

# The mathematical model of the "Bumby Bucket" movement transported on helicopter external cargo sling

Alexander Sviridenko  
CSTS "Dinamika", Zhukovksy, Russia

a.sviridenko@dinamika-avia.ru

This paper presents the results of calculations and theoretical work completed in the course of developing a simulator for an external cargo sling operator of a Mi-8 type helicopter, namely:

- the mathematical model of BB aerodynamic characteristics;
- the mathematical model of the main rotor's inductive stream influence upon BB's movement on hovering and low speed modes close to the screen;
- the phenomenological model of a synthetic sling's elastic characteristics;
- the results of mathematical modeling «the helicopter - BB» system for characteristic flight modes.

## Introduction

Aviation technologies, including the use of transport helicopters, have recently acquired a greater role in open fire suppression in domestic and international practice. The «vertical way» of suppression, which consists of water being dumped from a special tank ("Bumby Buckets" (BB)), transported on a helicopter external cargo sling, is the most commonly used one (Figure 1).

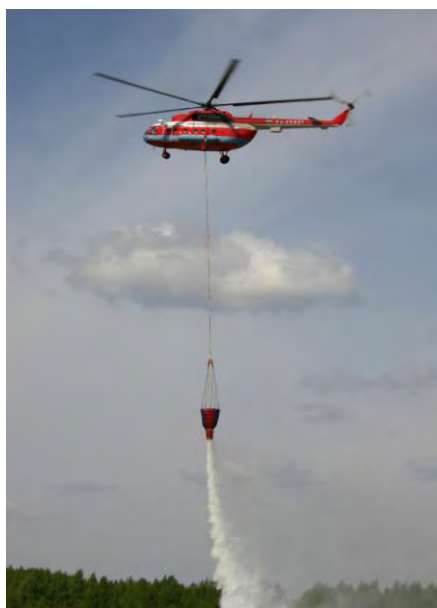


Fig.1 BB-5 on a helicopter external cargo sling

For the crew to acquire practical skills and to make their psychology more stable in particular situations it is reasonable to use a simulator, properly re-equipped for modeling fire extinguishing operations with a BB.

The crew's perception a "flight" on a simulator largely depends on the adequacy of external cargo sling and BB mathematical models. BB impact on helicopter dynamics, in its turn, depends on the accuracy of modeling BB spatial moving and external cargo sling elastic characteristics.

A BB on external cargo sling as a dynamic object possesses the following peculiarities:

- when loading/dumping the liquid, BB mass-inertial characteristics change considerably and intensively (while the BB maintains its form);
- aerodynamic forces influencing the BB depend not only on the approach flow parameters, but also on the flow induced by a helicopter's main rotor (on hovering and low speed modes)
- the elastic characteristics of BB synthetic slings are essentially non-linear.

## 1. Aerodynamic characteristics of a BB-5A with a foam former delivery out in doses

Defining of aerodynamic forces and moments that influence BB-5A is based on recalculating experimental data acquired while scavenging a BB-15 model in a wind tunnel [1] and on

generalized data on aerodynamic characteristics of high-drag bodies form the works [2,3] considering the difference in geometrical parameters and the amount of water in the tank. A BB scheme used to define the forces and moments, the coordinate system and the

conditional gravity centre location, against which the pitching moment is calculated are given in Figure 2. Considering that a BB is symmetrical, aerodynamic characteristics have been defined at the range of attack angles from  $-90^\circ$  to  $+90^\circ$ .

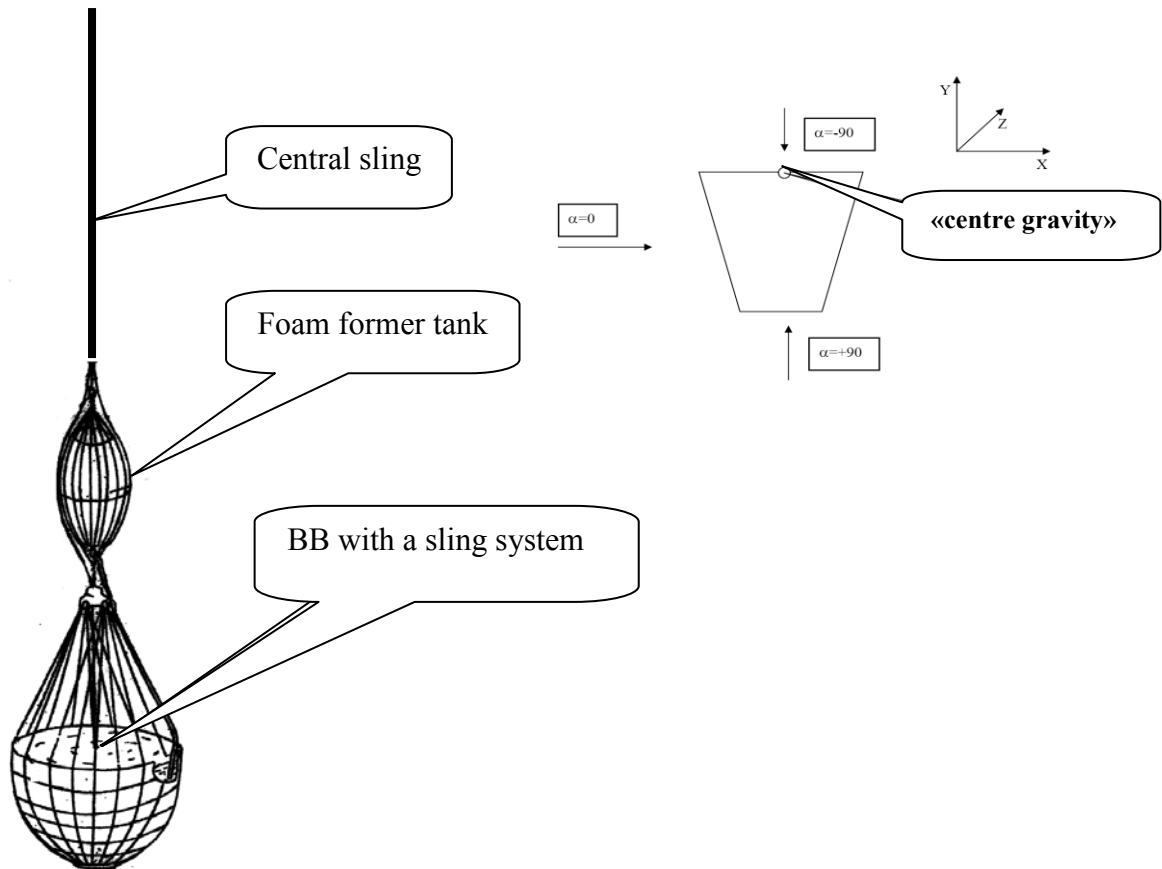


Fig.2 BB scheme, coordinate system and the conditional gravity centre location

The coefficients of aerodynamic forces and moments have been calculated separately for the central sling, foam former tank and for a BB with a sling system. BB configurations with and without tank channeling, empty and full have been considered.

To illustrate this Figure 3 presents the dependences of total coefficients of aerodynamic forces and moments of a full BB without channeling moment on the attack angle.

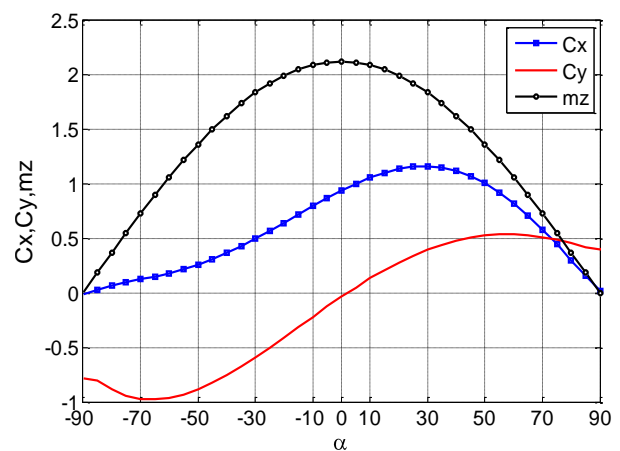


Fig.3 The dependences of aerodynamic forces and of the moment at a full BB without channeling circular scavenging

## 2. Calculating the velocities induced by the main rotor at low forward speed and hovering mode considering the position close to the screen

To define the field of velocities induced by the main rotor stream at a low speed mode considering the position close to the screen an

approximated mathematical model of interaction between incompressible liquid stream and the surface has been applied [4]. The main rotor related coordinate system used at calculating inductive velocities is shown in Figure 4, the Z axis is directed to the right alongside with helicopter flight.

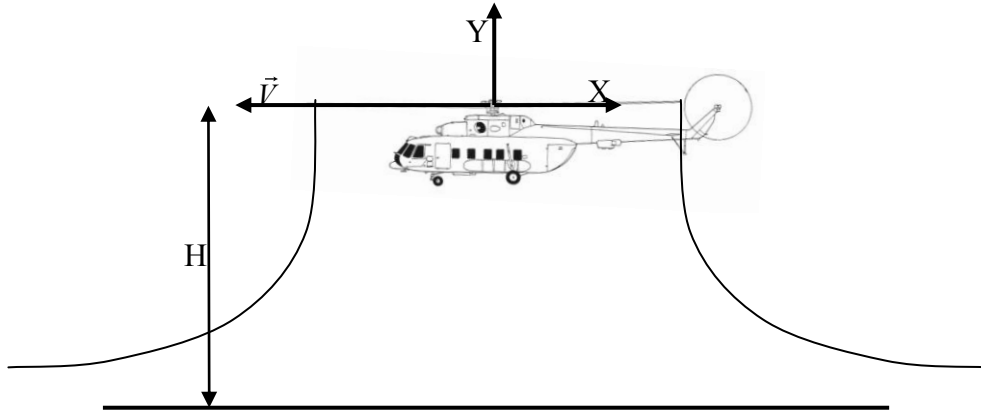


Fig.4 Main rotor coordinate system

Vertical velocity dispensing in the stream under the rotor in the plane with  $Y=0$  is specified with a parametrical dependence on the radius, air density and helicopter weight (Figure 5):

$$V(r) = A \cdot r \cdot (1 - r^{2N}),$$

$$\text{propeller trust } T = \rho \cdot R_0^2 \int_0^1 2\pi V^2(r) r dr \approx G$$

(at modes close to horizontal),

$$A = \sqrt{\frac{G}{2\pi R_0^2 (0.25 - 1/(N+2) + 0.25/(N+1))}} \quad (1)$$

$V(r)$  – velocity [m/s],  $r$  – relative rotor radius,

$\rho$  – air density [ $\text{kg/m}^3$ ],

$G$  – helicopter weight with cargo [N],

$R_0 = 10.625$  – main rotor radius [m]

$N = 3$  – parameter

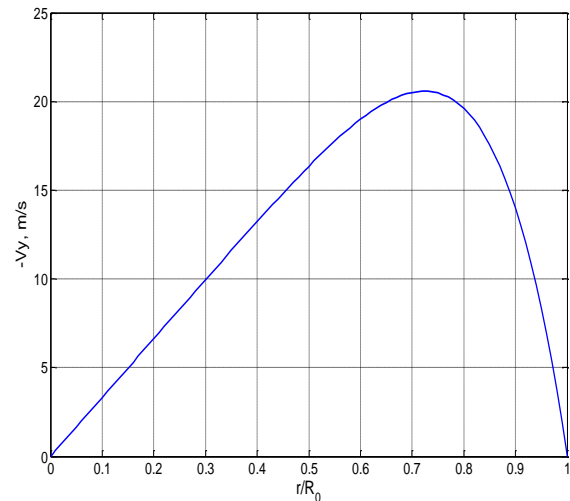


Fig. 5 Vertical velocity dispensing in the stream under the rotor

In the main rotor related coordinate system, the velocity component dispensing considering the close surface influence and the stream slope angle (the stream slope angle is defined with the helicopter airspeed) is expressed with the following relations:

$X, Y, Z$  – coordinates of the point  
in a main rotor related system [m]

$H$  – height [m]

$\alpha$  – angle of stream slope to the surface [deg]

$V_x, V_y, V_z$  – velocity components [m/s]

$$X_s = X + Y / \operatorname{tg}(\alpha), r = \sqrt{X_s^2 + Z^2}, r_e = (1 + Y/H) \cdot r$$

$$V_m = V(r_e), V_r = V_m \cos(\arctg((H + Y)/r)) \quad (2)$$

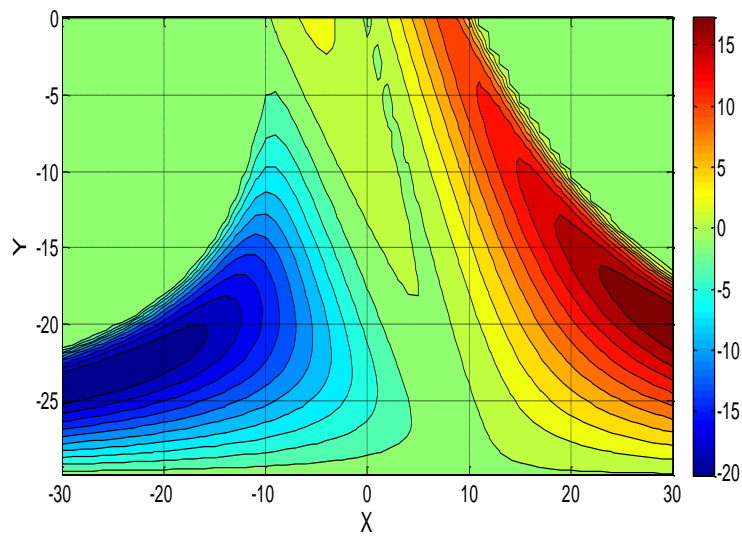
$$V_Y = -V_m \sin(\arctg((H + Y)/r))$$

$$V_X = V_r \cdot X_s / r - V_Y \cos(\alpha), V_Z = V_r \cdot Z / r$$

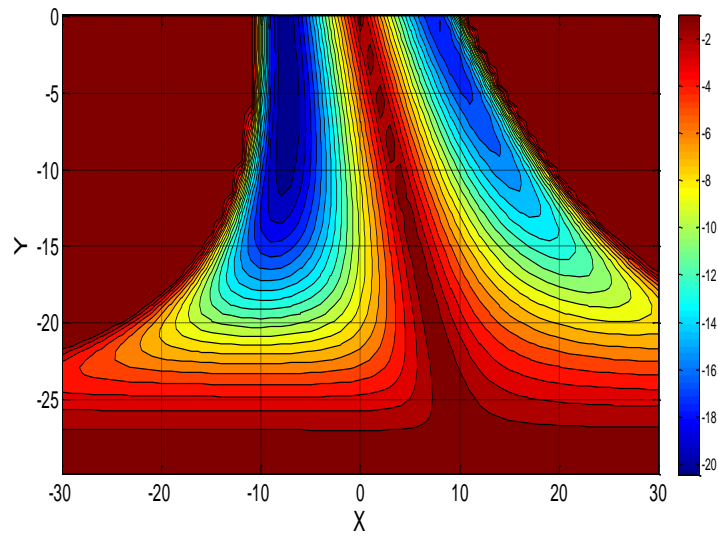
$$V_\Sigma = \sqrt{V_X^2 + V_Y^2 + V_Z^2}$$

$$V_X = V_m V_X / V_\Sigma, V_Y = V_m V_Y / V_\Sigma, V_Z = V_m V_Z / V_\Sigma$$

Figure 6 presents the isolines calculated using the formulas (2) of longitudinal and vertical velocities induced by the main rotor when the helicopter is flying at the height  $H=30\text{m}$  and the velocity  $V=4\text{m/sec}$  dispensing.



a)



b)

Fig.6 Induced velocity isolines in the plane under the rotor with  $Z=0$ :

a) longitudinal velocity  $V_x$  (m/sec)

b) vertical velocity  $V_y$  (m/sec)

### 3. BB geometrical, mass and inertial characteristics

Fig. 7 presents a BB calculation model. A soft waterproof cover consists of the lower hemispheric and the upper cylindrical parts. A metallic rim with the displaced gravity centre is attached to the upper part of the cover in order to prevent the BB from rotating around the longitudinal axis. In the lower part of the cover there is a hole for water dumping with a border of a lower metallic rim, which has a heavy ballast ring attached to it.

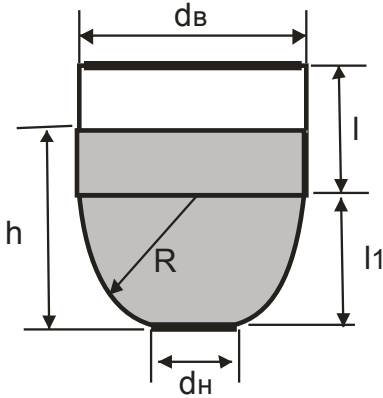


Fig. 7 BB calculation model

Depending on the quantity of liquid in a BB the liquid volume gravity centre location and respectively its inertia moments change (notably, a BB form during the flight hardly changes regardless of whether it contains water or no).

Figures 8 and 9 show how the fluid level, gravity centre location and BB inertia moments depend on the amount of liquid.

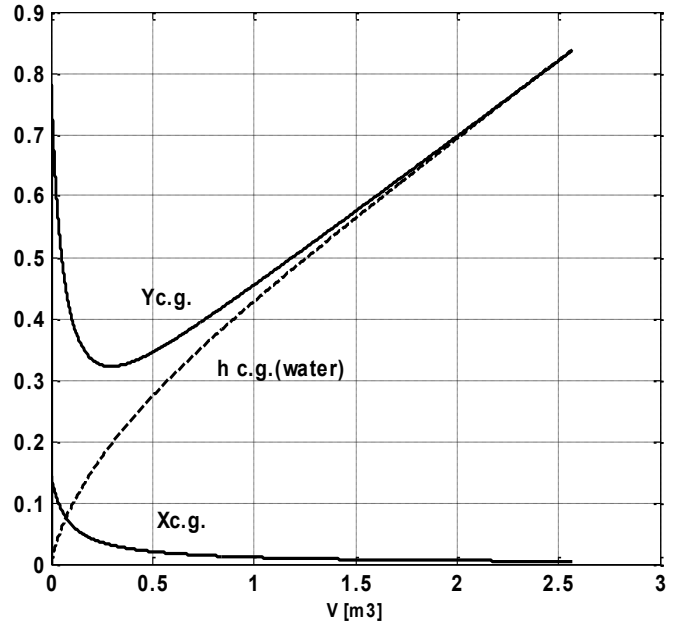


Fig.8 BB gravity centre location against the amount of liquid

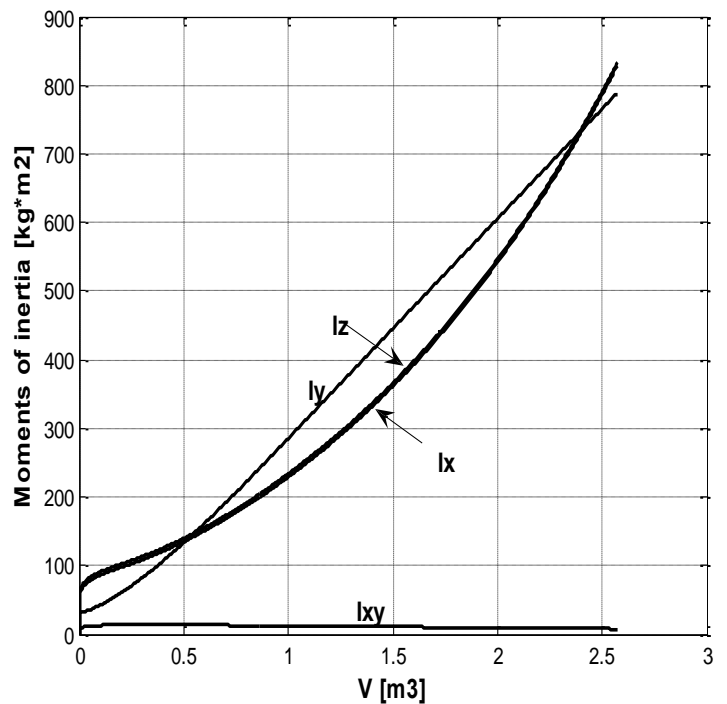


Fig.9 BB moments of inertia changing against the amount of liquid

#### 4. The phenomenological model of a synthetic sling

A BB is attached to a helicopter external cargo with a ribbon sling made of synthetic high-molecular textile bands. “Force-deformation” diagrams for these materials are non-linear (Figure 10 presents the diagram “force P – deformation  $\epsilon$ ” for capron).

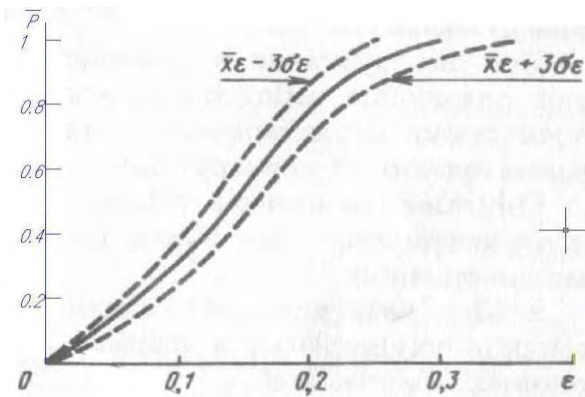


Fig.10 Capron stretching diagram [5]

Besides, there is great difference between the loading/unloading diagrams for these types of

materials, which can be accounted for by hysteresis. Figure 11 presents experimental data of a synthetic sling behavior during the free oscillation of the cargo attached to it.

Hysteresis loops are of different size and shapes. Gradually oscillation is reduced to a static value of force and deformation.

The fact that the slings have a nonlinear dependence of the force on deformation and that internal energy is widely dispersed when they are subject to a force with variable magnitude considerably complicates analyzing dynamic systems with elements made of synthetic fabric.

It must be said that though hysteresis is a common phenomenon both in nature and in technology, there is no universal mathematical tool to describe it exactly because of the diversity of its manifestations [6]. The idea of the phenomenological model [7] suggested to describe synthetic sling’s elastic characteristics is as follows (Figure 12).

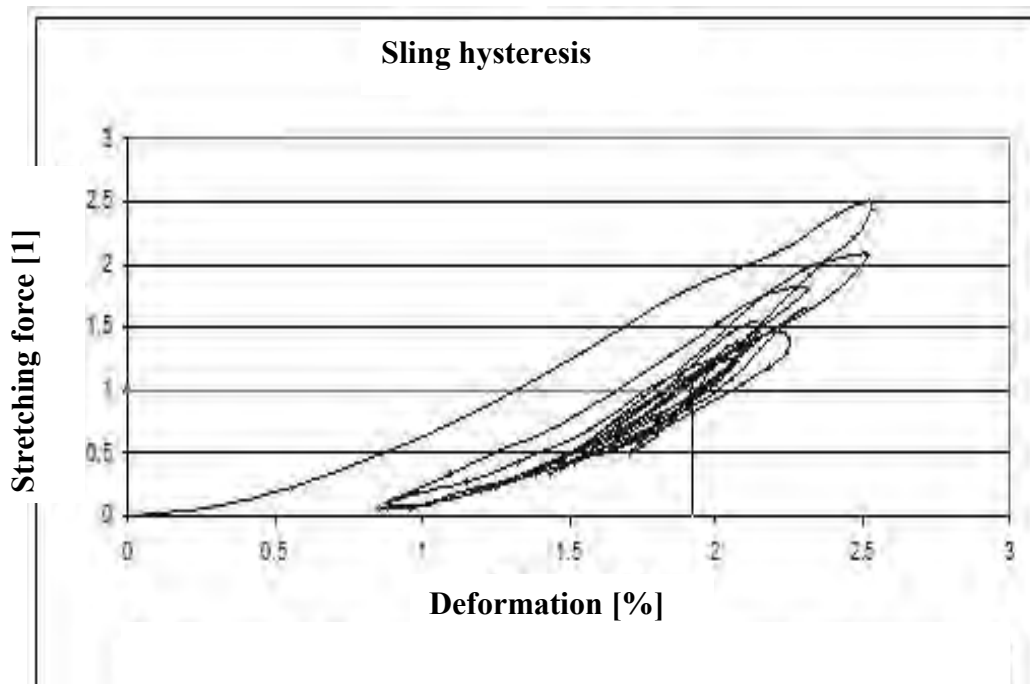


Fig.11 Force against deformation in the process of free oscillations (hysteresis)

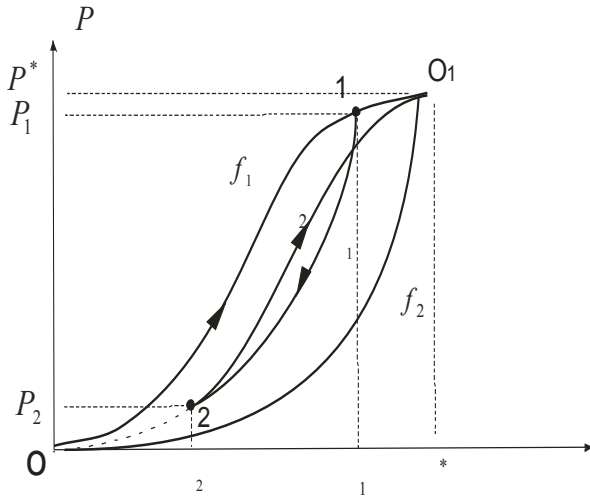


Fig.12 The elastic hysteresis mathematical model

Let the hysteresis loop be of the following form (Fig.12), with  $f_1$  and  $f_2$  being the functions that approximate experimental data  $P(\varepsilon)$  correspondingly for slow loading and unloading of the sling within the operational range  $[0, \varepsilon^*]$ . Supposing that the sling behavior during the unloading process is described with functions  $\varphi_1$  «similar» to  $f_2$ , and during the loading process with functions  $\varphi_2$ , «similar» to  $f_1$ . Considering the boundary conditions in points O and O1 and the fact that the deformation velocity in the reversal of sign points is continuous, a continuous analytical dependence of the sling reaction force on deformation  $P(\varepsilon)$  can be defined for any initial deformation  $\varepsilon \in [0, \varepsilon^*]$  value and loading value  $P \in [0, P^*]$ .

Figure 13 presents the mathematical modeling results of free longitudinal oscillation of a cargo on a synthetic sling.

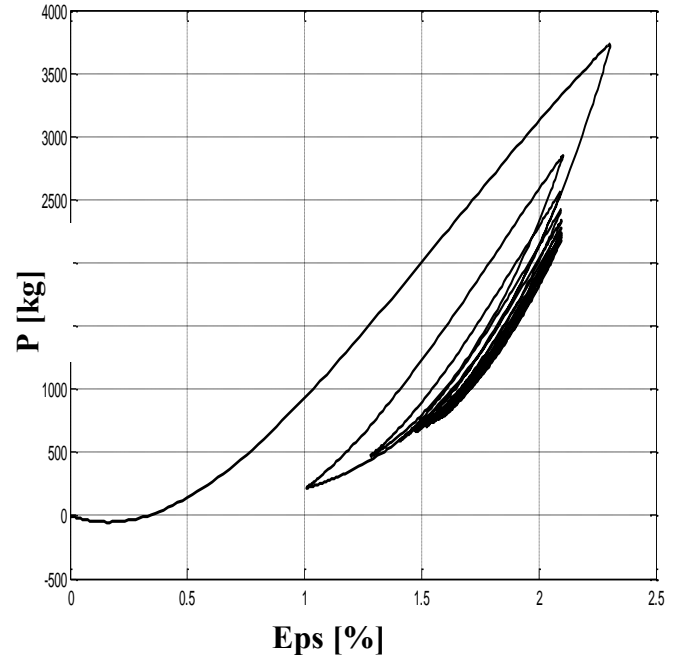


Fig.13 Elastic hysteresis of the sling LT-synthetic high-molecular fabric-48-600 (mathematical model)

The suggested sling mathematical model ensures good coinciding in terms of both quality and quantity with experimental data and can be applied to analyze dynamic systems with elements made of synthetic types of fabric.

## 5. The results of numerical simulation of the helicopter flying with a BB transported on an external cargo sling

### 5.1 Mathematical modeling of dumping the liquid from a BB

To illustrate the aspects of mathematical modeling of a BB movement on an external cargo sling mentioned above, the results of numerical simulation of a type Mi-8 helicopter flight during the liquid dumping from a BB are given below. The dumping starts at the moment of  $t=100$  sec, with the velocity  $V \sim 60$  km/h, a BB being  $h_{BB} \sim 30$ m above the earth, the initial amount of liquid is  $2.5 \text{ m}^3$ .

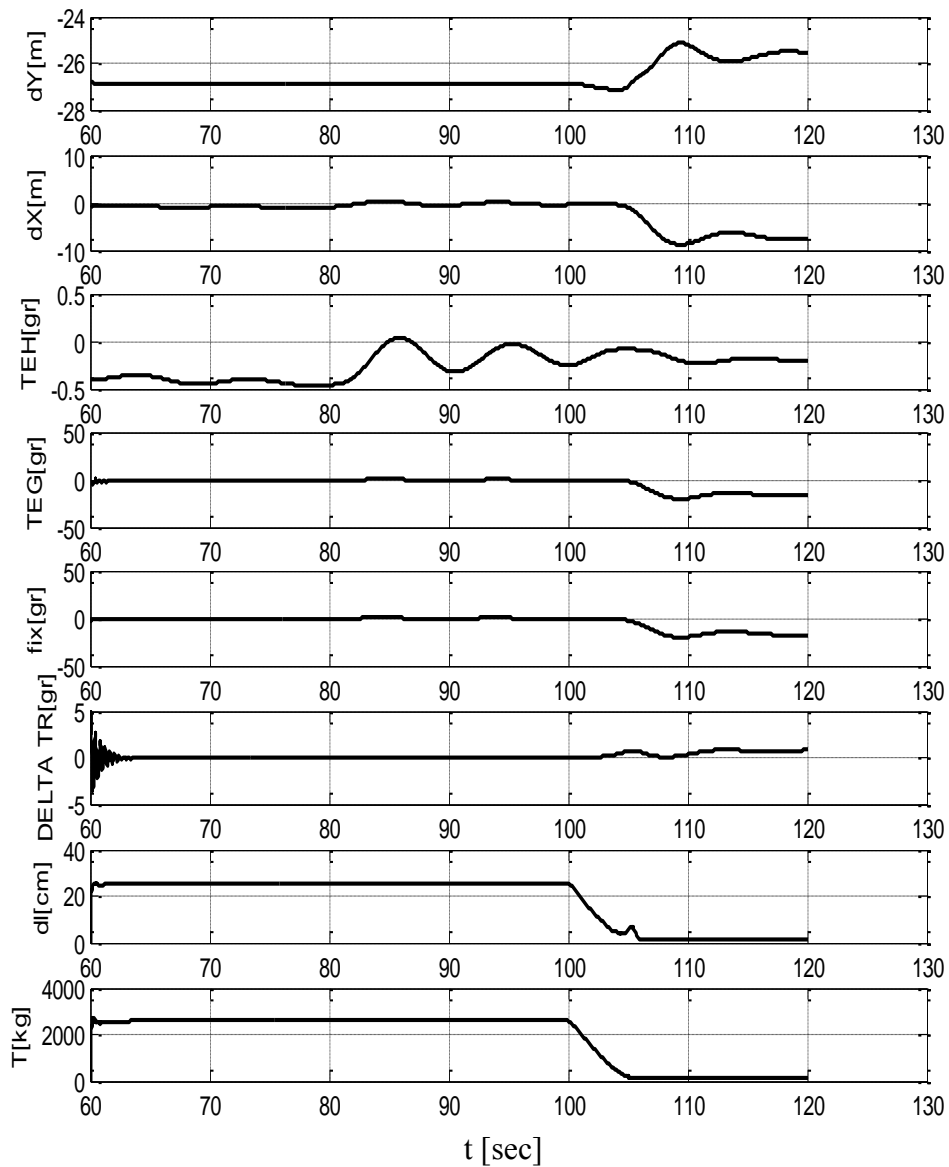


Fig. 14 Relative movement of a helicopter and a BB:

Key:

$$dY = Y_{BB} - Y_g;$$

$$dX = X_{BB} - X_g;$$

$X_g, Y_g$  – coordinates of the helicopter gravity centre [m];

TEH –helicopter pitch angle [deg];

TEG – BB pitch angle [deg];

fix –deviation angle of a central sling in the longitudinal plane [deg];

DELTA TR – angle between a BB longitudinal axis and the central sling[deg];

dl – central sling stretch [cm];

T – reaction force of the central sling [kg].



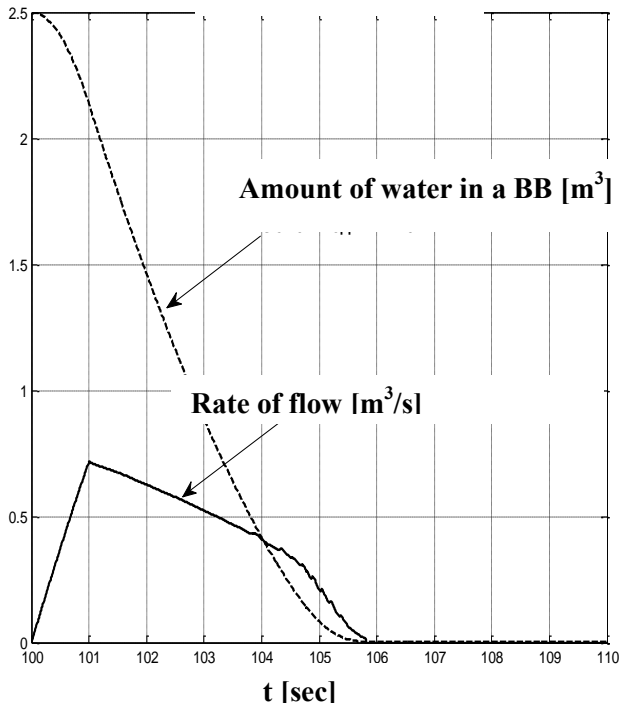


Fig.15 Liquid dumping from a BB

## 5.2 Mathematical modeling of the main rotor inductive stream influence

Figures 17 and 18 show the results of mathematical modeling of the inductive stream from main rotor influence on the system “helicopter - BB” movement. The calculation case scheme is given in figure 16:

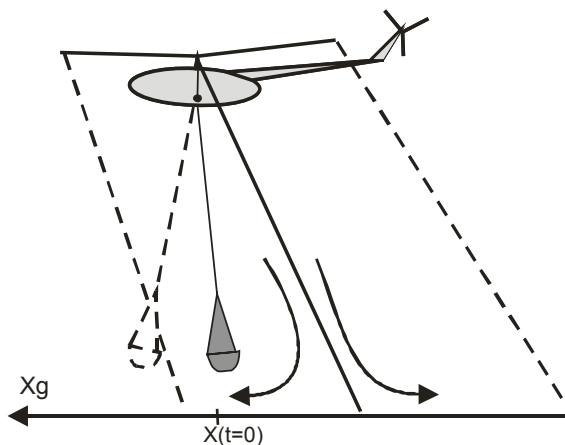


Fig. 16 The calculation case

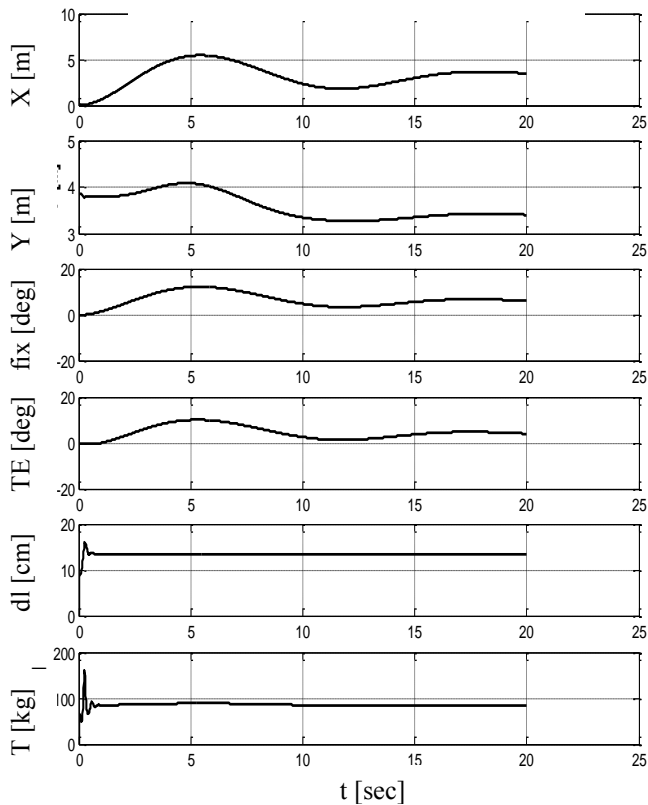
It is apparent that the developed mathematical model of BB movement reflects the major peculiarities of the phenomenon modeled sufficiently well:

- the force that influences the helicopter on the part of the external cargo sling changes and this results in a specific helicopter “swelling” in the dumping process;
- BB on a helicopter external cargo sling is retrimmed in the dumping process;
- the liquid dumping has a dynamic character with an average flow rate of  $\sim 500$  l/sec (Figure 15).

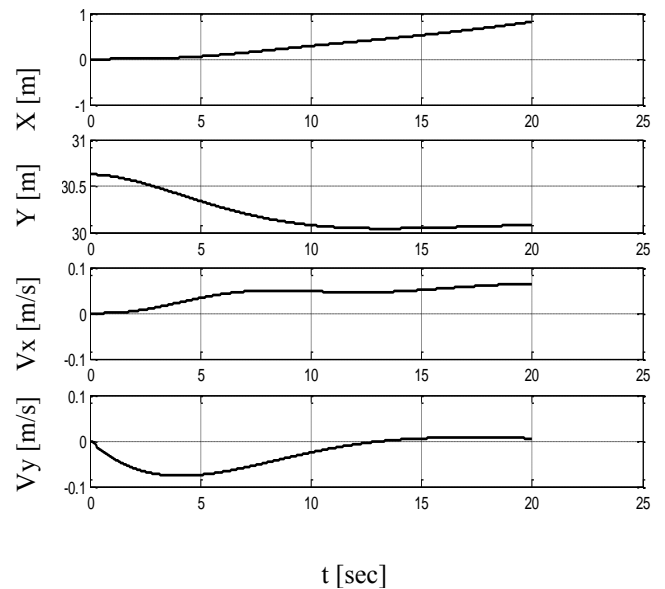
- a horizontally flying helicopter is transporting an empty BB that weighs  $\sim 80$  kg on an external cargo sling;
- at the moment  $t=0$  the helicopter hovers above the point  $X=0$ ;
- at the moment an inductive stream from the main rotor “reaches” the BB.

An airflow from a BB directed forward in relation to the flight direction (where the BB is located) tries to “push” the BB out to the fringe region (Figure 17), the BB moves forward against the helicopter which disturbs the balance of both forces and moments of the helicopter (Figure 18). It should be mentioned that in the general case the BB movement induced with the main rotor stream can be characterized as rather unpredictable, so the instruction for the flight personnel recommends that an empty BB is put on the ground or touches water during its loading

to the BB before the stream “comes” under the helicopter.



*Fig.17 The influence of the main rotor stream on the BB (hovering mode)*



*Fig.18 The influence of interaction between the main rotor stream and the BB on the helicopter movement (hovering mode)*

#### References

1. Baranikov S.N., Guverniuk S.V. et al. Study of the BB-15 on a Hanger Cable Dynamics under the Wind Influence. Report, Institute of Mechanics MSU, 2003 (in Russian)
2. Petrov K.P. Aerodynamics of Simple-shaped Bodies, “Factorial”, Moscow, 1998 (in Russian)
3. Denvin S.I. Aerohydrodynamics of High-drag Bodies: Reference book. – Leningrad, Sudostroenie, 1983. (in Russian)
4. Lavrentiev M.A., Shabat B.V. Hydrodynamic Problems and Their Mathematical Models, Moscow, Nauka, 1977 (in Russian)
5. Computer Research of Parachutes and Hang Gliders/S.M.Belotserkovky, M.I.Nisht, A.T.Ponomarev, O.V.Rysev; ed. S.M.Belotserkovsky. – Moscow.:Mashinostroenie, 1987. – 240p. (in Russian)
6. R.V. Lapshin. Analitical model for the approximation of hysteresis loop and its application to the scanning tunneling microscope. Review of Scientific Instruments, volume 66, number 9, pages 4718-4730, 1995.
7. Vasilchenko A.G., Sviridenko A.N. The Phenomenological Model of a Synthetic Sling, Nauchny Vestnik MSTU SA, series Aeromechanics and strength, №151, 2010 (in Russian)