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ROBUST MODEL-REFERENCE TRACKING CONTROL WITH A SLIDING MODE APPLIED TO AN ACT ROTORCRAFT

by

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Abstract

This paper presents preliminary results from an investigation into the application of a novel non-linear variable structure control law design to the robust stabilization and maintenance of performance objectives of a Helicopter over the flight envelope. The model-reference tracking controller is seen to enable good performance and to allow minimal cross-coupling to be attained, over the non-linear model's working air speed range. An ideal model specification using an H_∞ minimum entropy design, as opposed to eigenstructure assignment, is found to have increased robustness properties.

1 Introduction

The design of a rotorcraft flight control system, which will maintain system stability and performance over the aircraft's full flight envelope is receiving much attention from design engineers. Many control system design techniques have been applied, such as H_∞ robust optimization ([Yue and Postlethwaite,1990], [Walker and Postlethwaite,1991]), and eigenstructure assignment ([Manness and Murray-Smith,1992], [Samblancatt *et al*,1990]) The H_∞ frequency domain controller designs have been particularly successful ([Walker *et al*,1993]), and experience gained in the H_∞ methods has benefitted the current study. These methods apply linear techniques for controller design and then rely on controller switching and blending to achieve high performance, wide-envelope control. The design of a single controller which can satisfy performance objectives over the full flight envelope thus removing the need for scheduling is an open research question which is of particular interest.

The major difficulty in solving this problem arises from the dynamics of the helicopter which vary considerably as speed is increased. To overcome these speed-dependent dynamical nonlinearities, a nonlinear control law is designed in a model-reference framework ([Spurgeon and Davies,1993]). Here the controller acts on the error vector between the real plant and an ideal model. Two methods are applied for the design of these ideal dynamics: eigenstructure assignment and an H_∞ minimum entropy design. They are compared for their contribution to the overall controller performance.

A similar strategy has recently been applied ([Fossard,1993]). However, the proposed control configuration did not actively control aircraft height and was not shown to have been tested on a full non-linear model. Both these areas are considered, together with the application of an alternative nonlinear compensator.

Section 2 outlines the theoretical background to the nonlinear control strategy employed. The helicopter nonlinear model and description of the ideal performance requirements are discussed in Section 3. Nonlinear simulation results are presented and analysed in Section 4. Section 5 contains concluding remarks and an indication of the future direction of this project.

2 Control Law

Consider the following state space description of an uncertain plant:-

$$\dot{x}(t) = Ax(t) + Bu(t) + F(t, x, u) \tag{1}$$

where $x \in \Re^n$ and $u \in \Re^m$ represent the usual state and input, B is full rank, $n > m$ and (A, B) is a controllable pair. The unknown function F represents model uncertainties in the system. An associated linear model which has ideal response characteristics is defined by:

$$\dot{w}(t) = A_m w(t) + B_m r(t) \tag{2}$$

where $w \in \Re^n$, $r \in \Re^p$, are the state vector of the model and the reference input respectively. It is assumed that the ideal model is stable so that the poles of the system (2) have negative real parts. The associated control system design problem is thus that of determining a feedback strategy whereby the output variables of the plant, (1), faithfully follow those of the model. The following tracking error state is thus defined

$$e = x - w \tag{3}$$

Differentiating (3) with respect to time and substituting the plant and model dynamics from (1) and (2), the following model error dynamics are obtained.

$$\dot{e} = A_m e + (A - A_m)x + Bu - B_m r + F(t, x, u) \tag{4}$$

To satisfy the well-known model matching conditions for the nominal error system which will ensure asymptotic decay when $F(\cdot) \equiv 0$, the following structure is imposed upon the model.

$$A_m = A + BL_x \tag{5}$$

$$B_m = BL_r \tag{6}$$

The model is thus defined by a constant gain feedback matrix (L_x) for the nominal plant, and an input-output tracking precompensator gain matrix (L_r).

Note that if the control input, u, is defined by

$$u_1 = L_x x + L_r r \tag{7}$$

the nominal error dynamics are asymptotically stable. However, it has been noted that the helicopter is an extremely nonlinear, uncertain system and the problem of maintaining tracking performance in the presence of a broad class of uncertainty contributions $F(\cdot)$ is particularly pertinent. The design of an augmenting control effort to counteract the uncertainty $F(\cdot)$ is now considered. The methodology employed has its roots in the well known sliding mode approach to controller design, where the error state is constrained to lie on certain surfaces in the error state-space. This method possesses certain inherent robustness properties, and with appropriate switching surface selection, enables the designer to prescribe desired error transient behaviour. A set of switching surfaces are defined to be fixed hyperplanes in the error space passing through the origin

$$s = Ce \tag{8}$$

where $C \in \Re^{m \times n}$ is a constant design matrix which determines the ideal rate of decay of the error states.

A sliding mode is achieved when the error states are constrained to the intersection of the hyperplanes (8)

$$s = \{e : Ce = 0\} \tag{9}$$

The control required to achieve the desirable sliding mode condition, (9), was traditionally discontinuous in nature which was clearly undesirable for many applications. However, there are now well-established continuous nonlinear controllers which ensure (9) is satisfied in a completely robust fashion ([Ryan and Corless,1984], and [Spurgeon and Davies,1993]). Here the control effort (7) is augmented by

$$u_2 = L_e e + \rho(t, x, e, r)\frac{Ne}{\|M\|e + \delta} \tag{10}$$

so that

$$u = u_1 + u_2 \tag{11}$$

Here $L_e \in \Re^{m \times n}$ is an error-feedback to prescribe the rate of decay of the error states onto the switching surfaces. The matrices $N \in \Re^{m \times n}$ and $M \in \Re^{m \times n}$ are directly determined from the choice of switching surface C. The parameter $\delta > 0$ is a smoothing constant; for $\delta = 0$ an undesirable relay type control action would result. The nonlinear function $\rho(\cdot)$ is determined from worst case studies of possible uncertainty contributions $F(\cdot)$ in (4).

Although conceptually difficult at first, the control strategy employed is straightforward from the point of view of design. Selection of switching surfaces amounts to the solution of a full-state feedback sub-problem. Indeed, a prototype MATLAB toolbox is currently available which includes a number of routines to facilitate the above design and analysis.

Also the non-linear model and controller implementation was simulated in SIMULINK, which was found to be a flexible environment to build up the entire design.

3 Helicopter Model Description

The nonlinear model is representative of a single-main-rotor, Lynx-like, high performance military helicopter, and is known as the Rationalised Helicopter Model (RHM). This nonlinear model was developed at D.R.A. Bedford and contains eight rigid body states, three engine states, four simple actuator states, and three second order rotor dynamic states amongst other inherent model states. These rotor states include two flapping modes, and one coning mode. The overall model has been verified against flight test data, and tested in piloted simulations (Padfield,1981). The eight state rigid body linearizations involved in the controller design were in the state space form

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{12}$$

$$y = Cx(t) + Du(t) \tag{13}$$

The state vector x is tabulated as follows:

State	Description
Θ	Pitch Attitude
Φ	Roll Attitude
p	Roll Rate
q	Pitch Rate
r	Yaw Rate
u	Forward Velocity
v	Lateral Velocity
w	Vertiacl Velocity

and the outputs to be controlled are:

Controlled Output	Description	Pilot Input	Units
$\dot{H}$ (y1)	Heave Velocity	Coll.	Ft/sec.
Θ (y2)	Pitch Attitude	Long.	Rad.
Φ (y3)	Roll Attitude	Latt.	Rad.
$\dot{\Psi}$ (y4)	Heading Rate	Pedal	Rad/sec.

Two methods for presribing ideal helicopter performance will now be discussed. This is necessary in order to prescribe the desired model (2).

4 Eigenstructure Assignment Design

Through the selection of suitable eigenvalues for nominal performance objectives and eigenvectors for appropriate modal decoupling, an initial reference model was designed.

The chosen eigenstructure is similar to that employed by ([Manness and Murray-Smith,1992]), but incorporates knowledge of this particular helicopter's dynamics. The transmission zeros and their associated directions were included in the eigenstructure specification, to alleviate steady state errors.

Using a precompensator matrix ([O'Brien and Broussard,1978]) to match the control inputs with the controlled outputs the following nominal results were obtained.

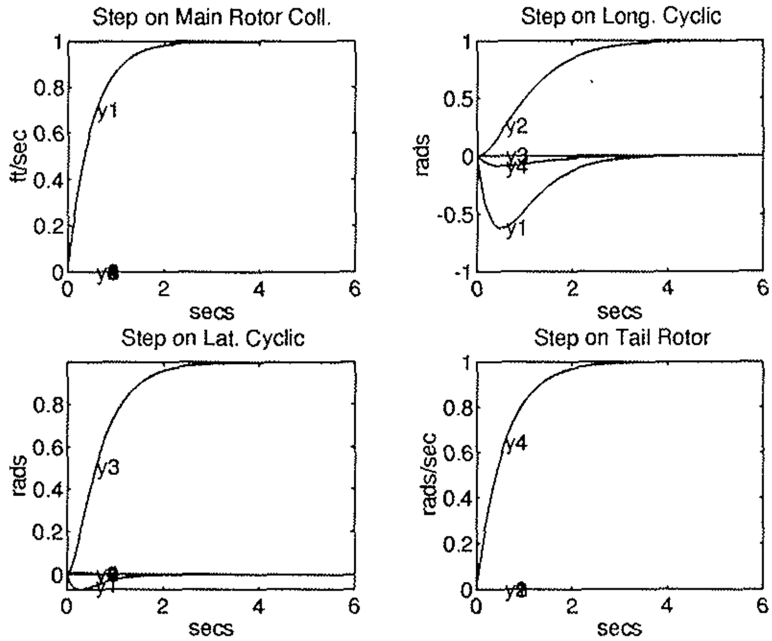

Figure 1: Nominal Hover Linear Eigenstructure Step Response Results

Figure (1) is unscaled, so for a very large pitch demand of 1 radian a coupling of only 0.6 ft/sec is seen in the height rate. After assembling the sliding mode controller, the results of testing the controller from zero knots to eighty knots were very promising. When tested on an 80 knots linearization the controller still maintained high performance objectives. However when this controller was tested on the full non-linear RHM model, there was a deterioration in the coupling when a pitch attitude input was applied. This was seen to be a robustness deficiency in the eigenstructure assignment technique applied. Therefore, a more robust method of specifying the ideal model dynamics was required.

5 H_∞ Minimum Entropy Design

A minimum entropy state feedback controller ([Boyd and Barratt,1991]) is derived by taking a Linear Quadatic Gaussian (LQG) controller, which has the H_∞ norm inequality specification:

$$||H||_\infty < \gamma \tag{14}$$

If this γ is such that the design specification above is feasible, then the two following algebraic Ricatti equations have unique positive definite solutions X_{me} and Y_{me} respectively.

$$A^t X_{me} + X_{me} A - X_{me}(BR^{-1}B^t - \gamma^{-2}W)X_{me} + Q = 0 \tag{15}$$

$$AY_{me} + Y_{me}A^t - Y_{me}(C^t V^{-1} C - \gamma^{-2}Q)Y_{me} + W = 0 \tag{16}$$

where Q, R,V, and W are design parameters. The state feedback controller then has the solution:

$$K_{sfb} = R^{-1}B^t X_{me}(I - \gamma^{-2}Y_{me}X_{me})^{-1} \tag{17}$$

In this case, the plant was augmented with an integrator state in each input channel to improve the decoupling and steady state performance. The nominal linear design results again showed fast response types with minimal cross coupling.

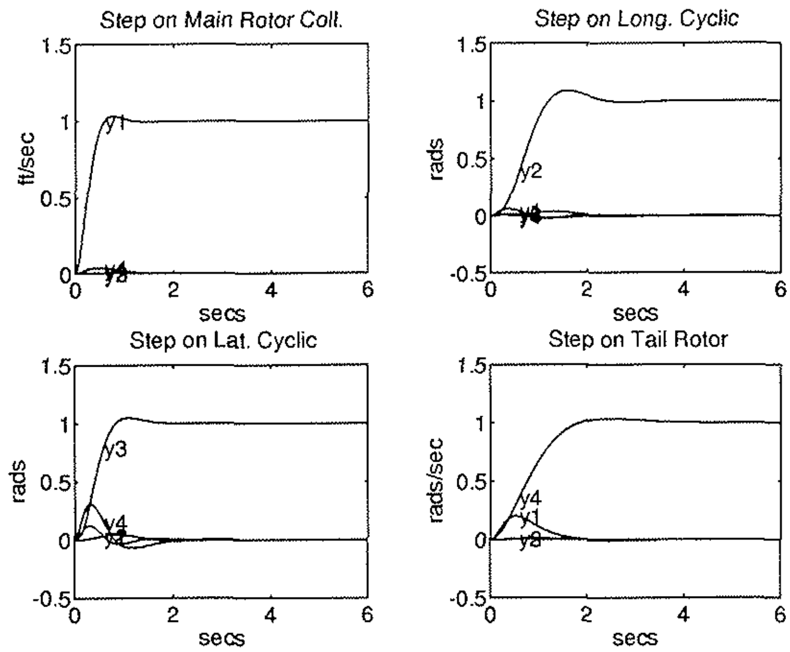

Figure 2: Nominal Hover Minimum Entropy Linear Step Response Results

The step responses of the H_∞ minimum entropy design shown in Figure 2 are seen to be very similar to those obtained for the eigenstructure assignment approach. The enhanced robustness of the minimum entropy design was already confirmed since the hover-designed controller alone stabilized a 20 knot linearization, while the eigenstructure controller alone failed to do so. Also the control action required to obtain these results was more than halved, which is not surprising as the minimization of control action is part of the H_∞ minimum entropy design formulation.

The next section will show the test results on the non-linear model.

6 Non-Linear Helicopter Model Simulation Results

The following Figures 3,5,6 & 7 show the full response of the four output channels to an input in each particular pilot control input. The low couplings and low coupling rates for large demands (5 ft/sec height rate, 10 degs pitch and roll rate) are evident.

All the following plots show that the actuator demands are well within their respective saturation limits, which future designs may be able to take advantage of. Also the rates of these actuator signals were low enough not to exceed any actuator rate limits.

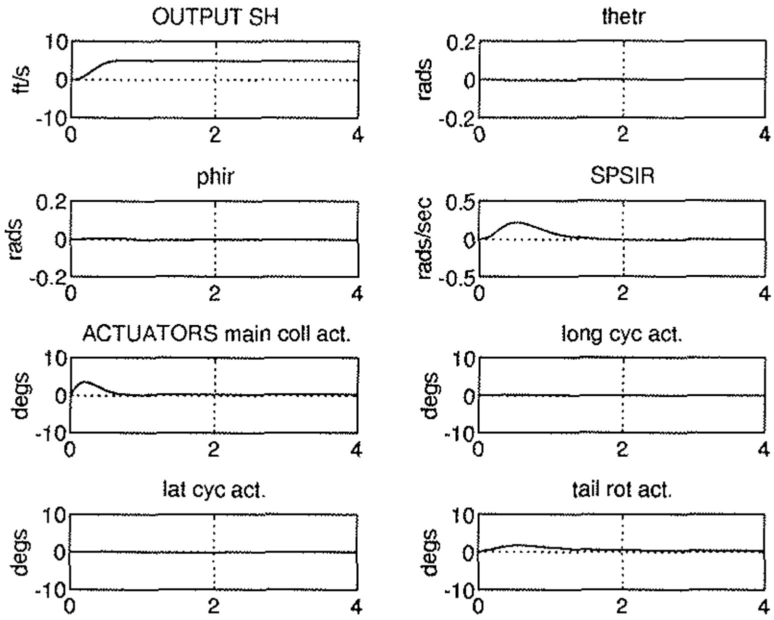

Figure 3: Non-linear Hover Response (Min. Ent.) to step demand on heave velocity

Below it is illustrated that the sliding mode condition (9) has been quickly achieved. Since the sliding mode is reached so rapidly, the properties of a system that is 'sliding' would be apparent if so excited. These include insensitivity to matched uncertainty, and a behaviour which is prescribed by the ideal model dynamics (2).

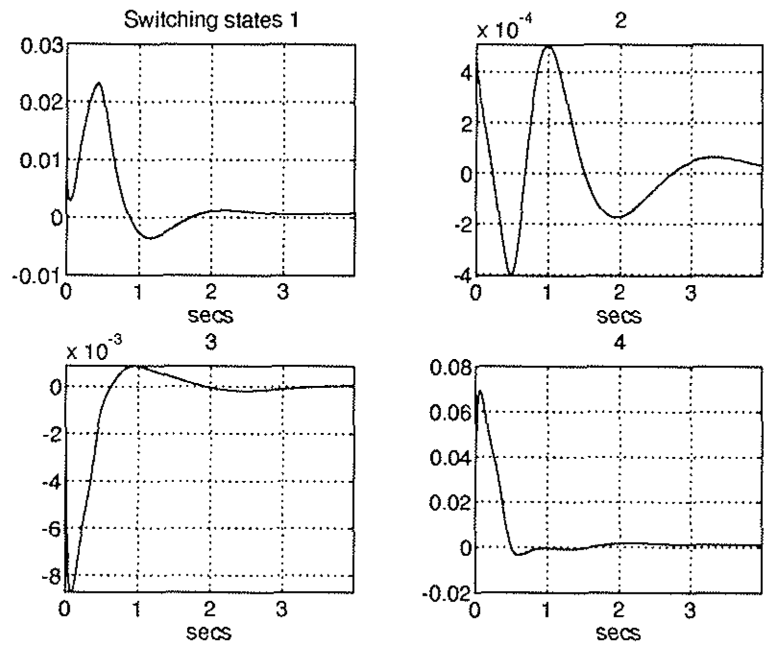

Figure 4: Non-linear Hover Switching States Response (Min. Ent.) to step demand on Heave Velocity

A step of -10 degs. on longitudinal cyclic (Figure 5) means the helicopter will pitch forwards, and after 4 seconds will be travelling at approximately 20 ft/sec, which will already include the different aerodynamical conditions of forward flight.

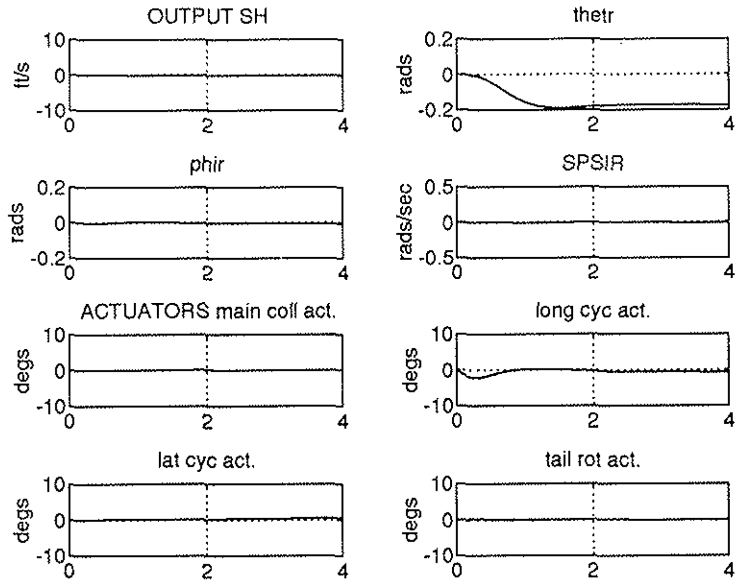

Figure 5: Non-linear Hover Response (Min. Ent.) to step demand on pitch attitude

The next two Figures 6 and 7 are included to show that the controller is able to give fast response types in both of the other channels.

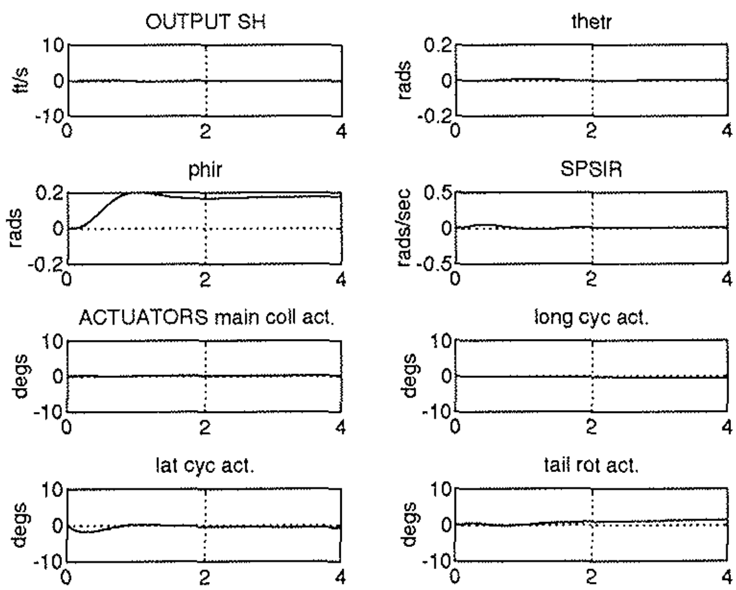

Figure 6: Non-linear Hover Response (Min. Ent.) to step demand on roll attitude

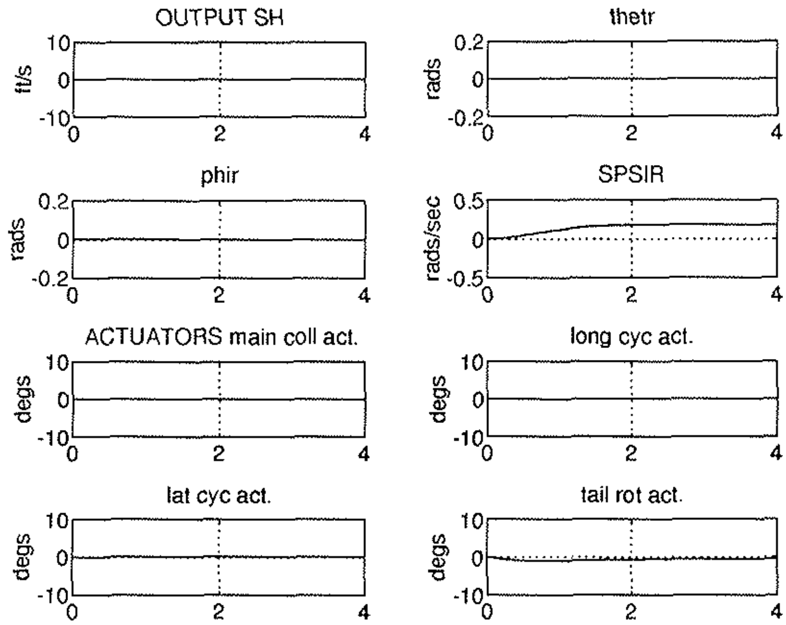

Figure 7: Non-linear Hover Response (Min. Ent.) to step demand on yaw rate

The following Figure 8 shows that when the design incorporated the eigenstructure feedback then the response deteriorated, and gave much justification for using a more robust method.

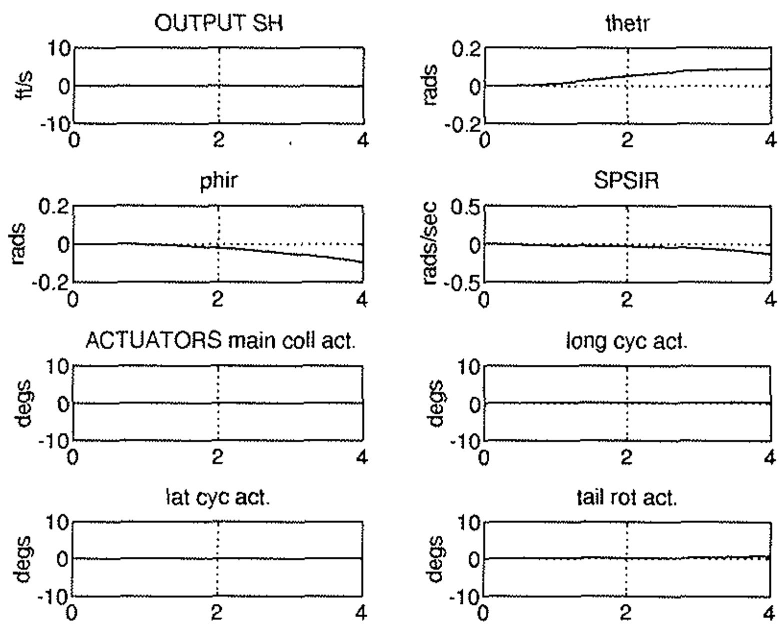

Figure 8: Non-linear Hover Response (Eig) to step demand on pitch attitude

An important test was to see if the controller could maintain a level of performance at 75 knots on the nonlinear model. For a step of 5 ft/sec on the height rate (Figure 9) the coupling was noticeably low, considering the size of height change desired when travelling at speed.

When comparing Figure 9 to the corresponding height rate demand at hover (Figure 3), the achievement of good performance in this channel is evident. Pitching nosedown 5

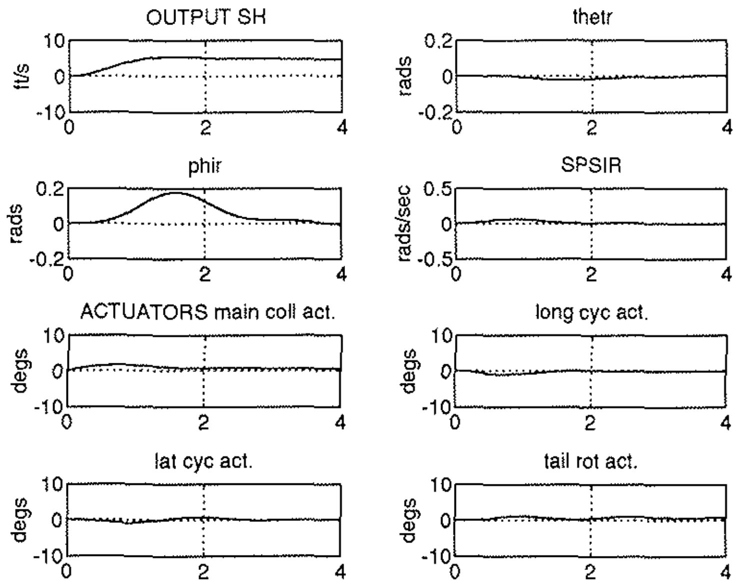

Figure 9: Non-linear 75 knot Response (Min. Ent.) to step demand on heave velocity

degs. increases the forward velocity by 6 knots, and Figure 10 below shows the smooth completion of this task as far as pitch is concerned. Unfortunately there is a slight tendency to drift in roll attitude as well, but this will hopefully be corrected in future designs. However, the ability of the nonlinear controller to keep a high level of performance away

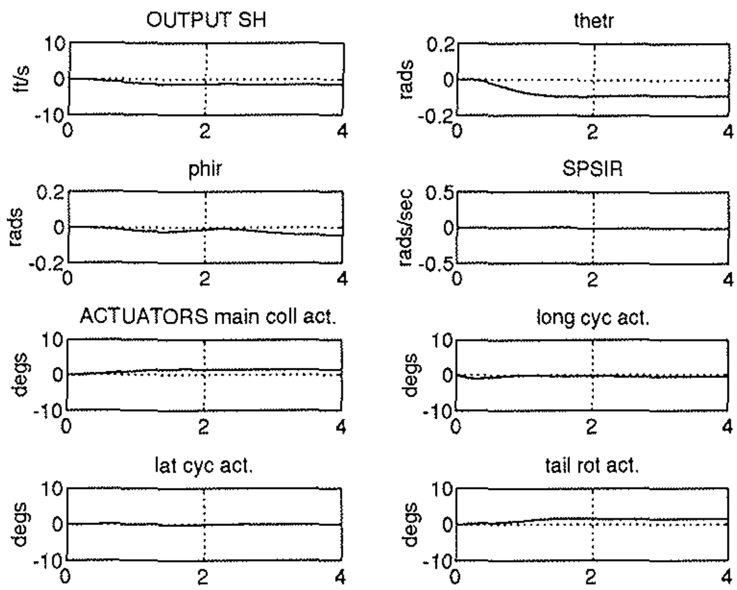

Figure 10: Non-linear 75 knot Response (Min. Ent.) to step demand on pitch attitude

from hover is apparent when comparing Figure 10 to Figure 5.

7 Conclusions

A novel nonlinear controller has been designed and tested on a complex nonlinear helicopter model. The non-linear simulation results show that a fast response type and decoupling in all the outputs was obtainable from a design that included the H_∞ minimum entropy for the specification of the ideal model. The increased robustness compared to a design involving eigenstructure assignment design was shown to be important in the overall controller configuration. The H_∞ minimum entropy design also gave lower required actuator action. With regard to the future directions of the project, a detailed consideration of the nonlinear controller's gust rejection properties will be undertaken. There is scope to improve the designs presented here, and further configurations will be looked at which incorporate the eigenstructure assignment method in a role that does not require such robustness. The non-linear simulation results are very promising, and the future tests will hopefully involve a piloted simulation.

8 Acknowledgements

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