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AUTOMATIC GENERATION OF EQUATIONS FOR ROTOR-BODY SYSTEMS WITH DYNAMIC INFLOW FOR A PRIORI ORDERING SCHEMES

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ABSTRACT

In helicopter dynamics research with interpretive models, the process of manually deriving state equations for a priori ordering schemes is tedious and of limited reliability. The feasibility of its computerization by a completely self contained symbolic processor in FORTRAN IV is discussed. The symbolic manipulation details are presented for a coupled rotor-fuselage system with dynamic inflow in forward flight. The coupled system refers to a rotor with rigid blades executing lag and flap motions and to a fuselage idealized as a simple rigid body executing roll and pitch motions. The feedback from dynamic inflow refers to a first-order model based on an unsteady actuator-disk theory. The use of such a processor offers considerable promise in that for an adequate model representation, state equations can be generated for a priori ordering schemes as required in stability, vibration and combined stability and vibration analyses.

1 Introduction

During the past ten years, the helicopter industries have been vigorously pursuing the development of 'hingeless-type' helicopters[1-4]. Still, most of the newly assembled rotorcraft did not meet their air resonance and vibration specifications[1-4]. Usually remedial measures were initiated after initial flight testing, involving "intensive, costly, time and payload consuming efforts"[3].

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Rather disquieting is the widely varying experience in the analysis, design and implementation of such measures[1-4].

Resonance and rotor induced vibrations have always been serious problems of rotorcraft development. Though much attention has been paid to these problems, the present state-of-the-art does not permit the development of future vehicles with the certainty of avoiding such problems. There are two main reasons for this situation[2-4]. First, most of the research including correlation with test data is confined either to highly complex global models that generate design data or to grossly idealized models that are based on a Coleman-type approach or on lumped-mass and hover-approximations[2-6]. A clearer and more complete understanding of the physics of these problems is precluded in the former case by model complexity and in the latter, by model crudeness. Second, it is recognised that "what is required is a third category of research"[2] with conceptual or interpretive models of "intermediate complexity"[2]. Such models provide a better understanding of air resonance and vibration phenomena with parameter and mode visibility and are better suited to parametric analyses of a wide range of configurations. However with consistent ordering schemes as required in stability, vibration and combined stability and vibration analyses[7,8], manual algebraic manipulation of their state equations is an awesome task and takes up bulk of the research effort. The necessity for the analyst to share the algebra with the computer has been emphasized[5,9,10], even e.g., for the rigid flap-lag model of the rotor with dynamic inflow and without the inclusion of body dynamics[9,10]. In these calculations the related multiblade equations involved hundreds of hours of algebra and a determination of their accuracy by independent means.

A symbolic processor (or manipulation) is probably the means of enabling the analyst sharing the algebra with the computer[11-14]. It reduces the tedium of algebraic manipulations, increases the reliability of generated state equations and thus "allows the analytical work to be pushed further before the computations start"[11]. Its limited applications have been reported in several branches of science and engineering for over thirty years[11-14]. As to its application to helicopter dynamics research, only the barest beginning[15] has been made and the available information is not comprehensive enough to assess its feasibility in such research. Recently it has been synthesized as general purpose packages or 'catholic systems'[16] such as FORMAC (FORmula MAnipulation by Compiler), MYCSYMA (project MAC'S SYmbolic MAnipulator), etc.; for an in-depth review see references 11 and 12.

With the preceding background, we now come to the mode of assistance that is expected of a symbolic processor in this "third category of research"[2]. The viability of this research requires that "the improved comprehension of physical phenomena from interpretive models must be integrated into the global models in a timely manner as they are verified"[17]. Given the necessity of such a time frame, given the less than adequate progress thus far in air resonance and vibration research, now is the time to explore the feasibility of generating state equations through a symbolic processor. The question also arises: why not explore the use of one of the general purpose catholic systems of symbolic manipulation as is done in several studies[11-14]? A good example is the generation of finite element stiffness matrices with the help of MACSYMA[14]. "The numerous services provided by such a system tend to slow it down[3] and to increase the user's learning difficulties"[16]. Also, there still is much research needed in a comparative assessment of different systems[11-16] and of their suitability in this "third category of research". Meanwhile it does not seem realistic to burden this exploratory study with the subtleties of such general purpose packages. The usage of such packages is also partly precluded by other considerations as well--cost effectiveness, limited accessibility, core storage constraints, the need of specially trained personnel, unlikely standardization of symbolic manipulation languages[14,17] etc. The present feasibility study examines how a completely self contained symbolic processor can be developed as a natural predecessor to programming for numerical computations, and as a viable adjunct of helicopter research. Shorn of such general purpose packages, it shows that the automatic generation of helicopter state equations is basically no more involved than programming for numerical results in forward flight. Thus, it attempts to provide a means of utilising the vast potential of symbolic manipulation that hitherto remains practically untapped in helicopter dynamics research.

2 General Features

The present exploratory study centres around the development of a symbolic processor, called HESL - Helicopter Equations for Stability and Loads. Before turning to the programming aspects, we present, preparatively, an overall descriptive view of HESL in several respects: 1) The two processes of generating state equations and numerical computations are carried out on the same basis in that both are programmed in the same language, in the present study in FORTRAN IV.

2) The symbolic processor is completely self contained, and has the facility for easy modifications and additions. It does not require the assistance from any specific features of the hardware and operating system of the host computer (high portability). Therefore the expertise in programming and soft-and hardware requirements are basically no different from those for numerical computations.

3) Compared to "catholic systems"[16] such as MACSYMA, the symbolic processor is tailored to a class of selected models for a priori ordering schemes. By proper organization of execution steps, there is hardly any problem of intermediate expression swell[11] and excessive core demand.

4) The automatic generation of state equations in multiblade coordinates is illustrated for the air resonance analysis of a coupled rotor-body system with dynamic inflow (Figure 1) in forward flight. For the selected model in stipulated rotating and non-rotating reference frames the input parameters are basically i) the position vector $\bar{\rho}$, ii) the rotational speed vector $\bar{\Omega}$, iii) the flow description over the rotor disk in terms of μ, $\bar{\lambda}$ and ν and iv) the ordering scheme.

5) The process of deriving state equations for a priori ordering schemes comprises: i) energy expressions, ii) generalised aerodynamic forces, iii) the Lagrangian formulation, iv) perturbed linear equations and v) multiblade coordinate transformation. Partial or complete literal expressions are generated at various stages for the purposes of spot-checking and qualitative study.

6) The combined stability and vibration analysis with an appropriate ordering scheme, provides "an excellent preliminary design tool"[8]. But the ordering scheme required in such a combined analysis is generally of higher order than that required in a separate stability or vibration analysis. The use of this symbolic processor offers considerable promise, since the related state equations are digitally derived for any stipulated ordering scheme.

3 Program Description

A detailed exposition of the symbolic processor HESL is beyond the scope of this paper and can be found in reference 18. However, for completeness, five essential aspects of the program are touched upon here. They are : 1) Algebraic manipulation capabilities, 2) Commands, 3) Input-

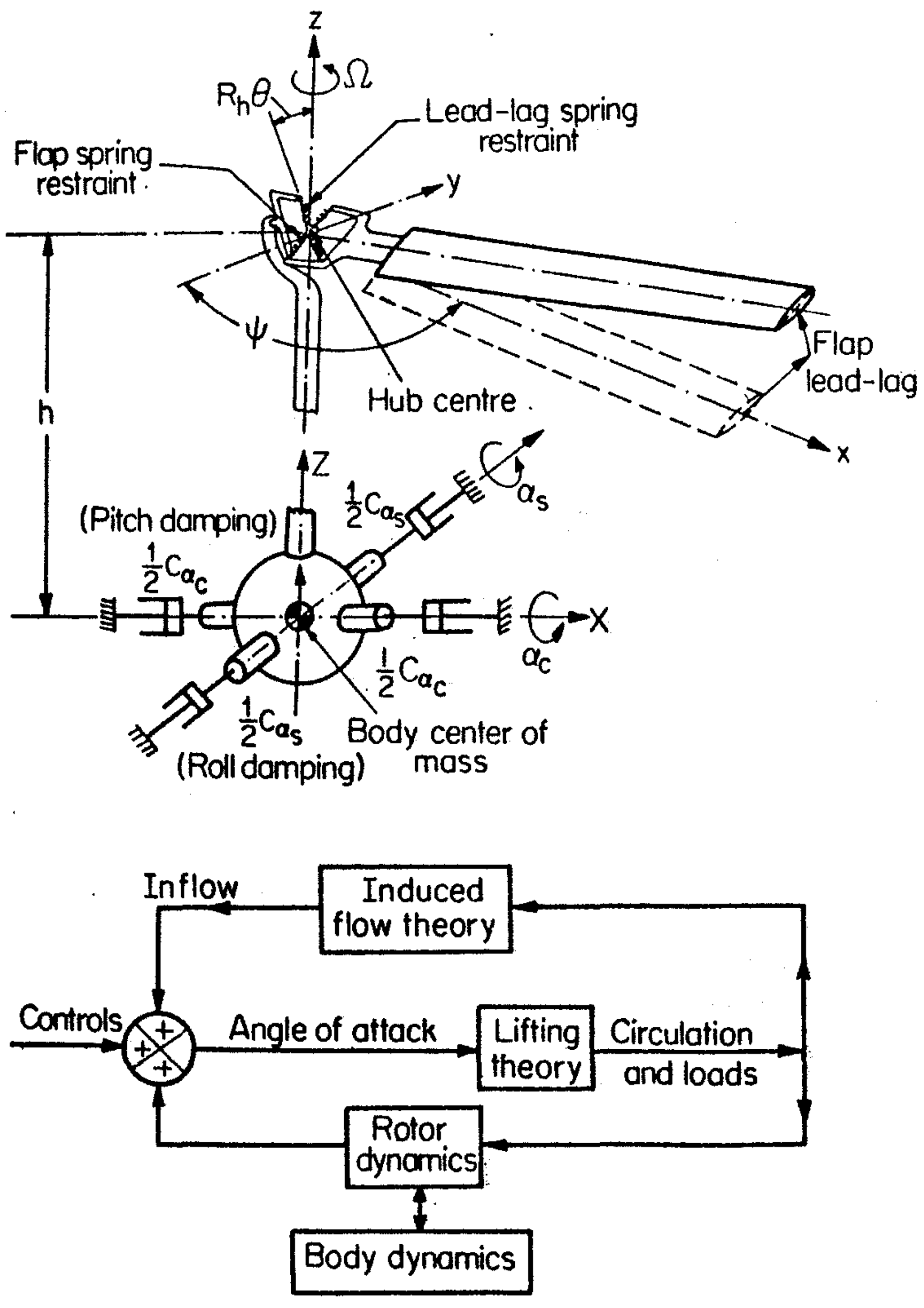

FIG.1. COUPLED ROTOR-BODY SCHEMATIC WITH BLOCK DIAGRAM OF INFLOW DYNAMICS.

output details, 4) Program structure and 5) Limitations.

3.1 Algebraic Manipulations

These manipulations consist of combining expressions, replacing variables in an expression by designated expressions (relations) and substituting numerical or logical values and tables into expressions. They also include the expansion of composite functions and expressions according to stipulated ordering schemes and the collection of coefficients of a specified variable in an expression. The algebraic manipulations of the partial differentiation and integration are carried out from the user supplied rules.

3.2 Commands

There are 13 commands built into the program. They are *classified into four groups:* input commands, general purpose commands, application oriented commands and special commands. While describing the mathematical model, the input commands are used to feed the data of expressions, relation/formula tables, variable strings etc. The general purpose commands generate expressions of $\dot{\bar{\rho}}$ $(= \partial\bar{\rho}/\partial\tau + \Omega X\bar{\rho})$, U_T, U_P, etc. and carry out the algebraic manipulations. The application oriented commands are designed to carry out the specific functions related to the general problems of rotor-body dynamics. For example, they generate multiblade functions (1, cos ψ, sin ψ for three bladed rotors) for the multiblade coordinate transformations, perturbed linear equations and final multiblade equations. The special commands look into the program management aspects such as the termination of the program execution and reappropriation of working core space for its optimal utilization. A listing of these commands and associated subroutines are given in Tables (1a) and (1b) which also include a brief description of the command functions.

The command 'FORM LAGRANGIAN' is the most important and merits special mentioning. As presented in the flow chart of figure 2 it carries out all the necessary analytical computations starting from the formation of the Lagrangian equation to the final multiblade equations. The operations involved are 1) Evaluation of the sub-elements of the Lagrangian i.e., expressions such as

$$\frac{\partial}{\partial\tau}\left\{\frac{\partial(T-U)}{\partial\dot{q}_i}\right\}, \quad \frac{\partial(T-U)}{\partial q_i}, \quad Q_{q_i}, \text{ etc.}$$

TABLE - 1a

BLOCK I SUBROUTINES

Sl No	COMMAND	SUBROUTINE TO BE LINKED AND EXECUTED	Brief description/ function of the subroutine
1	READ EXPRESSION	READEX	Inputs an expression into core
2	READ TABLE FOR SUBSTITUTION	READTB	Inputs a table of relations into core
3	READ GROUP AND ORDER OF VARIABLES	RDNTOR	Reads grouping and ordering of variables
4	READ ORDERING SCHEME	RDORSH	Inputs an ordering scheme
5	SUBSTITUTE TABLE INTO EXPRESSION	SUBTAB	Substitutes a relation table into a specified expression
6	DIFFERENTIATE EXPRESSION	DFRENT	Differentiates an expression
7	READ DIFFERENTIATION TABLE	DERTAB	Reads table of differentiation rules
8	READ VARIABLES FOR COLLECTION OF COEFFICIENTS	RDCCVR	Inputs a list of variables for collection of terms
9	RESET COUNTER	RESETC	Resets the terms counter
10	FORM LAGRANGIAN	LAGRAN	Forms perturbed linear equations and then converts them into multiblade coordinate equations
11	FORM EXPRESSION	SQMUAD	Generates a new function after performing the required additions and multiplications
12	INITIALISE MULTIBLADE	INTMUL	Generates multiblade factors
13	END OF DATA	To 'STOP'	Stops execution

TABLE - 1b

BLOCK II SUBROUTINES

Sl No	SUBROUTINES	Function of the program
1	PTEXDT	Registers the details of an expression
2	GTEXDT	Brings out the details of an expression
3	PUTROW	Registers the details of a term
4	GETROW	Brings out details of a term
5	WRTEXP	Prints expression details in Alpha numeric form
6	TABSRH	Identifies a table name and registers it in the list of tables
7	FLESRH	Identifies an expression name and registers it in the list of expressions
8	VARSRH	Identifies a variables name and registers it in the list of variables
9	IDVRFL	Identifies the group to which an input name belongs
10	TRNSFR	Transfers the term details from one position of the core to another position
11	MULTPY	Multiplies expressions
12	DIFREN	Differentiates a term of an expression
13	GTCLDT	Brings the variables' list details
14	COLECT	Inputs a variable for collection of terms containing the variable
15	RELATN	Substitutes a relation into the terms of an expression
16	USEREL	Identifies the relation to be used in a term form a specified table
17	MULEXP	Product of three expressions
18	EXPEXP	Product of terms
19	READVF	Reads a term of an expression
20	COMPCT	Shortens the length of an expression by adding identical terms
21	ORDEXP	Finds the order of magnitude of the term and identifies whether the term to be rejected or retained based on stipulated criteria

(Table - 1b continued)

22	GTORSH	Brings out ordering scheme details
23	MULTIB	Transforms terms of perturbed linear equations into terms of equations in multiblade coordinate equations
24	FORSTA	Transforms expressions into coded Fortran statements

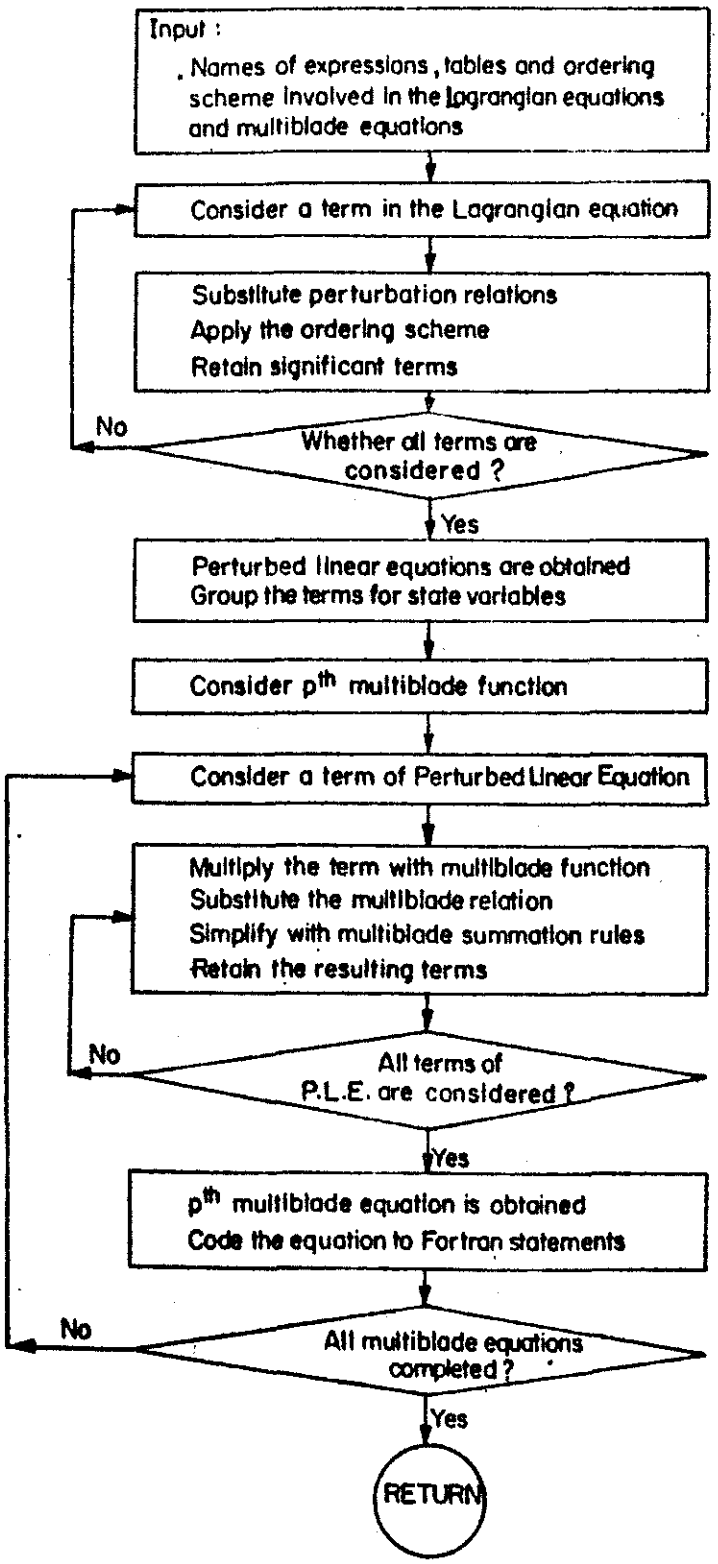

Fig. 2 : Flow chart of command FORM LAGRANGIAN

2) substitution of perturbation relations, 3) application of an ordering scheme to generate the perturbed linear equations, and 4) transformation of perturbed linear equations into multiblade equations. Operation (1), if executed completely at once leads to large number of terms burdening the computer core. Accordingly, the command FORM LAGRANGIAN does not evaluate complete sub-elements of operation (1). Instead, it carries out operations (2) and (3) successively with respect to each 'term' or components of the sub-elements and retains the resulting contributions. Then, the perturbed linear equations are generated by the summation of these contributions. Such a strategy of effecting an operation at term level rather than at complete expression level limits intermediate expression swell[11] to an absolute minimum and is also employed in forming the multiblade equations from the perturbed linear equations. Operation (4) is effected in four phases, a) substitution of multiblade relations, b) multiplication with the multiblade functions, c) usage of trigonometric identities and d) application of multiblade summation rules.

Additional features refer to modular construction and portability. The modular structure permits the introduction of new commands or modifications of the old commands to consider major modifications in the formulation. Thus, the same program can be utilised to consider a variety of modifications or extensions of the original analytical model. Usually the implementation of symbolic manipulation systems on another computer requires a major effort[11] in that it must take advantage of the specific features of the hardware and operating system of the host computer[11]. The present program written in FORTRAN IV, can be implemented with minimal assistance from the host computer, i.e. by utilising its Fortran compiler. As such, it is highly portable. A reset counter is also incorporated which erases all previous equations and saves core space for the next equation.

3.3 Input-Output

The inputs to the program comprise the command names and their parameters which are in alpha numeric format. The names of expressions, relation tables, the variable strings and ordering schemes are made of four alpha numeric characters. It is necessary to attach a special character with the names to signify the group (variables, expressions, tables, etc.) to which the name belongs. For example, variables β_0, $\ddot{\zeta}_k$, sin ψ_k and expressions F12, KE and differentiation table DERV are read as follows :

β_0 = b̸b̸b̸BT, $\dot{\zeta}_k$ = b̸ZEDD, $\sin\psi_k$ = b̸SNCY

F12 = % b̸F12, KE = % b̸b̸KE, DERV = @DERV

As seen from the above examples, the special characters 'b̸', '%' and '@' recognize respectively the variables, expressions and tables. We observe that all specific characters are to be fed to the program before any command name is read. The details of the terms in the expression are formated such that each card provides the details of one term. The program gives two sets of outputs. The first set contains the resulting expressions of algebraic manipulation commands, perturbed linear equations and multiblade equations. The expressions details are printed term by term one below the other for easy perusal by the user. The second set contains outputs which are coded Fortran statements of the equations as required in the subsequent numerical computations of Floquet transition matrices and forced responses.

3.4 Program Structure

As typified in figure 3 in a flow-chart form, the program has one main program and 36 subroutines. Control of the entire operation is done through the input commands. The main program initialises the internal data management parameters and reads the commands as data. Depending on the command, the required subroutines are called and the executions are performed. Each subroutine is like a building block, its size and scope being so designed that its function is 'obvious, logical and reasonable'[18].

The subroutines can be divided into two categories. The first category of subroutines represented by Block I in figure 3 and table (1a) are called by the main program for executing the command functions. The second category of subroutines represented by Block II in figure 3 and table (1b) are called by the subroutines of Block I in assisting its execution. The Block II-subroutines are the fundamental blocks in performing complex algebraic manipulations such as substitution of tables into expressions, composite-expression expansions etc.

3.5 Limitations

A restrictive aspect of the program is the necessity of providing all the relations needed for differentiation and integration and trigonometric identities. This can be easily overcome by building a data library into the program.

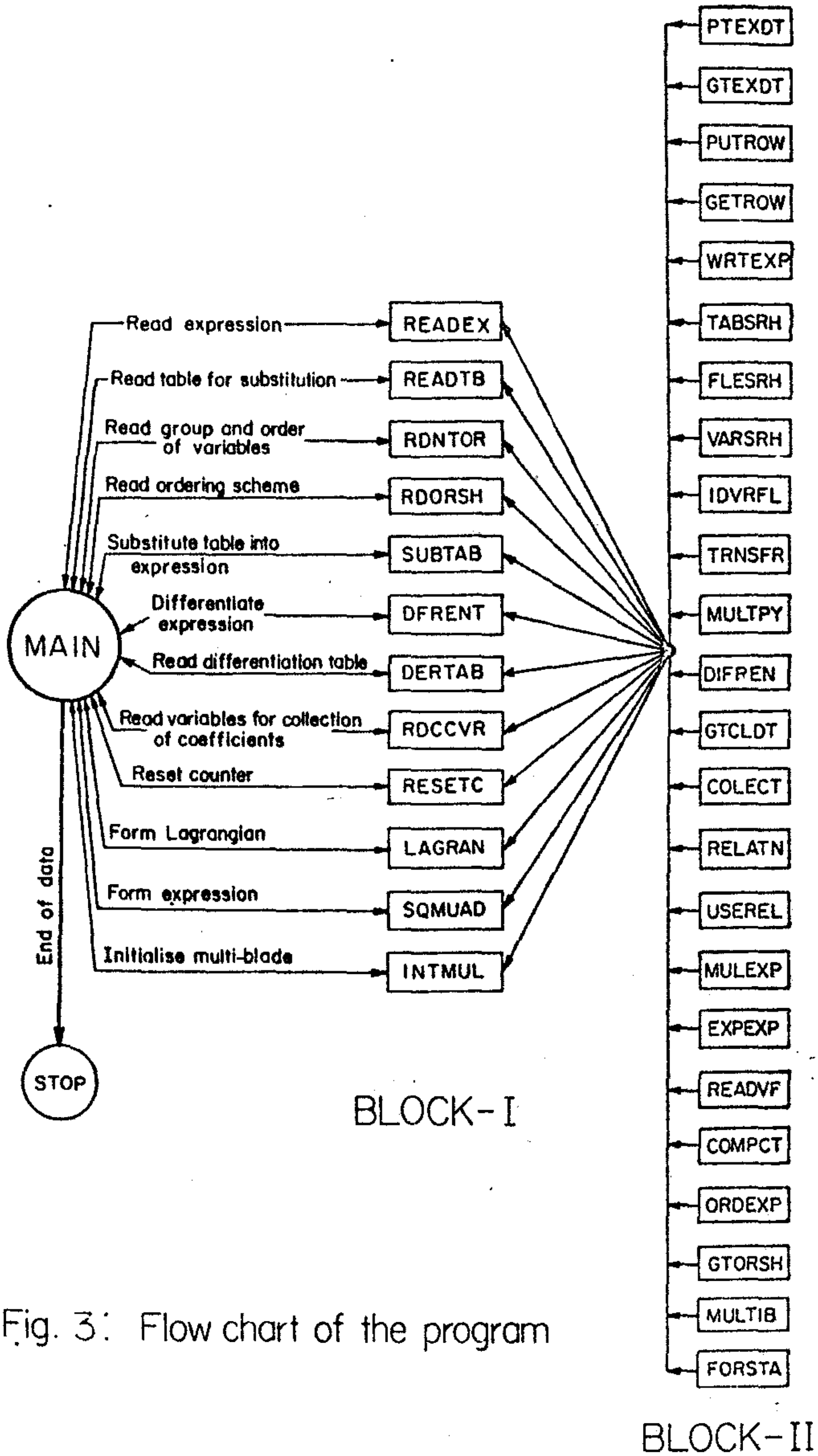

Fig. 3: Flow chart of the program

4 Equations of Motion

We now come to the symbolic manipulation details of generating the equations of motion for a priori ordering schemes. For illustrative purposes the coupled rotor-body model with dynamic inflow of Reference 4 is selected. A rotor-body schematic with inflow block diagram is sketched in figure 1. While the treatment of Reference 4 is restricted to hovering flight, these equations are presented here for the relatively more complex conditions of forward flight. The coupled model refers to a rotor system idealised as rigid blades executing flap and lag motions and to a fuselage system idealised as a simple rigid body executing roll and pitch motions. Hub elasticity and blade torsional flexibility are accounted for in a quasisteady manner[4]. Quasisteady aerodynamics is used for evaluating aerodynamic forces on the rotor blade. Effects of gravity, stall, reverse flow, compressibility and body aerodynamics are neglected. The processor HESL accepts dynamic inflow models based on both first and second order harmonic descriptions of inflow which respectively lead to three and five inflow distributions (uniform, fore-to-aft, etc.) or degrees-of-freedom[19]. The two matrices of inflow gain and time constants comprise the inputs to the inflow system[18,19]. They are based on an unsteady actuator disk theory[19]. Due to space limitations, the presentation is restricted to an inflow model with three degrees-of-freedom and to the flapping equations of $\partial\beta$ and β_0 for the ordering scheme $\varepsilon^2 << 1$.

Appendix I contains the input data for the generation of equations of motion, the corresponding flow chart being shown in figure 4. It is divided into eight parts. Each part corresponds to a particular aspect of the process of deriving the equations of motions. What follows is a brief account of input data in each part of Appendix I and corresponding formulation steps and outputs.

Part I pertains to tables of relations/formulaes/identities. For the present problem a total of seven tables of relations is required i.e. (a) perturbation relations @ PERT, (b) integrals of inertial terms @INRL, (c) integrals of aerodynamic force terms @ DYNM, (d) multiblade relations @ MULB, (e) integrals of terms in dynamic inflow equations @ DYIN, (f) trigonometric identities @ TRIG and (g) differentiation rules @ DERV. Due to space limitations we present the input details only for tables (a) and (g) in Part I. For a complete presentation, see reference 18. Typical relations in these tables are :

Perturbation relations $\beta = \bar{\beta} + \partial\beta$, $\ddot{\alpha}_c = \partial\ddot{\alpha}_c$, etc., and the differentiation rules $\partial\beta/\partial\tau = \dot{\beta}$, $\partial \sin\beta/\partial\tau = \dot{\beta}\cos\beta$, etc. (1)

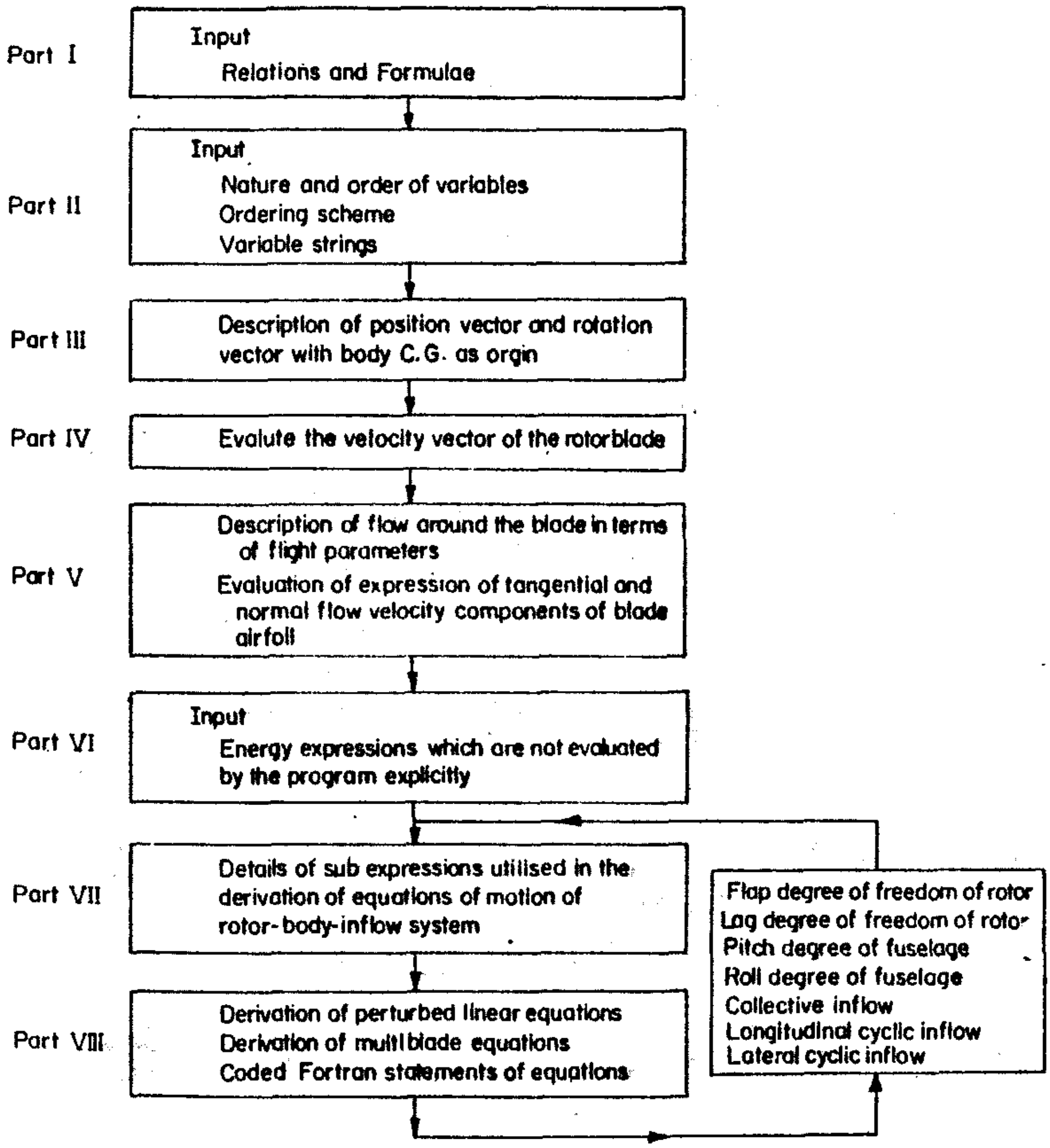

Fig. 4: Flow chart of Inputs to HESL

While implementing the stipulated ordering scheme, the variables are identified with two groups. In the first group they are assigned orders such as $0(1)$, $0(\varepsilon)$, $0(\varepsilon^2)$, etc. as in references 4 or 18. For example, θ_β is of order one while $\bar{\beta}_k$, $\bar{\theta}_k$, β_{pc}, $\bar{\lambda}$ etc. are of order ε and $^\beta c_d/a$, of order ε^2. In the second group we have the state variables and acceleration terms all of which have an order of δ. For the selected ordering scheme, typical expansions of input tables read:

$$\cos\beta_k = 1 - (1/2)\,\bar{\beta}_k^{\,2} - \bar{\beta}_k\,\delta\beta$$

and $$\sin\theta_k = \bar{\theta}_0 + \theta_\beta\delta\beta + \theta_\zeta\,\delta\zeta + \theta_\beta(\beta_k - \beta_{pc}) + \bar{\theta}_I\cos\psi_k + \bar{\theta}_{II}\sin\psi_k \qquad (2)$$

In the perturbation scheme state variable of order δ^2 are automatically deleted.

In part II, input data are presented in two divisions. Division one contains variables (state variables and acceleration terms) and equilibrium state parameters along with their orders of magnitude. The equilibrium parameters are incorporated in a general manner in that the generated equations can be used for a variety of control settings of moment trimmed ($\bar{f} = 0$), propulsive trimmed ($\bar{f} \neq 0$), untrimmed and roll trimmed conditions. In division two, input instructions are given to collect terms pertaining to a specific variable or a parameter.

The derivation of equations of motion starts with the definition of the rotor blade position vector and the rotation vector. As seen from figure 1

$$\bar{\rho}_x = r\cos\beta\cos\zeta, \quad \bar{\rho}_y = r\cos\beta\sin\zeta, \quad \bar{\rho}_z = r\sin\beta + h \qquad (3)$$

$$\bar{\Omega}_x = \dot{\alpha}_c\sin\psi - \dot{\alpha}_s\cos\psi, \quad \bar{\Omega}_y = \dot{\alpha}_c\cos\psi + \dot{\alpha}_s\sin\psi, \quad \bar{\Omega}_z = 1 \qquad (4)$$

which constitute the user supplied inputs as identified in part III of Appendix I.

Part IV in appendix I comprises the inputs to evaluate $\dot{\bar{\rho}}$. A representative program output of $\bar{\rho}_y$ is given below :

```
*************************************
*                                   *
*   DETAILS OF THE EXPRESSION  RYD  *
*                                   *
*************************************
1     -1.000000*  BTD*    SBT*    SZE*   RB*
2      1.000000*  CBT*    ZED*    CZE*   RB*
3      1.000000*  CBT*    CZE*     RB*
4     -1.000000*  SBT*    ACD*   SNCY*   RB*
5      1.000000*  SBT*    ASD*   CSCY*   RB*
6     -1.000000*  ACD*   SNCY*     HB*
7      1.000000*  ASD*   CSCY*     HB*
```

At a point (r,ψ) in the rotor disk, the dynamic inflow ν with components (ν_0, ν_I, ν_{II}) has the first order harmonic representation[19]:

$$\nu = \nu_0 + \nu_I\, r \cos \psi + \nu_{II}\, r \sin \psi \qquad (5)$$

The component ν_0 in the above equation refers to uniform inflow perturbation. The remaining two components ν_I and ν_{II} refer to the fore-to-aft and side-to-side perturbations. These components assume the role of degrees-of-freedom. (The dynamic inflow model with five degrees-of freedom is included on similar lines). The total induced flow λ is given by

$$\lambda = \overline{\lambda} + \nu \qquad (6)$$

where the dynamic inflow ν is perturbed with respect to the steady inflow $\overline{\lambda}$ such that $(4/3)\overline{\lambda}$ represents the trim inflow angle. Bypassing considerable algebraic details,we have the following expressions for tangential and normal velocity components on the air foil :

$$U_T = r\,(1 + \dot{\zeta}) \cos \beta - (r \sin \beta + h)\{\dot{\alpha}_c \sin (\psi + \zeta) - \dot{\alpha}_s \cos (\psi + \zeta)\} + \mu \sin (\psi + \zeta) \qquad (7a)$$

$$U_P = r\,\dot{\beta} + \lambda \cos \beta - (r + h \sin \beta)\{ \dot{\alpha}_c \cos (\psi + \zeta) + \dot{\alpha}_s \sin (\psi + \zeta)\} + \mu \sin \beta \cos (\psi + \zeta) \qquad (7b)$$

The description of equation (7) is given in part V of Appendix I which also includes the evaluation of the perturbed linear expressions U_{T_P} and U_{P_P} . As an illustrative example, the output corresponding to U_{T_P} follows :

```
***********************************
*                                 *
*  DETAILS OF THE EXPRESSION  UTP *
*                                 *
***********************************
```

1	1.000000*	RB*	DZD*		
2	-0.500000*	RB*	BB*	BB*	DZD*
3	1.000000*	RB*			
4	-0.500000*	RB*	BB*	BB*	
5	-1.000000*	RB*	BB*	DB*	
6	-1.000000*	SNCY*	RB*	BB*	DCD*
7	1.000000*	CSCY*	RB*	BB*	DSD*
8	-1.000000*	SNCY*	DCD*	HB*	
9	1.000000*	CSCY*	DSD*	HB*	
10	1.000000*	CSCY*	DZ*	MU*	
11	1.000000*	SNCY*	MU*		

Part VI in Appendix I contains details of expressions of strain energy of the blade-hub system, the equivalent viscous dissipating functions for the blade and fuselage and kinetic energy of the fuselage. The expression of the strain energy of the blade-hub system is computed from the user supplied relation :

$$S_{BL} = \frac{1}{2} \sum_{k=1}^{N} I_B\{ \omega_\beta^2 + R_h(\omega_\zeta^2 - \omega_\beta^2) \sin^2\theta_k\} (\beta_k - \beta_{pc} - \theta_\beta\beta_{pc})^2$$

$$+ I_B\{ \omega_\zeta^2 - R_h (\omega_\zeta^2 - \omega_\beta^2) \sin^2\theta_k\} \zeta_k^2 \qquad (8)$$

$$+ I_B R_h (\omega_\zeta^2 - \omega_\beta^2)(\sin 2\theta_k)(\zeta_k)(\beta_k - \beta_{pc} - \theta_\beta\beta_{pc})$$

Inputs of Part VII and VIII of Appendix I pertain to the generation of the equations of motion for flapping degree-of-freedom. (For input descriptions of other degrees-of-freedom motion see reference 18). The Lagrangian form of the flapping equation of motion of the i-th blade is written as

$$\int mR^3 \frac{\partial}{\partial\tau} \{ \dot{\bar{\rho}}_x (\frac{\partial\dot{\bar{\rho}}_x}{\partial\dot{\beta}}) + \dot{\bar{\rho}}_y (\frac{\partial\dot{\bar{\rho}}_y}{\partial\dot{\beta}}) + \dot{\bar{\rho}}_z (\frac{\partial\dot{\bar{\rho}}_z}{\partial\dot{\beta}})\} dr$$

$$- \int mR^3 \{\dot{\bar{\rho}}_x (\frac{\partial\dot{\bar{\rho}}_x}{\partial\beta}) + \dot{\bar{\rho}}_y (\frac{\partial\dot{\bar{\rho}}_y}{\partial\beta}) + \dot{\bar{\rho}}_z (\frac{\partial\dot{\bar{\rho}}_z}{\partial\beta})\} dr$$

$$+ \frac{\partial(S_{RB})}{\partial\beta} + \frac{\partial(D_{RB})}{\partial\beta} = \int R^2 r F_z dr$$

$$= \int \frac{PaCR^4}{2} \{ U_{T_p}^2 \sin\theta - U_{T_p} U_{P_p} (\cos\theta + \frac{C_{d_o}}{a})\} dr \qquad (9)$$

This equation is rewritten as consisting of 10 major steps or sub-elements which are

$$\int f_1 \frac{\partial}{\partial\tau}(f_6)\,dr + \int f_1\frac{\partial}{\partial\tau}(f_7)\,dr + \int f_1\frac{\partial}{\partial\tau}(f_8)\,dr + \int f_2\, f_6\, f_{12}\, dr$$

$$+\int f_2\, f_7\, f_{13}\, dr + \int f_2\, f_8\, f_{14}\, dr \tag{10}$$

$$+ f_3\frac{\partial}{\partial\beta_i}(f_3) + f_3\frac{\partial}{\partial\beta_i}(f_3) + \int f_4\, f_{17}\, f_{17}\, dr + \int f_5\, f_{17}\, f_{18}\, dr = 0$$

where the sub-elements f_1 to f_{18} are defined as follows :

$$f_1 = mR^3; \quad f_2 = -mR^3; \quad f_3 = 1; \quad f_4 = -\frac{\rho a C R^4}{2}\sin\theta$$

$$f_5 = \frac{\rho a C R^4}{2}\left(\cos\theta + \frac{C_{d_o}}{a}\right), \quad f_6 = \dot{\rho}_x\ ; \quad f_7 = \dot{\rho}_y; \quad f_8 = \dot{\rho}_z;$$

$$f_9 = \frac{\partial\dot{\rho}_x}{\partial\beta} \qquad f_{10} = \frac{\partial\dot{\rho}_y}{\partial\dot{\beta}} \qquad f_{11} = \frac{\partial\dot{\rho}_z}{\partial\dot{\beta}} \qquad f_{12} = \frac{\partial\dot{\rho}_x}{\partial\beta}$$

$$f_{13} = \frac{\partial\dot{\rho}_y}{\partial\beta}\ ; \quad f_{14} = \frac{\partial\dot{\rho}_z}{\partial\beta}\ ; \quad f_{15} = S_{BL}; \quad f_{16} = D_{BL}; \quad f_{17} = U_{TP};$$

$$f_{18} = U_{PP}. \tag{11}$$

While the sub-elements f_6, f_8, f_{15}, f_{16}, f_{17}, and f_{18} are evaluated by the computer, the remaining 11 sub-elements are fed as inputs which are identified in part VII.

Finally we come to the process of deriving the flapping equations in multiblade coordinates. This process starts with the evaluation of equation (10) in ten major steps. Necessary integral relations are available in @ INRL and @ DYNM. The nonlinear equation is converted to the perturbed linear equation according to the perturbation relations @ PERT and according to the stipulated ordering scheme *E2D1. The perturbed linear equations in turn are transformed into multiblade equations according to the multiblade relations @MULB. The multiblade equations are further simplified by the use of trignometric identities @ TRIG and multiblade summation rules. The terms of the perturbed and multiblade equations are grouped according to the listing of variables as specified in division 2 of part II in Appendix I. The inputs in Part VIII of Appendix I describe these major steps. The corresponding coded Fortran statements are derived from the multiblade equations for further numerical computations. While Appendix II presents the output of the perturbed linear equation for the flapping motion, Appendix III shows the multiblade equation for collective flapping mode β_0.

HESL was implemented on DEC-1090 to generate the equations of motion of the rotor-body inflow system. It takes about 15 minutes of CPU time to generate the equations for the rotor-body system with eight degrees-of-freedom (β_0, β_I, β_{II}, ζ_0, ζ_I, ζ_{II}, α_c and α_s), and about 18 minutes with the inclusion of dynamic inflow with components ν_0, ν_I, and ν_{II}.

5 Concluding Remarks

In this exploratory study, one of the simplest viable models[2-4,8-10] of coupled rotor fuselage systems with dynamic inflow has been assumed for describing the basic features of HESL, a completely self contained symbolic processor in FORTRAN IV. Such a study demonstrates the feasibility of using symbolic manipulation in the 'third category of research'[2] in helicopter dynamics. For the preceding coupled rotor-body model with a stipulated ordering scheme HESL requires basically the definitions of $\overline{\rho}$ and $\overline{\Omega}$ and the flow description parameters λ, μ and ν. It (i) generates perturbed linear equations from the nonlinear ordinary differential equations and (ii) transforms the perturbed linear equations in multiblade coordinates. The facility of direct coding into FORTRAN statements for subsequent numerical computations is included. The modular structure of the program allows the programmer to alter the existing modules and to add new subroutines. Thus, HESL is oriented towards flexibility of application and user modification. Unlike a general purpose processor or 'catholic system', it has high portability since it is written in FORTRAN IV and needs no special assistance from any other software systems of the host computer. Its application oriented commands make the user inputs minimal since the required formulation steps are built into the commands. The intermediate expression swell[11] is significantly minimised, since formulation procedures are carried out at term level rather than at expression level.

The continuing study concerns extensions in several respects: i) To refine the rigid roll-pitch model of the fuselage or rotor support to include rigid plunging motion. ii) To refine the rigid flap-lag model of the rotor to include elastic flap and lag modes. iii) To refine the rigid rotor-support model to include elastic beam modes.

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NOTATION

Symbol	FORTRAN Symbol	Description
h, r	HB,RB	Dimensionless distance from body centre of mass to rotor centre and blade radial coordinate (unit 1/rotor radius R)
$\bar{\rho}$, $\bar{\Omega}$	-,-	Position and rotational velocity vectors
τ	TAU	Dimensionless time (unit $1/\Omega$)
$\bar{\rho}_x, \bar{\Omega}_y, \dot{\bar{\rho}}_x, \partial\bar{\rho}_z/\partial\tau$	RX,OMGY,RXD,RZT	Typical components of $\bar{\rho}, \bar{\Omega}$ and the total and partial derivatives of $\bar{\rho}$ with respect to τ
ψ	CY	Rotor azimuth angle, $\psi = 0$, aft position
$\zeta, \beta, \alpha_c, \alpha_s$	ZE,BT,AC,AS	Blade lead-lag and flap and body pitch and roll motions
$I_\beta, I_{\alpha_c}, I_{\alpha_s}$	IB,IICC,IISS	Blade flap or lag inertia and body pitch and roll inertias
$\mu, \gamma, \gamma I_\beta$	MU,GAMA,GAMI	Advance ratio, blade Lock number and γI_β
$\omega_\beta^2, \omega_\zeta^2$	OMB2,OMZ2	Dimensionless and uncoupled flap (non-rotating) and lead lag frequencies squared (unit $1/\Omega$)
q_i	-	Generalized or quasi-generalized coordinate
Q_{q_i}	-	Generalized force in the q_i direction

C_{d_o}/a	CDoA	Blade profile drag coefficient over lift curve slope
$\theta_\beta, \theta_\zeta, \beta_{pc}$	THBT,THZE,BTPC	Pitch-flap and pitch-lag coupling ratios, and precone angle
$\overline{\theta}_o, \overline{\theta}_I, \overline{\theta}_{II}$	THBO,THB1,THB2	Collective, longitudinal and lateral blade pitch angles
$\nu, \nu_o, \nu_I, \nu_{II}$	-,NUO,NU1,NU2	Inflow perturbation, uniform, fore-to-aft, and side-to-side components of inflow
$\overline{\lambda}$	LAMB	Total steady inflow ratio
$\overline{f}$	-	Dimensionless helicopter flat plate drag area
F_z	-	Dimensionless force perpendicular to blade (unit $\Omega^2 I/R^2$)
λ	LAMD	$\lambda = \overline{\lambda} + \nu_o + \nu_I r \cos\psi_k + \nu_{II} r \sin\psi_k$
$U_T, U_P, U_{T_P}, U_{P_P}$	UT,UP,UTP,UPP	Dimensionless tangential and perpendicular velocity components of flow over the airfoil and their perturbed linear components
$\sin\beta, \cos\beta, \sin\zeta$	SBT,CBT,SZE	-
$\cos\zeta, \sin\psi, \cos\psi$	CZE,SNCY,CSCY	-
$\sin\theta, \cos\theta, \sin\alpha_s$	STH,CTH,SAS	-
$\dot{\beta} (= d\beta/d\tau)$	BDT	Flapping velocity
$\ddot{\beta} (= d^2\beta/d\tau^2)$	BTDD	Flapping acceleration
$\beta_o, \beta_I, \beta_{II}$	BO,B1,B2	Coning, longitudinal and lateral flapping angles
$\dot{\beta}_o, \ddot{\beta}_I, \dot{\overline{\beta}}$,	BDO,BDD1,BBD	Typical flapping derivatives with respect to τ

$\delta\ddot{\beta}, \delta\ddot{\zeta}_I, \dot{\bar{\zeta}}_o$	DBDD,DZDD,ZDO	Typical lead-lag derivatives with respect to τ
$\partial\dot{\bar{\rho}}_x/\partial\dot{\beta}, \partial\dot{\bar{\rho}}_y/\rho\beta$	RXDQ,RYQ	Partial derivative of $\dot{\bar{\rho}}_x$ with respect to $\dot{\beta}$ and of $\bar{\rho}_y$, with respect to β
$a\sigma/N$	ASGN	(Lift curve slope X solidity)/(number of blades)
$(-1)^k$	MOPK	$(-1)^k$ appears only for rotors with even number of blades
$\dot{\alpha}_c, \ddot{\alpha}_c$	ACD,ACDD	Body pitch velocity and pitch acceleration
$\delta\dot{\alpha}_c, \delta\ddot{\alpha}_c$	DCD,DCDD	α_c and $\delta\alpha_c$ are identical
D_{BL}, D_{FU}	DEBL,DEFU	Equivalent viscous dissipating functions of blade and fuselage
S_{BL}	SEBL	Strain energy of the blade-hub system (root springs)
K_{FU}	KEFU	Kinetic energy of the fuselage
T	-	Kinetic energy of the rotor-body system
U	-	Potential energy of the rotor-body system
R_h	ECF	Elastic coupling parameter
$\rho ac/2$	RAC2	(air density X lift curve slope X blade chord)/2
$\sin 7\psi, \cos 7\psi$	S7CY,C7CY	-
$2\eta_\beta\omega_\beta, 2\eta_\zeta\omega_\zeta$	CCBB,CCZZ	Blade flap and inplane structural damping ratios
$2\eta_{\alpha_c}\omega_{\alpha_c}, 2\eta_{\alpha_s}\omega_{\alpha_s}$	CCAC,CCAS	Body pitch and roll structural damping ratios.

```
****************************************************************************************
*                                    APPENDIX I                                        *
*     INPUT DATA FOR DERIVATION OF ROTOR BODY INFLOW EQUATIONS                         *
*                                                                                      *
****************************************************************************************
```

```
----------------------------------------------------------------------------------------
*    PART I: TABLES OF RELATIONS/IDENTITIES                                            *
----------------------------------------------------------------------------------------
```

```
READ TABLE FOR SUBSTITUTION
#PERT16
02   BT01
1.            BB
1.            DB
02  BTD01
1.            DBD
1.            BBD
02 BTDD01
1.           DBDD
1.           BBDD
03   SBT01
1.            BB
1.            DB
-.5           DB    BB    BB
03   CBT01
1.
-.5           BB    BB
-1.           BB    DB
01   ZE01
1.            DZ
01   ZED
1.            DZD
01 ZEDD01
1.           DZDD
01   SZE01
1.            DZ
01   CZE01
1.
01   ACD01
1.            DCD
01 ACDD01
1.           DCDD
01   ASD01
1.            DSD
01 ASDD01
1.           DSDD
07   STH01
1.           THBO
1.0          THBT    DB
1.           THZE    DZ
-1.          THBT BTPC
1.           THBT    BB
1.           THB1 CSCY
1.           THB2 SNCY
06   CTH01
1.
-.5          THBO THBO
-.5          THB1 THB1 CSCY CSCY
-.5          THB2 THB2 SNCY SNCY
-1.          THBO THB1 CSCY
-1.          THBO THB2 SNCY
```

```
READ DIFERENTIATION TABLE
#DERV22
  BT   BTQ1
1.
  BTD  BTDQ1
1.
  BT   CBTQ1
-1.          SBT
  BT   SBTQ1
1.           CBT
 TAU   BTQ1
1.           BTD
 TAU   BTDQ1
1.           BTDD
 TAU   CBTQ1
-1.          SBT   BTD
 TAU   SBTQ1
1.           CBT   BTD
  ZE   ZEQ1
1.
 ZED   ZEDQ1
1.
  ZE   CZEQ1
-1.          SZE
  ZE   SZEQ1
1.           CZE
 TAU   ZEQ1
1.           ZED
 TAU   ZEDQ1
1.          ZEDD
 TAU   CZEQ1
-1.          SZE   ZED
 TAU   SZEQ1
1.           CZE   ZED
 ACD   ACDQ1
1.
 TAU   ACDQ1
1.          ACDD
 ASD   ASDQ1
1.
 TAU   ASDQ1
1.          ASDD
 TAU CSCYQ1
-1.                SNCY
 TAU SNCYQ1
1.                 CSCY
```

```
*******************************************************************************
*   PART II: ORDERING SCHEME AND VARIABLES FOR COLL. OF COEFF.                *
*******************************************************************************

READ NATURE AND ORDER OF VARIABLES
42
   BB 0101    DB 0201   BBD 0101   DBD 0201  BBDD 0101  DBDD 0201    ZB 0101    DZ 0201
  ZBD 0101   DZD 0201  ZBDD 0101  DZDD 0201    CB 0101    DC 0201   CBD 0101   DCD 0201
 CBDD 0101  DCDD 0201    SB 0101    DS 0201   SBD 0101   DSD 0201  SBDD 0101  DSDD 0201
 CDOA 0102  LAMB 0101  THB0 0101   NU1 0201   NU2 0201  THBT 0101  THZE 0101   BBC 0101
 BBCD 0101   NU0 0201  THB1 0101  THB2 0101   BB0 0101   BB1 0101   BB2 0101  BTPC 0101
  NU3 0201   NU4 0201
READ ORDERING SCHEME
$B2D102
C1020201
READ VARIABLES FOR COLLECTION OF COEFFICIENTS
&BECF20
 DBDD  DBD   DB DZDD  DZD   DZ DCDD  DCD   DC DSDD  DSD   DS  NU0  NU1  NU2 BBDD
  BBD   BB  NU3  NU4
READ VARIABLES FOR COLLECTION OF COEFFICIENTS
&NUCF29
 BDD0  BD0   B0 BDD1  BD1   B1 BDD2  BD2   B2 ZDD0  ZD0   Z0 ZDD1  ZD1   Z1 ZDD2
  ZD2   Z2 DCDD  DCD   DC DSDD  DSD   DS  NU0  NU1  NU2  NU3  NU4

*******************************************************************************
*        PART III: BLADE POSITION VECTOR AND ROTATION VECTOR                  *
*******************************************************************************
```

```
READ EXPRESSION
$CMGX02
1.           ACD SNCY
-1.          ASD CSCY
READ EXPRESSION
$CMGY02
1.           ACD CSCY
1.           ASD SNCY
READ EXPRESSION
$CMGZ01
1.
```

```
READ EXPRESSION
$ RX01
1.                RB  CBT  CZE
READ EXPRESSION
$ RY01
1.                RB  CBT  SZE
READ EXPRESSION
$ RZ02
1.                RB  SBT
1.                HB
```

```
*******************************************************************************
*   PART IV: BLADE VELOCITY VECTOR                                            *
*******************************************************************************
```

```
DIFERENTIATE EXPRESSION
 TAU$ RX$ RXT
DIFERENTIATE EXPRESSION
 TAU$ RY$ RYT
DIFERENTIATE EXPRESSION
 TAU$ RZ$ RZT
FORM EXPRESSION
$ RXD03
1.        $ RXT
1.        $  RZ$CMGY
-1.       $  RY$CMGZ
```

```
FORM EXPRESSION
$ RYD03
1.           $ RYT
1.           $CMGZ$  RX
-1.          $CMGX$  RZ
FORM EXPRESSION
$ RZD03
1.           $ RZT
1.           $CMGX$  RY
-1.          $CMGY$  RX
```

```
*   PART V: FLOW DESCRIPTION OVER AEROFOIL                                *
```

```
READ EXPRESSION                         SUBSTITUTE TABLE INTO EXPRESSION
%LAMD04                                 @PERT%  UT% UTP   *E2D1
1.         TAMB                         FORM EXPRESSION
1.         NU0                          %  UE06
1.         NU1    RE CSCY               -1.        % RXD   CZE  SBT
1.         NU2    RE SNCY               -1.        % RYD   SBT  SZE
FORM EXPRESSION                         1.         %LAMD   CBT
%  UT04                                 1.            MU   SBT CSCY  CZE
-1.        % RXD   SZE                  -1.           MU   SBT SNCY  SZE
1.         % RYD   CZE                  1.         % RZD   CBT
1.           MU CSCY  SZE               SUBSTITUTE TABLE INTO EXPRESSION
1.           MU SNCY  CZE               @PERT%  UP% UPP   *E2D1
```

```
*   PART VI: SUPPLEMENTARY ENERGY EXPRESSIONS                             *
```

```
READ EXPRESSION                   FORM EXPRESSION
%BMBP02                           %SEBL05
1.          BT                    0.5        CMB2     IB%BMBP%BMBP
-1.         BTPC                  0.5        %CZCB    IB%BMBP%BMBP  ECF  STH  STH
READ EXPRESSION                   .5         CMZ2     IB    ZE    ZE
%CZCB02                           -0.5       %CZCB    IB    ZE    ZE  ECF  STH  STH
1.          CMZ2                  1.         %CZCB    IB    ZE%BMBP   ECF  CTH  STH
-1.         CMB2                  READ EXPRESSION
READ EXPRESSION                   %DEBL02
%KEFU02                           0.5        CCBB  BTD  BTD   IB
0.5         TICC  ACC  ACC        0.5        CCZZ  ZED  ZED   IB
0.5         TISS  ASL  ASL
```

```
*   PART VII: SUBFUNCTIONS FOR FLAP EQUATION                              *
```

```
DIFERENTIATE EXPRESSION                 READ EXPRESSION
  BTD% RXD%RXDD02                       %CCNS01
DIFERENTIATE EXPRESSION                 1.
  BT% RXD% RXD02                        READ EXPRESSION
DIFERENTIATE EXPRESSION                 %MUF101
  BTD% RYD%RYDD                         -0.5
DIFERENTIATE EXPRESSION                 FORM EXPRESSION
  BTD% RZD%RZDD                         %CCN201
DIFERENTIATE EXPRESSION                 1.          MAC2    RB  STH%MUF1
  BT% RYD% RYD                          FORM EXPRESSION
DIFERENTIATE EXPRESSION                 %CCN302
  BT% RZD% RZD                          -1.         MAC2    RB  CTH%MUF1
DIFERENTIATE EXPRESSION                 -1.         MAC2    RB CCOA%MUF1
  BT%SEBL%PEBL
FORM EXPRESSION
%CCN101
-1.
```

```
----------------------------------------------------------------------
*   PART VIII: DERIVATION OF MULTIBLADE EQUATION FOR FLAPPING MOTION     *
----------------------------------------------------------------------

INTILISE MULTI BLADE
103    WORK CSCY SNCY C2CY S2CY C3CY S3CY C4CY S4CY C5CY S5CY C6CY S6CY C7CY S7CY
FORM LAGRANGIAN
100302000001
$BTEO$BTM1$BTM2$BTM3
*E2D1*E2D1!FECF@MULE@TRIG!MUCF
$ RXD$ RXG$CCN1    BT       @INFL@PERT
$ RYD$ RYG$CCN1    BT       @INFL@PERT
$ RZD$ RZG$CCN1    BT       @INFL@PERT
$ UTP$ UTP$CCN2    BT       @DYNM@PERT
$ UTP$ UPP$CCN3    BT       @DYNM@PERT
$CCNS$PEBL$CCNS    BT       @DUMY@PERT
$CCNS$DEBL$CCNS    BT   BTD@PERT
$ RXD$RXDG$CCNS  BTD   TAU@INFL@PERT
$ RYD$RYDG$CCNS  BTD   TAU@INFL@PERT
$ RZD$RZDG$CCNS  BTD   TAU@INFL@PERT
END OF DATA
```

APPENDIX II

```
*****************************************
*                                       *
*    PERTURBED LINEAR EQUATION   BTEQ   *
*                                       *
*****************************************


-----------------------------------------
-   GROUP OF TERMS WITH VARIABLE   DBDD -
-----------------------------------------

1     1.000000*  IB*

-----------------------------------------
-   GROUP OF TERMS WITH VARIABLE    DBD -
-----------------------------------------

1     0.125000*GAMI*
2     0.166667*SNCY*GAMI*  MU*
3     1.000000*  IB*CCBB*

-----------------------------------------
-   GROUP OF TERMS WITH VARIABLE     DB -
-----------------------------------------

1    -0.125000*THBT*GAMI*
2    -0.333333*SNCY*THBT*GAMI*  MU*
3    -0.250000*SNCY*SNCY*THBT*GAMI*  MU*  MU*
4     0.166667*CSCY*GAMI*  MU*
5     1.000000*  IB*
6     0.250000*CSCY*SNCY*GAMI*  MU*  MU*
7     1.000000*  IB*OMB2*

-----------------------------------------
-   GROUP OF TERMS WITH VARIABLE    DZD -
-----------------------------------------

1     0.166667*LAMB*GAMI*
2     0.166667*CSCY*  BB*GAMI*  MU*
3     0.125000* BBD*GAMI*
4    -0.333333*SNCY*SNCY*THB2*GAMI*  MU*
5    -0.250000*THBO*GAMI*
6     0.250000*THBT*BTPC*GAMI*
7    -0.250000*  BB*THBT*GAMI*
8    -0.250000*CSCY*THB1*GAMI*
9    -0.250000*SNCY*THB2*GAMI*
10   -0.333333*SNCY*THBO*GAMI*  MU*
11    0.333333*SNCY*THBT*BTPC*GAMI*  MU*
12   -0.333333*SNCY*  BB*THBT*GAMI*  MU*
13   -0.333333*CSCY*SNCY*THB1*GAMI*  MU*
14    2.000000*  IB*  BB*

-----------------------------------------
-   GROUP OF TERMS WITH VARIABLE     DZ -
-----------------------------------------

1    -0.500000*CSCY*SNCY*THBO*GAMI*  MU*  MU*
2     0.500000*CSCY*SNCY*THBT*BTPC*GAMI*  MU*  MU*
3    -0.500000*CSCY*SNCY*  BB*THBT*GAMI*  MU*  MU*
4    -0.500000*CSCY*CSCY*SNCY*THB1*GAMI*  MU*  MU*
5    -0.500000*CSCY*SNCY*SNCY*THB2*GAMI*  MU*  MU*
6    -0.250000*SNCY*SNCY*THZE*GAMI*  MU*  MU*
7    -0.125000*THZE*GAMI*
8    -0.166667*SNCY*  BB*GAMI*  MU*
```

```
 9     0.250000*CSCY*LAMB*GAMI*  MU*
10     0.250000*CSCY*CSCY*  BB*GAMI*  MU*  MU*
11     0.166667*CSCY* BBD*GAMI*  MU*
12    -0.333333*CSCY*THB0*GAMI*  MU*
13     0.333333*CSCY*THBT*BTPC*GAMI*  MU*
14    -0.333333*CSCY*  BB*THBT*GAMI*  MU*
15    -0.250000*SNCY*SNCY*  BB*GAMI*  MU*  MU*
16    -0.333333*CSCY*CSCY*THB1*GAMI*  MU*
17    -0.333333*CSCY*SNCY*THB2*GAMI*  MU*
18    -0.333333*SNCY*THZE*GAMI*  MU*
19     1.000000*  IB*THB0*OMZ2* ECF*
20    -1.000000*  IB*THBT*BTPC*OMZ2* ECF*
21     1.000000*  IB*  BB*THBT*OMZ2* ECF*
22     1.000000*CSCY*  IB*THB1*OMZ2* ECF*
23     1.000000*SNCY*  IB*THB2*OMZ2* ECF*
24    -1.000000*  IB*THB0*OMB2* ECF*
25     1.000000*  IB*THBT*BTPC*OMB2* ECF*
26    -1.000000*  IB*  BB*THBT*OMB2* ECF*
27    -1.000000*CSCY*  IB*THB1*OMB2* ECF*
28    -1.000000*SNCY*  IB*THB2*OMB2* ECF*

------------------------------------------
-   GROUP OF TERMS WITH VARIABLE   DCDD   -
------------------------------------------

 1    -1.500000*CSCY*  IB*  BB*  HB*
 2    -1.000000*CSCY*  IB*

------------------------------------------
-   GROUP OF TERMS WITH VARIABLE    DCD   -
------------------------------------------

 1     0.333333*SNCY*THB0*GAMI*  HB*
 2     2.000000*SNCY*  IB*
 3    -0.125000*CSCY*GAMI*
 4     0.333333*CSCY*SNCY*THB1*GAMI*  HB*
 5    -0.250000*SNCY*LAMB*GAMI*  HB*
 6    -0.500000*CSCY*SNCY*  BB*GAMI*  HB*  MU*
 7    -0.166667*SNCY* BBD*GAMI*  HB*
 8     0.333333*SNCY*SNCY*THB2*GAMI*  HB*
 9     0.500000*SNCY*SNCY*THB0*GAMI*  HB*  MU*
10     0.500000*CSCY*SNCY*SNCY*THB1*GAMI*  HB*  MU*
11     0.500000*SNCY*SNCY*SNCY*THB2*GAMI*  HB*  MU*
12    -0.166667*CSCY*SNCY*GAMI*  MU*
13    -0.166667*CSCY*  BB*GAMI*  HB*

------------------------------------------
-   GROUP OF TERMS WITH VARIABLE   DSDD   -
------------------------------------------

 1    -1.500000*SNCY*  IB*  BB*  HB*
 2    -1.000000*SNCY*  IB*

------------------------------------------
-   GROUP OF TERMS WITH VARIABLE    DSD   -
------------------------------------------

 1     0.250000*CSCY*LAMB*GAMI*  HB*
 2     0.250000*CSCY*CSCY*  BB*GAMI*  HB*  MU*
 3     0.166667*CSCY* BBD*GAMI*  HB*
 4    -0.500000*CSCY*SNCY*SNCY*THB2*GAMI*  HB*  MU*
 5    -0.250000*SNCY*SNCY*  BB*GAMI*  HB*  MU*
 6    -0.333333*CSCY*THB0*GAMI*  HB*
 7    -0.333333*CSCY*CSCY*THB1*GAMI*  HB*
 8    -0.333333*CSCY*SNCY*THB2*GAMI*  HB*
 9    -2.000000*CSCY*  IB*
```

```
10    -0.500000*CSCY*SNCY*THB0*GAMI*  HB*  MU*
11    -0.166667*SNCY*SNCY*GAMI*  MU*
12    -0.166667*SNCY*  BB*GAMI*  HB*
13    -0.125000*SNCY*GAMI*
14    -0.500000*CSCY*CSCY*SNCY*THB1*GAMI*  HB*  MU*

 ******************************************
 *   GROUP OF TERMS WITH VARIABLE    NU0   *
 ******************************************

 1     0.250000*SNCY*GAMI*  MU*
 2     0.166667*GAMI*

 ******************************************
 *   GROUP OF TERMS WITH VARIABLE    NU1   *
 ******************************************

 1     0.166667*CSCY*SNCY*GAMI*  MU*
 2     0.125000*CSCY*GAMI*

 ******************************************
 *   GROUP OF TERMS WITH VARIABLE    NU2   *
 ******************************************

 1     0.166667*SNCY*SNCY*GAMI*  MU*
 2     0.125000*SNCY*GAMI*

 ******************************************
 *   GROUP OF TERMS WITH VARIABLE   BBDD   *
 ******************************************

 1     1.000000*  IB*

 ******************************************
 *   GROUP OF TERMS WITH VARIABLE    BBD   *
 ******************************************

 1     0.166667*SNCY*GAMI*  MU*
 2     1.000000*  IB*CCBB*
 3     0.125000*GAMI*

 ******************************************
 *   GROUP OF TERMS WITH VARIABLE     BB   *
 ******************************************

 1    -0.125000*THBT*GAMI*
 2     1.000000*  IB*
 3    -0.250000*SNCY*SNCY*THBT*GAMI*  MU*  MU*
 4     0.250000*CSCY*SNCY*GAMI*  MU*  MU*
 5    -0.333333*SNCY*THBT*GAMI*  MU*
 6     0.166667*CSCY*GAMI*  MU*
 7     1.000000*  IB*OMB2*

 ******************************************
 *        REMAINING TERMS IN EQUATION      *
 ******************************************

 1    -0.125000*CSCY*THB1*GAMI*
 2    -0.250000*SNCY*SNCY*SNCY*THB2*GAMI*  MU*  MU*
 3     0.250000*SNCY*LAMB*GAMI*  MU*
 4    -0.250000*SNCY*SNCY*THB0*GAMI*  MU*  MU*
 5    -0.125000*THB0*GAMI*
 6     0.250000*SNCY*SNCY*THBT*BTPC*GAMI*  MU*  MU*
 7    -0.250000*CSCY*SNCY*SNCY*THB1*GAMI*  MU*  MU*
 8     0.166667*LAMB*GAMI*
 9    -0.333333*CSCY*SNCY*THB1*GAMI*  MU*
```

```
10    -0.333333*SNCY*SNCY*THB2*GAMI*   MU*
11    -0.125000*SNCY*THB2*GAMI*
12    -0.333333*SNCY*THB0*GAMI*   MU*
13     0.333333*SNCY*THBT*BTPC*GAMI*   MU*
14     0.125000*THBT*BTPC*GAMI*
15    -1.000000*   IB*BTPC*OMB2*
```

APPENDIX III

```
*******************************************
*                                         *
*       MULTI-BLADE EQUATION   BTM1       *
*                                         *
*******************************************


-------------------------------------------
-   GROUP OF TERMS WITH VARIABLE   BDD0   -
-------------------------------------------

1      3.000000*  IB*

-------------------------------------------
-   GROUP OF TERMS WITH VARIABLE    BD0   -
-------------------------------------------

1      0.375000*GAMI*
2      3.000000*  IB*CCBB*

-------------------------------------------
-   GROUP OF TERMS WITH VARIABLE     B0   -
-------------------------------------------

1    -0.375000*THBT*GAMI*
2    -0.375000*THBT*GAMI*  MU*  MU*
3     3.000000*  IB*
4     3.000000*  IB*OMB2*

-------------------------------------------
-   GROUP OF TERMS WITH VARIABLE     B1   -
-------------------------------------------

1      0.187500*THBT*C3CY*GAMI*  MU*  MU*
2      0.187500*S3CY*GAMI*  MU*  MU*

-------------------------------------------
-   GROUP OF TERMS WITH VARIABLE    BD2   -
-------------------------------------------

1      0.250000*GAMI*  MU*

-------------------------------------------
-   GROUP OF TERMS WITH VARIABLE     B2   -
-------------------------------------------

1      0.187500*THBT*S3CY*GAMI*  MU*  MU*
2    -0.187500*C3CY*GAMI*  MU*  MU*
3    -0.500000*THBT*GAMI*  MU*

-------------------------------------------
-   GROUP OF TERMS WITH VARIABLE    ZD0   -
-------------------------------------------

1      0.500000*LAMB*GAMI*
2      0.250000* BB1*GAMI*  MU*
3    -0.500000*THB2*GAMI*  MU*
4    -0.750000*THB0*GAMI*
5      0.750000*THBT*BTPC*GAMI*
6    -0.750000*THBT* BB0*GAMI*
7    -0.500000*THBT* BB2*GAMI*  MU*
8      6.000000*  IB* BB0*

-------------------------------------------
```

```
-   GROUP OF TERMS WITH VARIABLE     Z0    -
--------------------------------------------

 1   -0.375000*THBT*S3CY* BB1*GAMI*  MU*  MU*
 2    0.375000*THBT*C3CY* BB2*GAMI*  MU*  MU*
 3   -0.375000*THB1*S3CY*GAMI*  MU*  MU*
 4    0.375000*THB2*C3CY*GAMI*  MU*  MU*
 5   -0.375000*THZE*GAMI*  MU*  MU*
 6   -0.375000*THZE*GAMI*
 7    0.375000*C3CY* BB1*GAMI*  MU*  MU*
 8    0.375000*S3CY* BB2*GAMI*  MU*  MU*
 9   -0.500000*THBT* BB1*GAMI*  MU*
10   -0.500000*THB1*GAMI*  MU*
11    3.000000*  IB*THB0*OMZ2* ECF*
12   -3.000000*  IB*THBT*BTPC*OMZ2* ECF*
13    3.000000*  IB*THBT* BB0*OMZ2* ECF*
14   -3.000000*  IB*THB0*OMB2* ECF*
15    3.000000*  IB*THBT*BTPC*OMB2* ECF*
16   -3.000000*  IB*THBT* BB0*OMB2* ECF*

--------------------------------------------
-   GROUP OF TERMS WITH VARIABLE    ZD1    -
--------------------------------------------

 1   -0.375000*THB1*GAMI*
 2   -0.250000*THBT*S3CY* BB1*GAMI*  MU*
 3    0.250000*THBT*C3CY* BB2*GAMI*  MU*
 4   -0.250000*THB1*S3CY*GAMI*  MU*
 5    0.125000*S3CY* BB2*GAMI*  MU*
 6    3.000000*  IB* BB1*
 7    0.187500* BB2*GAMI*
 8    0.250000* BB0*GAMI*  MU*
 9    0.250000*THB2*C3CY*GAMI*  MU*
10    0.125000*C3CY* BB1*GAMI*  MU*
11   -0.375000*THBT* BB1*GAMI*

--------------------------------------------
-   GROUP OF TERMS WITH VARIABLE     Z1    -
--------------------------------------------

 1   -0.500000*THBT*C3CY* BB1*GAMI*  MU*
 2   -0.500000*THBT*S3CY* BB2*GAMI*  MU*
 3   -0.500000*THB1*C3CY*GAMI*  MU*
 4    0.375000*THB2*GAMI*
 5   -3.000000*  IB* BB2*
 6   -0.375000*THB0*S3CY*GAMI*  MU*  MU*
 7    0.375000*THBT*BTPC*S3CY*GAMI*  MU*  MU*
 8   -0.375000*THBT*S3CY* BB0*GAMI*  MU*  MU*
 9    0.375000*C3CY* BB2*GAMI*  MU*
10   -0.187500*THBT* BB2*GAMI*  MU*  MU*
11   -0.187500*THB2*GAMI*  MU*  MU*
12    0.187500*THZE*C3CY*GAMI*  MU*  MU*
13    0.187500* BB1*GAMI*
14    0.375000*LAMB*GAMI*  MU*
15    0.375000*C3CY* BB0*GAMI*  MU*  MU*
16    0.187500* BB1*GAMI*  MU*  MU*
17   -0.500000*THB2*S3CY*GAMI*  MU*
18    1.500000*  IB*THBT* BB1*OMZ2* ECF*
19    1.500000*  IB*THB1*OMZ2* ECF*
20   -0.375000*S3CY* BB1*GAMI*  MU*
21    0.375000*THBT* BB2*GAMI*
22   -1.500000*  IB*THBT* BB1*OMB2* ECF*
23   -1.500000*  IB*THB1*OMB2* ECF*

--------------------------------------------
-   GROUP OF TERMS WITH VARIABLE    ZD2    -
```

```
-----------------------------------------

1     -0.125000*C3CY* BB2*GAMI*  MU*
2     -0.375000*THB2*GAMI*
3      0.250000*THB1*C3CY*GAMI*  MU*
4     -0.187500* BB1*GAMI*
5     -0.375000*THBT* BB2*GAMI*
6     -0.500000*THB0*GAMI*  MU*
7      0.500000*THBT*BTPC*GAMI*  MU*
8      3.000000*  IB* BB2*
9      0.250000*THB2*S3CY*GAMI*  MU*
10    -0.500000*THBT* BB0*GAMI*  MU*
11     0.250000*THBT*C3CY* BB1*GAMI*  MU*
12     0.250000*THBT*S3CY* BB2*GAMI*  MU*
13     0.125000*S3CY* BB1*GAMI*  MU*

-----------------------------------------
-   GROUP OF TERMS WITH VARIABLE     Z2   -
-----------------------------------------

1      3.000000*  IB* BB1*
2      0.375000*S3CY* BB0*GAMI*  MU*  MU*
3     -0.500000*THBT*S3CY* BB1*GAMI*  MU*
4      0.375000*S3CY* BB2*GAMI*  MU*
5     -0.187500*THB1*GAMI*  MU*  MU*
6     -0.187500* BB2*GAMI*  MU*  MU*
7     -0.187500*THBT* BB1*GAMI*  MU*  MU*
8     -0.375000*THBT* BB1*GAMI*
9     -0.500000*THZE*GAMI*  MU*
10     0.187500* BB2*GAMI*
11     0.500000*THB2*C3CY*GAMI*  MU*
12    -0.500000*THB1*S3CY*GAMI*  MU*
13     0.187500*THZE*S3CY*GAMI*  MU*  MU*
14     1.500000*  IB*THBT* BB2*OMZ2* ECF*
15     0.375000*THB0*C3CY*GAMI*  MU*  MU*
16     1.500000*  IB*THB2*OMZ2* ECF*
17    -0.375000*THB1*GAMI*
18     0.375000*C3CY* BB1*GAMI*  MU*
19    -0.375000*THBT*BTPC*C3CY*GAMI*  MU*  MU*
20     0.500000*THBT*C3CY* BB2*GAMI*  MU*
21    -1.500000*  IB*THBT* BB2*OMB2* ECF*
22     0.375000*THBT*C3CY* BB0*GAMI*  MU*  MU*
23    -1.500000*  IB*THB2*OMB2* ECF*

-----------------------------------------
-   GROUP OF TERMS WITH VARIABLE   DCDD   -
-----------------------------------------

1     -2.250000*  IB* BB1*  HB*

-----------------------------------------
-   GROUP OF TERMS WITH VARIABLE    DCD   -
-----------------------------------------

1     -0.375000*S3CY* BB1*GAMI*  HB*  MU*
2      0.375000*C3CY* BB2*GAMI*  HB*  MU*
3      0.500000*THB2*GAMI*  HB*
4      0.750000*THB0*GAMI*  HB*  MU*
5     -0.375000*THB1*C3CY*GAMI*  HB*  MU*
6     -0.375000*THB2*S3CY*GAMI*  HB*  MU*

-----------------------------------------
-   GROUP OF TERMS WITH VARIABLE   DSDD   -
-----------------------------------------

1     -2.250000*  IB* BB2*  HB*
```

```
-----------------------------------------
-   GROUP OF TERMS WITH VARIABLE    DSD  -
-----------------------------------------

1     0.375000*C3CY* BB1*GAMI*  HB*  MU*
2     0.375000*S3CY* BB2*GAMI*  HB*  MU*
3     0.375000*THB2*C3CY*GAMI*  HB*  MU*
4    -0.500000*THB1*GAMI*  HB*
5    -0.250000*GAMI*  MU*
6    -0.375000*THB1*S3CY*GAMI*  HB*  MU*

-----------------------------------------
-   GROUP OF TERMS WITH VARIABLE    NU0  -
-----------------------------------------

1     0.500000*GAMI*

-----------------------------------------
-   GROUP OF TERMS WITH VARIABLE    NU2  -
-----------------------------------------

1     0.250000*GAMI*  MU*

-----------------------------------------
-       REMAINING TERMS IN EQUATION      -
-----------------------------------------

1    -0.375000*THBT* BB0*GAMI*
2     3.000000*  IB* BB0*
3    -0.375000*THBT* BB0*GAMI*  MU*  MU*
4     0.187500*THBT*C3CY* BB1*GAMI*  MU*  MU*
5     0.187500*THBT*S3CY* BB2*GAMI*  MU*  MU*
6     0.187500*S3CY* BB1*GAMI*  MU*  MU*
7    -0.187500*C3CY* BB2*GAMI*  MU*  MU*
8    -0.500000*THBT* BB2*GAMI*  MU*
9     3.000000*  IB* BB0*OMB2*
10    0.187500*THB2*S3CY*GAMI*  MU*  MU*
11   -0.375000*THB0*GAMI*  MU*  MU*
12   -0.375000*THB0*GAMI*
13    0.375000*THBT*BTPC*GAMI*  MU*  MU*
14    0.187500*THB1*C3CY*GAMI*  MU*  MU*
15    0.500000*LAMB*GAMI*
16   -0.500000*THB2*GAMI*  MU*
17    0.375000*THBT*BTPC*GAMI*
18   -3.000000*  IB*BTPC*OMB2*
```