THIRD EUROPEAN ROTORCRAFT AND POWERED LIFT AIRCRAFT FORUM

Paper No. 20

MODAL CHARACTERISTICS OF A STRUCTURE USING PRINCIPAL EIGENVALUE METHOD

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September 7-9,1977

AIX-EN-PROVENCE, FRANCE

ASSOCIATION AERONAUTIQUE ET ASTRONAUTIQUE DE FRANCE

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Abstract

The method involves the use of a computer programme which forms part of continued research work undertaken to study the normal modes of a helicopter and other associated characteristics. The paper examines the application of the programme to a case where a structure is excited by one of the modern methods of excitation namely, the random technique.

Some important structures of a helicopter will be chosen for this task which will be subjected to random excitation. The experimental data thus obtained will be processed through the said computer programme which is in two parts. The first will provide the necessary frequency response function inputs, the second will further elaborate and analyse these processed experimental data yielding the modal characteristics of the structures chosen. The mode shapes, the mass, the stiffness and the damping matrices will be computed.

As a final step an appraisal of this programme will be made by comparing these results with those obtained using other programmes written in the course of this research effort.

1. INTRODUCTION

The present study is part of a long term research programme directed towards the study of the dynamic characteristics of mechanical structures. The study is being carried out under the following headings:

i) Experimental procedure

ii) Data acquisition

iii) Elaboration of experimental data

iv) Iteration procedure with theoretical data

Using the concept of "Total Dynamics" in which a small team is responsible for the whole project, it is endeavoured to perfect a system in order to furnish the information necessary for the development of new structures via sophisticated theoretical and practical solutions. A desirable requisite for a system is to adapt itself to the type of exigencies of single problems. This paper presents, in particular, a programme used for the evaluation of the elastomechanical characteristics of helicopter structures using experimental data obtained by random excitation.

As a sample case for study an experimental analysis of the flapping modes of a main rotor blade (without centrifugal field) is considered. A comparison of these results is made with those obtained using other analytical methods that employ finite element theory and other programmes which have been developed in the course of this research.

It has been found that the most efficient way to analyse a complex structure is to use a substructure approach in which each component part of the complete assembly is analysed individually. Methods for combining the mass, stiffness and damping matrices of these substructures in order to analyse the whole structure are being developed. An "Impedance-Matching" technique can then be applied to determine the overall dynamic behaviour of the complete helicopter.

2. GENERAL REMARKS ON GROUND VIBRATION TESTING

Mechanical structures transfer force and motion which by nature are inseparable. The transfer of one variable always accompanies the transfer of the other and in product they constitute mechanical energy. Structural mass, stiffness and damping characteristics all impede the transfer of forced dynamic motion. These structural characteristics tend to delay or distort the force or displacement quantity being transferred.

Understanding the nature of this transfer process is fundamental to understanding the behaviour of a structure under its environmental loading situation. Six transfer functions are frequently measured: compliance (x/F), mobility $(\dot{x}/F)$, inertance $(\ddot{x}/F)$, apparent stiffness (F/x), impedance $(F/\dot{x})$, apparent mass $(F/\ddot{x})$. To measure this series of transfer functions, characterising the structure, the structure itself is subjected to a force introduced at a point with a fixed orientation and the resultant motions are measured at one or more fixed points.

Until the mid 1960's transfer functions were measured by carefully controlling the amplitude of excitation to be constant and measuring the resultant response. That is, the denominator was held constant (for a range of frequencies) and the numerator was measured within the controlled frequency range.

The sweep sine method, being the most precise means of exciting a structure, has been successfully used to study the above. However this, as we know, is elaborate and time consuming by way of calibration, setting-up of equipment etc. It has also the drawback of not simulating the operating environment.

In recent times to eliminate the need for carefully controlled inputs and to excite all frequencies simultaneously random techniques have been developed. Unfortunately random techniques require more expensive instrumentation than the swept sine method, either to analyse on-line data using hardware instruments or software programmes to extract data from recorded tape.

To reduce the need of buying expensive and sophisticated equipment for on-line analysis of data we decided to build up a system as flexible as possible acquiring only the excitation part and writing the necessary interface and analysis programmes.

2.1 Test aim

We can describe a structure mathematically by the equation:

$$\left[M\right]\ddot{q}+\left[D\right]\dot{q}+\left[K\right]q=f \qquad (1)$$

We have to determine the transfer function data through ground vibration tests which provide us with information on natural frequencies, generalised masses, stiffness and damping characteristics. The experiment was designed to measure these properties and to test the validity of using these modes to predict forced response.

2.2 Test Method and Procedure

Three types of excitation can be used and these are:

i) Swept sine

ii) Random

iii) Transient or impulsive

Usually by measuring one of the following quantities: Compliance/Apparent stiffness, Mobility/Impedance, Inertance/ Apparent Mass the others can be obtained simply by a "switching technique" built into the software. That is, having measured real and imaginary inertance it is possible to compute the needed mobility data doing the following operations:

$$\text{COMPLIANCE} = C = x/F$$

$$\text{MOBILITY} = \Psi = \dot{x}/F$$

$$\text{INERTANCE} = \mathcal{J} = \ddot{x}/F$$

that is

$$|\Psi| = \omega\,|C| \quad ; \quad \vartheta_\Psi = \vartheta_C + 90^\circ$$

$$|\mathcal{J}| = \omega^2\,|C| \quad ; \quad \vartheta_\mathcal{J} = \vartheta_C + 180^\circ$$

and

$$|\Psi| = |\mathcal{J}|/\omega \quad ; \quad \vartheta_\Psi = \vartheta_\mathcal{J} - 90^\circ$$

or

$$\Psi^R = \mathcal{J}^I/\omega \quad ; \quad \Psi^\mathcal{J} = -\mathcal{J}^R/\omega$$

The sweep sine method as we know does not represent the real situation despite its optimal qualities. The Random and Impulsive techniques are a bit expensive, comparatively speaking, but have advantages over the former. The latter two are now extensively used on account of the rapidity with which they provide the required results. Whatever the method of testing, the procedure (fig. 1) is to excite the structure at a number of points (degree of freedom of mathematical model) and measure the driving point and transfer point functions at these points and record the same on tape. A number of records for each point is to be taken to get a good average of "points" to decrease random errors. Mass cancellation corrections have to be considered and incorporated into the results.

2.3 Processing of Test Results

A transfer function describes a cause/effect relationship between two measured signals. Experimentally exciting a structure with a measured force and simultaneously measuring its response motion permits a force/motion transfer function to be evaluated. This measurement can be used to determine the structure's resonant frequencies, damping characteristics, stiffness and inertia.

The Fourier Transform offers a method for converting a time history into its component frequencies. It takes the form:

$$S(f) = \int_{-\infty}^{+\infty} s(t)\, e^{-j2\pi f t}\, dt$$

where $s(t)$ is the time function and $S(f)$ the Descrete Fourier Transform, or in digital version:

$$F[jk] = \frac{1}{N} \sum_{i=1}^{n} x_n \left[\cos \frac{2\pi k_n}{N} - j \sin \frac{2\pi k_n}{N} \right]$$

which are illustrated in fig. 8.

It can be seen that the frequency resolution available is implicit in time, the length of a frame of data and the number "N" of data points in it. Limitations of core fix the upper limit in size of "N" and it can also be seen that an extremely short sampling of data should be chosen for a given duration T of the frame of data.

In vibration analysis this problem (plus the fact that one frame does not constitute a good statistical sample) has been overcome by averaging the values of the spectral coefficients over several frames of data. In fact this reinforces the quasi-periodic phenomenon and reduces random error (4% in the sample case presented).

A further problem arises from the finite length of a sample frame. This is referred to as truncation error and stems from the fact that each frame is treated as one cycle of a periodic phenomenon. To minimize this error the so called "hanning smoothing" procedure has been used. This means to weight the sample values to favour those in the centre of the frame and reducing those near the ends. One would expect this to introduce truncation errors in the vicinity of the frequencies occurring at the end of each frame, but this does not appear to be a problem provided that the frame period is long compared with the impulse response of the system being studied.

From these compound waveforms we can compute the Power Spectral Density for the input and output and the Cross Spectral Density between them using the FFT built into our ATSA programme (Agusta Time Series Analysis). We can then derive the transfer function in the form:

$$H(jk) = \frac{\text{Cross spectral density } (Y,X)}{\text{Power Spectral Density } (X)}$$

as shown in fig.3a/b, dealing with inertance.

After verifying that the coherence values are acceptable in the selected frequency range the analogue data is then digitised and stored on magnetic tape.

The real and imaginary parts (references 1 and 3) of the spectrum are then entered into subroutines, SDR (Structural Dynamic Research), which calculate the stiffness and the damping matrices, the associated natural frequencies and mode shapes. Other subroutine takes the parameters derived and mathematically reconstitutes the spectrum for comparison with the original one. This is illustrated in fig. 2 . The good agreement between the plots of experimental data and computed response allows us an acceptable degree of confidence in the evaluated mass, stiffness and damping matrices, as shown in fig. 10.

3. MODAL APPROACH TO THE EXTRACTION OF DOMINANT MODE EIGENVALUE

Equation (1) describes small motions of a linear elastic and lightly damped structure so that the Rayleigh assumption (reference 1) holds. With the procedure detailed in reference 4 defining the element mobility as the ratio between the velocity phasor and the force phasor along selected coordinates the following relation holds:

$$\overline{\Psi}^{R}_{i(\omega)} = \frac{1}{m_i \omega}\left(\frac{\omega}{\Omega_i}\right)^2 \frac{g_i}{g_i^2 + \left(\frac{\omega^2}{\Omega_i^2} - 1\right)^2} \tag{2}$$

with the associated imaginary component:

$$\overline{\Psi}^{I}_{i(\omega)} = -\frac{1}{m_i \omega}\left(\frac{\omega}{\Omega_i}\right)^2 \frac{\left(\frac{\omega^2}{\Omega_i^2} - 1\right)}{g_i^2 + \left(\frac{\omega}{\Omega_i^2} - 1\right)^2} \tag{3}$$

Each mobility is a complex function of frequency and it is necessary to measure the real and imaginary parts of the forces and moments relative to the motion of the structure over the range of interest. Plotting equation (2), fig. 4 , it is seen that real mobility associated with each natural frequency is almost unaffected by the presence of other modes especially when these modes are well separated from the one considered. It follows that a measure of real mobility at each natural frequency will have information only related to boundary modes. The problem is to extract the effective components from the measured mobility values to find the dominant mode.

The mobility data at a forcing frequency can be expressed as a summation,over the selected degrees of freedom,of each modal mobility:

$$\Psi^{R}_{(\omega)} = \sum_{i=1}^{DOF} \overline{\Psi}^{R}_{i(\omega)} \{\Phi\}_i \{\Phi\}_i^{T}$$

$$\Psi^{I}_{(\omega)} = \sum_{i=1}^{DOF} \overline{\Psi}^{I}_{i(\omega)} \{\Phi\}_i \{\Phi\}_i^{T} \qquad (4)$$

The sum of real mobility around each natural frequency considered is then a measure of the normal modes of the structure. Let the inverse of this sum be

$$\left[\sum_{\omega_f=\Omega_1}^{\Omega_{DOF}} \Psi^{R}_{(\omega_f)}\right]^{-1}$$

Multiplying this by the mobility value at a selected forcing frequency we have,

$$\Psi^{R}_{\omega^*}\left[\sum_{\omega_f=\Omega_1}^{\Omega_{DOF}} \Psi^{R}_{i(\omega_f)}\right]^{-1} = \Phi\left[\frac{\overline{\Psi}^{R}_{i(\omega^*)}}{\sum_{f=1}^{DOF} \overline{\Psi}^{R}_{i(\omega_f)}}\right]\Phi^{-1}$$

Now, considering the i-th component of the modal matrix vector Φ it is possible to express the dominant eigenvalue problem through an interation procedure based on the following expression:

$$\Psi^{R}_{(\omega^*)}\left[\sum_{\omega_f=\Omega_1}^{\Omega_{DOF}} \Psi^{R}_{(\omega_f)}\right]^{-1}\{\Phi\}_i = \frac{\overline{\Psi}^{R}_{i(\omega^*)}}{\sum_{\omega_f=\Omega_1}^{\Omega_{DOF}} \overline{\Psi}^{R}_{i(\omega_f)}}\{\Phi\}_i$$

Having thus determined the mode shape we also know the real and imaginary component of each mode.

3.1 Estimates of Modal Properties

The accuracy of the estimates depends at least on the following three:

i) Precision of measured data

ii) Validity of the viscous or hysteretic model for damping effects

iii) Coupling between modes

It has been shown that the response properties (eg. mobilities) of a system may be expressed in terms of contributions of each mode computed through the iterated mode shape. To compute the stiffness and mass elements it is useful, for systems with relatively few degrees of freedom, to model one with a "skeleton technique" (reference 5), fig. 5 . A great advantage of this is that the identified mass and stiffness terms are not sensitive to the accuracy of a damping estimate.

To evaluate modal damping and to have a comparison between results, the programme uses also the following quotations to determine generalised mass, stiffness and natural frequencies.

$$m_i = \frac{\omega_k \overline{H}^I_{i(\omega_k)} - \omega_j \overline{H}^I_{i(\omega_j)}}{\omega_k^2 - \omega_j^2}$$

$$\Omega_i^2 = \omega_j \omega_k \frac{\omega_j \overline{H}^I_{i(\omega_k)} - \omega_k \overline{H}^I_{i(\omega_j)}}{\omega_k \overline{H}^I_{i(\omega_k)} - \omega_j \overline{H}^I_{i(\omega_j)}}$$

$$\chi_i = \Omega_i^2 m_i \qquad g_i = \omega_j \overline{H}^R_{i(\omega_j)} * \chi_i^{-1}$$

The disadvantage of using the above formulation is that it implicity involves inverting the mobility matrix, and this is likely to emphasize experimental errors which, even with the efforts to obtain measurement accuracy, may be significant at the modal frequencies of the system.

To evaluate the mass, stiffness and damping matrices, if N is the number of measured points, the order of identified matrix will be N, even if it is possible to identify more than N natural frequencies from the transfer function plot. Sometimes these matrices may have negative terms and this means that the coupling between the various parts is to be taken into account although the structure is considered to be made up of isolated parts for our studies.

The physical meaning of i-j element of each matrix is that the real structure will generally give a partial derivative of the i-force in respect to j-response that has an effective mass component equal to the i-j element of the matrix itself.

4. TEST RESULTS

In order to verify the programme a test was designed in which the structure was excited by a narrow band signal having a Rayleigh distribution. This distribution is characterised by a certain bandwidth centred at a selected frequency. The consequence was, of course, a lack of coherence at both ends of the recorded signal as shown in fig. 6. Under these circumstances the best thing to do was to carefully select the input data where the coherence was acceptable. This was done by choosing the region as shown in fig. 10 but deleting the effect of deep "valleys" (fig 6).

A total of five points, fig. 7 , was chosen on the structure which was excited at each of these points in turn. For each point a ten-minute recording was obtained to enable a reasonable average of points to be made and to see also if this would be a good procedure to avoid the lack of coherence. The response of the structure was measured at these points to get the driving and transfer point functions, fig. 3 , enabling the determination of the modal characteristics shown in fig. 9a/b. In order to equalise the effect of each modal mobility it was necessary to introduce a number of normalization procedures. Of course, due to the Rayleigh distribution (fig. 6) it was not possible to identify the 2nd and the 6th mode shown in fig. 9a, 9b.

A comparison between the results of this test and the data obtained by using sinusoidal excitation, finite element method and transfer matrix method (reference 6) shows that there is generally a good agreement between theoretical and experimental methods. In this regard we have to say that using the finite element method we get more than three modes in the chosen region (fig. 10). The SDR programme identified three modes: this is due to the frequency resolution which is a function of the sampling rate of data.

The efficiency of this programme can be seen in fig.10 dealing with real and imaginary components of the measured mode.

It is now necessary to evaluate the physical meaning of the matrices computed by the programme and used to get the theoretical model giving the good response shown in fig.10. This model, according to the number of measurement points (equal to or higher than the degrees of freedom), give the same response of the physical structure at the selected point. The computed mass, stiffness and damping matrices are the elements with which it is possible to build up a system as the one shown in fig. 5 comprising of real masses, springs and dampers (even if some of these terms are negative).

The characteristics of real masses and springs differ, in general, from those of ideal elements in respect to non linearities and plastic deformation. Furthermore the force generated by a damper may not be exactly proportional to velocity, mainly with the use of elastomeric bearings or vibration isolation materials; in this case the damping coefficient may depend on the amplitude of motion.

Coupled with the above, as summarised in fig. 1 , there are factors, connected with the test itself, adding uncertainities into impedance data evaluation and use. This is a good reason to evaluate the coherence values before recording the measured signal and to perform several averages.

It has to be emphasized that the success of application and use of these data in any programme like the SDR depends largely on the accuracy of the measured data, and in order to ensure this, considerable care and attention to the details must be exercised in every aspect of the measurement technique. Furthermore high sampling rate in data digitalization and random noise generator with wideband signal are key points for higher reliability.

Conclusions

The goal of this test was to find the elastomechanical characteristics of a structure demonstrating that the reliability of a cheeper solution utilizing the software approach is acceptable in comparison to the more expensive hardware approach for data analysis. The goal is achieved.

For these tests we have utilized a hardware system sufficient for a demonstration of concept feasibility but not optimized for engineering purposes. This experience was utilized to define the hardware covering the desired spectra.

This work has demonstrated the validity of a modal approach for the definition of a mathematical model having the same response of the real structure through identified matrices of mass, stiffness and damping.

The reduction of vibratory level of a helicopter will be obtained tuning properly the coupling between each substructure. This will be obtained knowing the matrices of substructures (i.e. fuselage, pylon, engines, rotors, etc.) and connecting them by junction elements having known and easy to modify elastomechanical characteristics.

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NOTATION

x	Displacement
F	Input force
$\mathbf{M}$	Mass matrix
$\mathbf{K}$	Stiffness matrix
$\mathbf{D}$	Damping matrix
$\mathbf{q}$	Generalized coordinates, column or row vector
$\mathbf{f}$	Generalized forces, column or row vector
ω	Forcing frequency
Φ	Matrix of modal vectors
$\mathbf{H}$	Matrix of impedance
Ψ	Matrix of mobility
$\bar{H}_{i(\omega)}$	Generalized i-th modal impedance
$\bar{\Psi}_{i(\omega)}$	Generalized i-th modal mobility
$\lceil m \rfloor$	Generalized mass
$\lceil \varkappa \rfloor$	Generalized stiffness
g	Structural damping
Ω_i	Natural frequency of i-th mode
ϑ	Phase angle

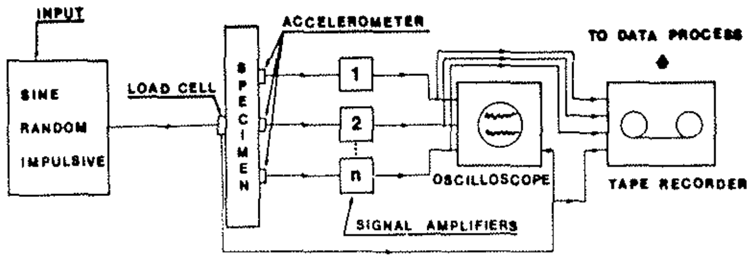
DEFINITION: A driving point is the measurement point where input and response coincide in location and direction.

SUPERSCRIPTS

T	Transpose matrix
$-T$	Inverse transpose
$\cdot$	Derivative with respect to time
R, I	Real, imaginary
$\| \ \|$	Modulus

SUBSCRIPTS

ω	Forcing frequency
j, k	Indices for frequencies near each resonance
DOF	Degree of freedom
NF	Natural frequencies



INFLUENCING FACTORS:

a – FIXTURE USED FOR COUPLING OF EXCITER & STRUCTURE.

b – MASS CANCELLATION EFFECTS.

c – LOCATION OF MEASURING POINT.

d – CALIBRATION OF TRANSDUCERS.

e – SPECIMEN NON LINEARITIES.

f – STIFFNESS OF CONCTACT AREA.

Fig.1

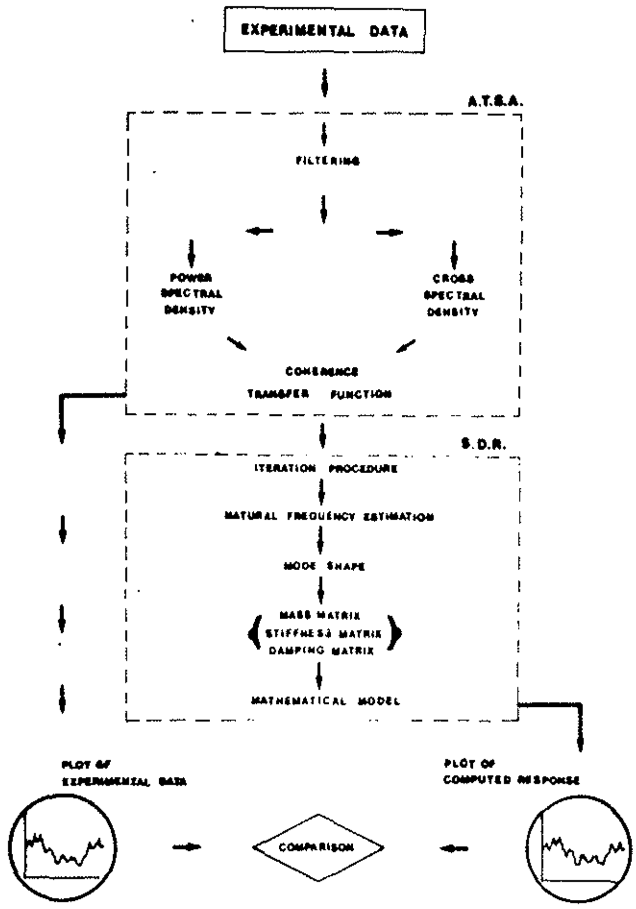
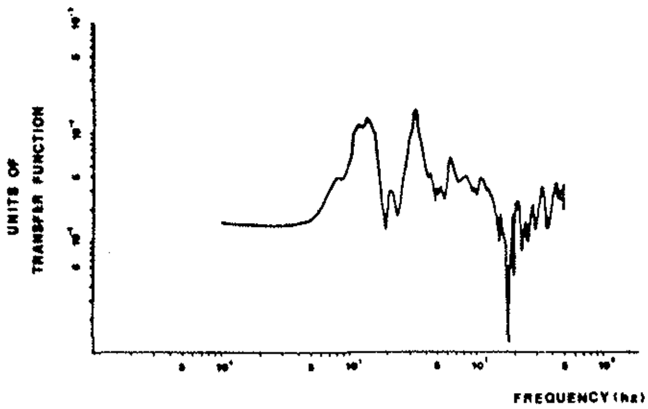


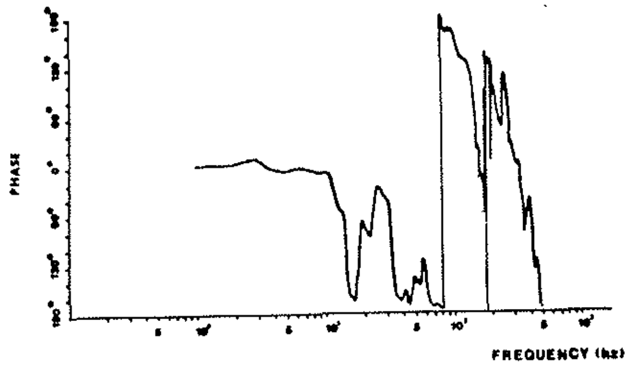
Fig.2

PROCESS OF EXPERIMENTAL DATA


UNITS OF
TRANSFER FUNCTION
FREQUENCY (hz)

a

Fig. 3


PHASE
FREQUENCY (hz)

b

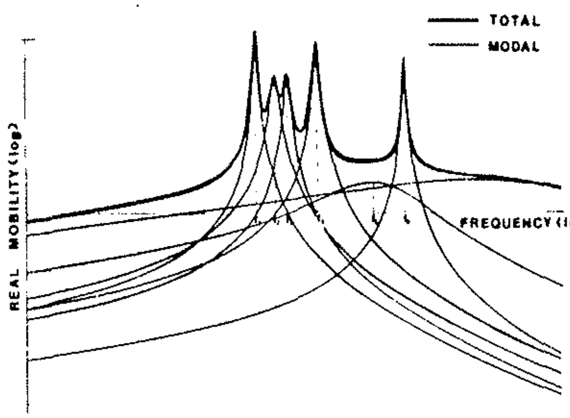

TOTAL
MODAL
REAL MOBILITY (log)
FREQUENCY (log)

Fig. 4

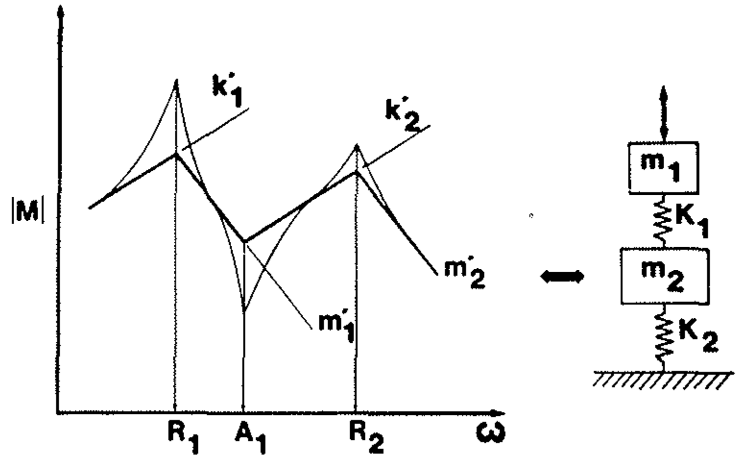


Fig.5

$$k_1 = (R_1^2 + R_2^2 - A_1^2)/m_2'$$

$$k_2 = k_1 k_1'(k_1 - k_1')$$

$$m_1 = m_2'$$

$$m_2 = (k_1 + k_2)/A_1^2$$

SYSTEM MODELLING VIA THE MOBILITY SKELETON

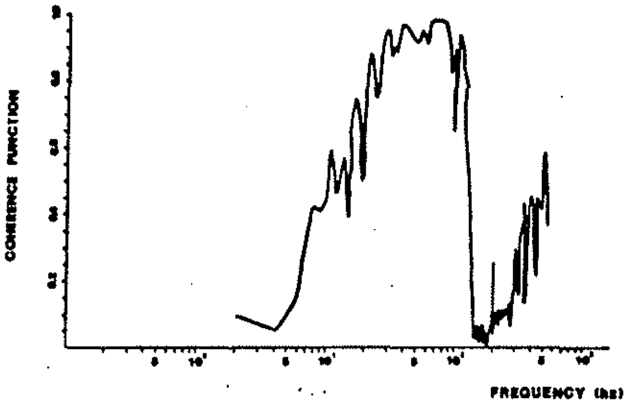
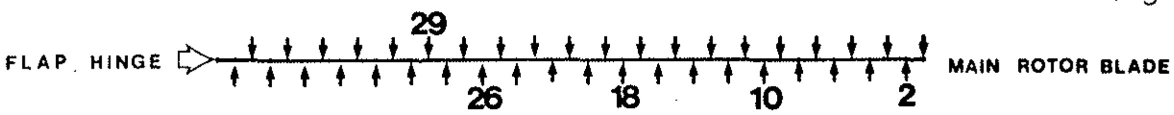


Fig.6

Fig.7



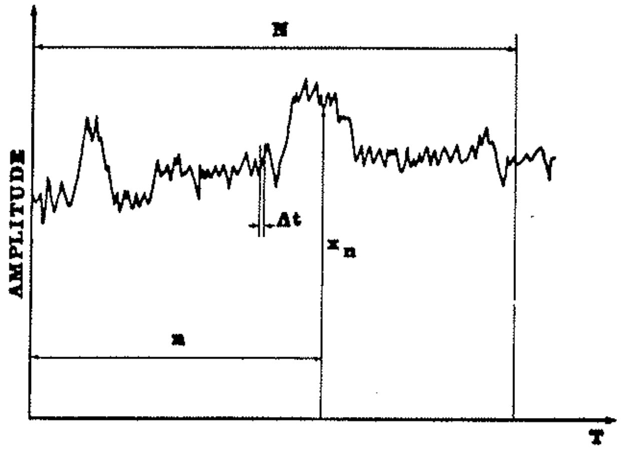
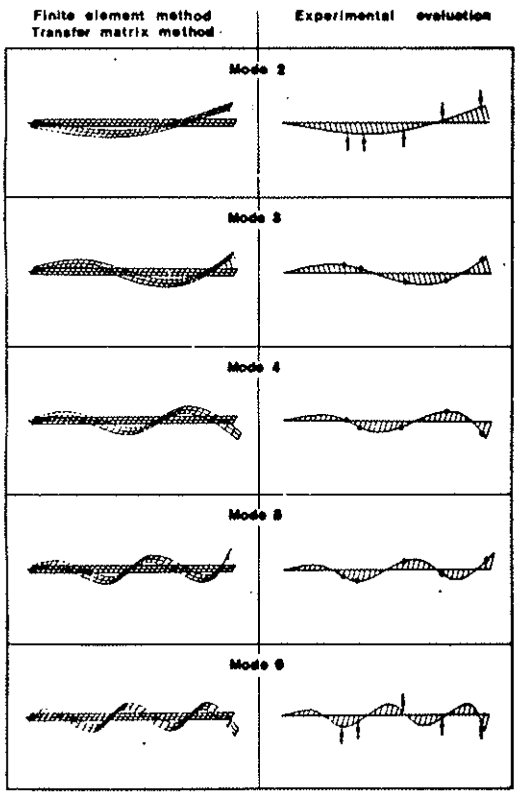


Fig.8

FRAME OF DATA IN TIME DOMAIN



* Measurement points

↑ Not identified by random excitation

a

b

1 Finite Element Programme

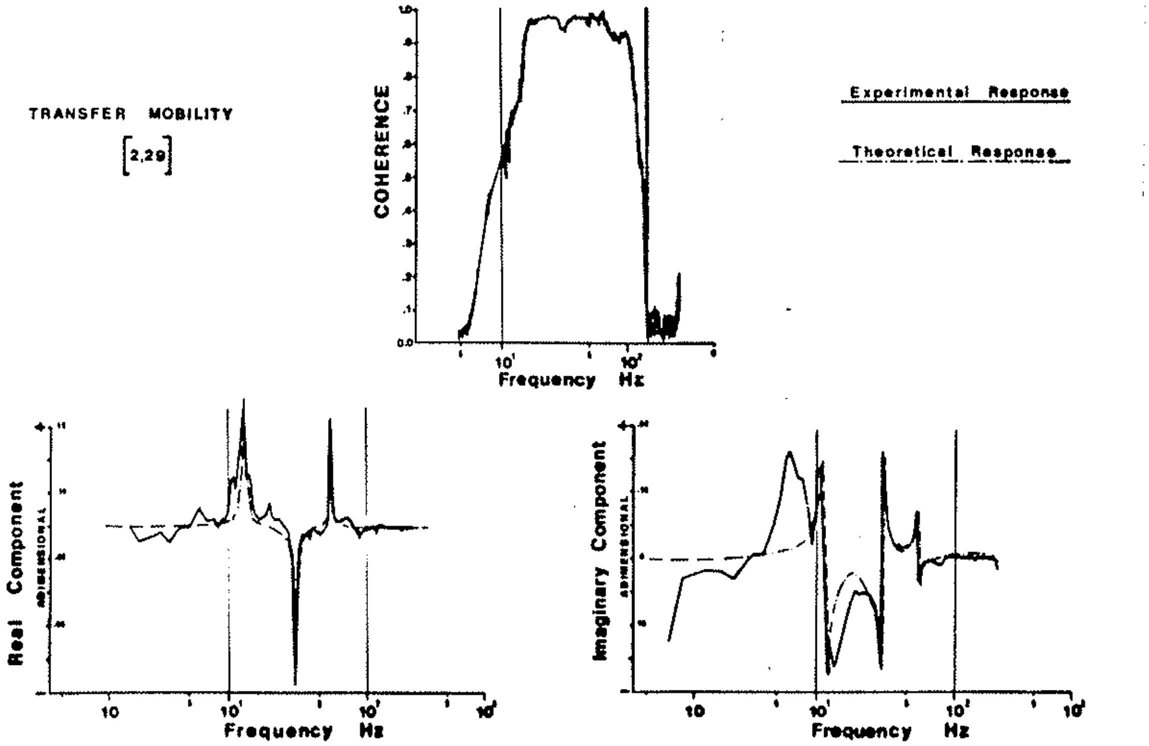
2 Sinusoidal Excitation

3 Random Excitation

4 Transfer Matrix Programme

		Measurement Point					
Prgm	rotor ord	2	10	18	26	29	mode
1	.78	-1.00	-.446	.505	.720	.658	2
2	.75	-1.00	-.461	.617	.738	.675	
3							
4	.76	-1.00	-.450	.573	.740	.649	
1	2.04	-1.00	.565	.690	-.324	-.801	3
2	1.99	-1.00	.533	.684	-.319	-.876	
3	2.07	-1.00	.540	.707	-.324	-.877	
4	1.96	-1.00	.545	.687	-.329	-.806	
1	5.59	-1.00	.859	-.536	-.542	.0291	4
2	5.08	-1.00	.852	-.618	-.640	-.0302	
3	5.18	-1.00	.850	-.621	-.548	-.0327	
4	5.14	-1.00	.862	-.548	-.338	-.0280	
1	9.28	.657	-.420	.776	-1.00	-.590	5
2	9.16	.840	-.418	.762	-1.00	-.560	
3	9.51	.826	-.437	.764	-1.00	-.530	
4	9.49	.837	-.452	.781	-1.00	-.60C	
1	14.67	-.675	-.522	.418	-.566	-1.00	6
2	14.75	-.601	-.517	.418	-.591	-1.00	
3							
4	14.41	-.647	-.501	.429	-.576	-1.00	

Fig. 9



TRANSFER MOBILITY
[2,29]
COHERENCE
Frequency Hz
Experimental Response
Theoretical Response
Real Component
Frequency Hz
Imaginary Component
Frequency Hz

Fig. 10