfection readings on the cylinder surface.

In addition, comparison were made at points of maximum imperfection (inwards/outwards). In general, the half-range sine series offered the best alternative in terms of accuracy and compactness.

The advantage of the methodology described above is that information on imperfection modes and amplitudes can be easily introduced in analytical and numerical predictions of shells with measured imperfections and their effect studied parametrically. However, this method is particularly useful when imperfections are recorded on groups of similarly manufactured shells. It is then possible to apply statistical techniques on the calculated coefficients to arrive at characteristic imperfection models that are associated with the particular manufacturing method used [4]. This approach can also be used for single mode imperfection sensitivity studies and for quantifying the effects of multi-mode imperfection patterns on cylinder buckling strength [5]. The use of probabilistic methods in calculating the reliability of shells with random imperfections has been extensively studied by Elishakoff and Arbocz [5].

The current test programme is well suited for this type of analysis since it includes two groups of 'nominally identical' cylindrical specimens, the first consisting of sixteen cross-ply models $(0^\circ/90^\circ)_s$ and the second comprising fourteen angle-ply models $(45^\circ/-45^\circ)_s$. Thus, comparison of the characteristic imperfection models resulting from the different lay-up configurations can lead to a rationalisation of geometric tolerance specifications and inspection methods for composite shell structures [7].

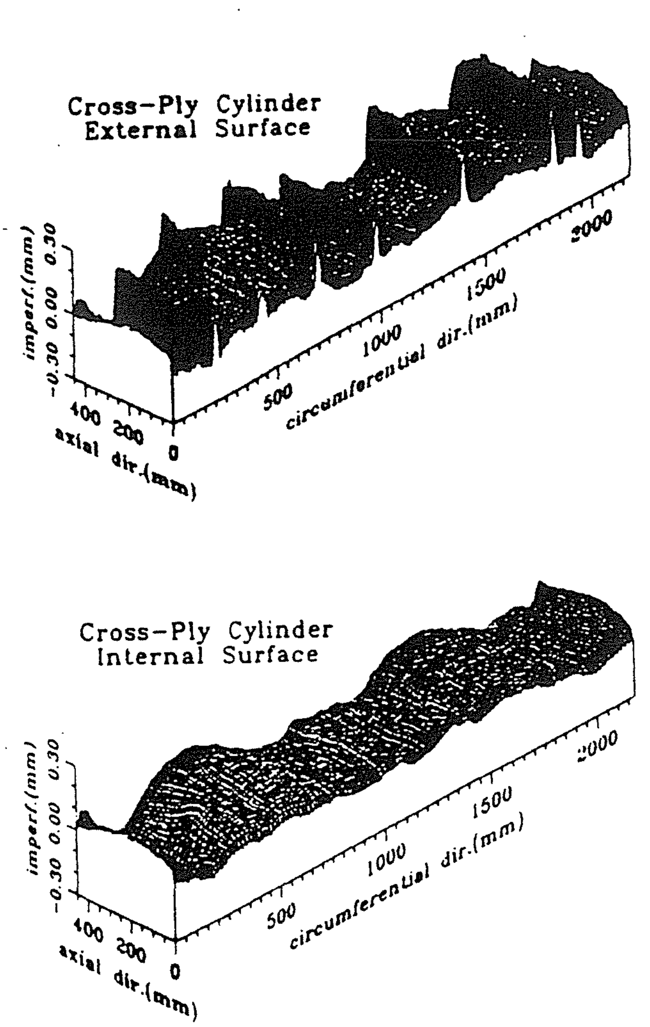


Fig.5a- Typical imperfection surfaces of cross-ply cylinders

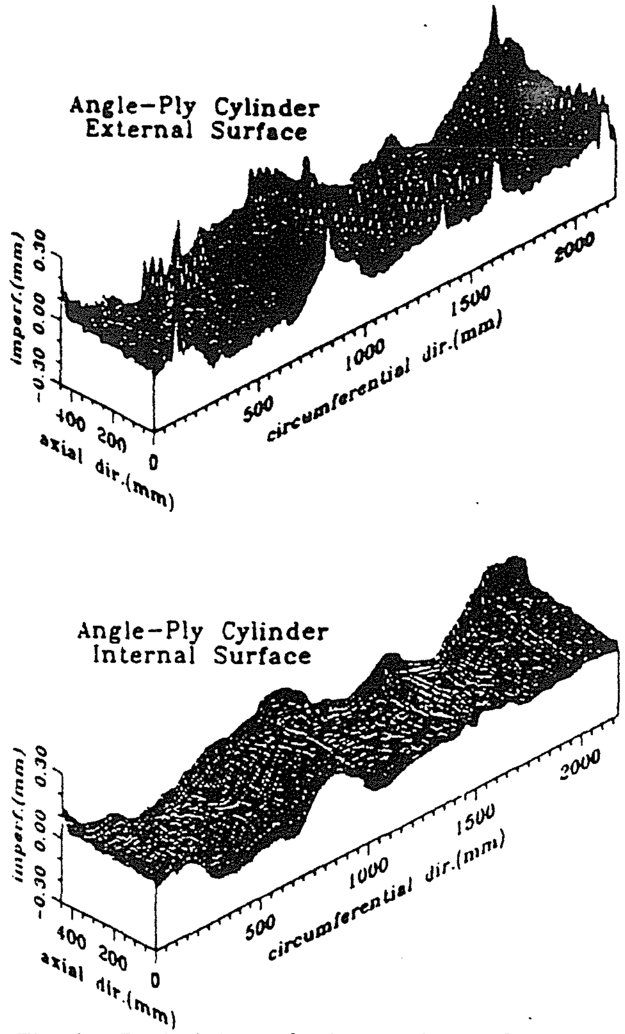


Fig.5b- Typical imperfection surfaces of angle-ply cylinders

Statistical analysis of geometric imperfections

Following the 'best-fit' analysis. two dimensional Fourier analysis was undertaken using the equation given in the previous section. Each surface was described by a set of coefficients. ξ_{mn} and ϕ_{mn}, with $\bar{m}=20$ and $\bar{n}=40$. These coefficients have subsequently been analysed using various statistical techniques in order to reveal common trends and identify important features that may be used in constructing suitable characteristic models [8].

Fig. 5a,b shows imperfection surfaces (after 'best-fit') obtained for typical cylinders in both Series A (cross-ply) and Series B (angle-ply). It is interesting to note that. although internal surfaces appear to have similar characteristics (dominance of long imperfections waves in both axial and circumferential directions), the external surface is strongly influenced by the orientation of individual layers. In fact. the sharp peaks obtained on the external surfaces are the result of local thickness variations due to overlapping layers and. hence. should not be treated as initial geometric imperfections.

In terms of Fourier coefficients. comparison of external and internal surfaces for the group of cross-ply models is made in Fig. 6a.b. As can be seen. the mean curves diverge when $n \geq 15$, demonstrating that short circumferential wavelength modes are only present on the external surface due to localised thickness variation. Similar observations were made on the second group consisting of angle-ply cylinder. On this basis. it may be concluded that in order to study the effect of geometrical imperfections. measurements on the internal surface have to be analysed. However, the effect of local overlapping should be noted in the analysis of experimental results.

Returning to the results for the internal surface shown in Fig. 6b, it is clear that dominant amplitudes are associated with long wavelengths in the circumferential direction. The extreme values associated with each mode are in agreement with the mean value trends. Angle-ply cylinders exhibit similar characteristic. This implies that the laminate configuration, provided that it is obtained using the same manufacturing method, does not have a significant influence on imperfection characteristics.

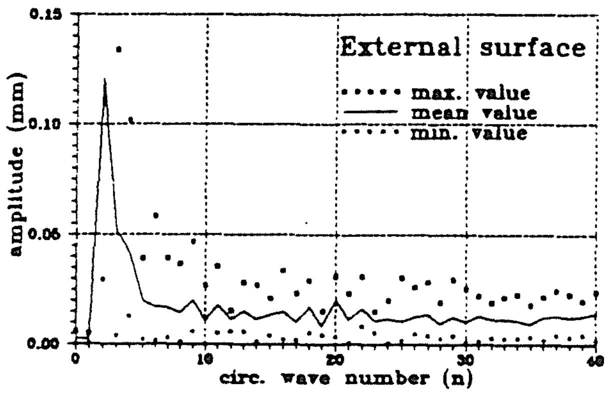


Fig.6a- Mean value analysis of imperfection modal amplitudes for external surface (Series A,m=1)

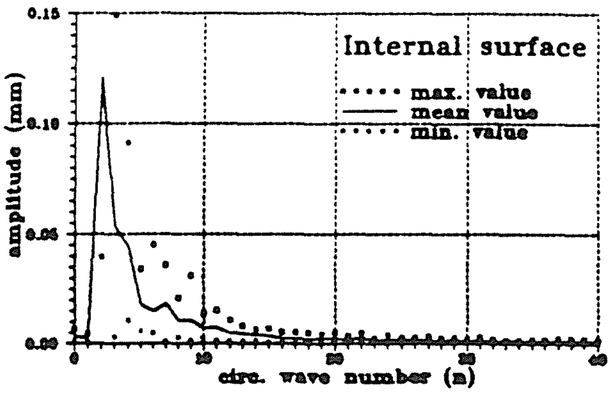


Fig.6b- Mean value analysis of imperfection modal amplitudes for internal surface (Series A,m=1)

It is of interest to note that the dominance of long circumferential wavelenghts has also been observed in isotropic cylinders [4]. On these grounds, it is suggested that the mean imperfection modal amplitudes can be modelled using simple expressions of the form

$$E(\xi_{mn}) = e^{\alpha n^{\beta}}$$

where α and β are constants evaluated from sample mean values.

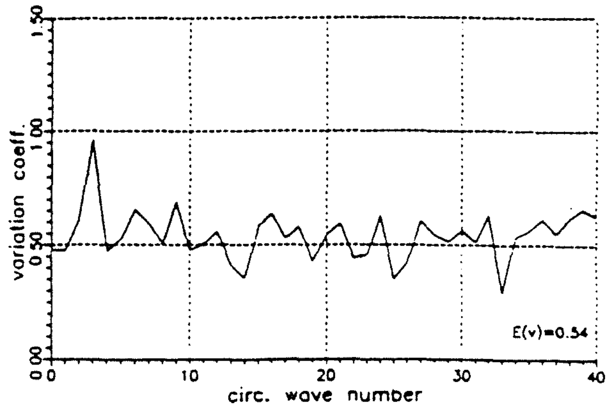


Fig.7- Varibility analysis of imperfection modal amplitudes (Series A, m=1)

Fig. 7 presents typical results of the variability analysis for cross-ply cylinders. It is seen that the coefficients of variation (standard deviation / mean value) does not exhibit significant varibility and, thus, a relationship that links the mean modal amplitude value to its standard deviation can be obtained using regression techniques, e.g.

$$\sigma(\xi_{mn}) = \gamma \, E(\xi_{mn})$$

where γ is a constant.

As demonstrated by Arbocz [2], the development of such simple expressions that contain the important features of imperfection amplitudes enables comparison of characteristic models due to different manufacturing methods to be readily undertaken.

Further to the univariate statistical analysis outlined above, correlation analysis between modal amplitudes was also undertaken. It was noticed that high correlation coefficients were obtained for modes with common circumferential wavenumber and odd axial wavenumber, i.e. $\rho(\xi_{1n},\xi_{mn})$ for m=3,5,.. .Correlation between modes with common axial wavenumber was generally much lower. Finally, in order to obtain a complete probabilistic description of modal amplitude variability, fitting of various probability distributions was examined and parameters of log-normal and Weibull distributions were estimated for the dominant modal amplitudes.

The results of the statistical analysis have revealed that, due to the common manufacturing process, several trends exist in the imperfection patterns. Various models have been developed that enable characteristic imperfection surfaces to be described. These will be used within numerical and analytical parametric studies to provide design recommendations for buckling of composite cylinders under combined compression and torsion loading.

TESTING RESULTS

The cylindrical models are tested under axial load, torsion and load combination [9]. So far 4 cylinders of the first series (cross-ply) have been tested under axial compression. A typical plot of the axial load versus average axial displacement of a tested cylinder is reported in Fig. 8. It is worth noting that the minimum value in the post-buckling curve is about 50% of the buckling load. Furthermore, the results from other nominally identical specimens are reported in Table 1 and compared to the theoretical buckling load. It is evident how all the results fall in the range between 27 and 29 KN. The plots relevant to the other cylinders present a linear prebuckling behaviour and a fairly flat post-buckling part similar to Fig. 8. It is worth noting that the detrimental effect due to imperfections is not very high for these laminae configuration (0°/90°) and corresponds to a "knock-down" factor of about 0.70. Fig 9 reports a typical buckling pattern of a tested cross-ply cylinder.

The results of the experimental programme will allow the definition of imperfection sensitivity curves for each different stacking sequence of these "nominally identical" cylinders [10]. The influence of the imperfection shape and amplitude of the thickness variation is being analysed also numerically for all the tested cylinders. The partial objective is to determine a general expression of the imperfection sensitivity for axially compressed composite cylindrical shells.

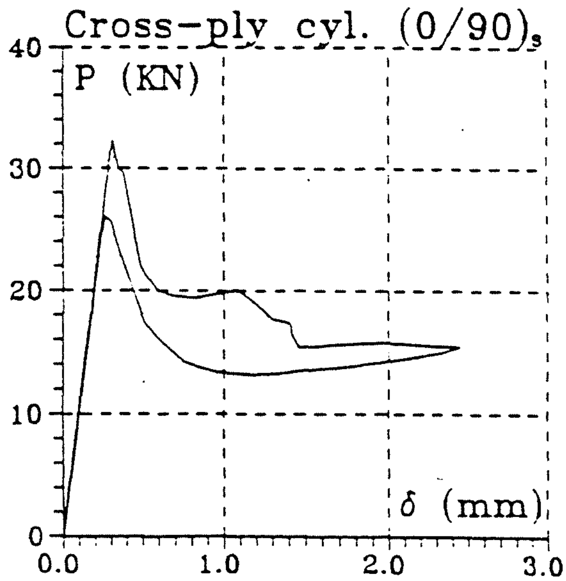


Fig.8- Load deflection curve of a tested cross-ply cylinder.

Fig.9- Buckling pattern of a tested axially compressed cross-ply cylinder.

Test results		
Lay-up	P_u (KN)	Exp/Theory
A) $(0^\circ/90^\circ)_s$	32.5	0.88
B) $(45^\circ/-45^\circ)_s$	31.8	0.86

Table 1 - Experimental results for axially compressed cross-ply cylinders compared to theory

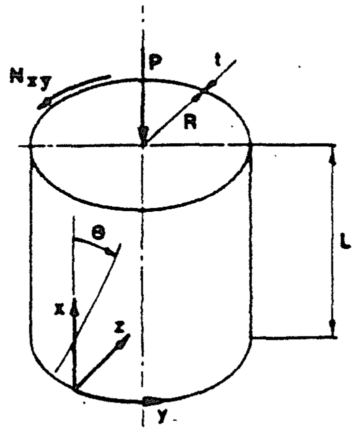


Fig.10- Cylinder geometry.

NUMERICAL AND ANALYTICAL PROGRAMME

The numerical study is being developed using two different and complementary numerical tools: a finite element program using composite shell elements and a specialised program to solve the buckling and post-buckling equations of composite cylinders. The latter has the advantage of allowing a description of the imperfections by means of analytical functions that, in particular, can be the same as those used in the biharmonic analysis of imperfections. Therefore it permits a better understanding of the influence of single imperfection modes, both axisymmetric and asymmetric, but, on the other hand, it is limited to shells with axisymmetric geometry.

In addition, numerical analyses using FE packages including the effects of the recorded imperfections of the cylinders are performed to simulate in detail the behaviour of the tested cylinders. The comparison will cover the load-deflection curves and the postbuckling deflections together with a check of the value of the stresses in certain points. The main objective of the comparison is to check the capabilities of the numerical tools in simulating and predicting the buckling behaviour of imperfect composite shells to allow a further extensive numerical parametric study to analyse the effect of all the geometric parameters. The effect of the boundary conditions will be also examined.

It is expected that the results of the parametric study will allow the formulation of analytical expressions and interaction diagrams suitable for design guidelines.

Governing equations and solution procedure

The Koiter's general theory of elastic stability has been applied to anisotropic shells. This theory allows to produce good indications of the nonlinear behaviour of imperfect composite shells but, on the other hand, the application of asymptotic procedures for the buckling and postbuckling analysis of shells involves manipulations of long and complicated expressions [11]. The use of modern symbolic manipulation programs facilitates this task and allows to derive error-free expressions in a quick and easy way [12].

A package for symbolic manipulation, recently adapted for personal computer, has been used to derive and solve the various sets of differential equations involved in the problem.

The Donnell type constitutive equations for axially compressed composite cylindrical shells may be expressed as follows

$$L_1(\phi) - L_2(w) = -1/2\, L_4(w,w)$$
$$L_3(w) + L_2(\phi) + \lambda\, w_{,xx} = L_4(w,\phi)$$

where w is the component of displacement normal to the shell surface (Fig. 10), ϕ is the stress function and λ is the normalized buckling load. The expressions of the operators are reported in Appendix 1.

Buckling and initial post-buckling analysis

Assuming that the eigenvalue problem for the buckling had a single solution, an asymptotic perturbation method has been applied to investigate the buckling and post-buckling behaviour of composite cylinder. A solution of the problem may be expressed in the form of the following asymptotic expansion

$$\lambda = \lambda_c + \lambda_1 a + \lambda_2 a^2 + \dots$$
$$w = w_0 + w_1 a + w_2 a^2 + \dots$$
$$\phi = \phi_0 + \phi_1 a + \phi_2 a^2 + \dots$$

The set of linear buckling equations results to be

$$L_1(\phi_1) - L_2(w_1) = 0$$
$$L_3(w_1) + L_2(\phi_1) + \lambda_c \, w_{1,xx} = 0$$

Assuming that the buckling mode is represented by the following functions

$$w_1 = \sin \frac{m\pi x}{L} \sin \frac{ny}{R}$$

$$\phi_1 = \gamma \sin \frac{m\pi x}{L} \sin \frac{ny}{R}$$

The coordinates and geometric parameters are those reported in Fig. 10.
The critical values of λ for given values of the longitudinal and circumferential numbers of waves (m,n) are given by

$$\lambda_c = -\frac{\gamma}{R} + \left(\frac{L}{m\pi}\right)^2 [\, \bar{D}_{(m,n)} + \gamma \, \bar{B}_{(m,n)}]$$

where: $$\gamma = \frac{-\frac{m^2\pi^2}{L^2 R} + \bar{B}_{(m,n)}}{\bar{A}_{(m,n)}}$$

and the algebric operators $\bar{A}$, $\bar{B}$ and $\bar{D}$ are reported in Appendix 1.

The governing equations of the second order are the following

$$L_1(\phi_2) - L_2(w_2) = -1/2 \, L_4(w_1,w_1)$$

$$L_3(w_2) + L_2(\phi_2) + \lambda \, w_{2,xx} = L_4,(w_1,\phi_1)$$

They admit solutions of the form

$$\phi_2 = \sum_{i=1}^{\infty} \left\{ B_{0i} + B_{2i} \cos \frac{2ny}{R} \right\} \sin \frac{i\pi x}{L}$$

$$w_2 = \sum_{i=1}^{\infty} \left\{ C_{0i} + C_{2i} \cos \frac{2ny}{R} \right\} \sin \frac{i\pi x}{L}$$

Finally the value of the second-order coefficient λ_2 can be worked out

$$\lambda_2 = \sum_{i=1}^{\infty} \frac{4\,n^2}{\pi R^2} \left[\frac{2i(\,B_{0i} + 2\,C_{0i}\gamma)}{(i^2 - 4m^2)} + \frac{B_{2i} + 2\,C_{2i}\gamma}{i} \right]$$

These coefficients will allow to produce the limit loads for cylinders with an initial imperfection similar to the corresponding buckling mode.

Analytical results

So far a study of the influence of the fiber orientation on the linear buckling load of an axially compressed cylinder has been undertaken but it is expected to extend the analysis to pure torsion and combinations of axial load and torsion [13]. The examined lamination geometries include all the angles between 0° and 45°.

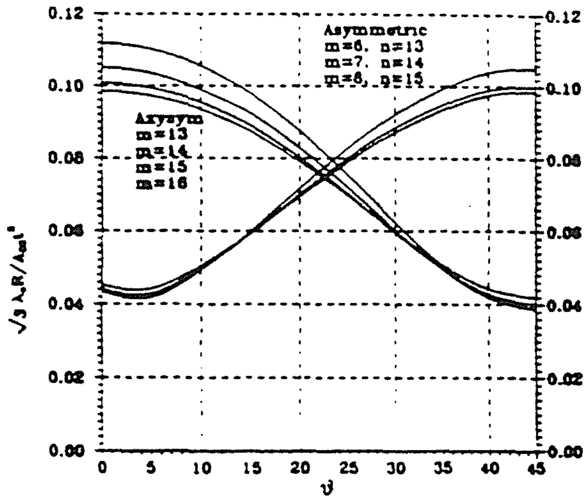


Fig.11-Buckling loads for a cylinder made with four +/- fabric plies.

The linear buckling loads of axially compressed cylinders have been calculated by means of the expression reported above. It was assumed that the coupling stiffnesses are very small and thus negligible. It is known that this is actually true only for α=0° and 45° or for symmetric angle-ply laminates made with many layers. The results of

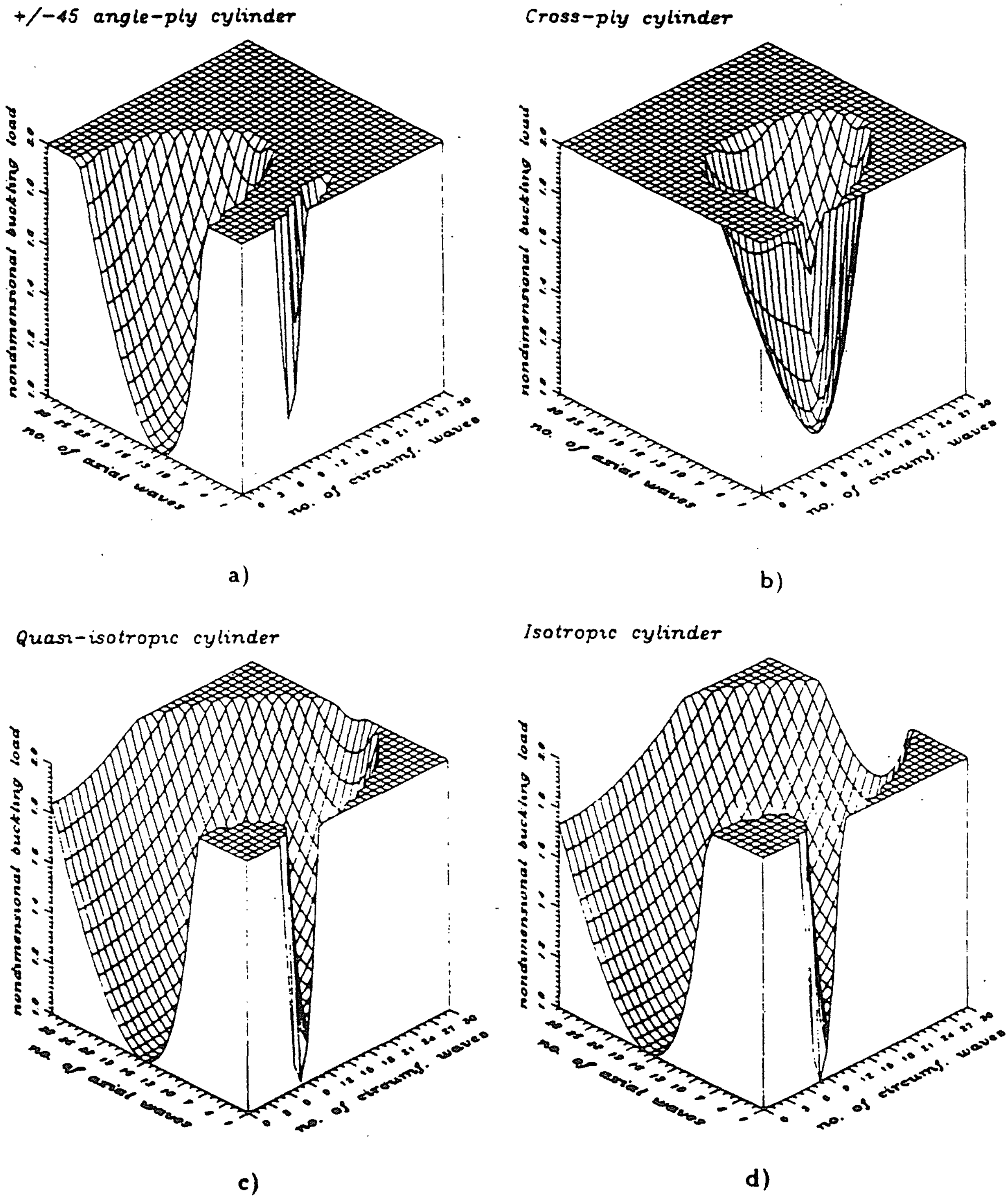


Fig.12a-d Normalized buckling load for composite and isotropic cylinders as a function of the circumferential and axial wavenumber.

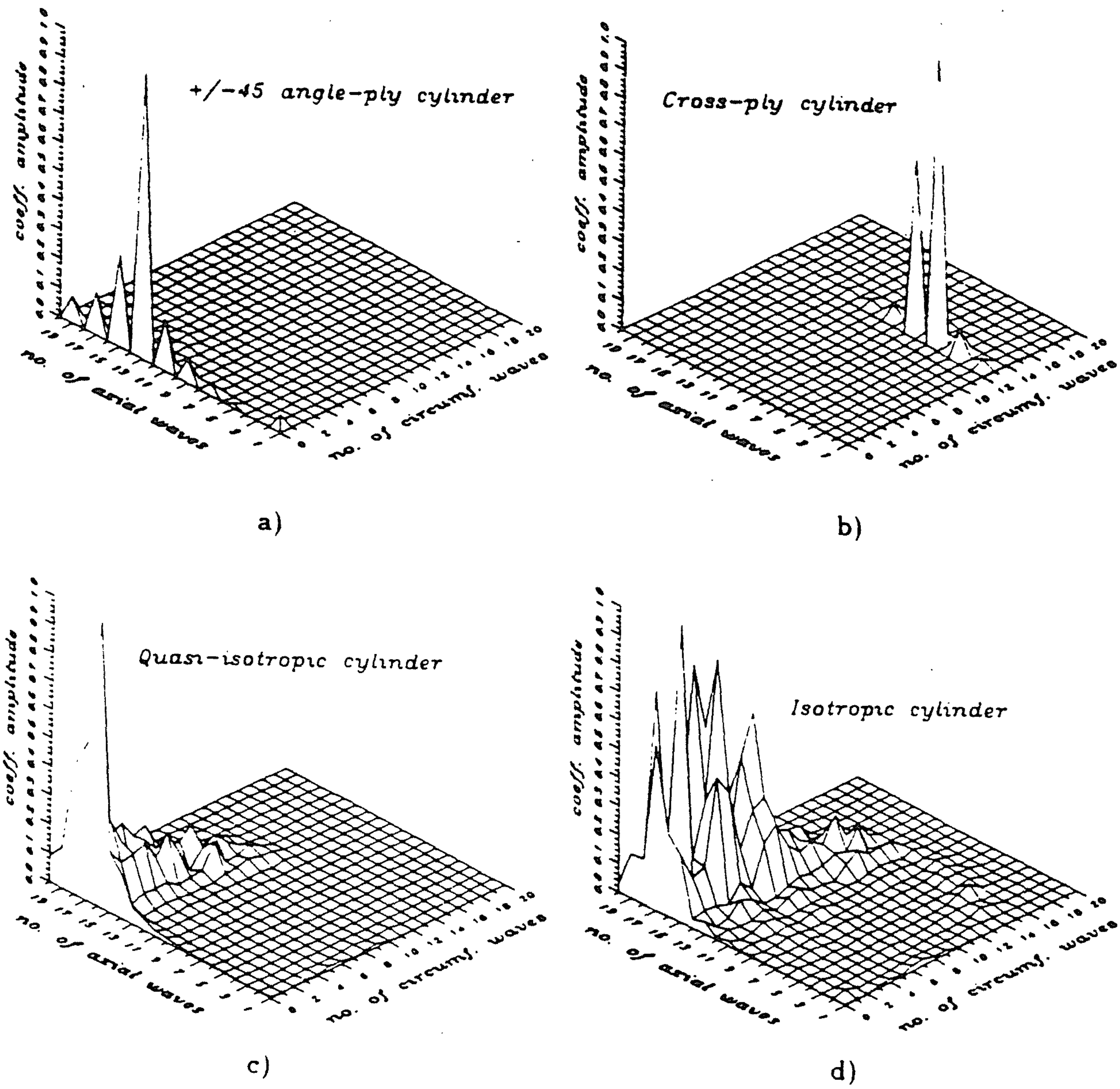


Fig.13 a-d Fourier analysis of the buckling modes obtained with a finite element model.

the analysis are reported in Fig. 11. The eigenvalue problem yields a single buckling load associated to axisymmetric modes with short axial wavelength for fiber orientation angles between 22.5° and 45° while for α angles in the range between 0° and 22.5° the buckling modes become asymmetric. Fig. 11 shows that the optimal stacking sequence in terms of buckling load is reached for an α value around 23° but it must be noted that in this region several different modes (both axisymmetric and asymmetric) correspond to almost equal buckling loads. Therefore a higher imperfection sensitivity is expected because of the nonlinear interaction of these modes.

A three dimensional map of the normalized eigenvalues relevant to various modes is reported in Fig. 12a,b for an angle-ply $(\pm 45°)_S$ and a cross-ply $(0°/90°)_S$ stacking sequence. It is evident how in both of them a single critical mode is well localized. On the contrary, a quasi-isotropic cylinder with the stacking sequence $(45°/-45°/0°/90°)_S$ shows various simultaneous buckling mode in a fashion very similar to the isotropic cylinder (Fig.12c,d). In both these figures the typical valley corresponding to the Koiter circle is clearly shown.

This characteristic can be very important in determining the imperfection sensitivity of each laminated geometry. In fact it is known that some composite cylinders may be as imperfection sensitive as an isotropic cylinder [13] and the quasi isotropic lamination geometry is expected to be very imperfection sensitive.

Finite element results

It is known that the actual buckling mode can be different from that obtained analytically with a single mode analysis mainly because of the coupling of several modes. For this reason and for comparison purposes, the same problem has been studied also using a finite element model. After having verified that the results are very similar in terms of buckling loads, the attention has been focused mainly on the shape of the eigenmodes. To compare with the theoretical results, a Fourier analysis was performed on the finite element eigenmodes. The plots in terms of Fourier coefficients are reported in Fig. 13a-d. In particular, Fig. 13a.b are relevant to the angle-ply (±45°) and cross-ply cylinders respectively. It is clear that the same buckling modes identified with the theoretical model have been picked up. The dominant mode with highest Fourier coefficient for the cross-ply cylinder (Fig. 13a) includes 6 axial half waves and 13 circumferential full waves while the dominant mode for the angle-ply cylinder is axisymmetric with 13 axial half-waves. In Fig. 13c,d the results of similar analyses for the quasi-isotropic cylinder and for an equivalent isotropic steel cylinder are reported. It is evident that the distribution of the active modes for the quasi-isotropic composite cylinder includes much more modes than the two previously examined composite configurations and is very similar to the isotropic.

The research will include the analysis of single imperfection modes and their combinations on the buckling load. The choice of the imperfection modes will be also suggested by the statistical imperfection analysis performed on each series of nominally identical cylinders. Both commercially available programs [14] and in house packages [15] will be used for this study.

CONCLUSIONS

A research project aiming at improving the knowledge of the buckling behaviour of composite shell structures has been presented. The experimental programme has been described and the first test results obtained for axially compressed composite cylinders made with a cross-ply stacking sequence have been presented. Geometrical imperfections have been extensively analysed using statistical methods in order to quantify their effect on buckling strength. The analytical procedure derived to analyse the buckling and post-buckling of the cylinders has been summarized and the available results have been compared to finite element solutions. The research will include other stacking sequences and loading cases such as torsion and combination of axial load with torsion.

Appendix

The operators have the following form

$$L_1(\cdot) = A^*_{22}\,(\cdot)_{,xxxx} + 2\,(A^*_{12} + \tfrac{1}{2}A^*_{66})\,(\cdot)_{,xxyy} + A^*_{11}(\cdot)_{,yyyy}$$

$$L_2(\cdot) = B^*_{21}\,(\cdot)_{,xxxx} + (B^*_{11} + B^*_{22})\,(\cdot)_{,xxyy} + B^*_{12}\,(\cdot)_{,yyyy} + \frac{1}{R}\,(\cdot)_{,xx}$$

$$L_3(\cdot) = D^*_{11}\,(\cdot)_{,xxxx} + 2\,(D^*_{12} + 2D^*_{66})(\cdot)_{,xxyy} + D^*_{22}(\cdot)_{,yyyy}$$

$$L_4(H,K) = H_{,xx}K_{,yy} - 2H_{,xy}K_{,xy} + H_{,yy}K_{,xx}$$

$$\bar{A}_{(m,n)} = A^*_{22}\left(\frac{m\pi}{L}\right)^4 + 2\,(A^*_{12} + \tfrac{1}{2}A^*_{66})\left(\frac{m\,n\,\pi}{LR}\right)^2 + A^*_{11}\left(\frac{n}{R}\right)^4$$

$$\bar{B}_{(m,n)} = B^*_{21}\left(\frac{m\pi}{L}\right)^4 + (B^*_{11} + B^*_{22})\left(\frac{m\,n\,\pi}{LR}\right)^2 + B^*_{12}\left(\frac{n}{R}\right)^4$$

$$\bar{D}_{(m,n)} = D^*_{11}\left(\frac{m\pi}{L}\right)^4 + 2\,(D^*_{12} + 2D^*_{66})\left(\frac{m\,n\,\pi}{LR}\right)^2 + D^*_{22}\left(\frac{n}{R}\right)^4$$

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