

EIGHTH EUROPEAN ROTORCRAFT FORUM

DEVELOPMENT OF THE BASIC METHODS NEEDED
TO PREDICT HELICOPTERS AEROELASTIC BEHAVIOUR

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SUMMARY

The problems of structural dynamics, blade unsteady aerodynamic forces, stability and forced vibrations of the coupled rotor-fuselage structure are discussed. The paper shows the peculiarities of the basic calculation methods developed at ONERA in cooperation with the AEROSPATIALE. Some of these methods have been derived from the formulations used by the fixed wing specialists.

RESUME

La discussion porte sur les problèmes de dynamique de la structure, d'aérodynamique instationnaire des pales, de stabilité et de réponse forcée de l'ensemble couplé fuselage-rotor. Le texte montre les particularités des méthodes de calcul de base développées à l'ONERA en coopération avec l'Aérospatiale. Certaines de ces méthodes ont été déduites des formulations utilisées par les spécialistes de l'aile fixe.

INTRODUCTION

The fixed wing flutter phenomenon has long motivated researches in the fields of Structural Dynamics, Unsteady Aerodynamics and Stability Analysis. This activity resulted in the Development (up to operational stage) of many computational and experimental technics : computation of structures by finite elements (in view of modelizing the structure by its normal modes), calculation of coupled unsteady aerodynamic forces based on the linear lifting surface theory, ground and flight vibration testing and flutter model wind-tunnel testing. Other methods, such as, for instance, the methods of calculation based on the resolution of the small transonic disturbance equation are still in the process of development.

Except for the difficulties specific to transonic flow, or to the presence of dry friction in the structure, the linear methods are satisfactory for a wide range of applications to airplanes, but they cannot be applied directly to helicopters because the generation of lift and thrust by rotating blades is a formidable source of complexity to the aerodynamic and structural dynamics problems (presence of periodic coefficients in the Lagrangian equations, structural non linearities due to flap and pitch large amplitude oscillations, aerodynamic non linearities due to transonic flow on the advancing blade and to unsteady stall on the retreating blade).

Consequently, the manufacturers and research organizations had to develop methods suitable for rotary wing aircraft.

In France this task is performed mainly by ONERA, in cooperation with the AEROSPATIALE. Among the methods which were proposed, some are specific to rotary wing, others have been derived from methods used for the fixed wing flutter analysis. Whenever possible, this last procedure was preferred even when the fixed wing methods had to be deeply modified in order to cope with the difficulties peculiar to helicopters.

The various aspects of that research are discussed in the paper.

1. STRUCTURAL DYNAMICS

Different approaches have been used in order to develop the mathematical model which is needed by the dynamics specialists to predict helicopters instabilities and forced vibration, as, for instance, [1] and [2].

1.1 Modal representation

In this chapter it is shown that the modal representation, which is extensively used by the specialists of fixed wing aircraft aeroelasticity, can be applied also to helicopters.

The modelization used for the fixed wing aircraft is based on the linear theory and is valid for the investigation of small amplitude vibrations. The structure deflections due to external loads are approximated by a superposition of a finite number of normal mode shapes of the conservative structure :

$$r(P,t) = \sum_{k=1}^N \Phi_k(P) q_k(t) \quad (1)$$

where

$r(P, t)$ is the deflection vector at the point P of the structure

$\Phi_k(P)$ the spatial vector field describing the k^{th} mode shape

$q_k(t)$ the generalized coordinate associated to mode k .

The internal dissipation forces, which are small compared to inertial and elastic forces, are generally approximated by viscous forces and the Lagrangian equations which govern the response to dynamic loads may be written as :

$$[\mu](\ddot{q}) + [\beta](\dot{q}) + [\gamma](q) = (Q) \quad (2)$$

where

$[\mu]$ is the diagonal matrix of generalized masses

$[\gamma]$ the diagonal matrix of generalized stiffnesses

$[\beta]$ the symmetric matrix of generalized dissipation coefficients

(Q) the column of external generalized forces.

The matrix $[\beta]$ is often taken as diagonal, as well as $[\mu]$ and $[\gamma]$. This approximation, which is justified for weakly dissipative structures, makes the system of equations diagonal, so that each generalized coordinate is determined by a separate equation and responds like a single degree of freedom system with low damping. The selectivity of the response in the frequency domain, which results from the low damping, allows one to investigate the aeroelastic problems in limited ranges of frequency, with a relatively small number of degrees of freedom. This peculiarity of the modal representation simplifies the analysis and makes the interpretation of the results easier.

Another advantage lies on the fact that the modal characteristics which enter into the dynamic model, i.e. the mode shapes and generalized masses, stiffnesses and dissipation coefficients, can be determined experimentally by a ground vibration test. They can also be predicted by structural calculations (finite element method), except for the dissipation coefficients which are usually evaluated empirically.

But the reason why the modal representation is so easy to apply in the case of the fixed wing aircraft, is that the motion of the structure is limited to small amplitude vibrations superimposed to a uniform translation.

1.2 Application to the coupled rotor-fuselage

If a part of the structure performs a non uniform motion at finite velocity, as it is the case for helicopter blades, the structural dynamic analysis is much more complex. The helicopter with blades rotating at constant speed may be assimilated to a set of two coupled substructures : one performing small amplitude vibrations in a fixed frame (fuselage and hub), the other one performing small movements in a rotating frame (rotor).

In order to modelize the full helicopter structure by a discrete system, the small deflections can be defined as functions of a set of fuselage generalized coordinates, $q_k(t)$, and a set of blade generalized coordinates, $s_{k\alpha}(t)$, $s_{k\beta}(t)$

where $\alpha, \beta \dots$ stand for blade α , blade $\beta \dots$. Then the Lagrangian equations can be derived from the kinetic and potential energies, or from a direct application of the theorem of virtual works. As a consequence of the assumption of small motions, the generalized coordinates $q_k, s_{k\alpha}, \dots$ are considered as first order small quantities and the Lagrangian equations are linearized.

But the linearized Lagrangian equations are determined by the development up to the second order of the kinetic and potential energies as functions of the generalized coordinates ; and this development depends on the second order approximation of certain blade deflections. For instance if Ω is the rotor velocity, r the radial coordinate of a point P of the undeformed rotor and u the radial deflection, the term $\Omega^2(r+u)^2$ has a major contribution to the rotating blade kinetic energy. Since r and Ω are finite quantities, the evaluation of the second order approximation of the product $\Omega^2 r u$ necessitates the development of the deflection u itself up to the second order :

$$u(-s_k) = \sum_k \frac{\partial u}{\partial s_k} s_k + \frac{1}{2} \sum_{k\ell} \frac{\partial^2 u}{\partial s_k \partial s_\ell} s_k s_\ell$$

The representation of the deflections by a linear superposition of prescribed spatial fields, which is used for the fixed wing (formula (1)) gives, only the first order term $(\sum_k \frac{\partial u}{\partial s_k} s_k)$ which is obviously not sufficient in this case.

But this approximation can be completed by a separate evaluation of the second order terms based on appropriate kinematic assumptions.

For instance, if the blade lies in the plane of the rotor, the second order radial deflection results from the first order bending deflection as shown in fig. (1). If we assume that the length of the neutral fiber is not modified by the bending deflection, we see that the first order bending $w(r,t)$ induces a second order radial displacement given by :

$$u(r,t) = -\frac{1}{2} \int_0^r \left(\frac{\partial w}{\partial r_0} \right)^2 dr_0$$

and if the bending deflection is approximated by a superposition of functions $W_k(r)$ by the formula

$$w(r,t) = \sum_k W_k(r) s_k(t)$$

we have :

$$u(r,t) = -\frac{1}{2} \sum_{k\ell} \int_0^r \frac{\partial W_k}{\partial r_0} \frac{\partial W_\ell}{\partial r_0} dr_0 s_k(t) s_\ell(t)$$

This term is particularly important since it defines the virtual displacement of the centrifugal force and determines the centrifugal stiffening effect which makes the blade natural bending and flapping frequencies increase with Ω .

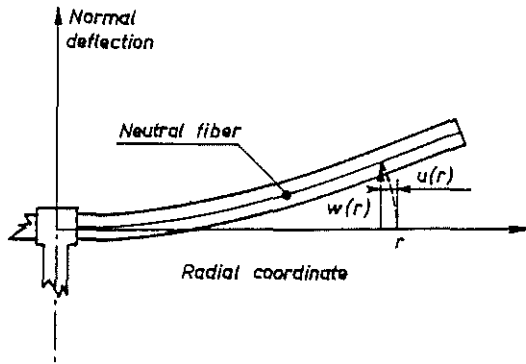


Fig. 1 — Evaluation of the second order radial deflection of a blade (u) induced by the first order bending deflection (w).

With this procedure, the fuselage and blade generalized coordinates can be defined by stating that the fuselage and hub deflections in the fixed frame, and the first order blade deflections in the rotating frame, are linear superpositions of prescribed spatial fields defined respectively on the fuselage and hub, and on the blades.

The choice of the basic fields is particularly important. By analogy with the method of branch-modes, which is applied to non rotating structures, we can use the normal mode shapes of each separate sub-structure, i.e. the fuselage and the hub on one hand, and one blade on the other hand. These modes can be calculated or determined experimentally by a ground vibration test (fig. (2)).

But if the free-modes of the sub-structures are used, we must keep in mind that the mode shapes so determined are unable to represent the deflections due to the concentrated reactions which are present, at the hub and blade attachment in the coupled rotor-fuselage structure. In order to get a satisfactory representation of these deformations, it is necessary to use additional static modes describing the deflections due to the different components of the reactions.

The use of these additional static modes is not necessary if instead of the mode shapes of the non modified free sub-structures, one takes the mode shapes determined on modified configurations. For instance a mass can be fixed on the hub in order to get fuselage modes with loads at the hub ; on the other hand we can use the modes of the blades fixed on a wall by their attachment.

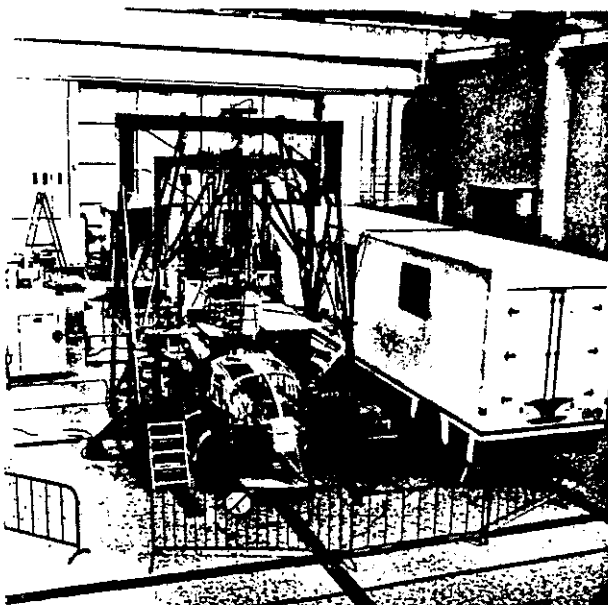


Fig. 2 — Ground Vibration test of a SA 349 helicopter fuselage. Measurement of normal modes and transfer function for forces and moments applied at the hub.

1.3 Form of the Lagrangian equations

The Lagrangian equations derived with the fuselage and blade coordinates defined in this manner are a set of second order differential equations, linear but with coefficients which depend on the blade azimuth and are periodic functions of time.

If the number of blades of the helicopter is larger than two, the multi-cyclic symmetry of the rotor makes it possible to cancel the periodic coefficients with an appropriate choice of coordinates. These coordinates σ_k may be defined as :

$$s_{k\alpha} = \sigma_{k0} + \sigma_{kc} \cos \theta_\alpha + \sigma_{ks} \sin \theta_\alpha$$

where $s_{k\alpha}(t)$ is a generalized coordinate of blade α

$\theta_\alpha(t)$ the azimuth coordinate of blade α in the fixed frame

σ_{k0} , σ_{kc} and σ_{ks} are the new coordinates, also called rotor coordinates or Coleman coordinates.

If the variables σ_k are substituted to the blade variables s_k , the periodic coefficients cancel in the azimuth integration and the resulting Lagrangian equations have only constant coefficients and may be written as :

$$[M](\ddot{X}) + \Omega [G](\dot{X}) + [[K] + \Omega^2 [R]](X) = 0 \quad (3)$$

where (X) is the column of generalized coordinates : $(X) = \begin{pmatrix} \theta \\ \sigma_{k0} \\ \sigma_{kc} \\ \sigma_{ks} \end{pmatrix}$

[M] the symmetric positive generalized inertia matrix,
 [K] the symmetric positive generalized stiffness matrix,
 [G] the anti symmetric gyroscopic matrix,
 [R] the symmetric matrix of centrifugal stiffness.

The matrix [R] is symmetric but usually not positive, so that the matrix sum $[K] + \Omega^2 [R]$ is semi negative when Ω is sufficiently large (i.e. one or several of its eigen values are negative), and the system may become unstable (divergence or dynamic instability). The energy which is necessary for the divergent motion to occur is taken from the rotor kinetic energy.

For lower values of Ω , the system is stable and the natural modes are harmonic vibrations, as one would expect since the system is conservative.

The Lagrangian equations may also be used to predict the response of the structure to loads applied to the fuselage or the blades. But in the problems of forced response, it is necessary to take into account the internal dissipation forces. The dissipation characteristics of the fuselage and the blades may be determined in the vibration tests performed on the two substructures, and the dissipation at the blade attachment may also be derived from an appropriate vibration test.

The transfer functions relating the structural responses to the loads, in the frequency domain, can be derived from the Lagrangian equations or measured directly in a vibration test.

It is interesting to notice that the problem of assembling two sub-structures can also be formulated directly in the frequency domain. In that case, the coupling conditions at the hub and blade attachment are determined by the fuselage and blade transfer functions of the two sub-structures (for forces and moments applied at the hub and at the blade attachment).

1.4 Comparison with experiment

Generally, the natural frequencies and dampings and the forced responses predicted with the modelization described here are in good agreement with the measurements performed on models (see for instance [3]).

A comparison performed in cooperation with the US Army Aeromechanics Laboratory, at Ames (USA), is illustrated by fig. (3) and (4). The model built and tested by the US Army is a three bladed rotor mounted on a support simulating fuselage degrees of freedom. The natural frequencies and dampings were determined while the blades were rotating at a stabilized r.p.m. The tests were performed in still air. The blades aerodynamic forces were small compared to inertial and elastic forces and had a negligible effect on the natural modes.

The calculations were performed with an ONERA computer code using the Coleman transformation, with the fuselage and blades characterized by their normal modes measured in appropriate configurations.

The comparison of theoretical and experimental frequencies, shown in fig. (3), is quite satisfactory. At the frequency coalescence which is observed at the point A of the diagramme, the calculation predicted a ground resonance instability which was confirmed by experiment.

Consequently the structural dynamics problem of the coupled rotor-fuselage seems to have been solved satisfactorily.

Nevertheless, the mathematical model is linear and its application to helicopters must be carried out cautiously because the amplitude of the low frequency blade oscillations (flapping and cyclic pitch) is relatively large and the non linear effects cannot be neglected. Usually the resulting non linear problem can be solved by an iterative procedure in which each step is a linear calculation which may be performed with the model discussed here.

1.5 Interpretation of the Coleman transformation

The Coleman transformation, which makes use of the multicyclic symmetry of the rotor to cancel the periodic coefficients from the Lagrangian equations, deserves a special attention.

Its physical meaning appears more clearly if instead of the actual multi-bladed rotor, one considers a continuous axisymmetric rotor.

The small deflections at a point P of the rotor, in the rotating frame, may be defined by a vector U with three components (radial, azimuthal and normal deflection). The vector U is function of the time and of the coordinates of the point P in the rotating frame :

$$U = U(r, z, \nu, t)$$

where r is the radial coordinate of the point P

ν its azimuth in the rotating frame

z its ordinate in the direction normal to the plane of the rotor.

But an observer standing in the fixed frame and looking at the rotor refers the azimuth to a fixed axis and so defines the rotor deflection as function of θ instead of ν , where $\theta = \Omega t + \nu$ is the azimuth in the fixed frame :

$$U = U(r, z, \theta, t)$$

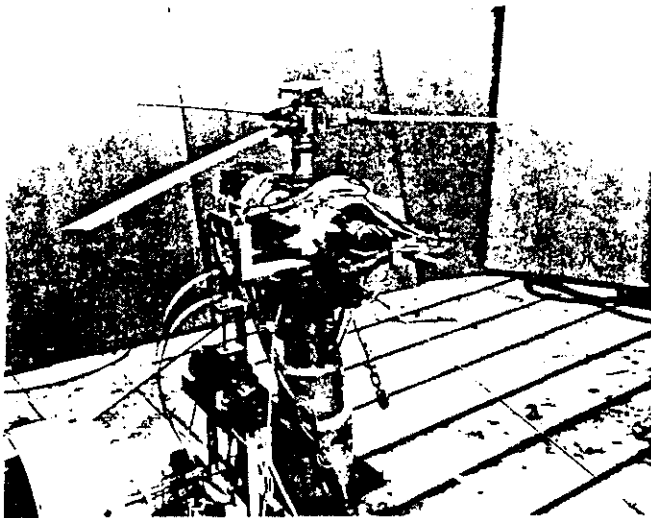


Fig. 3 – Simplified helicopter model.

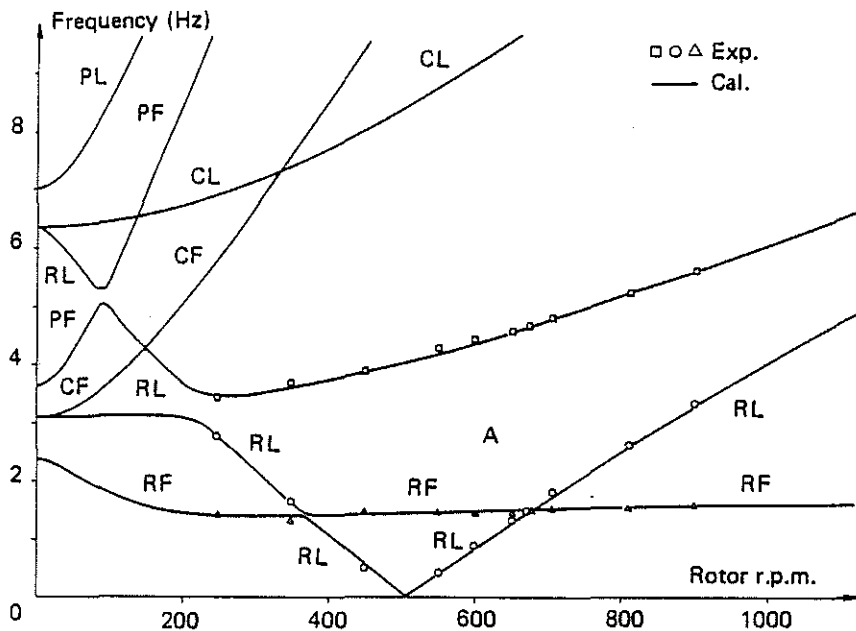


Fig. 4 – Variation of natural frequencies with rotation speed. Identification of the modes.

- | | |
|--------------------------------|--------------------------------|
| PL, blade progressive lead-lag | PF, blade progressive flapping |
| CL, blade collective lead-lag | CF, blade collective flapping |
| RL, blade regressive lead-lag | RF, blade regressive flapping. |

U is periodic in θ and its Fourier series development may be written as :

$$U = U_0 + \sum_k (U_{kc} \cos k\theta + U_{ks} \sin k\theta)$$

U_0, U_{kc}, U_{ks} are deflection vectors depending on r, z, t but independent of the azimuth θ .

If now these vectors are substituted to U in the equation giving the developments of the kinetic and potential energies up to the second order we find that, as a consequence of the axisymmetry of the rotor, the periodic coefficients cancel in the integrations along the azimuth coordinate.

Furthermore, the analysis of the couplings, which are determined by the cross products in the kinetic and potential energies, show that :

- the deflections of order k (i.e., $U_{kc} \cos k\theta + U_{ks} \sin k\theta$) are independent and may be analysed separately, with the Lagrangian equations, for each value of k , for $k \geq 2$.

- the deflections of order zero (i.e. U_0) are coupled only with the hub axial degree of freedom

- the deflections of order one (i.e., $U_{1c} \cos \theta + U_{1s} \sin \theta$) are coupled only with pitch, roll and in-plane degrees of freedom of the hub.

Therefore, the fuselage rotor induced vibrations can be investigated with a Fourier series limited to the only terms which are coupled with the hub degrees of freedom, i.e. : U_0 , U_{1c} and U_{1s} :

$$U = U_0 + U_c \cos \theta + U_s \sin \theta$$

The representation of U_0 , U_c and U_s with a basis of prescribed spatial fields leads to the already defined Coleman coordinates σ_{k_0} , σ_{k_c} and σ_{k_s}

Since the rotor deflections of order zero and one are not coupled together, the natural modes resulting from the resolution of the Lagrangian equations may be divided into modes resulting from the combination of fuselage deflections and rotor deflections of order zero (U_0), and modes resulting from the combination of fuselage deflection and rotor deflection of order one (U_c and U_s).

For an observer standing in the fixed frame, the modes derived with the deflections U_0 are axisymmetric rotor vibrations (collective modes) ; as for the modes resulting from U_c and U_s , they take on the form of traveling waves, as a result of the combination of the $\cos \theta$ and $\sin \theta$ deflections (progressive and regressive modes, also called cyclic modes).

The possibility of canceling the periodic coefficients from the Lagrangian equations with the Coleman transformation is not restricted to the structural dynamics problem. It also applies to any force field coupled with the blades, provided that the coupled forces are linear and have the appropriate symmetries. Consequently it applies to a helicopter in stationary flight and to an aircraft with a propeller in forward flight, but it does not apply to a helicopter in forward flight.

2. BLADE UNSTEADY AERODYNAMICS

The accuracy with which the blades unsteady aerodynamic forces should be evaluated depends on the dynamic phenomena which are investigated. For instance, the blade flap-lag-torsion instability is often investigated with a quasi-steady model whereas the periodic loads in forward flight must be evaluated with a more accurate aerodynamic model.

But the combination of the blade advancing and rotating speeds is a formidable source of complexity for the flow. At large values of the advance ratio, the aerodynamic field around the blades undergoes such variations that there are problems of transonic flow, with shock waves, at the advancing blade tip, problems of speed reversal and low speed unsteady stall on the retreating blade, and problems of oblique attack in the fore and aft blade positions. Furthermore the geometry of the wake, which is an important vibration and noise generator, is much more complicated than the fixed wing wake geometry.

The problem of wake interaction has long been the object of experimental and theoretical researches. Several authors have developed calculation methods in which the velocity induced by the shed and trailing vortices is determined by the Biot-Savart law. They relate the strength of the vortices to the variations of circulation around the blades (time variations for the shed vortices and spanwise variations for the trailing vortices) and use an iterative procedure to determine the blade spanwise load distribution [4] and [5]. With this approach it is possible to take into account the wake distortions as they are measured in the wind tunnel or calculated separately. On the other hand the method is flexible only in its application to incompressible fluid and so the compressibility effects are usually taken into account with the Prandtl Glauert correction.

The approach used at ONERA, which will be described here, is different. It proceeds from the basic formulation used for the fixed wing flutter calculations and gives, except for the errors due to the discretization, the solution of the linearized problem of the lifting surface in a compressible inviscid 3-D flow.

But, since it is linear, the method is valid only for a rotor moderately loaded and moving at relatively low advance ratio, in order for the transonic and unsteady stall effects to be relatively small.

The problem of unsteady stall on the retreating blade is solved in a semi-empirical manner, with a model of 2-D unsteady flow made with a system of differential equations whose coefficients are identified by wind tunnel tests on oscillating airfoils.

The 2-D model and the 3-D linear theory can be combined together, in order to predict the simultaneous effects of 3-D flow and unsteady stall, with a coupling algorithm based on the assumption that the linear theory is valid, even in presence of stall, provided that it is utilized only to evaluate the induced velocity resulting from the 3-D interactions.

For the transonic flow problems, the approach based on the small transonic disturbance equation (STD) is generally preferred to a resolution of the full equations because the computing time is an important feature for aeroelastic calculations. A significant effort is being made in this direction in view of the applications to fixed wing and to helicopters [6], [7]; but the discussion of these methods would be outside the scope of this Dynamics session which is limited to methods peculiar to aeroelastic and dynamic investigations.

1.1 Linear lifting surface theory

The classical fixed wing flutter calculations are based on the linear lifting surface theory.

In this application, the lifting surface is assimilated to a thin plate lying in an (x, y) plane as shown in figure (5) and performing small amplitude harmonic vibrations which create a small disturbance to the uniform flow.

The disturbance velocity potential verifies the linear equation of acoustic waves and the solution of the boundary value problem depends linearly on the distribution or normal velocity (or downwash) over the lifting surface, through the boundary condition of normal derivative.

In the derivation and interpretation of the basic formulation used by the aerolasticians, it is interesting to consider the acceleration potential ψ defined as the substantial derivative of the velocity potential :

$$\psi = \frac{d\varphi}{dt} = \frac{\partial\varphi}{\partial t} + U \frac{\partial\varphi}{\partial x}$$

Ψ is proportional to the pressure perturbation :

$$p - p_\infty = -\rho_\infty \Psi$$

Let h be the function which describes the harmonic vibration of the lifting surface at a frequency ω :

$$h(x,y,t) = \tilde{h}(x,y) e^{i\omega t} \quad \text{over (A)}$$

The downwash which determines the normal derivative of φ on the lifting surface is given by :

$$\tilde{w}(x,y) = U \frac{\partial \tilde{h}}{\partial x} + i\omega \tilde{h} \quad \text{over (A)}$$

In the aerodynamic problem which is considered here, the lifting surface motion is given, and so are h and w . On the other hand, the unknown function which must be determined is the pressure jump through the lifting surface, $\Delta \tilde{p}(x, y)$.

The basic formulation used by aeroelasticians is the singular integral equation relating the downwash to the pressure jump. This equation proceeds directly from the two following equations :

$$\tilde{\varphi}(x,y,z) = \frac{1}{4\pi\rho_\infty U} \iint_{(A)} z K(x-\xi, y-\eta, z, \omega) \Delta \tilde{p}(\xi, \eta) d\xi d\eta \quad (4.1)$$

$$\tilde{w}(x,y) = \lim_{z \rightarrow 0} \frac{\partial \tilde{\varphi}}{\partial z} \quad (4.2)$$

Lifting surface deflection

$$z = h(x,y,t) = \tilde{h}(x,y) e^{i\omega t}$$

Downwash:

$$w = U \frac{\partial h}{\partial x} + \frac{\partial h}{\partial t}$$

Velocity potential

$$\varphi(x,y,z,t) = \tilde{\varphi}(x,y,z) e^{i\omega t}$$

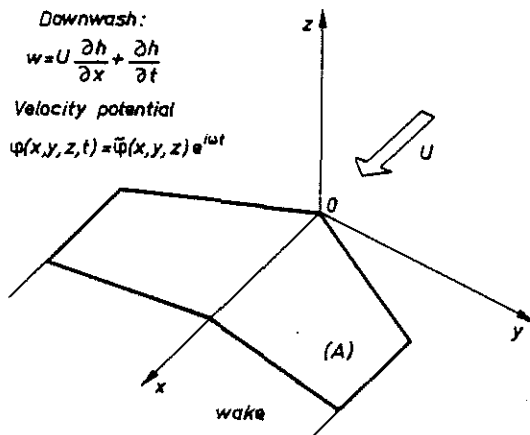


Fig. 5 - Lifting surface.

The singularity of the equation relating directly w to Δp results from the z differentiation for $z = 0$.

In equation (4-1), $\tilde{\varphi}$ is given as an integral superposition in which the Kernel function zK is the potential induced by an element of pulsating lifting surface. But an element of lifting surface is essentially a pressure discontinuity along the axis defined by its normal vector, and such a discontinuity can be created by a pressure doublet, or a doublet of acceleration potential (since the pressure disturbance and the acceleration potential are proportional).

Indeed the Kernel function is shown to give the velocity potential induced by a pulsating acceleration doublet of strength $q = \Delta \tilde{p} / \rho_\infty$. It contains an integral resulting from the resolution of the differential equation $d\varphi/dt = \Psi$ which is integrated in order to derive the velocity potential from the expression of the doublet's acceleration potential. The integration is performed along the path of the lifting surface and so determines the effect of time history of the flow resulting from the wake.

The numerical solutions of the equations (4) may be determined by a collocation method, with collocation points distributed over the lifting surface, the pressure distribution being approximated by a superposition of prescribed pressure functions. But the method which is generally preferred at the present time is the "doublet lattice method" in which the lifting surface is divided into trapezoidal boxes as shown in fig. (6). In each box the doublet distribution is concentrated on a segment equivalent to a small bound and trailing vortex. The box also contains a collocation point on which the boundary condition of normal derivative is verified.

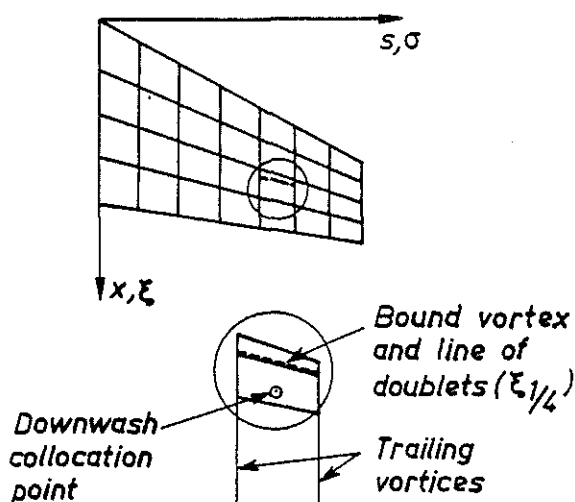


Fig. 6 - Surface idealization into boxes and location of vortices, doublets and collocation points.

The helicopter blades may also be assimilated to thin lifting surfaces, but for the determination of aerodynamic forces we need an integral equation which is not restricted to the combination of a small amplitude harmonic vibration and a uniform translation.

In view of this application, the integral equation was formulated in ref. [8] for a lifting line and in ref. [9] for a lifting surface performing an arbitrary motion in a fixed frame with the only limitation that the angle of attack should remain small. The lifting surface was assimilated to a distribution of acceleration doublets. The resulting equation, which is written here, shows that the potential at point P and time t depends on the time history of the movement and lift of the lifting surface elements.

$$\varphi(P, t) = \iint_{(A)} \frac{\Delta p(\vec{P}_0(\tau)) [\vec{P} - \vec{P}_0(\tau)] \cdot \vec{n}_0(\tau) d\sigma_0}{4\pi\rho_\infty c |\vec{P} - \vec{P}_0(\tau)|^2 \left[1 - \frac{[\vec{P} - \vec{P}_0(\tau)] \cdot \vec{V}_0(\tau)}{c |\vec{P} - \vec{P}_0(\tau)|} \right]} + \iint_{(A)} \int_{-\infty}^{\tau} \frac{\Delta p(\vec{P}_0(\tau_0)) [\vec{P} - \vec{P}_0(\tau_0)] \cdot \vec{n}_0(\tau_0)}{4\pi\rho_\infty |\vec{P} - \vec{P}_0(\tau_0)|^3} d\tau_0 d\sigma_0 \quad (5)$$

where τ is given by the equation :

$$t - \tau - \frac{|\vec{P} - \vec{P}_0(\tau)|}{c} = 0.$$

$d\sigma_0$ is the element of area associated to the integration point P_0 .

For the derivation of this equation, see preferably ref. [9] or [10].

The effect of time history results from the integration in τ_0 , which is performed along the path of the lifting surface, i.e. over the underformed wake as shown in fig. (7).

The integral equation is particularly convenient for the determination of the coupled unsteady aerodynamic forces on propellers and for the calculation of the periodic loads on a helicopter rotor in forward flight.

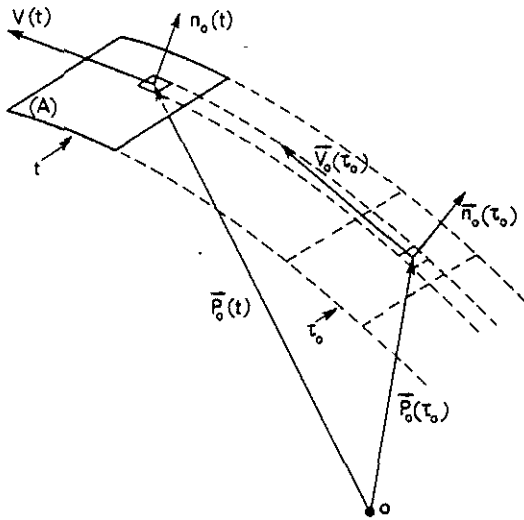


Fig. 7 — Path of the elements of lifting surface.

The numerical solutions are determined with a collocation method. The distribution of pressure jump through the blades is approximated by a superposition of prescribed pressure functions and the collocation is performed points distributed over the rotor disk. The normal derivative at a point in contact with a blade can be derived by a finite difference from the values of φ at two points located in the vicinity of the blade, on the same normal vector.

For the sake of simplification, the blade lift can be concentrated on the blade forward quarter chord line and in that case, the solution gives the lifting line approximation.

The applications performed at ONERA are described by J.J. Costes [11] [12].

As the acceleration potential is used for the derivation of the integral equation (5), the method is often called the "acceleration potential method" by helicopter specialists.

The integral equation gives the velocity potential as a superposition of fundamental solutions of the acoustic wave equation and so it takes account rigorously of the small disturbance compressibility effects. But it suffers from the limitations proper to linear methods.

On the other hand, it does not provide a convenient means of taking into account the wake distortion, and this may be a real source of difficulty when the wake remains in the vicinity of the rotor as it is the case for a helicopter in hover flight.

The comparisons with experiment are usually satisfactory except when the non-linear effects are predominant. Results obtained on an experimental 4m-diameter rotor, tested in the large S1 wind tunnel of Modane, are shown in fig. (8). The blade local lift is plotted against azimuth for different values of the blade section radial coordinate. The agreement between the linear theory and experiment is satisfactory except on the retreating blade (azimuth 270°) where the effect of stall is predominant.

Therefore the linear theory may be used as the basic theoretical tool to evaluate the unsteady aerodynamic loads, but non linear tools are also needed to take into account the strong non-linear effects. The fig. (8) also shows the result of a calculation combining, in a consistent manner, the linear theory and a model of 2-D unsteady stall [13] which was developed previously to the ONERA model described in the following paragraph. As it can be seen the combination of the linear 3D and non-linear 2D models results in a satisfactory agreement in the whole range of azimuth angle.

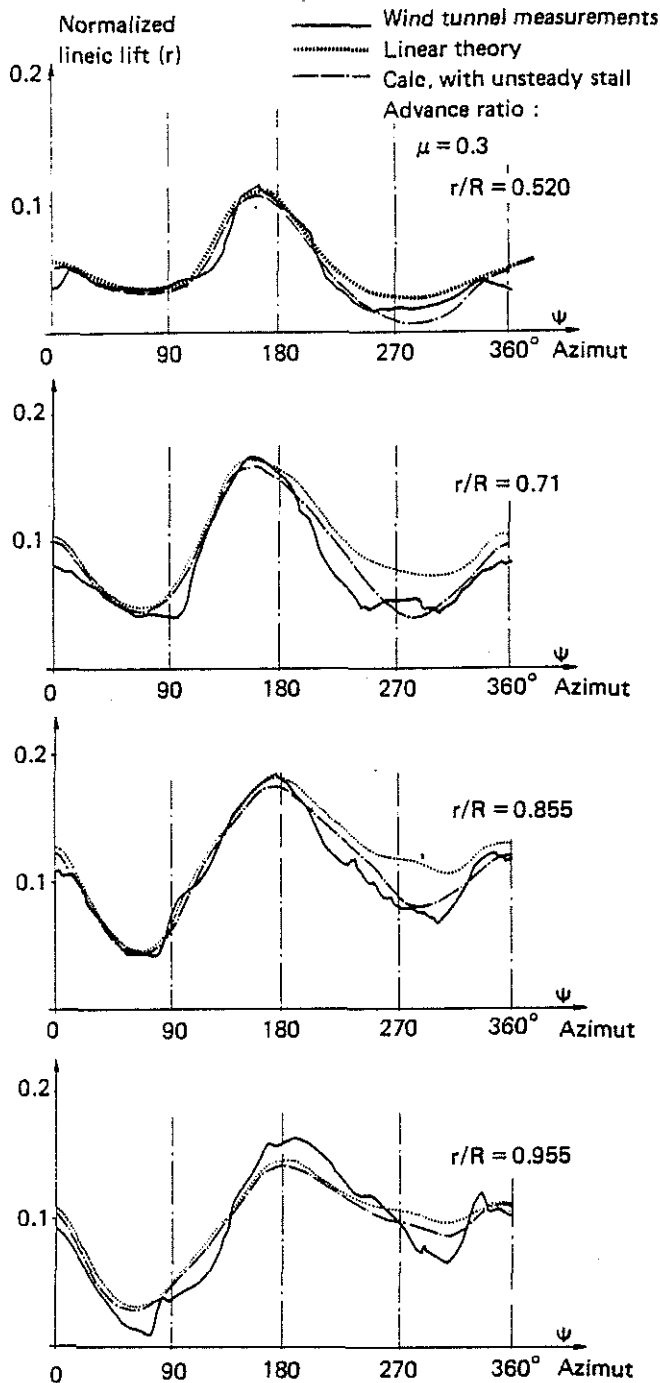


Fig. 8 – Blade lift on an experimental rotor for different values of the radial coordinate r/R .

2.2 Non-linear model of unsteady aerodynamic loads

In the preceding paragraph it is shown that the use of a 2D unsteady stall model can improve significantly the prediction of the blade aerodynamic loads.

But the model of ref. [13], which was used in the calculation illustrated by fig. (8), does not represent explicitly the effect of time history and consequently its validity is limited to a small range of frequencies.

In 1975, Beddoes [14], presented a model in which the unsteady behavior of the flow is simulated by the introduction of a time delay.

The ONERA model, which is briefly described here, is a more general formulation which makes use of the properties of differential equations to simulate the different effects which can be identified on an oscillating airfoil, i.e. : pseudo elastic, viscous and inertial effects, and the effect of the flow time history. The model is formulated in the time domain and is valid for an airfoil performing an arbitrary motion in the whole range of angle of attack, i.e. in the range of small incidences (or linear range) as well as the range of incidences beyond stall. It was presented in 1979 in ref. [15] and its application is illustrated in ref. [16] [17] [18].

The range of small incidences will be discussed first in order to justify the choice of the differential equations.

If the angle of attack remains in the linear range, the oscillating airfoil and the flow may be assimilated to a linear system in which the input variables are the deflection and pitch angle, h and θ , defined in fig. (9) and the output functions are the aerodynamic lift and moment non dimensionalized coefficients ϕ_F and ϕ_M .

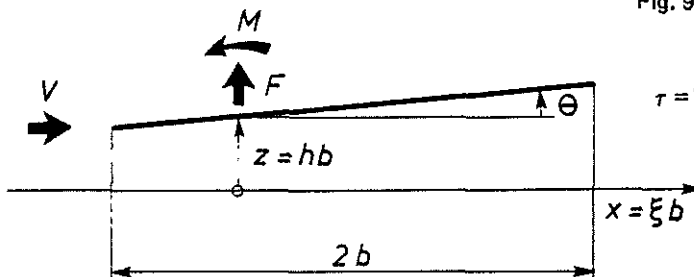


Fig. 9 - Definition of input variables and output functions.

$$F = -\pi\rho V^2 b L \phi_F$$

$$M = \pi\rho V^2 b^2 L \phi_M$$

$$\tau = Vt/b : \text{reduced time} ; \nu = \omega b/V : \text{reduced frequency}$$

Harmonic vibration \rightarrow

$$\begin{pmatrix} \tilde{\phi}_F \\ \tilde{\phi}_M \end{pmatrix} = [A(\nu)] \begin{pmatrix} \tilde{h} \\ \tilde{\theta} \end{pmatrix}$$

The dynamic characteristics of such a system may be defined in the time domain by its impulse responses, or in the frequency domain by its transfer functions.

For instance, the Kussner, Theodorsen or Van de Vooren coefficients resulting from the linear theory of 2D flow around a thin airfoil oscillating harmonically [19], [20] [21] are the aerodynamic transfer functions of the airfoil.

These transfer functions, like the transfer functions of most linear systems (even continuous systems), can be reasonably approximated by rational fractions in a relatively wide frequency range. For instance, fig. (10) shows that the coefficient relating the non dimensionalized lift ϕ_F to the pitch variable θ may be approximated by the rational fraction:

$$A_{12} = \frac{0,125}{i\nu + 0,1} + 1,551 + 1,65 i\nu + 0,35 \nu^2 - 0,53 i\nu^3$$

The resulting differential equation relating ϕ_F to θ in the time domain is equivalent to the system of equations :

$$\phi_F = \phi_1 + \phi_2 \quad (6.1)$$

$$\dot{\phi}_1 + 0,1 \phi_1 = 0,125 \theta \quad (6.2)$$

$$\phi_2 = 1,551 \theta + 1,65 \dot{\theta} - 0,35 \ddot{\theta} + 0,53 \theta''' \quad (6.3)$$

where ϕ_1 and ϕ_2 are dummy functions of the non-dimensionalized time τ .

For the sake of simplification, the two input variables h and θ are replaced in the model by a single, almost equivalent input variable defined as : $\alpha = \dot{h} + \theta$, where h is non dimensionalized and $(\dot{})$ stands for reduced time derivative. The variable α is the angle of attack (or downwash) at the forward quarter chord.

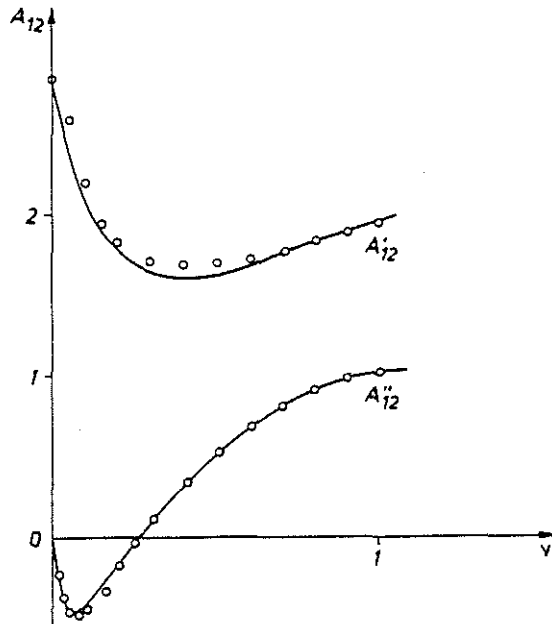


Fig. 10 - Lift due to pitch oscillation : $A_{12} = A_{12}^i + A_{12}^r$
 — Linear theory (ref. [20]) \circ Rational fraction :
 $A_{12} = (0.125/i\nu + 0.1) + 1.551 + 1.65i\nu + 0.35\nu^2 - 0.53i\nu^3$

The differential equation is a suitable formulation for the determination of the loads due to an arbitrary motion defined as function of time. The general solution is a convolution integral. For instance, the lift coefficient is given by :

$$\phi_F(\tau) = \int_{-\infty}^{\tau} a(\tau - \sigma) \alpha(\sigma) d\sigma$$

where $a(\tau)$ is the impulse response, i.e. the solution of the differential equation for an input α given by $\alpha = \delta(\tau)$, where $\delta(\tau)$ is the Dirac impulse function.

By this integral, $\phi_F(\tau)$ is shown to depend on the values of the input variable α in the past time (time history).

The differential equations may be applied to an arbitrary variation of the angle of attack. But, since the approximation of the transfer functions is valid only in a limited range of frequency the formulation in the time domain is valid only if the high frequency terms contained in the Fourier transform of the input function are negligible.

If, for instance, we could perform a step variation of the angle of attack, $\alpha = H(\tau)$, the error of the transfer function in the high frequency range would induce a misrepresentation of the aerodynamic response $\phi(\tau)$ near the origin. But actually, α is determined by the motion of the airfoil and there are technological limitations which prevent us from performing a truly step variation. It is possible to perform only smoothed step variations in which the high frequency content of the Fourier transform appears to be negligible. Therefore, one can expect that the proposed model is valid for all the motions that can be actually performed on an airfoil or on a wing or helicopter blade section.

Obviously, the linear model is not valid if the range of angle of attack exceeds the stall incidence and if the airfoil is experiencing unsteady flow separation.

In order to build a non-linear model in a consistent manner, it is necessary to define the input variables and output functions.

Let us consider an airfoil moving at large incidence as shown in fig. (11).

V is the resultant velocity of the reference point relative to the undisturbed fluid ;

α is the angle of attack at the reference point ;

θ is the pitch angle of the airfoil relative to a fixed reference axis Ox .

The motion of the whole airfoil relative to the undisturbed fluid is defined by V , α and the torsional or pitch oscillation velocity $d\theta/dt = \theta'$. Therefore, V , α and θ' may be taken as input variables.

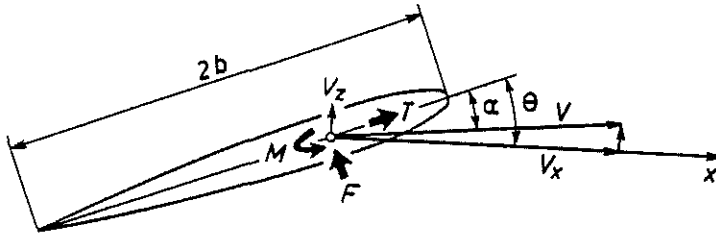


Fig. 11 -

On the other hand, the output functions are the lift normal to the airfoil, F , the moment at the reference point, M , and the tangent force, T .

Let us assume that F , M , T , V , α , θ' and their derivatives, up to an order which is still not determined, can be related by a set of non-linear equations.

In order to determine a simplified formulation of these equations, we notice that the oscillating motions that can be performed on a wing structure are always limited in amplitude or frequency by structural strength : large amplitude oscillations can be performed only at low frequency and high frequency oscillations can be performed only at small amplitude.

This implies that the time derivatives of F , M , T , V , α and θ are small quantities and also that the difference between θ' and α' can be neglected, so that θ' needs not be taken as an independent variable and the input variables can be reduced to two : V and α .

If we assume that the non-linear equations are differentiable, we can replace them by their first order approximation with respect to the small quantities. The resulting differential equations are linear with respect to the derivatives of F , M , T , V , α and θ , with variable coefficients which are functions of the angle of attack and eventually of the Mach number and Reynolds number. Furthermore the equations must give the steady flow lift and moment characteristics when α and V are constant.

Therefore, the above considerations show that the model should be a system of differential equations containing unsteady linear terms whose coefficients are functions of the angle of attack, and steady flow non-linear terms. But the order to which the equations can be limited and the number of terms which must be taken can be determined only empirically after comparing model results with unsteady wind tunnel test results.

On the other hand, the coefficients of the equations must be derived, for each profile, from systematic wind tunnel test results through an identification procedure.

The methods of identification which have been used so far are briefly described in ref. [15] and [16].

The steady flow characteristics result from classical steady flow tests. But unsteady flow tests are necessary to determine the coefficients in the linear differential terms. Since these coefficients are derivatives, their value, for a given value of α , can be derived from measurements of unsteady lift and moment on the airfoil performing small amplitude oscillations around the incidence α . If the amplitude of oscillation is sufficiently small (for instance 1°), the lift and moment contain a constant term and an unsteady term which depend linearly on the oscillating motion. As in any linear system, the unsteady terms can be related to the variations of incidence by a transfer function matrix.

Small amplitude harmonic oscillations are often used but pseudo random oscillations may also be utilized. Small amplitude step variations of incidence are also interesting because the evolutions of aerodynamic forces after the step variation determines the poles of the transfer functions and the coefficients of the derivatives of F and M in the differential equations.

The equations obtained after identification on a helicopter profile are presented in ref. [15] and [16].

The variation with frequency of the aerodynamic forces measured in the harmonic tests at low incidence showed that the transfer functions should contain only one real and negative pole, which implies a first order differential equation in the time domain, like equation (6.2).

But at incidences beyond stall, the evolution of the transfer functions was modified, as a consequence of the less stable character of the flow, and it was necessary to introduce two complex conjugate poles, which implies second order equations in the time domain.

In order to take account of these observations it was found convenient to separate each one of the non dimensionalized force and moment coefficients Φ_F and Φ_M into two terms, one governed by a first order equation and the other by a second order equation.

Taking the lift coefficient as an example, we write :

$$\begin{aligned} \Phi_F &= \Phi_1 + \Phi_2 \\ \dot{\Phi}_1 + \lambda \Phi_1 &= \lambda \Gamma_0 \alpha + [\lambda s + \sigma] \dot{\alpha} + s \ddot{\alpha} \\ \ddot{\Phi}_2 + 2\eta\gamma \dot{\Phi}_2 + \gamma^2 [1 + \eta^2] \Phi_2 &= -\gamma^2 [1 + \eta^2] [\Delta C_F + c \Delta \dot{C}_F] \end{aligned}$$

$\lambda, s, \sigma, \gamma, \eta, c$ are nondimensionalized coefficients depending on α and on the Mach number.

λ, η and γ determine the effect of time history and the resulting time delays.

Γ_0 and $\Delta C_F(\theta)$ describe the steady flow lift curve of the profile : Γ_0 is the slope in the linear range of incidences and ΔC_F is the decrease in lift due to stall :

$$C_F(\theta) = \Gamma_0 \alpha - \Delta C_F$$

where $\Delta C_F \equiv 0$ for α smaller than the stall incidence.

It is easy to check that Φ_F meets the steady flow value, $\Phi_F = C_F(\theta)$, when the derivatives with respect to t are made equal to zero.

On the other hand, if α remains smaller than the stall incidence, $\Phi_2 = \Phi_1$; is equal to zero and the lift is determined by the first order equation in the pole of the transfer function is equal to $-\lambda$.

Beyond stall, the term Φ_2 modifies the transfer functions corresponding to small amplitude oscillations and introduces the two complex conjugate poles given by $-\eta \pm i\zeta$.

The moment coefficient was given by a similar equation.

The validity of the model for large amplitude oscillations is shown in ref. [15] and [16]. The fig. (12) from ref. [16] shows the lift-incidence hysteresis loops for harmonic pitch oscillations with an amplitude of 6° .

It should be remarked that the model was identified with harmonic tests performed with an amplitude of oscillation smaller than 1° .

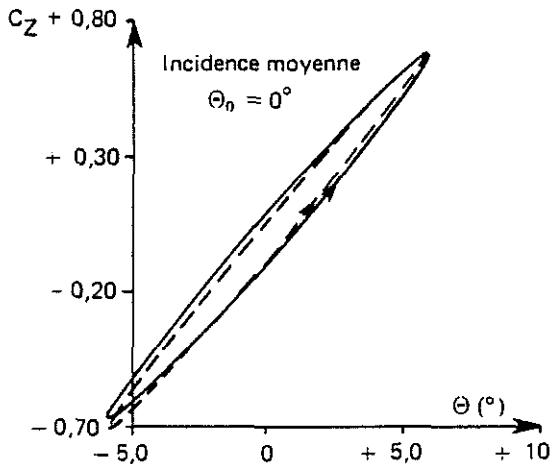
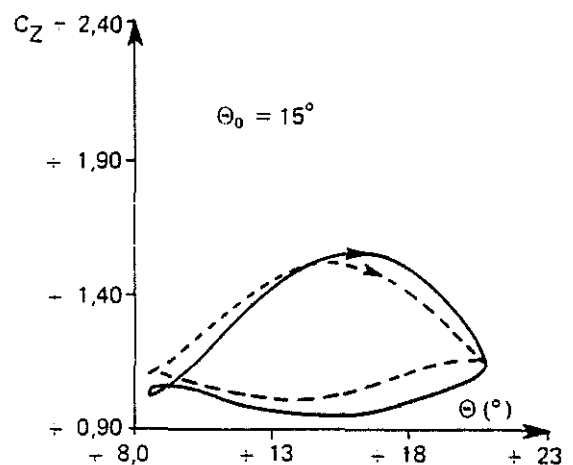
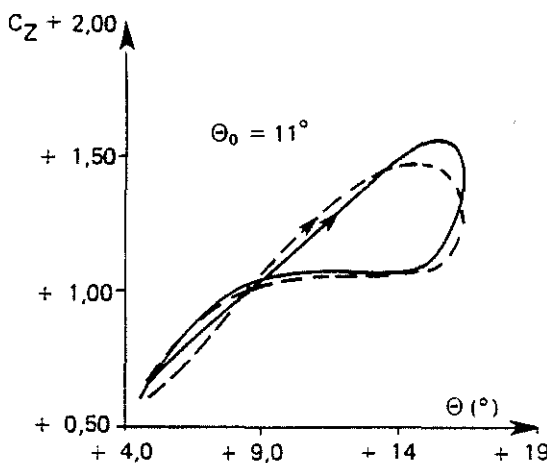


Fig. 12 - OA 213 AIRFOIL, lift-incidence hysteresis loops. $\nu = 0.049$; $M = 0.3$; $\tilde{\theta} = 6^\circ$.



2.3 Simultaneous effects of 3D flow and unsteady stall

The combination of the 3D linear theory and a 2D non-linear flow model, which is necessary to predict the simultaneous effects of 3D flow and unsteady stall, must be based on well defined assumptions and performed in a consistent manner.

The coupling procedure makes use of the concept of induced velocity, which was introduced in the lifting surface theory of Prandtl and was extended here to unsteady flow. For the evaluation of the aerodynamic loads in a blade section we assume that the profile is moving in a 2D flow, but its angle of attack is modified by the induced velocity resulting from the 3D interactions.

The induced velocity considered to evaluate the angle of attack of a particular blade section is the difference between the velocity induced by the actual 3D lift, distributed over the rotor blades, and the velocity which would result in 2D flow with the same lift on the same blade section.

When there are non-linear effects, the aerodynamic loads must be evaluated with a 2D non-linear model (or theory), but the induced velocity may be calculated with the 3D linear theory.

In order to justify the utilization of the 3D linear theory let us consider a blade section with separated flow (fig. (13)). The airfoil and its wake form a relatively thick layer within which the flow is deeply disturbed by vorticity. But outside the layer, the assumptions of potential flow and small disturbance are valid. Furthermore, for an observer located at a point P sufficiently far from the blade section, the lifting surface can be assimilated to a surface of pressure discontinuity, and consequently we can rely on the integral equation (5) for the calculation of the velocity field induced at the point P by the lift F. This conclusion holds for the calculation of the induced velocity resulting from 3D interactions.

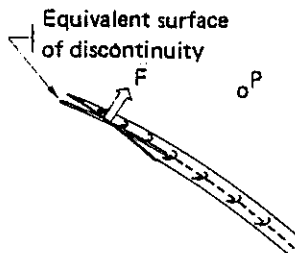
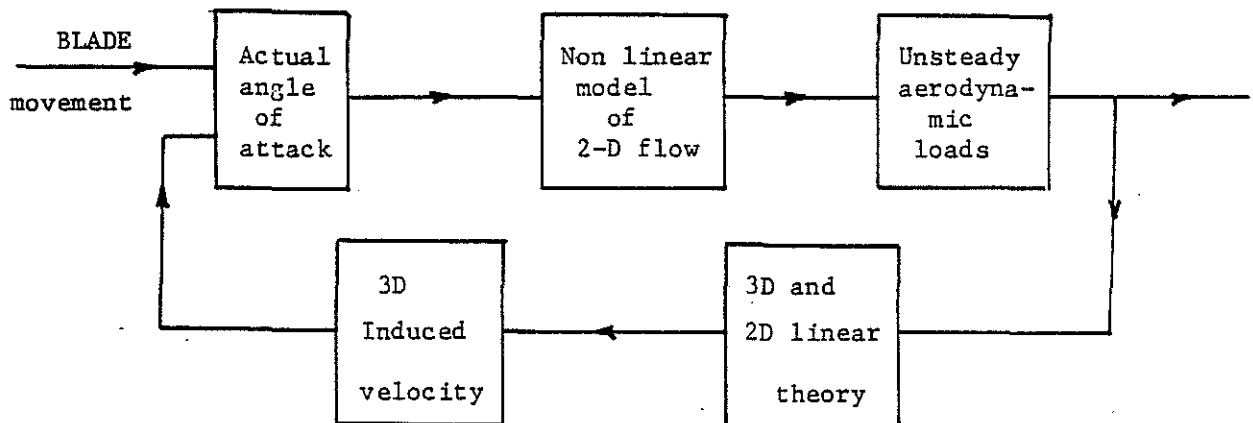


Fig. 13 - Separated flow.

Finally, the unsteady blade aerodynamic loads can be determined with the iterative procedure shown in the following flow diagram



The validity of the iteration procedure has been checked with experiment only with the model of ref. [13] (fig. (8)). Comparisons of theoretical and experimental results have not yet been performed with the ONERA 2D model for the lack of experimental results on rotors equipped with the airfoils for which the model has already been identified.

But the iterative procedure itself is valid for a wing as well as for a rotor, and its application is not restricted to the unsteady stall investigation : it may also be used, for instance, with a 2D transonic flow model to predict the transonic loads.

Fig. (14) illustrates the application to a rectangular wing model with a supercritical profile in transonic flow. The calculations of unsteady pressure distributions are based on the small transonic disturbance approximation. The figure compares the pressure distribution given by a 3D transonic flow calculation [6] and by a 2D transonic flow calculation coupled with a 3D linear code.

The comparison is satisfactory.

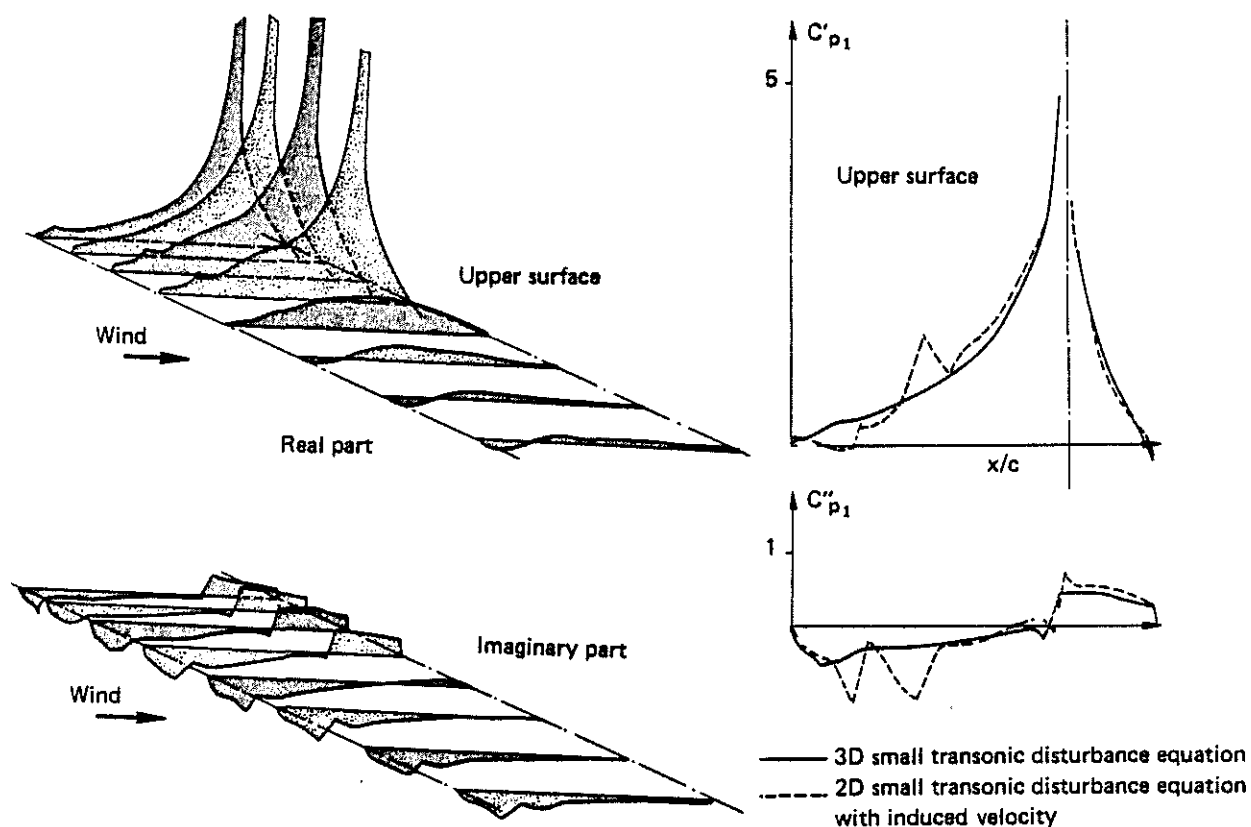


Fig. 14 a) - Straight wing oscillating control surface, pressure distribution on wing. $M = 0.73$; $f = 42$ Hz.

Fig. 14 b) - Straight wing median section of control surface. $M = 0.73$; $f = 42$ Hz.

3. STABILITY AND FORCED RESPONSE ANALYSIS

The modelizations used to predict the structural dynamic characteristics and the blade unsteady aerodynamic forces are described in the preceding paragraphs.

The calculation of forced vibrations and stability of the coupled rotor-fuselage aeroelastic system contains in itself a considerable amount of difficulties resulting from the non-linearities and from the presence of periodic coefficients in the Lagrangian equations.

The aeroelastic problems are numerous and varied, but some of them can be treated with a simplified modelization.

For instance in ref. [23] and [24], the blade flap-lag-torsion instability in hover is investigated with quasi-steady aerodynamic forces and the hub degrees of freedom are neglected, so that each blade is studied separately and the small blade deflections in the rotating frame are governed by Lagrangian equations with constant coefficients.

With the Coleman variables, the coupled rotor-fuselage instabilities and forced vibrations are also governed by Lagrangian equations with constant coefficients in the case of a helicopter in hover or a propeller in forward flight, provided that the rotor possesses the appropriate multicyclic symmetry (the number of blades should be larger than 2).

In the case of a helicopter in hover, the wake remains in the vicinity of the rotor and its interaction with the blades must be taken into account in the calculations of dynamic loads. Furthermore, the wake distortion may significantly influence this interaction.

But in the case of a propeller, which is also governed by equations with constant coefficients, with the Coleman transformation, the wake interaction is generally less important because the shed vortices separate from the trailing edge at a relatively high velocity. Since, on the other hand, the reduced frequencies of the blade vibrations are usually small, the quasi-steady assumption is valid for the evaluation of the blade unsteady aerodynamic loads.

This conclusion was confirmed in an investigation of the whirl flutter instability on the half model of a tilt rotor configuration which was tested in a low speed wind tunnel of ONERA (fig. (15)). The propeller was mounted on a nacelle at the wing tip with pitch and yaw degrees of freedom. The natural frequencies and dampings of the pitch, yaw and blade bending modes were measured in the wind tunnel and compared to the computed values (fig. 16). The agreement is quite satisfactory. Furthermore, no significant difference was found when the blade aerodynamic forces were evaluated with the 3D linear theory or with a quasi-steady model.

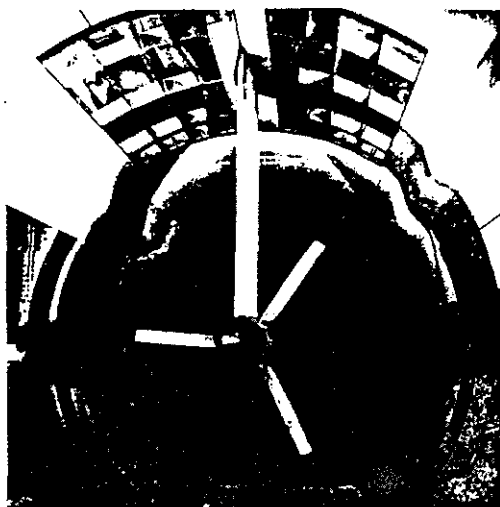


Fig. 15 — Tilt rotor model in the S2 ONERA wind tunnel in Chalais Meudon.

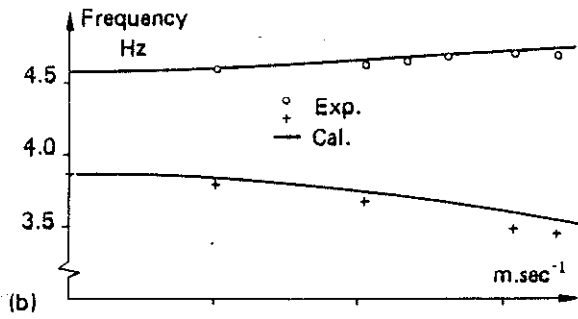


Fig. 16 a) – Natural frequencies and damping of the nacelle-propeller pitch and yaw modes.

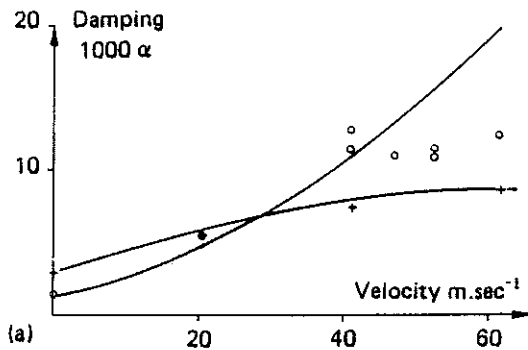
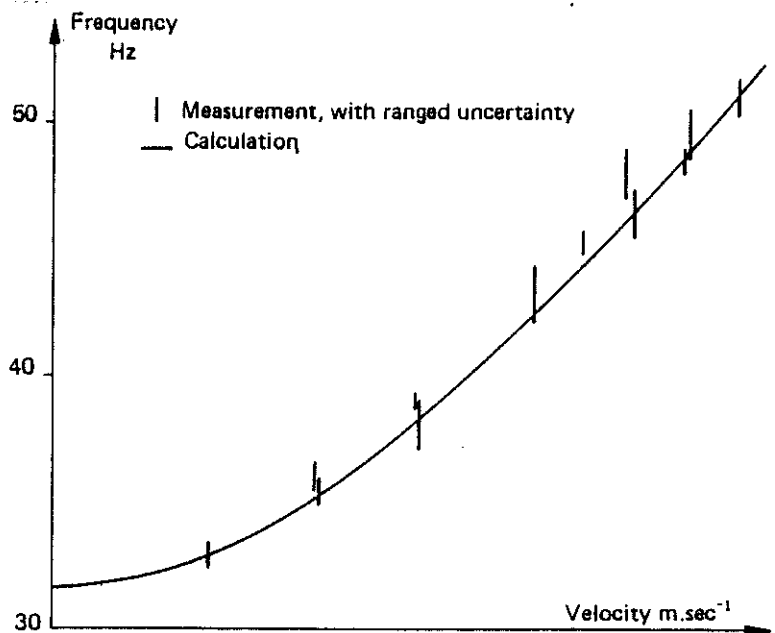


Fig. 16 b) – Natural frequency of the first blade bending mode (the rotation speed of the propeller increases with the wind velocity).

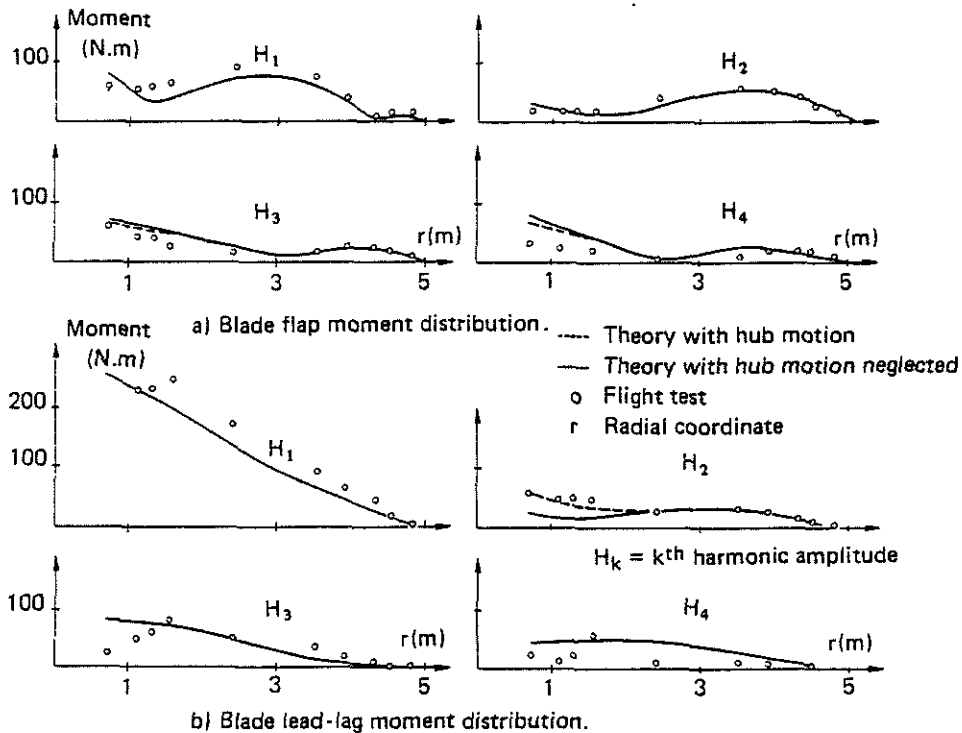


In forward flight, the aerodynamic forces lose the property of multi-cyclic symmetry and the periodic coefficients of the Lagrangian equations cannot be cancelled with the Coleman transformation. This makes the calculation and interpretation of the results more difficult.

For the calculation of periodic responses due to the periodic variation of the flow field around the blades, it is necessary to take into account the wake interaction. Therefore the use of the 3D linear theory, eventually coupled with a 2D model, as discussed in paragraph (2), is necessary. But on the other hand, the resolution of the equations can be performed relatively easily if the loads and responses are developed in Fourier series.

A comparison of measured and computed loads is shown in fig. (17) from ref. [25]. The experimental blade flapping moment was measured in flight on a SA 349 helicopter at different blade radial stations, and the calculations were

performed with a Fourier series development limited to the 5th harmonic. The 3D linear theory was used for the calculations of unsteady aerodynamic forces, but the use of a 2D non-linear flow model was not necessary in that case.



Helicopter SA349Z - harmonic analysis : $M = \sum_k H_k \sin(k\Omega t + \phi_k)$ - Advance ratio = 0.25.

Fig. 17 - Periodic loads on a helicopter blade in forward flight.

But the development in Fourier series is valid only for the prediction of periodic loads. For the prediction of instabilities and responses to arbitrary loads, such as gusts or turbulence, it is necessary to use the Floquet's theory of equations with periodic coefficients.

A computer code has been implemented at ONERA for the solution of these equations.

- It determines the transition matrix and the periodic matrix defining the Floquet's transformation which leads to a system of equations with constant coefficients.

- Then it solves the eigen value problem in order to get the Floquet's modes which characterize the stability of the system, and it determines the responses to external loads varying arbitrarily with time.

The calculations can be performed with a quasi-steady aerodynamic model or with a more sophisticated aerodynamic formulation like in paragraph (2). The non-linear 2D model is used in particular to investigate the blade stall flutter ; but, for the sake of simplification it is applied with a reduced configuration of only one blade with seven degrees of freedom (blade flap and pitch and 5 flexible modes). This limitation to 7 structural DOF was necessary because the differential equations contained in the 2D model introduce additional degrees of freedom and so increase the size of the problem.

Calculation results are presented in ref. [17] and [26]. The Floquet's modes are shown and discussed in ref. [17].

When non-linear effects are important the problem may be linearized around an approximate solution. Let $X(t)$ be the vector representing the fuselage and rotor responses. X may be divided into two vectors :

$$X = X_0 + X_1$$

where

$X(t)$ is the solution of the full non-linear aeroelastic problem with the full unsteady aerodynamic model

$X_0(t)$ is the solution of the non-linear aeroelastic problem with a quasi-steady approximation of the aerodynamic loads.

Generally, the difference between X_0 and X_1 is relatively small and X_0 may be considered as an approximate solution. Therefore X_1 may be considered as a first order small vector: If we substitute $X_0 + X_1$ in the Lagrangian equations and neglect the second order terms we find that X_1 is determined by a system of linear equations whose periodic coefficients depend on $X_0(t)$.

Consequently the solution of the full non-linear aeroelastic problem can be divided into two steps.

- The first step is the calculation of the non-linear quasi-steady approximation. This calculation is relatively easy to perform because the coupled blade airforces are formulated in a simple manner.

- The second step is the resolution of the linear equations giving X_1 with the computer code based on the Floquet's theory.

These two steps may be considered as the beginning of an iteration procedure. At high advance ratio it is often necessary to proceed to a third step.

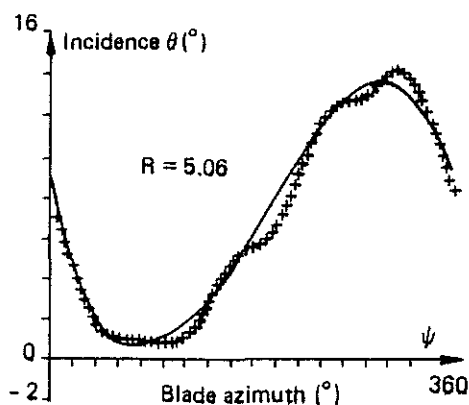


Fig. 18 a) - Calculation of periodic responses of a blade in forward flight. Evolution of incidence as a function of azimuth in the section $R = 5.06$ m (blade tip 5.3 m from hub). + quasi-steady model ; — unsteady model.

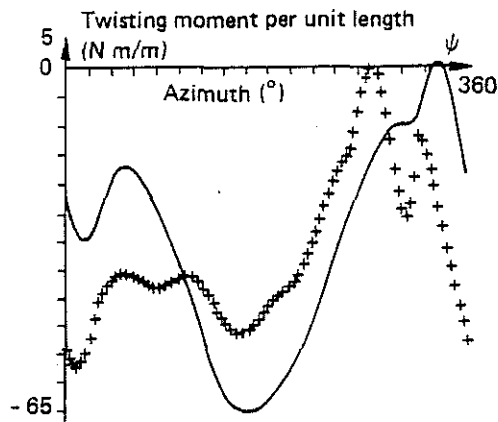


Fig. 18 b) - Calculation of periodic responses of a blade in forward flight. Evolution of moment per unit length as a function of azimuth in the section $R = 5.06$ m (blade tip 5.3 m from hub). + quasi-steady model ; — unsteady model.

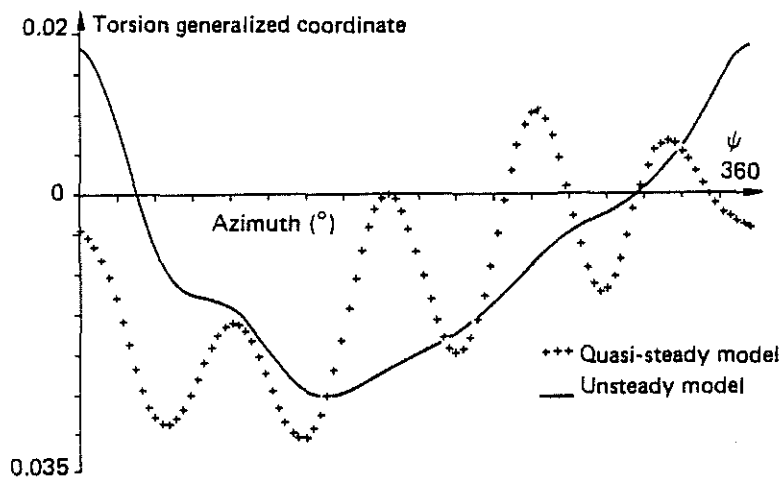


Fig. 18 c) - Calculation of periodic responses of a blade in forward flight. Evolution of torsion generalized coordinate.

The fact that the additional term X_1 is small means that the difference between the full solution and the quasi-steady approximation is relatively small when we compare the large amplitude blade pitch and flap oscillations. This is shown in figure (18-1) which compares the large amplitude variations of blade angle of attack. But, on the other hand, the difference is often relatively large when the comparison is made on the small amplitude blade vibrations. This is illustrated by fig. (18-2) and (18-3) for the twisting moment and the torsional vibrations. These results show that the quasi-steady approximation is valid only for the evaluation of the blade large amplitude oscillations and that the structural vibrations must generally be predicted with a more sophisticated aerodynamic modelization.

CONCLUSION

The basic methods which are presented are potentially able to provide a good evaluation of the vibrations and risks of instability of helicopters. In spite of the non-linear phenomena, which are relatively important on helicopter rotors, the linear methods are shown to be efficient when they are utilized with an appropriate strategy, in an iterative procedure, or coupled with a simplified non linear model. Nevertheless many improvements are still needed on the theoretical formulations and on the computer codes in order to improve the accuracy of the results, to reduce computing times and to make the computer codes more flexible for industrial applications.

But in order to contribute efficiently to the design and development of new helicopters, it is perhaps more important to define calculation strategies adapted to the different problems which can be met on helicopters. And this can be done only with the help of an extended wind tunnel research programme providing the knowledge of the practical problems and data to check the calculations.

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