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EXPERIMENTAL EVALUATION OF THE NONLINEARITIES IN A  
HELICOPTER GROUND VIBRATION TEST

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# EXPERIMENTAL EVALUATION OF THE NONLINEARITIES IN A HELICOPTER GROUND VIBRATION TEST

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## Abstract

The paper summarizes the experimental modal analysis on a normal production helicopter with the aim to investigate experimentally the dynamic behaviour of such aerospace structures and to set a general methodology when the structure presents nonlinearities in order to have high quality data for a successive analytical ground stability analysis comparison. The excitation has been limited to one single location of the structure at the mast of the helicopter, two directions lateral and longitudinal. The choice of the excitation signal is one of the factors that can affect the quality of the FRF's obtained so the stepped sine excitation has been used to minimise the errors and to detect the nonlinearities. The main purpose of the paper is to show that many structures such as a helicopter are nonlinear and the excitation, instrumentation and estimation can accentuate or obscure the nonlinear response if not properly used.

## 1.0 Introduction

The importance of the modal analysis and his diffusion is increased in the last few years with the increasing complexity of the aerospace structures and the necessity of an accurate dynamic characterization. The dynamic characterization is usually carried out

identifying the structure's modal parameters. The object of this type of analysis is different and in particular:

- a) experimental determination of structure parameters such as the natural frequencies, mode shapes, damping ratios, modal mass and stiffness.
- b) to substantiate FEM structural analysis activities for the accuracy of the analytical model which would predict accurately the dynamic response characteristics of the actual structure.
- c) complement to the analytical activities for the ground stability analysis.

In order to avoid possible errors in the data acquired, it is necessary that every test phase is as accurate as possible. The experimental result consists of a set of complex functions, the Frequency Response Functions (FRF)  $H_{ij}(\omega)$  obtained from an excitation in point  $i$  and response in point  $j$ .

The FRF matrix  $[H(\omega)]$  obtained permits to extract the modal parameters but the applicability of the results depends on their quality. So, attention must be taken first of all to the accuracy on the measurement process and second verifying the major assumptions of the modal analysis technique, linearity, reciprocity and repeatability.

Therefore it is necessary to detect the existence of the nonlinearities present in a mechanical structure that can cause errors in the estimation of the response [1].

## **2.0 Instrumentation and Test Equipment**

The quality of the results and consequentially the validity of the modal parameters extracted cannot be compensated using sophisticated analysis methods but attention should be used to the measurement chain.

It is infact essential for a modal survey testing that the transducers are calibrated before the beginning of the test particularly when the experimental results will be used for correlating an analytical model [2]. Radio communication was set up between test area and the analysis system terminal engineers to control all aspects and inconvenients that could occur during the test.

### **2.1 Suspension System**

The correlation between modal and analytical analysis can be correctly performed when the suspension system modes are separated from those of the aircraft. The results obtained show a very good separation between the structure and the isolation system.

Often the suspension system used for a complicated and big structure like an helicopter is a compromise between cost, complexity, time and benefits in terms of isolation. The new system used during these tests consists of an air spring composed by a toroidal shell of rubber with two loops between which there is a metallic ring in order to contain the trasversal deformations.

The air for the spring is furnished through two rubber pipes connected to a reservoir of 500 litre capacity and pressures up to 13 bar.

The reservoir is then connected to the central system of compressed air. The cut-off frequency of the system is 0.8 Hz.

The advantages of the air spring are the constant static height for all load conditions, constant frequency changing the load, no maintenance and noiselessness.

The only disavantage noticed is a possible small pressure reduction in the air spring ( $\approx 1\%$ ) for a long period test depending on the airborne condition. If the pressure is re-establish furnishing air slowly, the FRF's are not affected by errors.

For the tests at different airborne the helicopter was suspended at the main rotor hub by a cable connected to this suspension system and opportunely electrically isolated.

### **2.2 Excitation System**

The aircraft has been excited by an oleodynamic exciter normally used for endurance tests. This type of exciter permits to have at low frequencies a better force range or displacements normally not permitted by the electromagnetic shakers. Compared to these has the disavantage of the complexity and a more limited frequency range if frequencies on the order of kHz are needed. In these tests the frequency range was between 1 Hz to 9 Hz.

Before starting it has been verified that the load cell gave zero load and there was not pre-load. The actuator was controlled by the stepped sine software.

### **2.3 Acquisition System**

The front-end used was the DIFA Scadas II system with 24 input channels with a 12 bit analog-digital converter and a total maximum capacity of 800 ksamples.

The anti-aliasing filters are analogical of elliptical type 80 dB per octave. The filter modules incorporate a range amplifier and overload detector. The phase difference between the channels is below 0.2 degrees. The connection with the computer is done through two interfaces HPIB and GPIO, the first used for the selection of the parameters and the second for the data transfer at high speed. This front-end have a digital-analog converter 4 channels, 16 bit used by the software for the generation of the function excitations.

## 2.4 Transducers

The primary transducers were B&K accelerometers type 4367, 4369, 4370, 4381 and 4382 with a sensitivity range between 2 and 11 pc/ms<sup>2</sup> mounted on alluminium blocks to measure the acceleration in the three or two axis. The accelerometers were connected to the B&K 2635 pre-amplifiers and the measurement chain obtained could measure signals with an harmonic content greater than 0.2 Hz.

The calibration of the transducers has been made using the B&K 4291 type calibrator. The excitation at the helicopter mast was measured by a force transducer B&K type 8201 and the force control was made by a load cell in axis with the oleodynamic actuator.

## 3.0 Measurements

The measurement points on the helicopter were selected to obtain the dynamic response in those points essential to describe the dynamics of the aircraft.

The measurements were performed using a stepped sine signal with a relative filter cutoff

ratio which was 80% of Nyquist frequency for both force and displacement.

The excitation was applied over a frequency spectrum of interest during the ground vibration test from 1 Hz to 9 Hz with a frequency resolution of 0.03 Hz, 128 were the number of acquisition samples per sine period and 4 the number of sine periods for data acquisition. The total frequency spectrum successively was divided into two bandwidths each surrounding one of the natural frequencies of the system.

### 3.1 Stepped Sine Excitation

Different techniques of excitation are available today for testing system response.

The excitation signals that normally are used can be divided in two categories: the single frequency and broadband signals.

The sinusoidal excitation is the more indicated signal to reveals the eventual nonlinearity present in a structure [3] and for this reason the stepped sine excitation has been used.

The stepped sine is a typical single frequency signal which varies in a discrete way. A sine wave is generated at a certain frequency by the computer using a predefined number of points per wave, 128 in these tests, and sent out through a digital-analog converter to the excitation system. The advantages of this type of excitation are:

- very high signal-to-noise ratio which results in coherence values close to one
- high RMS value-pick ratio
- precision of signal excitation and magnitude

the disadvantage is the time for the acquisition partially reduced by the possibility to used different steps in the various frequency ranges,

for example high resolution close to the resonances and lower far from them.

#### 4.0 Analysis

The FRF's acquired have been successively analyzed to extract the modal parameters using the Least Squares Complex Exponential method for the functions acquired in the range 1-9 Hz with a sufficient number of acquisition points and the Frequency Domain Parameter Identification method in the other cases when the first method could not be used.

#### 4.1 Nonlinear Behaviour

The nonlinear dynamic behaviour was observed directly from the distorted FRF's obtained using different excitation force and displacement levels. Many are the possibilities for a nonlinear dynamic system to show a nonlinearity such as the amplitude jump phenomena. The effects of a nonlinear force-deflection characteristic with linear hysteretic damping for a hardening behaviour is shown in figure 1.

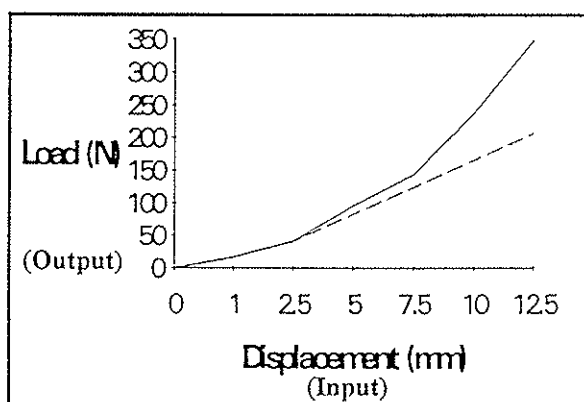


Figure 1 Hardening spring rate

The presence of a nonlinear response can be concentrated locally in a helicopter component or distributed and induced by bolted

connections, deformations in riveted as well as within in the structure themselves.

#### 4.2 Effect of Excitation Parameters: Force and Displacement

A series of measurements with a constant harmonic excitation level both  $D=\text{const}$  and  $F=\text{const}$  were made in the vicinity of one mode and the results are shown in figure 2 and figure 3 respectively.

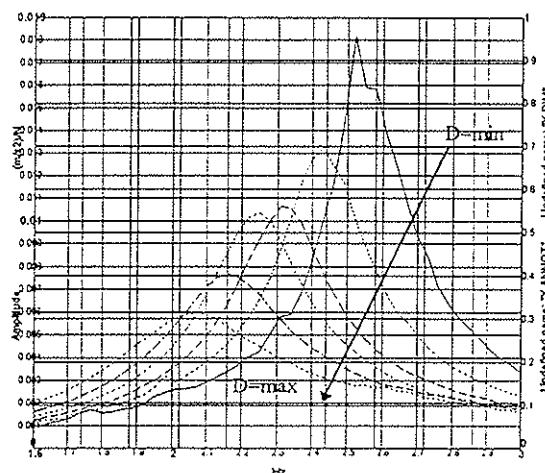


Figure 2 Lateral excitation with constant displacement

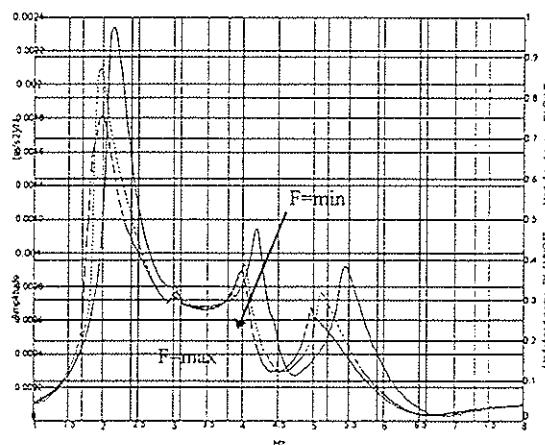


Figure 3 Longitudinal excitation with constant force

In both cases the result of different levels of excitation force or displacement shows that the resonant magnification decreases when the excitation parameter increases.

### 4.1.2 Finding the Linear Range

The method for the determination of the linear range consists of performing identical measurements but increasing the levels of force or displacement. The linear range of the structure will be reached when two mobility spectra are identical.

The reason for testing in the linear range is to reduce the nonlinear effects and to obtain constant values of acceleration with force but unfortunately the levels of force or displacement are too high and problems during the tests can arise.

During these tests the linear range has been reached for more than 8 mm constant displacement excitation and 600 Newton constant force excitation but the magnitude of the excitation parameters were too high to perform for a long period.

The figure 4 shows that increasing the excitation force the frequency changes with a rapid decreasing of magnitude and then the frequency variation is proportional to the magnitude.

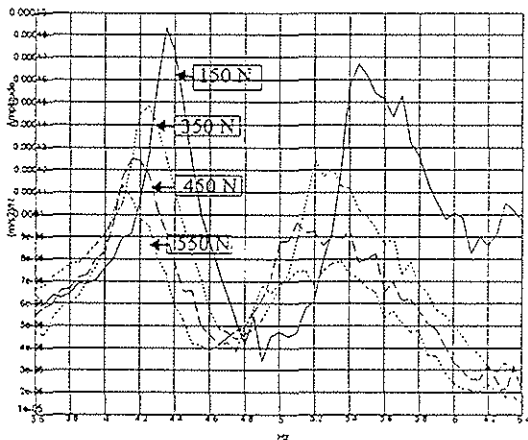


Figure 4 FRF increasing excitation force finding linear range

In figure 5 is shown as the resonant frequency decreases increasing the excitation at constant displacement at the beginning following a

quadratic law and finally, after 8 mm excitation, linear law.

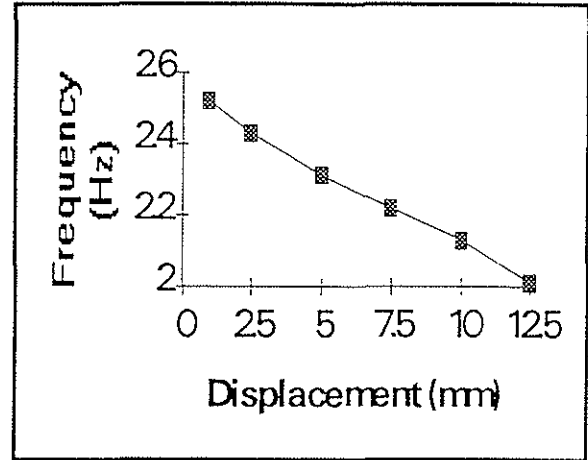


Figure 5 Frequency vs excitation displacement finding linear range (lateral excitation)

Finding the linear range with an excitation along the longitudinal axis at constant force, as the level of force increased the frictional forces at the joints continue to reduce in magnitude causing a nonlinear relationship of acceleration and the level of force.

Finally when the frictional forces at the joints have been reduced to a constant value there is the true linear region and so the acceleration increases linearly with the force.

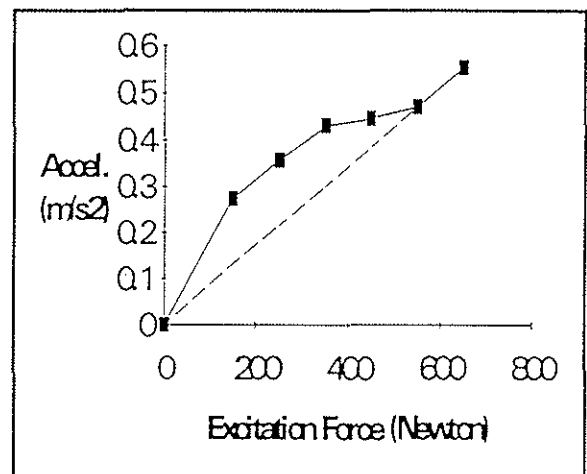


Figure 6 Acceleration vs excitation force finding linear range (longitudinal excitation)

### 4.3 Landing Gear

The landing gear plays an important role in the helicopter dynamics and the damping and stiffness characteristics of its elements, shock absorber and tire, are the basic means against ground resonance. When the natural frequencies of the helicopter are calculated it is assumed that the shock absorber and tire of the landing gear have linear characteristics and for small vibrations this approximation can be used. In reality the tire and the oleodynamic strut are nonlinear components. The helicopter examined has a tricycle landing gear arrangement. Two landing gears with different shock absorber were tested and compared.

#### 4.3.1 Tires

A typical case of nonlinearity in mechanical vibrations is that of an elastic member as a tire. Two configurations have been analyzed with two tire pressures 6 and 8 bar which meaning about 6÷7% in stiffness variation. The result is shown in figure 7 where substantial no variations can be noticed on the FRF. To obtain a substantial difference the pressure of the tire have to be increased in the order of about 20% in stiffness.

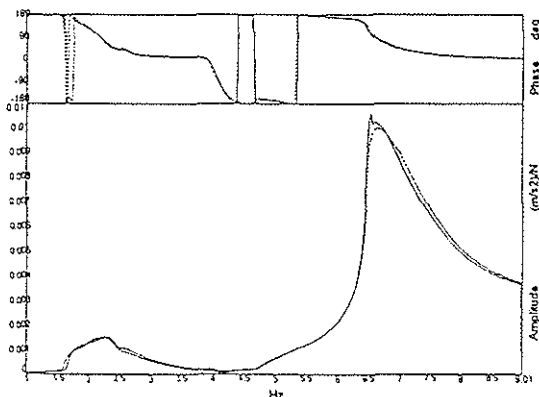


Figure 7 Effect of tire pressure- lateral excitation

The excitation along the longitudinal axis confirms the previous lateral excitation results as shown in figure 8.

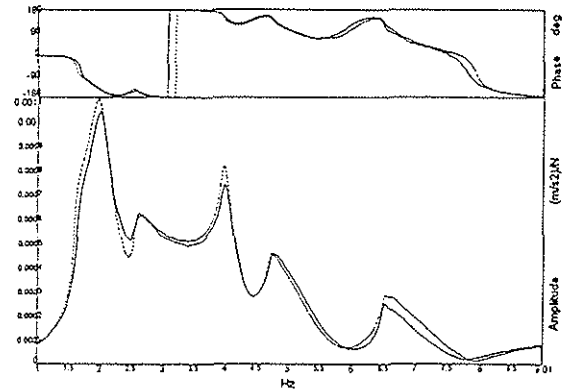


Figure 8 Effect of tire pressure- longitudinal excitation

The effect of the tire on the amplitude reduction and nonlinearity is clearly shown in figure 9 where from the FRF obtained with a lateral excitation at constant force of 100 Newton from the reference point (line 1) there is an amplitude reduction at the fuselage bulkhead (line 2) and a further amplitude reduction on the wheel-hub (line 3) with an increased softening effect.

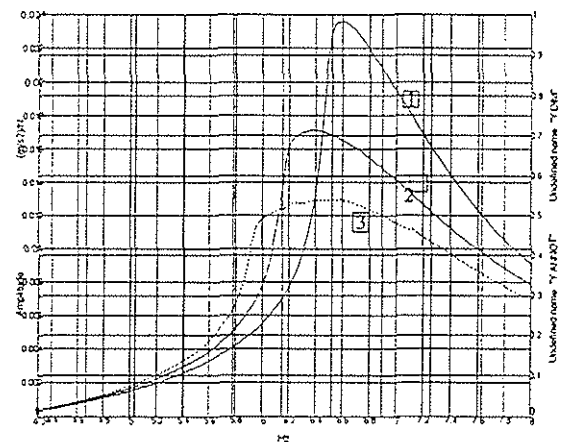


Figure 9 Tire damping effect excitation with constant force

The lateral excitation induces the wheels to flutter through a displacement which is a function of the lateral flexibility of the tires and for a low excitation magnitude this



parameter dominates the landing gear behaviour.

### 4.3.2 Shock Absorber

A new landing gear was design to support heigher loads due to the increased helicopter weight. Moreover, it was arranged to increase the damping of the shock absorber.

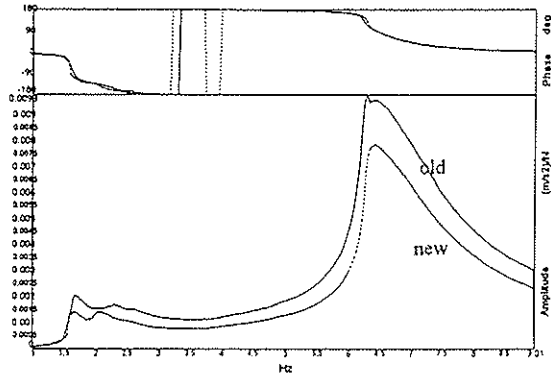


Figure 10 Shock absorber effect on FRF

The effect of these changes on the response functions is shows in the figure10 where the amplitude of the response for the new landing gear is reduced with a more softening nonlinear behaviour.

### 4.4 Stinger

To transmit the force from the excitation system to the structure it is necessary to connect these two elements by means of a stinger that can obviously contaminate the test's results [4]. It is important to ensure that pure translational force is applied along the axis so careful must be taken for the alignment.

#### 4.4.1 Type of Stinger

The effects of this component on the response functions are very important as shown in figure 12. The three FRF's are relative to

three different type of stingers. A rigid rod type (#3) a spherical bearing (#1) and a universal joint (#2) as shown in figure 11.

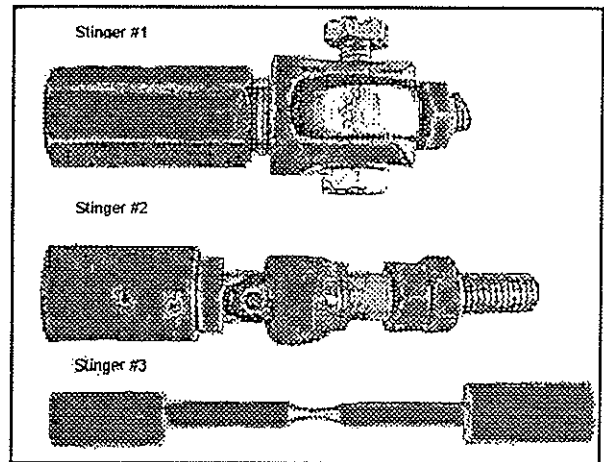


Figure 11 Type of stingers

Analyzing the figure FRF's in figure 12 it is possible to see hardening (stinger 1), softening (stinger 3) and linear (stinger 2) behaviours. It is important to notice that the effect shown in figure 12 at the driving point come out only in the excitation direction and not on the two other axis. Increasing the excitation force to 200 Newton the effects of backlashes are reduced and the FRF are almost equal both amplitude and frequency.

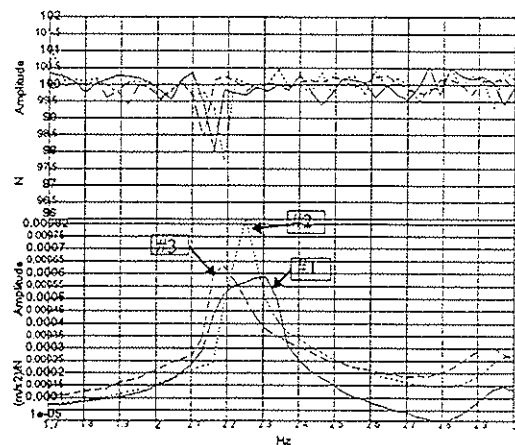


Figure 12 Stinger effect on FRF

At higher frequency (the second mode) 5-8 Hz the FRF's are equal using low excitation forces too.

#### 4.4.2 Stinger Effect on Nonlinearities

The boundary conditions can introduce nonlinearities in the examined structure otherwise linear.

The interaction exciter-structure can cause on the excitation force autospectrum the notches when the excitation frequency is close to the structural resonance and to introduce nonlinearities on the corresponding FRF as shown in figure 12.

In the past the depth of a notch has been attributed to the mass of the armature of excitation system.

Unholtz [5] from his research concluded that the depth is reduced if the mass is increased while Olsen [6] support an antithetical thesis.

The experimental results of the tests show that changing only the stinger different depth of notch was obtained. The stinger stiffness seems to be the parameter that drive the notch depth considering that the mass of the three stingers were identical. This can be verified using the relationship between stinger's axial stiffness and stinger mass which is given as

$$K_r = \frac{m_r \omega^2}{\alpha^2} \quad (1)$$

where

$$\alpha = \frac{\omega l_s}{c} \quad (2)$$

Using the measured values of  $m_r$ ,  $l_s$ , the resonance frequency  $\omega$  of the FRF and  $c$  which is the longitudinal wave propagation velocity, it is possible to find that the deeper glitch correspond to the stinger with the higher stiffness.

The characterization of a glitch is done by two parameters, width and depth. When the depth  $D_f$  is more than 6 dB as in the figure 13 is

defined "deep glitch" otherwise "flat glitch". The width  $\eta_G$  is defined in percentage as the difference between the pick and glitch frequencies over the glitch frequency  $\eta_G = \Delta\omega/\omega_N$  in the figure 13  $\eta_G = 7\%$ .

Since  $\eta_G > 1\%$  this is a "wide irregularity" while for  $\eta_G < 1\%$  is "narrow irregularity".

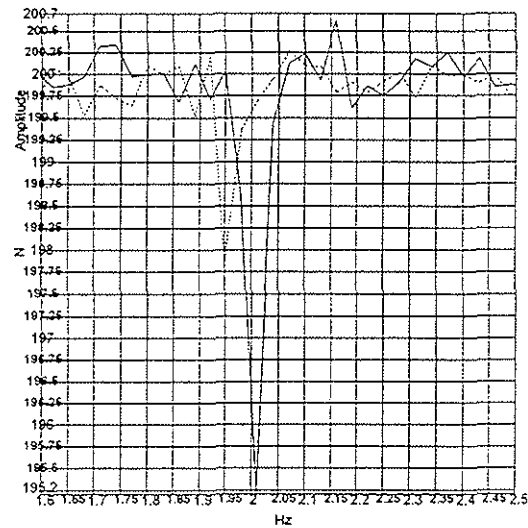


Figure 13 Typical force autospectrum with glitches

#### 4.5 Effect of Helicopter Airborne

The suspension system was used to simulate different airborne configurations. Using an excitation at constant displacement of 1.0 mm four airborne conditions from 0% to 75% with 25% step were tested. The FRF's obtained shown a response similar to those obtained in figure 2 and 3 with the constant displacement or force excitation respectively.

No particular nonlinearities were observed due to the reduced wheel contact with the ground surface and different shock struts position. It is interesting to compare figure 14 and figure 15 where as the airborne increases, at the driving point the resonance frequency decreases as the amplitude while at the wheel-hub the resonance frequency decreases but not the amplitude until 75% of airborne when the tire is slightly in contact with the ground.

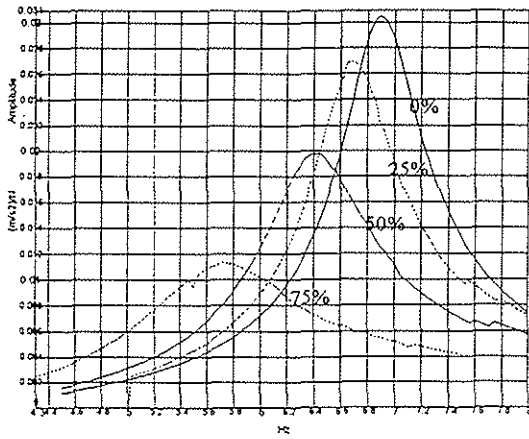


Figure 14 Effect of airborne on FRF's (driving point)

This is true only for a low excitation parameter.

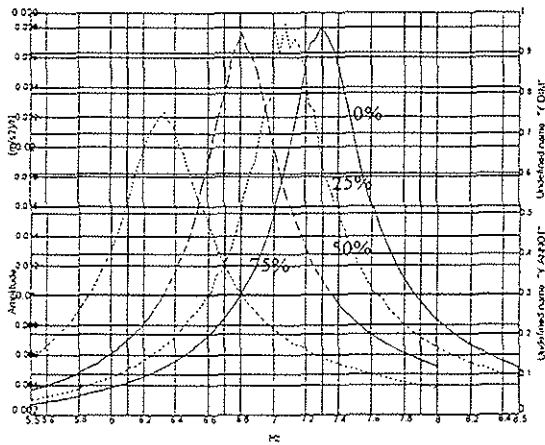


Figure 15 Effect of airborne on FRF's (wheel-hub)

#### 4.6 Identification of Nonlinear Systems

For the investigation of a nonlinear behaviour several techniques have been developed as Hilbert transform, direct time stepping method, the harmonic detection technique, the damping plot and the complex stiffness method.

Following the simple approach as proposed in [7] it is possible to analyze an FRF obtained with a sinusoidal excitation signal without the requirement that  $D=\text{const}$  or  $F=\text{const}$ .

Under these conditions, for a SDOF linear system, the real and imaginary parts are given as:

$$\Re(1/\alpha) = k(x) - \omega^2 m \quad (3)$$

$$\Im(1/\alpha) = i\omega c \quad (4)$$

Using the FRF obtained during a test with a constant force excitation  $F=150$  N and  $F=300$  N at the wheel-hub and plotting the real part of the inverse of receptance is a straight line for a linear case or curved for a stiffness nonlinearity (Figure 16)

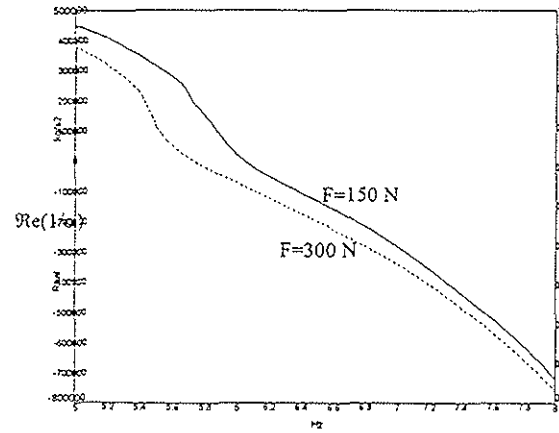


Figure 16 Real part of inverse receptance with softening cubic stiffness

Figure 16 shows that close to the resonance, the curve is affected slightly by the nonlinearity and far from it become a straight line

When the imaginary part of the inverse of receptance is a straight line as for the linear case for the presence of nonlinear damping become a curve (Figure 17).

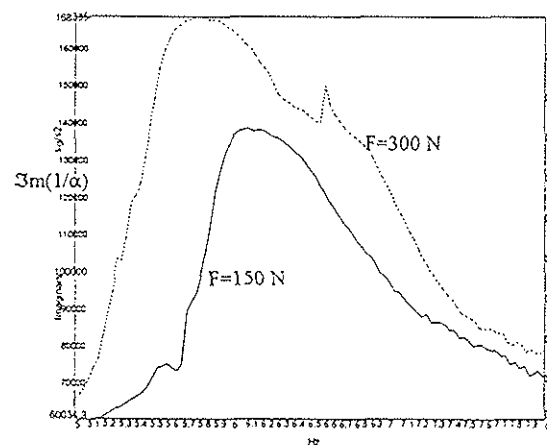


Figure 17 Imaginary part of inverse receptance with quadratic damping

#### 4.6.1 The Complex Stiffness Method

In many cases the linearisation of the model is not acceptable because the nonlinearity is severe.

The complex stiffness method is a frequency domain method which permits using a FRF data set obtained with a sinusoidal excitation to identify and quantify approximately, the mass  $m$ , stiffness  $k$  and damping  $c$  of a nonlinear single degree-of-freedom system. The hypotheses assumed using this method are:

- the mass is constant
- the damping is a function of the velocity
- the stiffness is a function of the displacement

With the previous hypotheses a SDOF can be described by the equation

$$m\ddot{x} + c'(x)\dot{x} + k'(x)x = F \quad (5)$$

where the damping  $c'$  and stiffness  $k'$  are estimated from a linearisation in each frequency point.

This method has been applied to the set of the measured frequency response functions acquired during these tests [8] and obtained using the force as input.

Following the theory described in [9] it is easy to show that the experimental inertance is related to the system parameters as

$$\frac{F}{x} = (m - k_{eq})\omega^2 - j\omega c_{eq} \quad (6)$$

First of all must be calculated the equivalent mass (modal mass) which is not the helicopter mass, using FRF's obtained with different force level excitations (Figure 18).

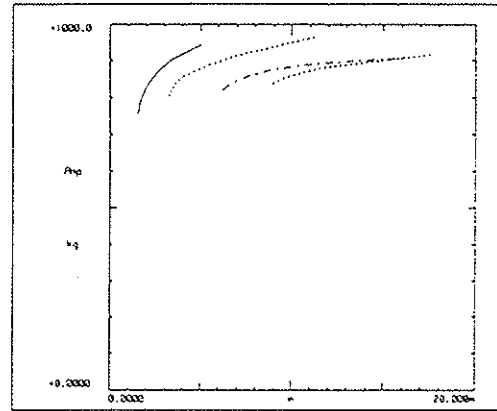


Figure 18 Equivalent Mass

Then, from the equation (6) it can be written

$$k_{eq} = \left[ m - \text{Re} \left( \frac{F}{x} \right) \right] \omega^2 \quad (7)$$

$$c_{eq} = -\omega \text{Im} \left( \frac{F}{x} \right) \quad (8)$$

The equivalent stiffness and damping are then calculated as described in the literature of this method and the results are shown in the following two figures 19 and 20.

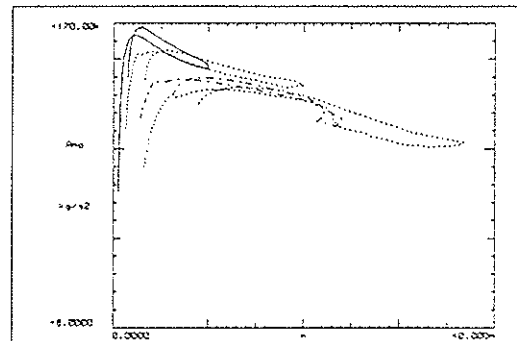


Figure 19 Equivalent stiffness

The first part of the figure 19 is attributable to the backlash in the main gear box or/and landing gear. The second part indicates a softening stiffness due to the landing gear and for high excitations there is a sudden drop corresponding to the beginning of the shock absorber displacement.

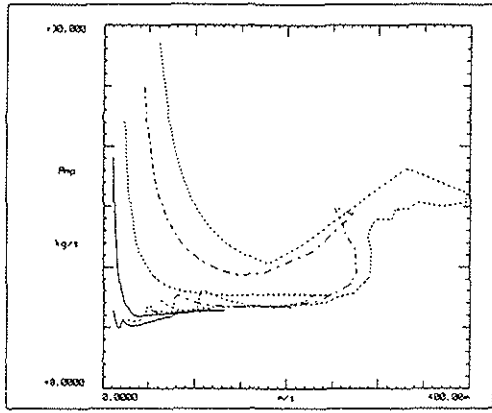


Figure 20 Equivalent damping

The evolution of the equivalent damping in figure 20 is characterized by a rapid decrease for which no physical meanings have been found. The small peak increases and decreases are typical of friction damping. The final damping increases correspond with the start of the motion of landing gear struts. As the FRF's are calculated using the fundamental component of the input and output signal, it follows that the goodness of the results obtained using this method depends on the harmonic contents.

The concept of the describing function (DF) is often used to characterize nonlinear systems. Omitting the theory which is available in [10], it is possible to demonstrate [9] that if the harmonic content of the response is low compared to the fundamental component, then the results of DF and complex stiffness method are similar.

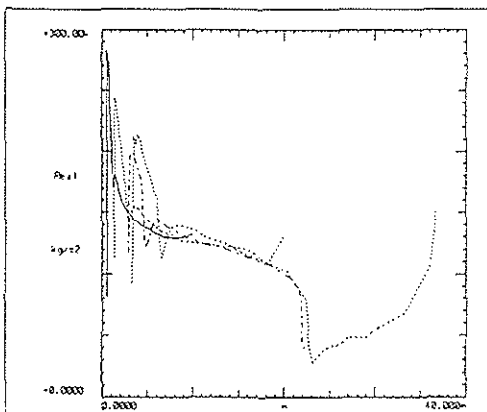


Figure 21 Original nonlinear stiffness

The original nonlinear stiffness and damping are obtained by inverting the describing function. In this case only the function obtained for increasing displacement (Figure 21) or velocity (Figure 22) are plotted.

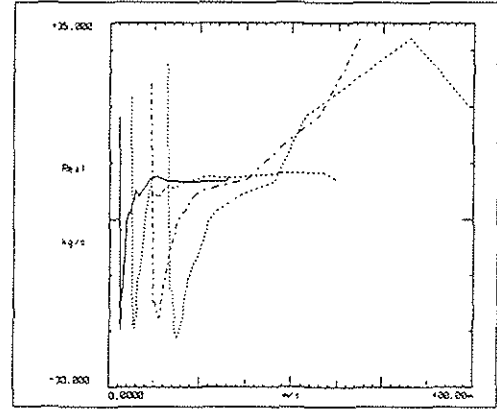


Figure 22 Original nonlinear damping

## 5.0 Conclusions

The intention of this paper was to show the experience and first evaluations about a ground vibration test and modal analysis of a structure like an helicopter. This complicated structure presents nonlinearities that can be derived from the structure but also can be introduced by the analyst using a non correct excitation signal, stinger, boundary conditions and other features that can show nonlinearities when the structure is linear and viceversa. The effects of the landing gear components on the helicopter response have been shortly analysed. The use of identification techniques of nonlinearity like the complex stiffness method on the data set have been done and analysed. The use of this method do not means that is the ideal method available but it allows the detection and identification of a nonlinear behaviour easily. The multiple input measurement technique can, for sure, help to detect some false nonlinearities. The nonlinear behaviour of this airframe needs to be further

investigated to understand deeper the effects of local nonlinearities due to the components on the whole nonlinear response.

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