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EFFECT OF STRUCTURAL COUPLING PARAMETERS ON THE FLAP-LAG FORCED RESPONSE OF A ROTOR BLADE IN FORWARD FLIGHT USING FLOQUET THEORY

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1. Introduction.

In forward flight, when the main rotor provides both lift and propulsive force for the helicopter, the flow over the blades is asymmetric due to the velocity differential over the advancing and retreating blades (Figure 1). Rotor control is obtained by "cyclic pitch" change, which is the name given to the first harmonic variation applied to the blade pitch angle as it rotates. Since the relative air velocity over the blade also has a first harmonic variation and since aerodynamic forces are proportional to the square of the relative velocity, we may expect to find at least three harmonics in the force fluctuations acting on the blades. This would be true if the airflow through the rotor were uniform, however, due to the proximity of the rotor to its own vortex wake, which is swept backwards under the rotor disc, the flow is far from uniform, and velocity fluctuations are induced which give rise to very many harmonics of blade loading. The aerodynamic characteristics of a rotor in forward flight give rise to shear forces and moments at the blade root which are then transmitted to the rotor hub where they are combined and sent through the rotor shaft into the airframe. The necessary procedure for a vibration analysis is illustrated in Figure 2.

During the design of a helicopter, accurate prediction of main rotor blade and hub oscillatory loading is important for fatigue, vibration, and forward flight performance. Critical dynamic components are designed for fatigue based on these predicted loads. Vibration characteristics of the airframe and the need for vibration reduction devices are determined from these loads. Since vibratory response and rotor loads usually determine limiting speed and load factor operational envelopes, the predicted loads greatly influence forward flight performance. Recent Army helicopter development programs, Utility Tactical Transport Aircraft System (UTTAS) and Advanced Attack Helicopter (AAH), have revealed that these loads cannot be predicted accurately (Reference 1). A similar conclusion was drawn from a comparison study (Reference 2) where the helicopter industry's major state-of-the-art rotor loads analyses were independently exercised on an identical hypothetical helicopter problem. The comparison of results illustrated significant differences, particularly in structural dynamic modeling, between the various analyses. A strong recommendation as a result of this study was to conduct computer experiments to study specific isolated aspects of the solution methods and structural dynamics (Reference 2). It was the results of this study and the UTTAS/AAH Programs that provided the impetus for this research.

ROTOR BLADE AZIMUTHS AND VELOCITIES

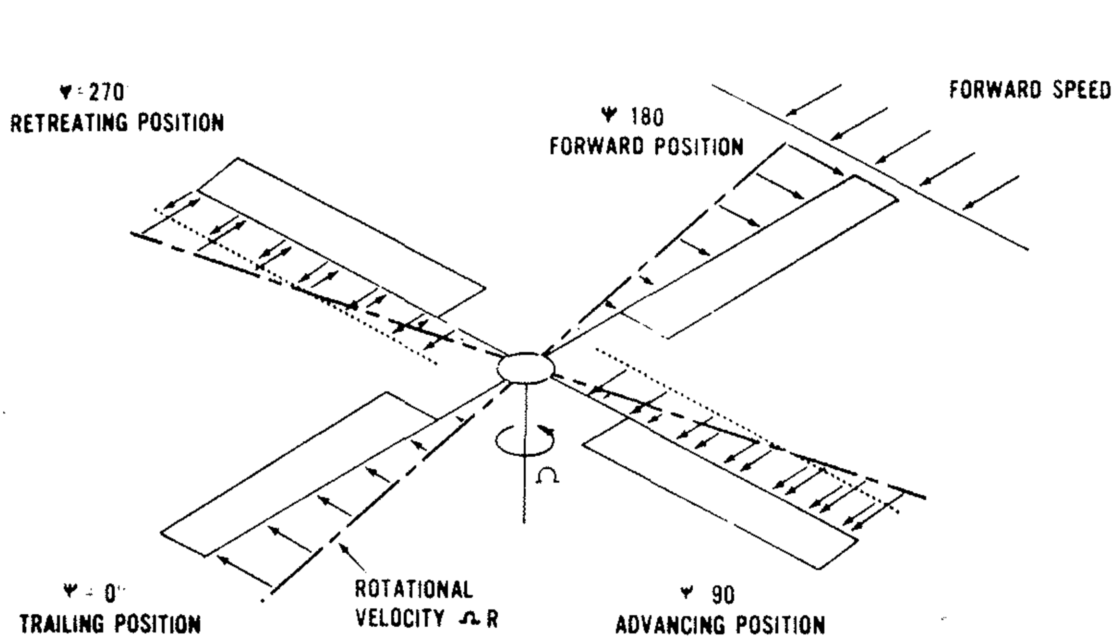

FIGURE 1

VIBRATION ANALYSIS

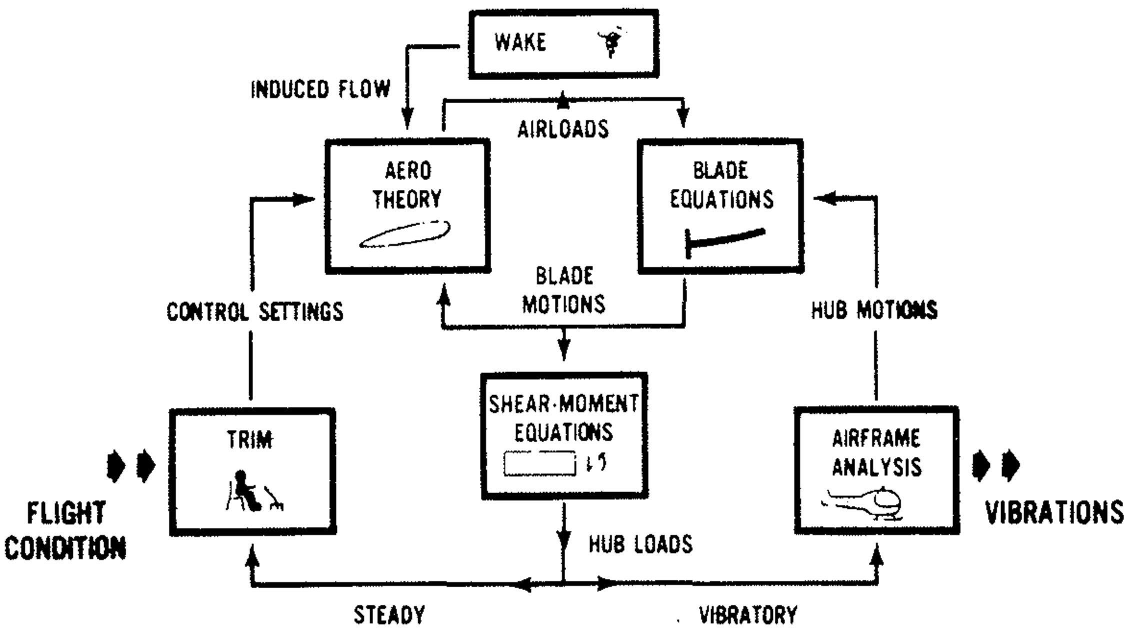

FIGURE 2

2. The Mathematical Model.

The model is illustrated in Figure 3. It consists of a slender rigid blade, hinged at the center of rotation, with spring restraint at the hinge. The orientation of the flap (out-of-plane) and lag (in plane) restraint springs which are parallel and perpendicular to the blade chord line, respectively, simulate the elastic coupling characteristics of the actual elastic blade. The spring stiffnesses are chosen so that the uncoupled rotating flap and lag natural frequencies coincide with the corresponding first mode rotating natural frequencies of the elastic blade. This allows the model to represent a hingeless rotor treated by virtual hinges or a fully hinged rotor (Reference 3).

The model chosen has been used to evaluate rotor aeroelastic stability and can include the incorporation of flap-lag elastic coupling, pitch-flap and pitch-lag coupling, and a blade lag damper. These were some of the fundamental dynamic mechanisms found important in the stability studies of References 3 and 4. Their importance with regards to forced response is a major objective sought in this research.

CENTRALLY HINGED, SPRING RESTRAINED, RIGID BLADE REPRESENTATION

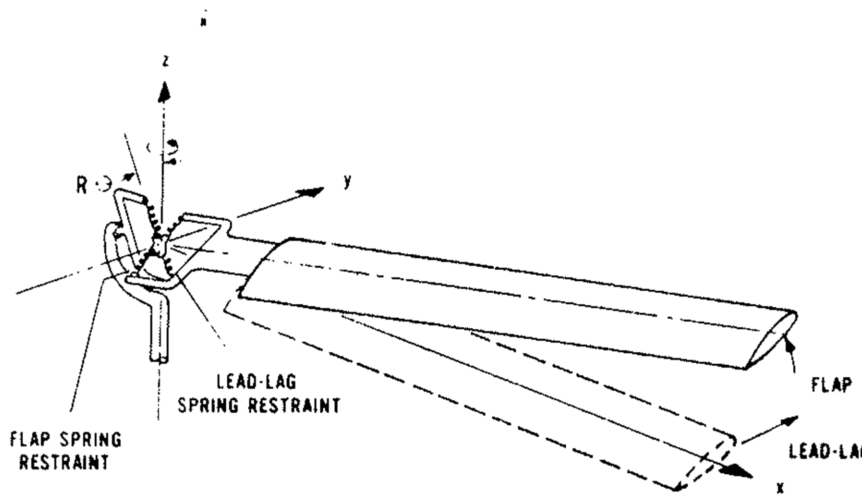

FIGURE 3

To include flap-lag structural coupling and be able to vary it to determine its effect on blade vibrations, the parameter R is defined. Geometrically, it is defined as the ratio of the blade pitch angle inboard and outboard of the pitch bearing. Physically, it represents how the flap-lag structural coupling is dependent on the relative stiffness of the blade segments inboard and outboard of the pitch bearing. This is because the principal elastic axes of the outboard segment rotate through an angle as the blade pitch varies while the inboard segment principal axes do not. The method of implementation is accomplished by replacing θ by $R\theta$ in the structural terms in the flap-lag equations while the mass and inertial terms are unchanged. Thus, when $R=1$, the original equations are retained, but as R is reduced to zero, the flap-lag structural coupling terms diminish and eventually vanish (Reference 3).

Torsion is included in a quasi-steady manner as pitch-flap and pitch-lag coupling. This is accomplished by defining the pitch angle, θ, as:

$$\theta = \theta_o + \theta_s \sin\psi + \theta_c \cos\psi + \theta_\beta \delta\beta + \theta_\zeta \delta\zeta$$

Pitch-flap coupling, θ_β, is intentionally built into some rotors to reduce the flapping motion. If negative θ_β is used, then as the blade flaps upwardly the pitch is decreased, which reduces the aerodynamic forces on the blade and the flapping motion is somewhat suppressed. Pitch-lag coupling, θ_ζ, is sometimes used in a similar way to reduce the lagging motion.

As stated previously, the model has been used to evaluate aeroelastic stability. These studies, (References 3 and 4), have shown the model to give quite reasonable accuracy at low advance ratios when compared with more complicated models. Since all pure helicopters currently developed operate at a $\mu < 0.5$ and the aeroelastic stability of a rotor is considered more sensitive to system parameters than forced response the model is believed to be an adequate representation for parametric studies.

3. Equations of Motion.

The general procedure followed was to derive the nonlinear flap-lag equations of a helicopter rotor in forward flight. The left hand side of the equations was made linear by considering small perturbation motions about a periodic equilibrium position (trimmed condition) of the nonlinear system. The nonlinear terms were consolidated on the right hand side to form the following flap-lag perturbation equations of motion:

$$\begin{Bmatrix} \delta\ddot{\beta} \\ \delta\ddot{\zeta} \end{Bmatrix} + \left[C(\psi)\right] \begin{Bmatrix} \delta\dot{\beta} \\ \delta\dot{\zeta} \end{Bmatrix} + \left[K(\psi)\right] \begin{Bmatrix} \delta\beta \\ \delta\zeta \end{Bmatrix} = \left[L(\psi)\right] \begin{Bmatrix} \dot{\bar{\beta}} \\ \bar{\beta} \end{Bmatrix} + \left[M(\psi)\right] \begin{Bmatrix} \bar{\theta} \\ \bar{\Phi} \end{Bmatrix} + \left[N(\psi)\right] \begin{Bmatrix} \ddot{\bar{\beta}} \\ \frac{Cdo}{a} \end{Bmatrix}$$

There are two forms of periodic coefficients in the equations. The explicit periodic coefficients are of the form $\mu \sin\psi$ or $\mu \cos\psi$. These types are only associated with aerodynamic terms. The second type are implicit periodic coefficients that result from the fact that θ and $\bar{\beta}$ may have cyclic components. These types are found in both structural and aerodynamic terms.

A damping term is included in the $C(\psi)$ matrix to account for the lead-lag damper included on many rotors, particularly hinged rotors. These equations are put into state variable format to reduce them to first order so that an efficient computer library differential equation solver can be utilized.

4. Method of Solution.

The method used to solve the linearized flap-lag equations with periodic coefficients is called an eigenvalue and modal decoupling analysis. It takes advantage of Floquet Theory with its application to linear systems with time varying coefficients. It is unique in that it extends the Floquet Transition Matrix (FTM) Method (Reference 5) to include the calculation of forced response. The eigenvalue and modal decoupling method is summarized in Figure 4. Once the flap-lag equations have been put into state variable format, the state transition matrix and FTM are obtained by step-wise integration over one period. This period consists of one rotor revolution from time zero to 2π. Once the FTM, $\Phi(T)$, is obtained it can be shown by application of the Floquet-Liapunov Theorem (Reference 5) that the problem of stability reduces to solution of an eigenvalue problem. Stability is determined from the real part of the complex eigenvalue, $\Lambda = \eta + i\omega$. Therefore, when η is negative the system is stable, with $\eta = 0$ representing the stability limit or boundary.

The extension of the FTM Method to determine forced response requires considerable manipulation although the mathematics are relatively simple. The eigenvectors of the FTM and the state transition matrix at each integration step increment must be obtained and saved. This is necessary in order that the characteristic functions, $A(t)$, can be calculated and used as a variable substitution to decouple the system of equations. This procedure is completely analogous to the modal analysis method (Reference 6) used to uncouple a system of equations with constant coefficients. Hence, the name eigenvalue and modal decoupling analysis is given to the method of solution.

The information obtained from the analysis are the flap and lag displacements and velocities. These quantities are combined in the time domain to obtain flap and lag shears and moments in both the rotating and non-rotating systems. Once calculated the shears and moments are harmonically analyzed through Fourier Series expansion to determine the relative strengths of the rotor harmonics. In the rotating system the primary concern is fatigue of root end and hub components so that the first harmonic (1/REV) is the major oscillatory source. In the fixed system, the rotor acts as a filter and only allows rotor harmonics that are integral multiples of the number of blades to be transmitted. Therefore, for a two bladed helicopter only the even harmonics, 2/REV, 4/REV, etc., are of major concern in the fixed system. The root shears and moments for each blade in the rotating coordinate system are expressed as the integrals of the blade aerodynamic and inertial loading. The single bladed results in the fixed system are valid for rotors with any number of blades so long as the appropriate solution harmonics are set to zero. Therefore, if a three bladed rotor is being analyzed only the 3/REV loads are of interest in the fixed system.

EIGENVALUE AND MODAL DECOUPLING ANALYSIS

Step	Description
1	Put Flap-Lag Equations into State Variable Format: $X(t) + D(t)X(t) = f(t)$
2	Determine the State Transition Matrix, $\phi(t)$, and Floquet Transition Matrix, $Q = \phi(T)$, By Numerical Integration Over One Period. The Solution Can Be Written as: $X(t) = \phi(t)\, X(0)$
3	Find Eigenvalues and Eigenvectors of FTM, Q. Floquet's Theorem States that a System of Equations Having Periodic Coefficients Has Transient Solutions of the Form: $X(t) = A(t)\, [\ulcorner e^{\Lambda t} \lrcorner]\, \{\alpha\}$. At $t=0$: $\{\alpha\} = A(0)^{-1} X(0)$
4	Using 2 and 3 an Eigenvalue Problem is Developed: $A(0)^{-1} Q\, A(0) = [\ulcorner e^{\Lambda t} \lrcorner]$. The Characteristic Functions, $A(t)$, Can Be Obtained From: $A(t) = \phi(t)A(0)\, [\ulcorner e^{-\Lambda t} \lrcorner]$
5	Through Variable Substitution, $X(t) = A(t)Y(t)$, the Equations in 1 Can Be Decoupled to Obtain: $Y(t) - [\ulcorner \Lambda \lrcorner]\, Y(t) = g(t)$
6	The Uncoupled Response, $Y(t)$, Can Be Obtained From Complex Fourier Series Expansion. The Coupled Response $X(t)$, Can Be Obtained From Matrix Multiplication.

FIGURE 4

5. Results and Discussion.

As with any new analytical method correlation with other proven analyses or measured data must be obtained for validation. The hypothetical rotor of Reference 2 and the UTTAS tail rotors were used for this purpose. The input parameters for the four rotors analyzed are presented in Table 1. Results for the first two rotors in Table 1 are presented in Reference 7. Results for the two UTTAS tail rotors, Rotor No. 3 and Rotor No. 4, will be emphasized in this paper for several reasons. First, they include significant variations in the important structural coupling parameters that are significant to both rotor stability and forced response, but are difficult to analyze by conventional methods. Second, they come closest to the assumed blade model with most of their flexibility at the root of the blade. Third, their development and problems encountered have been well documented so that adequate correlation and comparison could be achieved.

ROTOR BLADE INPUT PARAMETERS

No.	1	2	3	4
TYPE	HINGED	HL(SOFT)	HL(STIFF)	HL(STIFF)
p	1.031	1.136	1.16	1.08
ω_ξ	0.25	0.75	1.67	1.51
γ	7.5	9.3	2.9	2.4
R (elastic)	0	0	0	0
RFL	0	0	1	1
DPL	.15	0	0	0
θ_β	0	-.15	-.70	-2.1
θ_ζ	0	0	0	0
β_{pc}	0	.06	0	0
C_T	.0063	.0062	.0068	.0068
Cd_0	.010	.010	.013	.013
R (feet)	25.0	24.5	5.5	5.0
b	3.0	4.0	4.0	4.0
c (feet)	1.83	1.92	0.81	0.59
σ	.07	.10	.19	.19
FUNCTION	MAIN ROTOR	MAIN ROTOR	TAIL ROTOR	TAIL ROTOR
CONTROLS	SPECIFIED	MOMENT TRIMMED	UNTRIMMED	UNTRIMMED
θ_f	0	0	.131	.157
μ	.33	.36	.42	.42

TABLE 1

Correlation with measured loads for Rotor No. 3 is presented in Figure 5. The curve labeled TEST was obtained during a flight load survey which was part of the UTTAS development program. The hump in the test data at low speed flight can be attributed to nonuniform, transition inflow. The differences at high speed flight can be attributed to blade stall and compressibility effects.

Since these effects have been neglected in the analysis it is felt that excellent correlation has been achieved for the flapping moments.

CORRELATION WITH TEST RESULTS

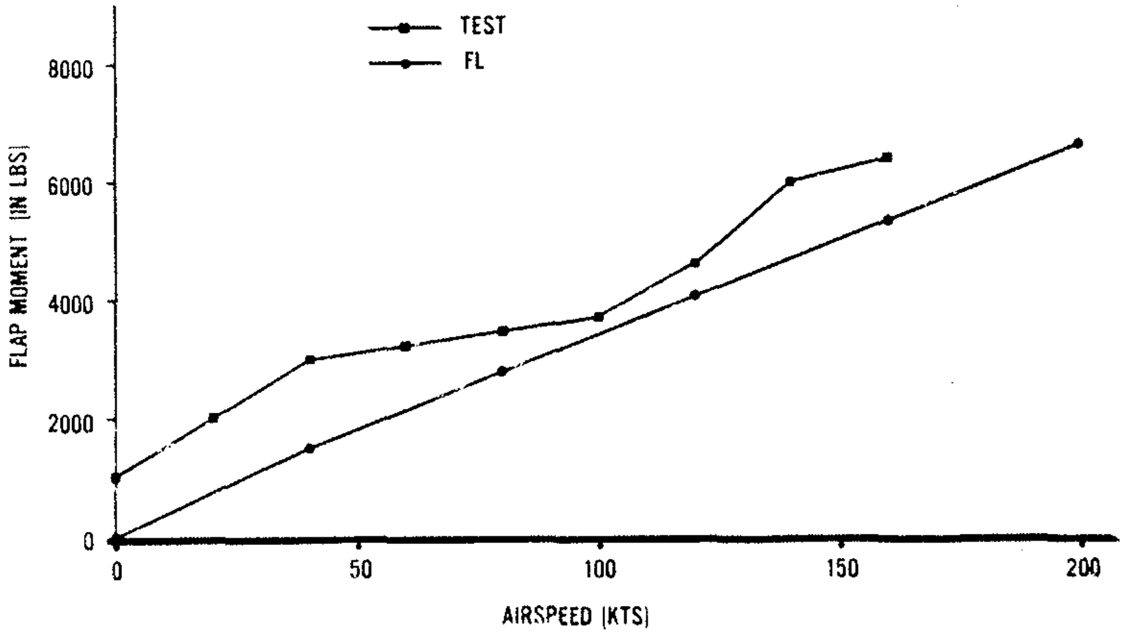

FIGURE 5

Flap-lag elastic coupling, R, is an important parameter for hingeless rotors. While no attempt was made to give a specific value to the baseline configuration on Rotor No. 3 or No. 4, its effect on forced response and stability was obtained by sweeping R from zero to one. The effect of R is illustrated in Figures 6 and 7. Lag damping is a minimum for R equal to zero although it remains stable (Figure 6). Flap-lag elastic coupling has a significant influence on 1/REV lag shears and moments (Figure 7). Comparing the lag moments with those obtained during the UTTAS flight loads survey it would appear that an R value of approximately 0.1 would properly model Rotor No. 3. Both inplane shears and moments can be greatly reduced by setting the elastic coupling close to zero. The flap shears and moments, however, are unchanged by moving to R = 0; and the lag damping is deteriorated. Therefore, elastic coupling alone could not be used to minimize loads and yet increase damping.

VARIATION OF DAMPING WITH ELASTIC COUPLING

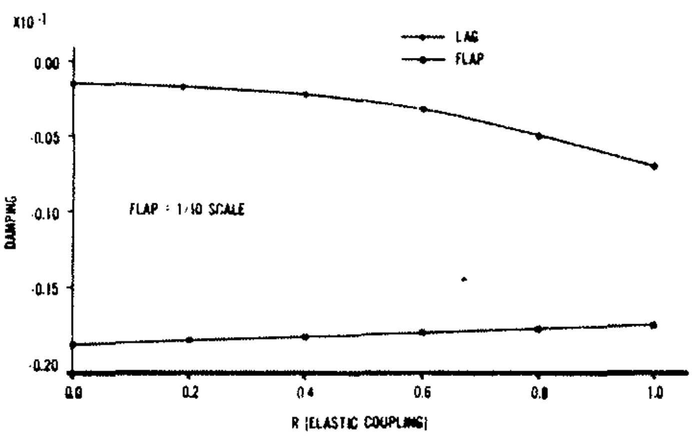

FIGURE 6

VARIATION OF ROTOR LOADS WITH ELASTIC COUPLING

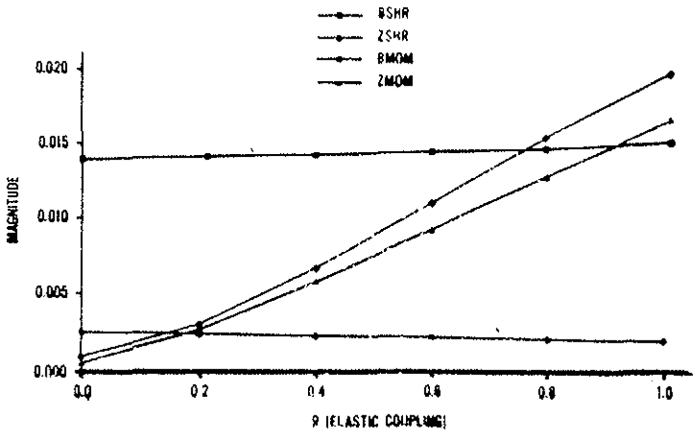

FIGURE 7

Pitch-flap coupling sweeps are illustrated in Figures 8 and 9 for Rotor No. 4. Similar trends were found on both tail rotors. The large effect that pitch-flap coupling has on flap-lag stability can be attributed to its shift in flap mode frequency. Negative coupling raises the flap mode so that it eventually coalesces with the lag mode (Figure 10), and when combined with a high thrust setting can produce a flap-lag instability. Although no instability is illustrated in Figure 8, an instability was produced at coalescence, $\theta_\delta = -3.5$ (Figure 10), at a thrust setting, C_T, of approximately .023. Positive pitch-flap coupling produces a parametric instability at 0.5/REV, similar to other instabilities associated with systems of linear equations with periodic coefficients. The benefits of negative pitch-flap coupling on flap shears and moments are clearly illustrated in Figure 9. Therefore, negative θ_δ could be a valuable design tool for reducing 1/REV flapping shears and moments on flexstrap tail rotors while leaving inplane or lag loads unchanged. For high thrust settings, however, coalescence or near coalescence could produce a flap-lag instability.

The last significant structural coupling parameter analyzed was pitch-lag coupling. On both rotors pitch-lag sweeps had little influence on 1/REV rotor loads, but had a significant impact on rotor stability. This trend is illustrated in Figure 11 for Rotor No. 4. A flap-lag instability occurs at a negative pitch-lag coupling of -0.6 (-31°). While the slope of the curve was similar for Rotor No. 3, sufficient lag damping was available. It appears that the large negative pitch-flap coupling of Rotor No. 4 when combined with sufficient negative pitch-lag coupling will produce a flap-lag instability. The destabilizing effects of pitch-lag coupling and its coupling with pitch-flap coupling are consistent with the predictions of Reference 8.

VARIATION OF DAMPING WITH Θ$_\beta$

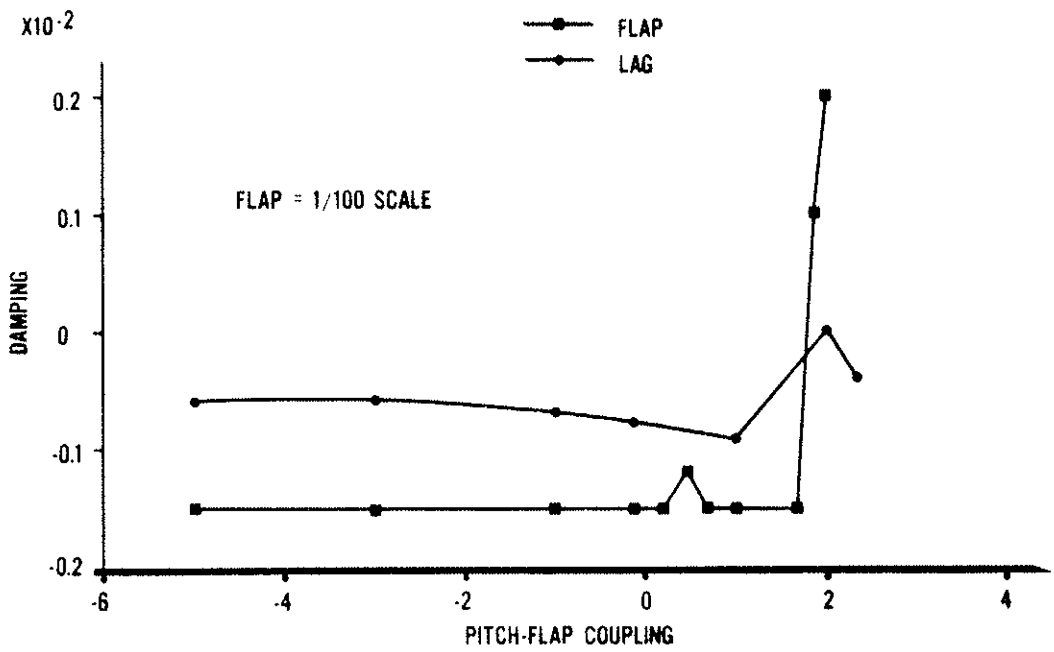

FIGURE 8

VARIATION OF ROTOR LOADS WITH Θ$_\beta$

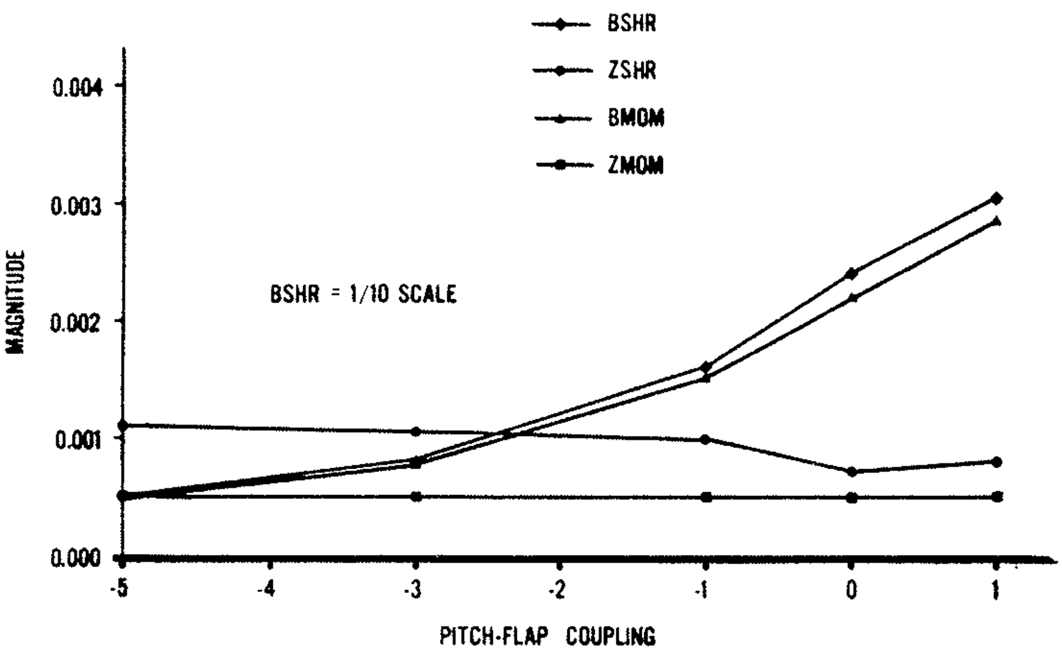

FIGURE 9

VARIATION OF FREQUENCIES WITH Θ_β

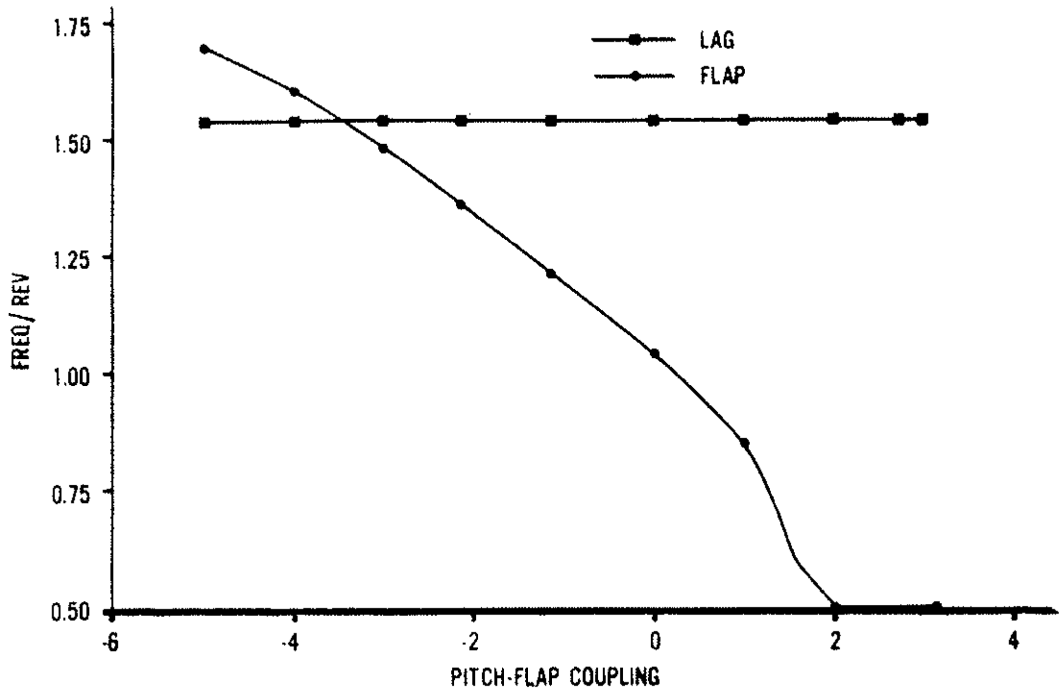

FIGURE 10

VARIATION OF LAG DAMPING WITH Θ_ζ

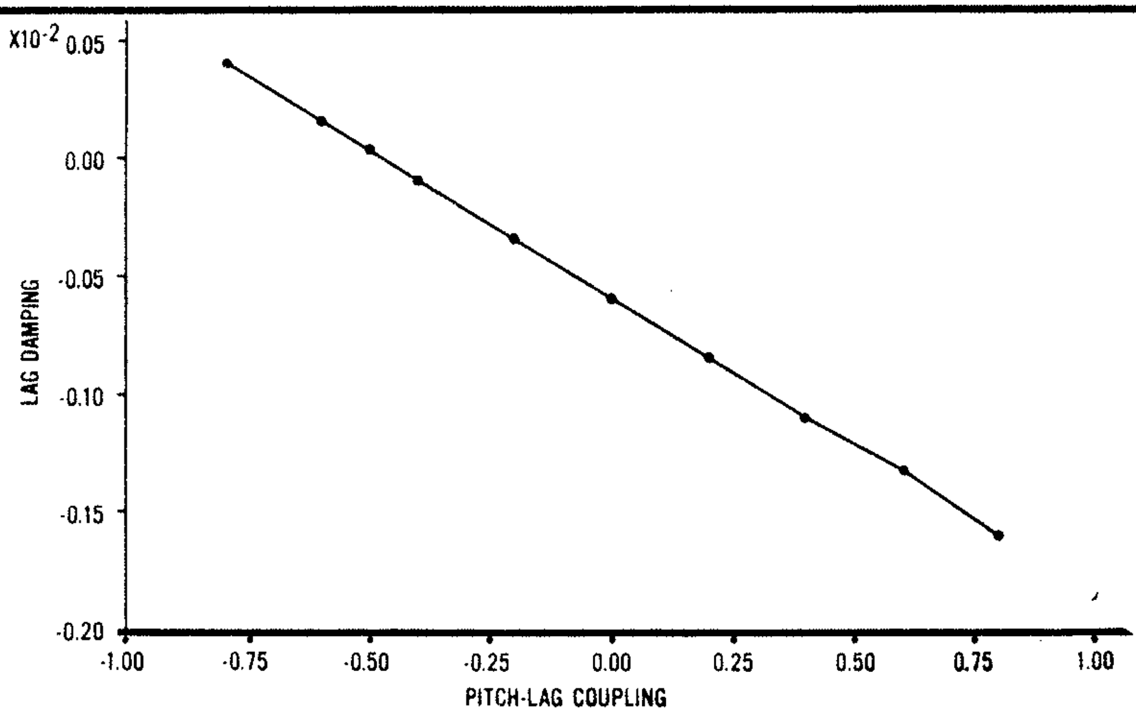

FIGURE 11

6. Conclusions.

An eigenvalue and modal decoupling method to predict helicopter rotor stability and forced response has been successfully developed. The advantages of this method are:

1. Stability and forced response can be obtained from the same analysis making it an excellent preliminary design tool.

2. Since only one rotor revolution of numerical integration for an initial condition of unity imposed on each degree of freedom is necessary to define the Floquet Transition Matrix, the method is efficient and yet retains the periodicity of forward flight equations of motion.

The results of this study illustrate that the major rotor coupling parameters can be chosen in a systematic way to achieve stability and low rotor loads. The following sequence indicates the manner in which the parameters could be chosen during preliminary design:

1. The first parameter to choose should be the elastic coupling, R. The results for the rotors analyzed show that zero elastic coupling gives the smallest inplane loads. Although R also affects the damping, other parameters can be used to counter any destabilizing effects.

2. The second parameter to be chosen is pitch-flap coupling, θ_β. This parameter should be given a large negative value in order to greatly reduce the 1/REV flapping loads, although practical limits exist on main rotors due to flying quality considerations. Another practical limitation on negative θ_β is the coalescence of flap and lag frequencies at high collective pitch.

3. The third parameter to be chosen is the pitch-lag coupling, θ_ζ. Pitch-lag coupling has been shown to have little effect on loads but a large effect on stability. Thus, once R and θ_β have been chosen so as to minimize inplane and flapping loads, θ_ζ can be chosen to provide stability without affecting the airloads.

NOTATIONS

a	=	slope of lift curve
b	=	number of blades
c	=	blade chord, ft.
c_{do}	=	blade profile drag
$C(\psi)$	=	damping matrix
C_T	=	thrust coefficient
$K(\psi)$	=	stiffness matrix
p	=	nondim. rot. flapping frequency at $\theta=0$, $p=(1+\omega_\beta^2)^{1/2}$
r	=	blade radial coordinate, ft.
$\bar{r}$	=	nondim. coordinate r/R
R	=	blade radius, ft., or elastic coupling parameter
t	=	time, sec
T	=	rotor thrust
V_i	=	uniform induced velocity in negative Z direction, fps
V_x, V_z	=	components of helicopter speed in negative X and Z directions, respectively, fps
X, Y, Z, x, y, z	=	aircraft coordinates and rotating blade coordinates respectively
$\bar{X}$, $\bar{Y}$	=	Forward and Lateral Shears respectively
$L(\psi)$	=	forcing coefficient matrix
$M(\psi)$	=	forcing coefficient matrix
$N(\psi)$	=	forcing coefficient matrix
BSHR, BMOM	=	**nondim. 1/REV flap shear and mom.**
ZSHR, ZMOM	=	**nondim. 1/REV lag shear and mom.**

β	=	flap angle, pos up, rad
$\bar{\beta}$	=	equilibrium flap angle, $\beta_o + \beta_s S\psi + \beta_c C\psi$, rad
β_{pc}	=	precone angle, rad
α	=	Locke number, $\rho a c R^4/I$
$\delta\beta$, $\delta\zeta$	=	perturbation flap and lag angles, rad
ζ	=	lead-lag angle, pos. fwd, rad
$\bar{\zeta}$	=	equilibrium lead-lag angle, rad
η	=	neg real portion of lead-lag eigenvalue
θ	=	pitch angle $\theta + \theta_\beta\delta\beta + \theta_\zeta\delta\zeta$
$\bar{\theta}$	=	equilibrium pitch angle $\theta_o \pm \theta_s S\psi + \theta_c C\psi + \theta_\beta(\bar{\beta}-\beta_{pc})$, rad
θ_β, θ_ζ	=	pitch-flap and pitch-lag coupling ratios
λ	=	inflow ratio $(V_i+V_z)/\Omega R$
Λ	=	complex eigenvalue, $\Lambda = \eta+i\omega$
μ	=	advance ratio, $V_x/\Omega R$
ρ	=	air density slugs/ft^3
σ	=	rotor solidity, $bc/\pi R$
T	=	period of one revolution, $T = 2\pi/\Omega$ sec
$\bar{\Phi}$	=	inflow parameter, $\bar{\Phi} = 4/3\lambda$
ψ	=	rotor azimuth angle, $\psi = 0$ aft, $\psi = \Omega t$, rad, dimensionless time
ω	=	imaginary portion of lead-lag eigenvalue
ω_β, ω_ζ	=	dimensionless nonrotating flap and lag frequencies at $\theta = 0$
Ω	=	rotor angular velocity
$S\psi$, $C\psi$	=	sine (ψ), cosine (ψ)
$(\dot{\ })$	=	$\frac{d}{d\psi}(\ ) = \frac{1}{\Omega}\frac{d}{dt}(\ )$

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