EXPERIENCE WITH OPTIMAL INPUT DESIGN FOR HELICOPTER PARAMETER IDENTIFICATION

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Abstract

This paper investigates a technique for the optimal design of helicopter control inputs, in order to provide the most accurate parameter estimates. An existing software package for optimal input design for fixed wing aircraft was modified to design control inputs for rotary wing aircraft. The design issues are discussed with some inherent problems of helicopter dynamics in mind.

Helicopters differ from fixed wing aircraft by being typically unstable, and have more dynamic modes (translation, rotation, blade modes plus inflow dynamics). A high degree of coupling also exists between the blades, longitudinal and lateral forces/moments.

The design technique encompasses the salient features of Mehra's design technique in the frequency domain (using Convex Analysis) with the Two Step method for decoupling state and parameter estimation. The input design is performed for power constrained inputs by optimising a norm of Fisher's information matrix. The results are presented for the BO-105 helicopter, using data supplied by DLR, Germany.

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Notation

A	State matrix
B	Input matrix
C	Output (measurement) matrix
D	Input matrix
0	Zero matrix

δ_lon	Longitudinal Cyclic Input	} Measured
δ_lat	Lateral Cyclic Input	} in % of
δ_ped	Pedal Input	} total
δ_col	Collective Input	} control
	movement as measured at the pilot station	

det	Determinant of any matrix
eig	Eigenvalue of any matrix
e_i	Basis vector of Information Space $\Re_M$
I	Identity matrix
I_{xx}	Moment of Inertia about x (body) axis
I_{xy}	Cross moment of Inertia about x-y (body) axes
J	Cost Function of Fisher information matrix
ln	Natural logarithm
L ($\tilde{L}$)	Moment (Specific moment) about x (body) axis
M ($\tilde{M}$)	Moment (Specific moment) about y (body) axis; Fisher information matrix
N ($\tilde{N}$)	Moment (Specific moment) about z (body) axis; number of samples
m	Number of outputs, measured data.
n	Order of system model.
p ($\dot{p}$)	Angular roll velocity (acceleration) about x (body) axis
q ($\dot{q}$)	Angular pitch velocity (acceleration) about y (body) axis
r ($\dot{r}$)	Angular yaw velocity (acceleration) about z (body) axis; total number of parameters
$\Re_M$	Information space
S_{uu}	Power spectral density matrix
s	Number of inputs to system model
T	time length of input
tr	Trace of any matrix
$\underline{u}$	Input vector
u ($\dot{u}$)	Linear velocity (acceleration) along the longitudinal body (x) axis
v ($\dot{v}$)	Linear velocity (acceleration) along the lateral body (y) axis
V_{vv}	Noise covariance matrix
w ($\dot{w}$)	Linear velocity (acceleration) along the normal body (z) axis
$\underline{y}$	Output (measurement) vector
X ($\tilde{X}$)	Force (Specific force) in x direction, body axes
Y ($\tilde{Y}$)	Force (Specific force) in y direction, body axes
Z ($\tilde{Z}$)	Force (Specific force) in z direction, body axes

Y_u	Aerodynamic (stability) derivative, $\frac{\partial Y}{\partial u}$
X_{δ_lon}	Control derivative, $\frac{\partial X}{\partial d\_lon}$
Δu	Change in forward velocity
λ	Eigenvalue of any matrix
$\theta\ (\dot{\theta})$	Pitch angle (of rate of change); Parameter estimate
$\phi\ (\dot{\phi})$	Roll angle
ψ	Yaw angle; Vector within Information Space
σ, σ^2	Standard deviation, variance
ω	Frequency
ζ	Damping ratio (2nd order system)

Acronyms

AFDD	Aero-Flight Dynamics Directorate (US Army)
AGARD	Advisory Group for Aerospace Research and Development (NATO)
CRLB	Cramer-Rao Lower Bound
DLR	Deutsche Forschungsanstalt fur Luft- und Raumfahrt (German National Aerospace Laboratory)
NLR	National Aerospace Laboratory, The Netherlands
TUDelft	Delft University of Technology
WG-18	Working Group 18 (Part of AGARD)

Introduction

System and Parameter Identification

Aerodynamic model identification is the process of obtaining a mathematical description of the aerodynamic forces and moments acting on an aircraft from measurements in flight. Identification includes the selection of an appropriate model structure, and the estimation of the parameter values of the model, in this case the *stability and control derivatives*. The model can then be used in the optimisation of a flight control system, evaluation of flying qualities, or the validation of experimental results.

Model identification is often referred to as parameter identification, of which parameter estimation is a subset. Parameter estimation is the problem of estimating the numerical value of parameters, given the form or structure of the mathematical model. Various techniques are available, whereby recorded flight data is analysed against an estimation criterion (e.g. Least Squares) using an iterative algorithm in order to improve previous estimates.

This paper seeks to apply recent work done in the field of Optimal Input Design, where *optimal* means the input signal that will result in the best (most accurate) estimates. The work done at Delft University (Refs 1-3) has proved significant in this particular area, with application to fixed wing aircraft.

Parameter Identification for Rotorcraft

Parameter identification is inherently more difficult in the field of rotorcraft due to the large degree of coupling from the effects of main and tail rotors, and the high level of additional signal noise present.

Under the direction of AGARD, Working Group 18 was set up to investigate the various techniques for rotorcraft identification. The study included three helicopter types, one of which was taken as the model for this paper, the MBB (Eurocopter) BO-105 (Fig 1). Their work has also been published (Ref 4), and it is against these results that the optimal inputs are compared.

Input Design

Design of Optimal Input Signals

The role played by optimal input design as a part of parameter identification is highlighted in Fig 2, taken from Ref 5. The first step is to obtain *a priori* values of the parameters to be estimated, either from wind tunnel data or previous experimental results, in order to derive an appropriate order model. This model can then used as the basis for designing a flight test input, subject to experimental objectives and conditions.

The input is then evaluated (by simulation initially), by comparing results with previous values, in order to successively refine the input signal. Once an optimal signal has been designed and tested, a flight test can be conducted, the data from which should (ideally) yield better parameter estimates. Such identification can make use of one of several possible techniques, since the input design aims to produce a globally optimal signal. These new parameter values can be used to replace the *a priori* values, and hence improve the accuracy of the rotorcraft model.

There are a number of important criteria which must be considered in the design of optimal inputs, which must all be satisfied if such design is to be successful (Ref 5):

- The signal should be as short as is practical and yield the highest attainable accuracy of parameter estimates.
- Flight condition. Control inputs must not cause structural loads to exceed safe levels, and must ensure that the aircraft remains well within its flight envelope.
- Instrumentation. The inputs must be designed with regard to the measurement instruments on-board, in particular their accuracy and dynamic range.
- Pilot acceptability. A simple form of input may allow it to be performed manually. A complex type of multiple input will require a flight control system which can accept a composite signal.

The design of optimal input signals may be conducted in the time or frequency domain, and a good summary of input design techniques may be found in Ref 6.

Frequency Domain

By transforming the model equations to the frequency domain (using Fourier Transforms), the need to differentiate the state equation, as required for design in the time domain, is removed. The same process can be achieved by simple multiplication in the frequency domain, which thus reduces the computation time. A further advantage is the improvement in data analysis, since it is now easier to select a frequency range of interest. It is also easier to avoid a particular band of frequencies, around one of the high order modes, for example.

The disadvantages in using the frequency domain are the approximation error entailed in using Fourier Transforms, and the truncation error, due to the finite time available for measuring data (see Klein, Ref 7). The technique is only suitable for linear dynamic systems.

Much work has been done in this field by Mehra and Gupta (Refs 8, 9), and also Goodwin and Payne (Ref 10), and it is in the frequency domain that the current work is based. The design process used here is identical to that used by van der Linden, Sridhar et al (Refs 3, 5):

- Design in frequency domain.
- Application of the Two-Step Method for parameter estimation, to enhance Mehra's original design technique.
- Use of Convex Theory to minimise the number of elementary signals within the optimised signal.

Two Step Method

During his research in the field of Parameter Estimation, Gerlach (Ref 11) proposed that, due to the high degree of accuracy obtainable from digital measurement systems, the estimation problem could be separated into two distinct phases. These two de-coupled estimation problems are then solved consecutively, in what has since been called the Two-Step Method (Ref 1), or Estimation Before Modelling (EBM). The two steps are:

- **State Estimation** (flight path reconstruction). This assumes a high accuracy of flight test measurements from accelerometers and rate gyros. Data analysis using Kalman filter will then yield the exact aircraft states, which are then used in the second step.
- **Parameter Estimation** - The estimation problem is now linear-in-the-parameters, and a regression technique may be applied.

State vector $\underline{\mathbf{x}}$ is now considered as independent of the parameters θ (model is linear-in-the-parameters), and can therefore be used as inputs to the Estimation Model.

The two-step method is applied during the calculation of the Fisher information matrix, and enables the computational load to be reduced significantly.

Helicopter Model

Rotorcraft represent a very high order system, and an accurate model must of necessity become quite complex. The high degree of coupling between longitudinal and lateral motion, plus the high order rotor blade modes imply a high order of equivalent system. The rotor drive is governed to maintain a constant speed, and this adds further states.

In fixed wing aircraft, a 6 DoF model can be shown to give a very accurate model for deriving system parameters, involving only the rigid body states u, v, w, p, q and r. Separate 4th order longitudinal and lateral models also produce accurate results, and certain modes of motion can be isolated, e.g. Short period pitching motion (SPPO), as a 2nd order mode.

The helicopter is a dynamically more complex aircraft, requiring additional states and auxiliary dynamic equations. However, the model may be simplified by assuming the eigenvalues of these additional modes to be significantly higher than the rigid body values. By limiting the flight control inputs to relatively gradual excitations, to prevent excitation of the rotor modes, it is possible to separate rotor and body modes.

For the purpose of this study, an 8th order model has been used, as for the study of AGARD WG 18 (Ref 4). This consists of a fully coupled, 6 degrees-of-freedom rigid body system.

Inputs were also designed for a reduced order (4th order longitudinal) model, extracted from the 8th order model. This simplified the design process, and helped to build experience of using the software.

8th Order Model

The helicopter model developed is based on equations in Ref 4, with the following assumptions:

- $I_{xy} = 0$; $I_{yz} = 0$
- Angular rates are small, and are neglected in the moment equations.
- Gyroscopic reactions due to rotating elements of the helicopter are neglected.
- Small values of angular velocities (p, q, r).
- Small variations of Euler angles ϕ and θ.
- Small variations of the translational velocities (u, v, w).

The state and input matrices, and vectors are represented in the following form:

State vector

$x = [\Delta u \;\; \Delta v \;\; \Delta w \;\; \Delta p \;\; \Delta q \;\; \Delta r \;\; \Delta\theta \;\; \Delta\phi]^T$

Input vector

$u = [\Delta\delta lon \;\; \Delta\delta lat \;\; \Delta\delta ped \;\; \Delta\delta col]^T$

$$A = \begin{bmatrix} X_u & X_v & X_w & X_p & X_q - w_0 & X_r + v_0 & -g\cos\theta_0 & 0 \\ Y_u & Y_v & Y_w & Y_p + w_0 & Y_q & Y_r - u_0 & 0 & g\cos\theta_0 \\ Z_u & Z_v & Z_w & Z_p - v_0 & Z_q + u_0 & Z_r & -g\sin\theta_0 & 0 \\ L_u & L_v & L_w & L_p & L_q & L_r & 0 & 0 \\ M_u & M_v & M_w & M_p & M_q & M_r & 0 & 0 \\ N_u & N_v & N_w & N_p & N_q & N_r & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} X_{\delta\_lon} & X_{\delta\_lat} & X_{\delta\_ped} & X_{\delta\_col} \\ Y_{\delta\_lon} & Y_{\delta\_lat} & Y_{\delta\_ped} & Y_{\delta\_col} \\ Z_{\delta\_lon} & Z_{\delta\_lat} & Z_{\delta\_ped} & Z_{\delta\_col} \\ L_{\delta\_lon} & L_{\delta\_lat} & L_{\delta\_ped} & L_{\delta\_col} \\ M_{\delta\_lon} & M_{\delta\_lat} & M_{\delta\_ped} & M_{\delta\_col} \\ N_{\delta\_lon} & N_{\delta\_lat} & N_{\delta\_ped} & N_{\delta\_col} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Output Equation - Two Step Method

The linear form of the specific aerodynamic forces and moments acting on the helicopter may be written:

$$\begin{bmatrix} \tilde{X} \\ \tilde{Y} \\ \tilde{Z} \end{bmatrix} = \begin{bmatrix} \tilde{X}_0 \\ \tilde{Y}_0 \\ \tilde{Z}_0 \end{bmatrix} + \begin{bmatrix} \Delta\tilde{X} \\ \Delta\tilde{Y} \\ \Delta\tilde{Z} \end{bmatrix} \qquad (1)$$

where X_0 represents steady state force, and ΔX the transient force.

For the purpose of input design, the standard output equation ($\underline{y} = C\underline{x} + D\underline{u}$) is modified, and the output C and D matrices are filled with the aerodynamic stability and control parameters.

The aerodynamic forces are the only external forces in Equation 1, so it is their effect that will be measured by the accelerometers. Therefore:

$$\begin{bmatrix} \tilde{X} \\ \tilde{Y} \\ \tilde{Z} \end{bmatrix} = \begin{bmatrix} ax \\ ay \\ az \end{bmatrix} \qquad \text{and}$$

$$\begin{bmatrix} \tilde{X} \\ \tilde{Y} \\ \tilde{Z} \end{bmatrix} = \begin{bmatrix} \tilde{X}_0 \\ \tilde{Y}_0 \\ \tilde{Z}_0 \end{bmatrix} + \begin{bmatrix} \Delta\tilde{X} \\ \Delta\tilde{Y} \\ \Delta\tilde{Z} \end{bmatrix} = \begin{bmatrix} ax_0 \\ ay_0 \\ az_0 \end{bmatrix} + \begin{bmatrix} \Delta ax \\ \Delta ay \\ \Delta az \end{bmatrix}$$

Hence

$$\begin{bmatrix} \Delta ax \\ \Delta ay \\ \Delta az \end{bmatrix} = \begin{bmatrix} \Delta\tilde{X} \\ \Delta\tilde{Y} \\ \Delta\tilde{Z} \end{bmatrix} \qquad (2.1)$$

Similarly, the angular accelerations may be expressed as:

$$\begin{bmatrix} \Delta\dot{p} \\ \Delta\dot{q} \\ \Delta\dot{r} \end{bmatrix} = \begin{bmatrix} \Delta\tilde{L} \\ \Delta\tilde{M} \\ \Delta\tilde{N} \end{bmatrix} \qquad (2.2)$$

The observation vector now takes the form

$\underline{y} = [\Delta ax \;\; \Delta ay \;\; \Delta az \;\; \Delta\dot{p} \;\; \Delta\dot{q} \;\; \Delta\dot{r}]^T$

From Equations 2.1 and 2.2, the output equation becomes:

$$\begin{bmatrix} \Delta ax \\ \Delta ay \\ \Delta az \\ \Delta\dot{p} \\ \Delta\dot{q} \\ \Delta\dot{r} \end{bmatrix} = \begin{bmatrix} X_u & X_v & X_w & X_p & X_q & X_r & 0 & 0 \\ Y_u & Y_v & Y_w & Y_p & Y_q & Y_r & 0 & 0 \\ Z_u & Z_v & Z_w & Z_p & Z_q & Z_r & 0 & 0 \\ L_u & L_v & L_w & L_p & L_q & L_r & 0 & 0 \\ M_u & M_v & M_w & M_p & M_q & M_r & 0 & 0 \\ N_u & N_v & N_w & N_p & N_q & N_r & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta v \\ \Delta w \\ \Delta p \\ \Delta q \\ \Delta r \\ \Delta\theta \\ \Delta\phi \end{bmatrix}$$

$$+ \begin{bmatrix} X_{\delta\_lon} & X_{\delta\_lat} & X_{\delta\_ped} & X_{\delta\_col} \\ Y_{\delta\_lon} & Y_{\delta\_lat} & Y_{\delta\_ped} & Y_{\delta\_col} \\ Z_{\delta\_lon} & Z_{\delta\_lat} & Z_{\delta\_ped} & Z_{\delta\_col} \\ L_{\delta\_lon} & L_{\delta\_lat} & L_{\delta\_ped} & L_{\delta\_col} \\ M_{\delta\_lon} & M_{\delta\_lat} & M_{\delta\_ped} & M_{\delta\_col} \\ N_{\delta\_lon} & N_{\delta\_lat} & N_{\delta\_ped} & N_{\delta\_col} \end{bmatrix} \begin{bmatrix} \Delta\delta lon \\ \Delta\delta lat \\ \Delta\delta ped \\ \Delta\delta col \end{bmatrix}$$

Eigenvalues of model

The eigen values of the coupled 8th order model are given in Table 1, and compared with the results of the AFDD. As can be seen, the values are almost exactly the same, the small discrepancies arising because the AFDD model includes four time delays (one representing each control). These time delays were included in order to represent the time lag present in the helicopter actuators.

Table 1: Eigen frequencies of BO-105 Model

Mode	Eigen values $[\zeta, w_n]$ or $(1/T)$	AFDD values
Phugoid	[-0.37, 0.32]	[-0.36, 0.30]
Dutch Roll	[+0.23, 2.66]	[+0.22, 2.60]
Roll	(8.2)	(8.32)
Pitch mode 1	(6.17)	(6.04)
Pitch mode 2	(0.43)	(0.49)
Spiral	(0.025)	(0.03)

ζ represents the damping ratio
w_n represents the undamped natural frequency
$1/T$ represents $(s + 1/T)$ where T is the time constant in sec

BO-105 Research Work

The helicopter upon which these studies are based is the Eurocopter BO-105, a multi-purpose, twin engine design with four fibre-reinforced composite rotorblades.

For the purpose of system identification, DLR conducted a series of flight tests with a suitably equipped test helicopter, and provided data to a number of research institutions (AGARD Working Group 18). From the work undertaken by this group Ref 4 was published, which gives a description of the techniques used for identification, and the results obtained.

Details of the instrumentation used for data collection is given in Ref 4. Further information regarding the test and identification procedures employed are also given by Kaletka et al (Ref 12).

For the AGARD tests, three types of input were used, applied to each of the four controls:

- **Doublet** : A "bang-bang" type signal, duration 2 seconds.
- **Modified "3211"** : A multi-step signal, with duration 7 seconds (3-2-1-1).
- **Frequency Sweep** : A sinus type input of increasing frequency (from about 0.08 Hz up to 5-8 Hz).

Results from the AFDD were obtained from identification performed in the frequency domain. Since the optimal input design process also takes place in this domain, the AFDD results were in fact used as *a priori* values in the models used here.

The flight tests for the BO-105 were carried out in trim configuration in steady horizontal flight at 80 knots, and at a density altitude of approximately 3000ft. Helicopter weight was between 2250 kg and 2100 kg (total), and the tests took place in calm air flight conditions.

The measurement noise is assumed to be white, additive with Gaussian distribution, with values for the standard deviations supplied by NLR. The data recordings of p, q, and r, supplied by DLR, were used to calculate standard deviations for $\dot{p}$, $\dot{q}$ and $\dot{r}$, although these recordings were themselves differentiated from p, q and r measurements.

For helicopter instrumentation, the assumption of zero measurement noise bias may be hard to justify however, due to the high frequency structural modes present within rotorcraft.

Optimal Input Design in Frequency Domain

This section contains an overview of the theory involved in the optimal input design process. The technique is founded on Mehra's technique for design in the frequency domain, subsequently developed and applied by van der Linden (Ref 2). For a more in-depth presentation of the theory behind the design, the reader is directed to Ref 2, 8-10.

Input Design Criteria (Two Step Method)

The optimal input design is based on an 8th order state space model, represented by the linear equations:

$$\dot{x}(t) = A(\theta)x(t) + B(\theta)u(t)$$
$$x(0) = x_0 \qquad (3)$$

$$y(t) = C(\theta)x(t) + D(\theta)u(t) \qquad (4)$$

$$z(i) = y(i) + v(i) \qquad i = 0,1,\ldots,N-1$$

where x(t) is the $(n\times1)$ state vector, u(t) is the $(s\times1)$ input vector and y(t) is the $(m\times1)$ output signal. θ represents a $(r\times1)$ vector in which all aerodynamic parameters are located. The measured output signal z is obtained at N discrete instants of time and includes a noise signal v, caused by model and measurement errors.

The measurements are assumed to be corrupted by white noise, which is additive, mutually independent and normally distributed. The noise is represented by the $(m\times1)$ vector v(i), with Gaussian distribution:

$$E\{v(i)\} = 0$$

$$E\{v(i)v^T(j)\} = V_{vv}\delta_{ij} = \mathrm{diag}(\sigma^2,\ldots,\sigma^2{}_m)\delta_{ij}$$

Having specified the type of system, the design process begins, which aims to *calculate the signal, or composition of signals, which will produce the best parameter estimates of the linear state space system.* The technique to be summarised here makes use of the Two Step Method, as described previously.

First of all, the amplitude of the signal must be limited due to practical considerations:

- The (linearised) helicopter model will only be valid for small perturbations about a certain trim condition
- The rotorcraft structure must not be overstressed.
- Although a larger amplitude should give better parameter estimates (higher signal/noise ratio), this would not give a fair indication of the optimality of the input signal.

A boundary must therefore be set on either the amplitude or the power of the input signal. As for the fixed wing aircraft case, a power constraint was chosen, since this also indirectly limits the input signal amplitude. The total power of an s-dimensional input signal is given by:

$$P_u = \sum_{i=1}^{s} \frac{1}{T}\int_0^T u_i^2(t)dt = \frac{1}{T}\int_0^T \mathrm{tr}\{u(t)u^T(t)\}dt$$

An additional constraint on the input design relates to the frequency content and shape of the input signals. The inputs must not excite system resonant frequencies, since this would produce large system responses and non-linearities. The input spectrum must also be chosen to exclude high frequencies to prevent aliasing in the data analysis stage.

The different classes of signals which can be considered are "bang-bang" type signals, with full or zero amplitude only, or signals composed of elementary sinus, square or pulse functions. For this design process, only signals composed of sinus functions will be considered, although this can extended for square-type signals by representing the latter as the sum of sinus functions.

The optimality of the signal will be evaluated with respect to the accuracies of the parameters to be estimated. So that the quality of the input signals alone can be evaluated, the influence of the estimator has to be eliminated. This is achieved by assuming an efficient, unbiased estimator, which results in the absolute minimum parameter covariance matrix obtainable from a type of input. This is the so-called Cramer-Rao Lower Bound (CRLB), and is equal to the inverse of Fisher's Information matrix, M.

The matrix M may be calculated using the sensitivities of the outputs with respect to all the unknowns, i.e. the aerodynamic parameters θ, the initial conditions x_0 and the elements of the output noise covariance matrix V_w. Since the states are estimated from different output components than the parameters, the cross products of sensitivities with respect to x_0 and θ become zero.

Also, the sensitivities with respect to x_0 and V_w are not functions of u(t). This means that M will have a blockdiagonal structure, with blocks corresponding to θ, x_0 and V_w:

$$M = \begin{bmatrix} M_{\theta\theta} & M_{\theta x_0} & 0 \\ M_{x_0\theta} & M_{x_0 x_0} & 0 \\ 0 & 0 & M_{\eta\eta} \end{bmatrix}$$

In order to perform the optimisation itself, a scalar norm of M must be evaluated as the so-called Cost Function, J. When using a norm of M in this way, the matrix blocks containing x_0 and V_w lead only to additional constants. It is therefore sufficient to consider purely the sensitivities with respect to θ (Mulder, Ref 1):

$$M_{\theta\theta} = \sum_{i=0}^{N-1} \frac{\partial y^T(i)}{\partial \theta} V^{-1}{}_{vv} \frac{\partial y(i)}{\partial \theta^T} \qquad (5)$$

For any input signal leading to a nonsingular matrix M, the value of M will tend to a zero matrix as N, the number of samples, tends to infinity. It is therefore best to calculate the average information matrix *per sample* as:

$$\overline{M} = \frac{1}{N} M_{\theta\theta}$$

Cost Functions

The optimality of the input signal was evaluated using the following scalar norm, applied to the Fisher information matrix.

$$\mathbf{J = tr\ [(M/N)^{-1}].}$$

The minimisation of the trace of M^{-1} in effect minimises the (unweighted) sum of the variances of the parameter estimation errors - these make up the diagonal elements of cov(θ), the Cramer-Rao Lower Bound. It is therefore also possible to give priority to certain important parameters by multiplying M^{-1} by a certain weighting matrix.

Fisher's Information Matrix (Frequency Domain)

The next step is to differentiate Equations 3 and 4 with respect to θ_k, which gives:

$$\frac{\partial \dot{x}(t)}{\partial \theta_k} = \frac{\partial A}{\partial \theta_k} x(t) + A \frac{\partial x(t)}{\partial \theta_k} + \frac{\partial B}{\partial \theta_k} u(t)$$

$$\frac{\partial x(0)}{\partial \theta_k} = 0$$

$$\frac{\partial y(t)}{\partial \theta_k} = \frac{\partial C}{\partial \theta_k} x(t) + C \frac{\partial x(t)}{\partial \theta_k} + \frac{\partial D}{\partial \theta_k} u(t) \qquad (6)$$

Applying the "Two-Step" method, the state vector $\underline{x}$ is regarded as independent of the parameters. Hence, Equation 6 becomes:

$$\frac{\partial y(t)}{\partial \theta_k} = \frac{\partial C}{\partial \theta_k} x(t) + \frac{\partial D}{\partial \theta_k} u(t) \qquad (7)$$

In order to perform the optimisation in the frequency domain, Equation 7 is expressed using Fourier transforms as:

$$\frac{\partial Y(\omega)}{\partial \theta_k} = \left[\frac{\partial C}{\partial \theta_k} H(\omega) + \frac{\partial D}{\partial \theta_k} \right] U(\omega) \qquad (8)$$

H(w) is the frequency response matrix, where

$X(w) = H(w)\, U(w)$ and

$H(\omega) = [sI - A]^{-1} B\big|_{s=j\omega}$

In the frequency domain, the input signal can be represented by the power spectral density matrix $S_{uu}(w)$. This may be written as

$$S_{uu}(w) = \frac{1}{T} U(-\omega) U^T(\omega)$$

Now, the expression for $\underline{y}$ derived in Equation 8 is substituted in Equation 5. Defining $W_{kl}(w)$ as follows:

$$W_{kl}(\omega) = \left[\frac{\partial C}{\partial \theta_k} H(\omega) + \frac{\partial D}{\partial \theta_k}\right]^T V_{vv}^{-1}\left[\frac{\partial C}{\partial \theta_l} H(-\omega) + \frac{\partial D}{\partial \theta_l}\right]$$

Then the element of $\overline{M}$ on the kth row and lth column can be written as:

$$\left[\overline{M}\right]_{kl} = \frac{1}{2\pi}\int_{-\infty}^{\infty} tr\{W_{kl}(\omega)S_{uu}(\omega)\}d\omega \qquad (9)$$

k=1(1)r }where r is the number
l=1(1)r }of parameters

$\overline{M}$ is now a function of power spectral density matrix of input signal

Optimisation using Convex Theory

In order to derive the optimal input signal, a search is made for the information matrix leading to the lowest value of the cost function J (= tr[M/N]$^{-1}$). First the matrix is transformed to "Information Space" $\Re_M$ (with basis vectors e_i), where the optimisation is performed. The average information matrix is then represented by an information vector y, with components y_i:

$$\overline{M} \mapsto \psi := \sum_{i=1}^{d} \psi_i(u).e_i \qquad \text{where d is the dimension of}$$

the space $\Re_M$.

For a power constrained input signal, a set of all possible information matrices is designated M, which is a convex set. This means that an optimal input signal can be built-up from elementary signals $u^{(i)}(t)$, represented by a convex combination of information matrices. Any element M of the set M can be represented by:

$$\overline{M} = \sum_k \alpha^{(k)} \overline{M}^{(k)} \qquad , \overline{M}^{(k)} = \overline{M}(u^{(k)})$$

$$1 = \sum_k \alpha^{(k)} \qquad , a^{(k)} > 0$$

A search is then made to find the elementary signals which will yield the smallest cost function, J, and the optimal signal takes the following form:

$$\boxed{u(t) = \sum_k \sqrt{\alpha^{(k)}}\, u^{(k)}(t)}$$

The convex hull used may be visualised in the case of a 2nd order, single input system (see Fig 3). This figure shows, in two and three dimensions, the attainable cost functions for a 2nd order model taken from BO-105 data. The surrounding hull represents the information matrices of all sinus frequencies from 0 to 10 rad/sec, while the contours give lines of constant cost function. The optimal cost function is represented by the greatest depth of the three-dimensional hull.

Results and Discussion

Using the linear models as previously described, the Optimal Input Design program was used to design and evaluate test inputs for the BO-105. The signals so designed are based upon *a priori* values taken from the results of WG-18 (AFDD).

First of all, two of the input constraints, the length of the input signal, and the total power content, had to be decided. In order to perform an acceptable comparison with the results obtained by the AGARD working group, both similar power and signal duration were used. However, the input signals used by AGARD WG-18 for identification (the 3211 and frequency sweep) differ greatly in terms of input length and power.

Also, the optimal design process distributes power across all the inputs, hence a four input signal is designed for the 8th order model. This is in contrast to the WG-18 tests, where only one primary control was used, the others being held close to zero to prevent the helicopter diverging from the flight envelope.

Design Procedure

The optimal input signals are designed based on the assumption that the Two-Step Method is used for the parameter estimation, i.e. the parameters to be estimated occur only in the output equation. This also implies that the measured output data includes the linear accelerations (specific forces) and angular accelerations.

The input design length was chosen as 15 seconds, long enough to observe the Phugoid mode, but short enough to prevent any large divergence from normal flight conditions.

The input power was increased up to a selected value of Pu = 7 %^{2}, the same power as that of the longitudinal cyclic frequency sweep.

Both square and sinus signals were designed using the Optimal Input Design program. The cost function tr(M/N)$^{-1}$ of the Fisher information matrix was chosen as the optimising criterion, as this represents the sum of the parameter variances, $\sum \sigma^2$.

Frequencies are all given in rad/sec, phase in radians and block lengths in sec. Block length is defined as the time for a full cycle of the square wave form.

Evaluation Procedure

The signals designed for each model are then used as the inputs to the same model, but with the original C and D matrices, **C = I**, **D = 0**.

The Fisher information matrix is then calculated as a function of time via the system responses, with the output noise added, as for the design. This will then give the lowest attainable parameter variances at a given sampling rate.

The signal can then be assessed by comparing the expected standard deviations of parameters, and the simulated responses, to check that they are not excessive.

Signals optimised for the 4th order models (i.e. two-input signals) were also evaluated in the full 8th order model. In this case, the other two inputs for the full order model are set at zero. The reasons for this are:

- A better comparison can be made with the AGARD signals, which have only one primary control input during each test.
- The validity of a 4th order (longitudinal or lateral) model of a helicopter, *for the purpose of input design*, can be tested.
- A two input signal will be easier to implement.

The parameter variances predicted by the software package are then compared with values calculated by the members of WG18. Since the *a priori* values for the BO-105 model were taken from the AFDD, most of the comparisons are made against these variances.

Longitudinal 4th Order Model (de-coupled)

For the purpose of optimal input design, a longitudinal 4th order model of the BO-105 was considered, since this simplifies the computational task. The two inputs are longitudinal cyclic (δ_lon) and collective (δ_col), with states Δu, Δw, Δq and $\Delta\theta$. Several two-input signals were designed and then applied to the full 8th order model.

Use of Results from 4th Order Model

Signals optimised in the 4th order models were applied to the full 8th order case, to see if this would allow better estimation results and easier implementation of the two-input signal. The results are presented for a fixed-frequency square-type signal (δ_lon/δ_col), optimised in the 4th order longitudinal model (Table 2, Fig 4a/b).

The sampling interval has been reduced to 0.05 sec (20 Hz), *due to memory limitations when calculating the Fisher information matrix. This will have some effect in degrading the accuracy of parameter estimates, and should be taken into consideration when comparing results.* Most the real flight data was sampled at 50 or 100 Hz (300 Hz for linear accelerations).

Signal Optimised for 4th order (longitudinal) model, applied to 8th order model: Signal **8th310** (Fig 4a)

The four input Fixed Square Signal, with length 15 sec, sampling interval dt = 0.05 sec, was evaluated. The Cost Function, J is quoted for all the longitudinal stability and control derivatives.

Table 2 : Fourth Order Signal applied to 8th Order Model (Pu = 7 $\%^2$): 8th310

Fixed block length (sec)	Power $(\%)^2$	Ampl δ_lon (%)	Phase δ_lon (rad)	Ampl δ_col (%)	Phase δ_col (rad)
0.625	3.417	1.34	0	1.27	-0.25
2.5	3.058	1.60	0	0.69	-0.89
5	0.131	0.25	-1.26	0.25	0
7.5	0.394	0.48	0	0.4	-0.62

Cost Function J = 788.7 and J/N = 2.62* = $\sum \sigma^2$

* Sampling rate 20 Hz (dt = 0.05 sec)

The results were also compared with those gained by evaluating the modified 3211 signal as used in AGARD test numbers 09 and 14.

The AFDD results were used to calculate the cost function of 1.569 (the sum of parameter variances). The parameter estimate results from the optimised signal are given in Fig 6, and are compared with DLR results, since the latter used 3211 input signals for identification.

The power content of both signals has been increased to Pu = 7 $\%^2$, which is the lowest power of any comparable AGARD signal. Although the cost functions are not as good as the AFDD results, Fig 6 shows that the new signals perform well against the 3211 inputs. The chart shows the difficulty in estimating helicopter "cross-derivatives" such as X_p and Z_p, which were in fact excluded from the model used by DLR.

The new signals also have the advantage that both δ_lon and δ_col control derivatives can be identified simultaneously, unlike the single test input.

The responses (Fig 4b) appear to be within limits for most of the 15 second input, and show the initiation of the phugoid mode. However, the simulated responses for pitch (θ) and roll (ϕ) angles departs from normal flight after 10 seconds, due to the lack of compensating lateral input.

Design for 8th Order Model: Signal **8th202** (Fig 5a)

A total of 47 stability and control derivatives were considered for the optimisation of the multi-axis input signals. This in itself leads to some difficulties with the amount of computer memory needed and the speed of computation, due to the large size of matrices. The cost function for the AFDD results is 2.668, compared with 2.6 for the optimised signal (Table 3).

The four inputs are δ_lon, δ_lat, δ_col and δ_ped. Input signal length was 15 sec, with sampling interval dt = 0.05 sec (20 Hz). The Cost Function, J is quoted for all 8th order parameters, and direct comparison can be made using Fig 7.

Table 3 : Coupled 8th Order Model - Optimised Sinus Signals Pu = 7 %^{2}: 8th202

Freq (rad/sec)	Power $(\%)^2$	Ampl δ_lon (%)	Ampl δ_lat (%)	Ampl δ_ped (%)	Ampl δ_col (%)
0.42	0.480	0.48	0.48	0.48	0.48
0.84	0.480	0.48	0.48	0.48	0.48
1.68	0.480	0.48	0.48	0.48	0.48
2.09	0.480	0.48	0.48	0.48	0.48
2.7426	0.785	0.23	0.70	0.98	0.21
3.0188	0.646	0.37	0.80	0.61	0.37
3.0418	0.003	0.03	0.05	0.05	0.01
4.19	0.480	0.48	0.48	0.48	0.48
5.03	0.480	0.48	0.48	0.48	0.48
7.54	0.480	0.48	0.48	0.48	0.48
9.62	1.728	1.11	1.29	0.48	0.54
9.63	0.480	0.48	0.48	0.48	0.48

Freq (rad/sec)	Power $(\%)^2$	Phase δ_lon (rad)	Phase δ_lat (rad)	Phase δ_ped (rad)	Phase δ_col (rad)
0.42	0.480	0	0	0	0
0.84	0.480	1	0	1	0
1.68	0.480	0	0	0	1
2.09	0.480	1	1	0	0
2.7426	0.785	0	-2.32	1.509	0.449
3.0188	0.646	0	-2.87	1.738	0.39
3.0418	0.003	0	-2.88	1.661	0.176
4.19	0.480	0	0	0	0
5.03	0.480	0	0	0	0
7.54	0.480	0	0	0	0
9.62	1.728	0	-2.85	2.595	0.092
9.63	0.480	0	0	0	0

Cost Function J = 781.6 and J/N = 2.6* = $\sum \sigma^2$

* Sampling rate 20 Hz (dt = 0.05 sec)

Fig 7 *shows that optimised signals perform well across the board against the AFDD results*, with one or two exceptions. This is remarkable considering the relatively short duration of the signals (15 seconds) and the low sampling rate, the latter due to lack of computer memory during evaluation. The figure again highlights the difficulty in estimating the "cross-derivatives" e.g. X_p and Y_w, but also shows a problem in estimating Y_r and N_p. One possible reason for this may be that the dutch roll mode is not excited during the test input, and hence derivatives relating to this mode will be hard to identify.

The new signals gave average results for the control parameters, but Y_{δ_ped} and Y_{δ_col} results can be seen to be unsatisfactory. This may be due the relatively low frequency content of the new signals (0 - 10 rad/sec), since the AFDD input signal included higher frequencies (up to 5-8 Hz, 31-50 rad/sec). This higher frequency content is particularly useful for identifying control derivatives.

The simulated responses for input signal **8th202** can be seen in Fig 5b. This again shows the start of the phugoid, and divergence from normal flight, highlighting the need for a stabilising loop.

The use of optimal input signals seems to hold much promise in shortening the time taken to identify parameters in a flight test. Although there may not seem to be much of an improvement in accuracy of estimates, the results given here were for only limited time duration and power. The values against which they were compared were identified using a series of concatenated test runs, taking much longer than the 15 seconds employed here.

The complexity of a four-input, multi-frequency signal would preclude its use in a helicopter with manual pilot input only. Given the advent of a rotorcraft automatic flight control system however, such complicated inputs may be possible in the future. There still remains work to be done in reducing the number of constituent frequencies in an optimal input signal, but it will surely yield improved results in the future.

Model Verification

In order to verify that the model used in this simulation is indeed a valid one, a check can first be made of the eigen frequencies. As shown in Table 1, the values from the 8th order model lie very close to those of the AFDD model.

The model was further validated by using a typical AGARD 3-2-1-1 (δ_lon) input signal to generate simulated response time histories, using the program software. These simulated responses were then compared with the actual (recorded) responses, and were found to have a close resemblance.

Approximation errors are present due to the inability of the model to describe the high order blade modes, plus the exclusion of the (equivalent) control time lags.

Conclusions and Recommendations

Although rotorcraft are a more complex dynamic system than fixed wing aircraft, optimal input design likewise holds much promise for parameter estimation, with the same advantages:

- by reducing the time length of inputs for identification.
- by an increase in accuracy over heuristic signals.

The results gained from the input design for the BO-105 compare favourably with those published by AGARD, and were of much shorter duration, 15 seconds. The results would have been better with a higher sampling rate, since this was limited to 20 Hz to allow the computer program to evaluate all parameters at once.

However, the number of elementary signals need not reach this theoretical maximum number, as shown here, where realistic signals can be obtained with as few as four elementary signals.

Some difficulty was experienced in simulating the BO-105 responses with a purely open loop system, since the phugoid is unstable. The next steps in this work would be to:

- increase the input length to 20 sec, with a simple stabilising loop.
- increase power to a higher level, 10-15 $\%^2$, comparable with highest powers of the AGARD signals.
- attempt to simplify the signal to allow pilot manual input and/or apply designed signal in a simulator with an automatic flight control system. A BO-105 simulator could be used in the future to test inputs, before any actual flight test takes place.
- reduce the number of simultaneous inputs,
  - to facilitate implementation, and
  - to gain better estimates for pre- determined groups of parameters.

The last point can be achieved by designing inputs with a reduced order model, say 4th order, as was performed here.

Some drawbacks do exist with this technique of input design. The Two-Step Method assumes a very high data accuracy, which may not be justifiable with a helicopter probe system, and its attendant high noise level.

The input design also assumes stationary Gaussian, white noise, with zero mean. In this case (helicopters), the measured output will contain the response, plus noise and higher (non-modelled) order dynamics. The noise will therefore be non-white, and would require the use of a filter. This will then have an effect of biasing the noise level, perhaps rendering the white noise assumption invalid.

Other factors which could give improvements are an increase in frequency range, since 10 rad/sec was found to be somewhat limiting, particularly for identification of control parameters. A maximum of 20-30 rad/sec could give better results, while still avoiding the excitation of higher order, e.g. blade modes.

A weighting matrix should also be used to increase/decrease the importance of parameters, depending on their sensitivity. This should be taken into account during the optimisation.

Lastly, the strengths and weaknesses of sinus and square-type signals should be noted. Sinus-type signals have exactly known frequencies, and steps can therefore be taken to avoid certain frequency bands, e.g. resonant frequencies. This is more difficult to achieve with a square-type signal, which contains a much wider range of frequencies (Fourier series).

However, square-type signals are often better for finding control derivatives, which are best identified at higher frequencies.

The aim of this paper was to demonstrate the application of optimal input design for helicopter control inputs, in order to provide the most accurate parameter estimates. It had been shown that optimal input design does indeed have much to offer, just by comparing their performance against the "standard" 3211. It is certain that it will prove a significant feature of rotorcraft identification for the future.

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Fig 1 : BO-105 Helicopter used at DLR

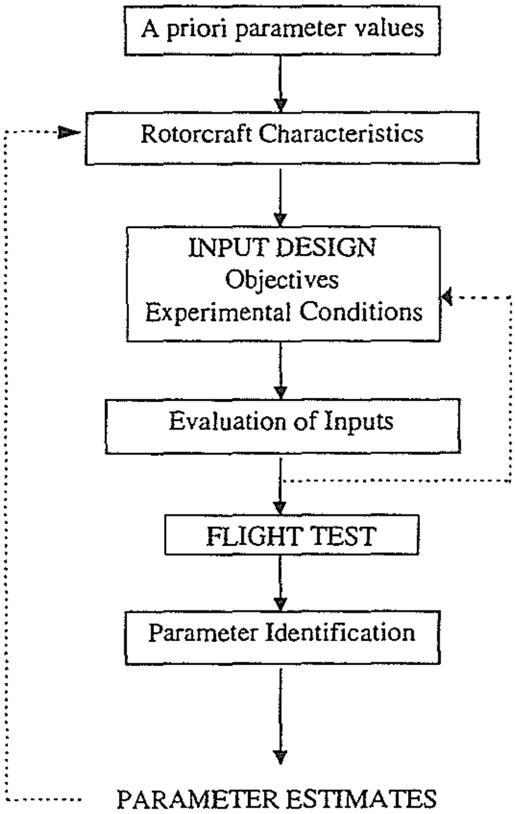


Fig 2 : Identification Prodecure

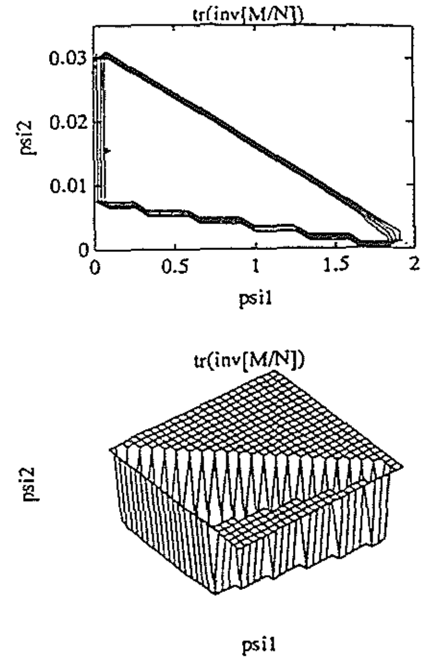


Fig 3 : Convex Hull for Optimisation (2-D and 3-D)

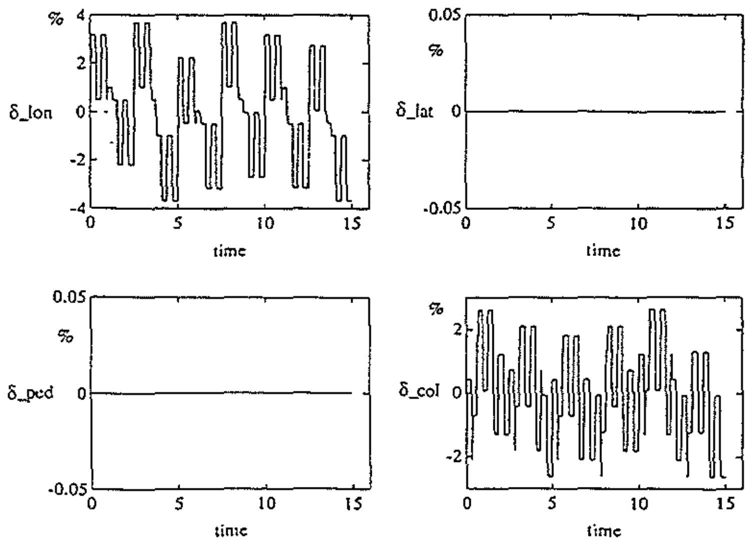


Fig 4a : Square Two-input Signal - 8th310

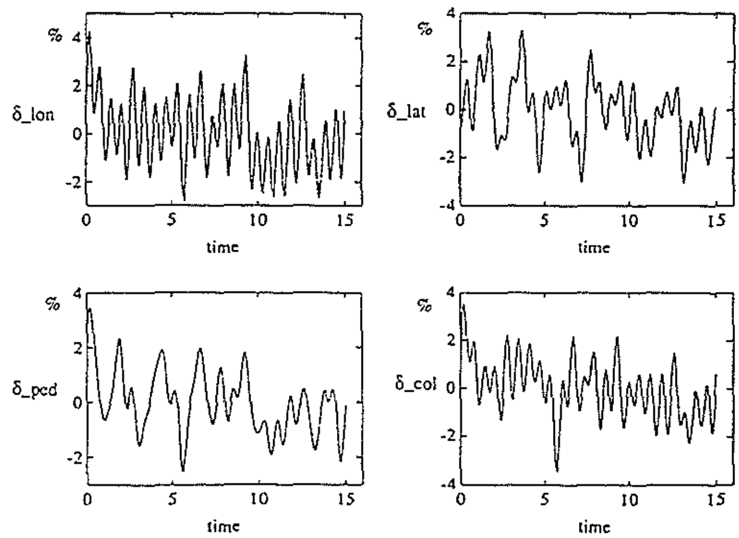


Fig 5a : Optimised Sinus Input Signal - 8th202

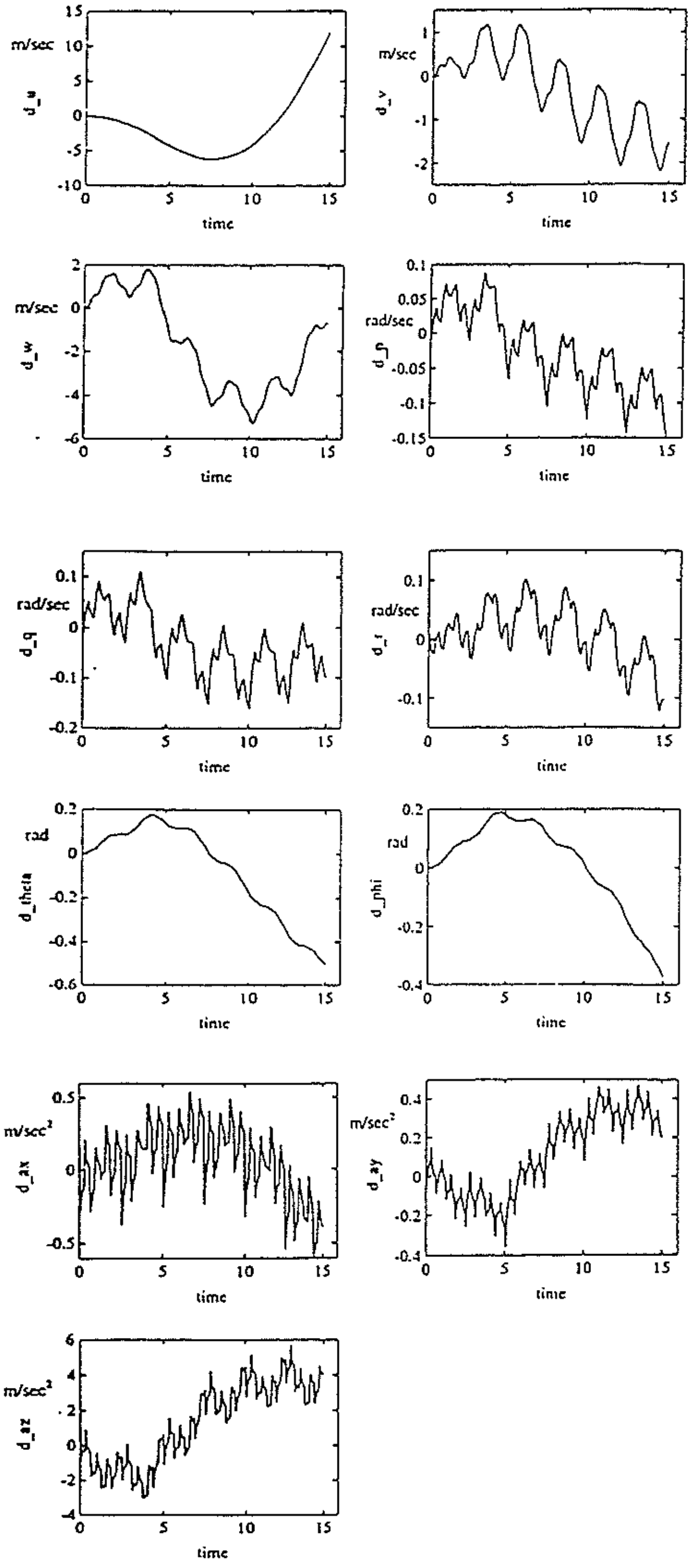


Fig 4b : Simulated Responses to Input Signal 8th310

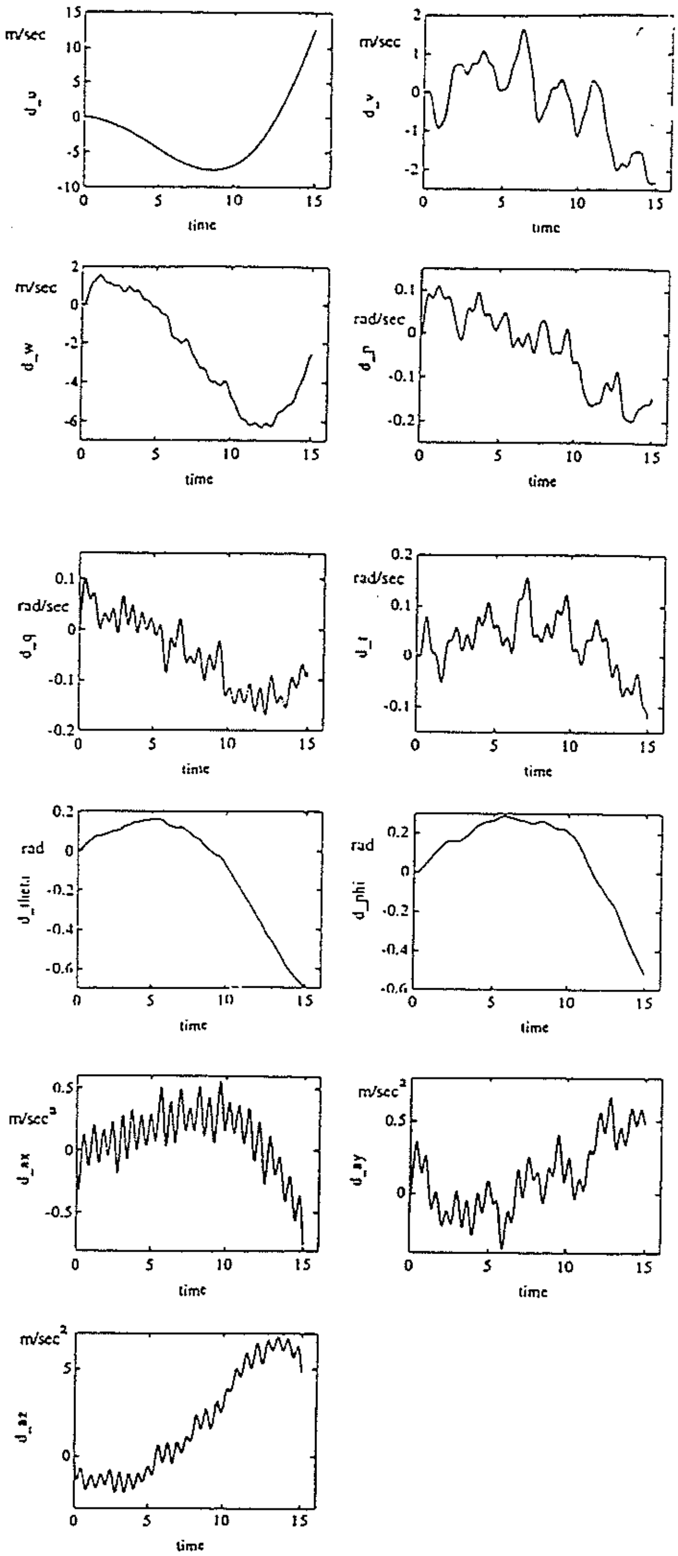


Fig 5b : Simulated Responses to Input Signal 8th202

Time base units : seconds

Fig 6 : Longitudinal Parameter Results - 4th Order Model

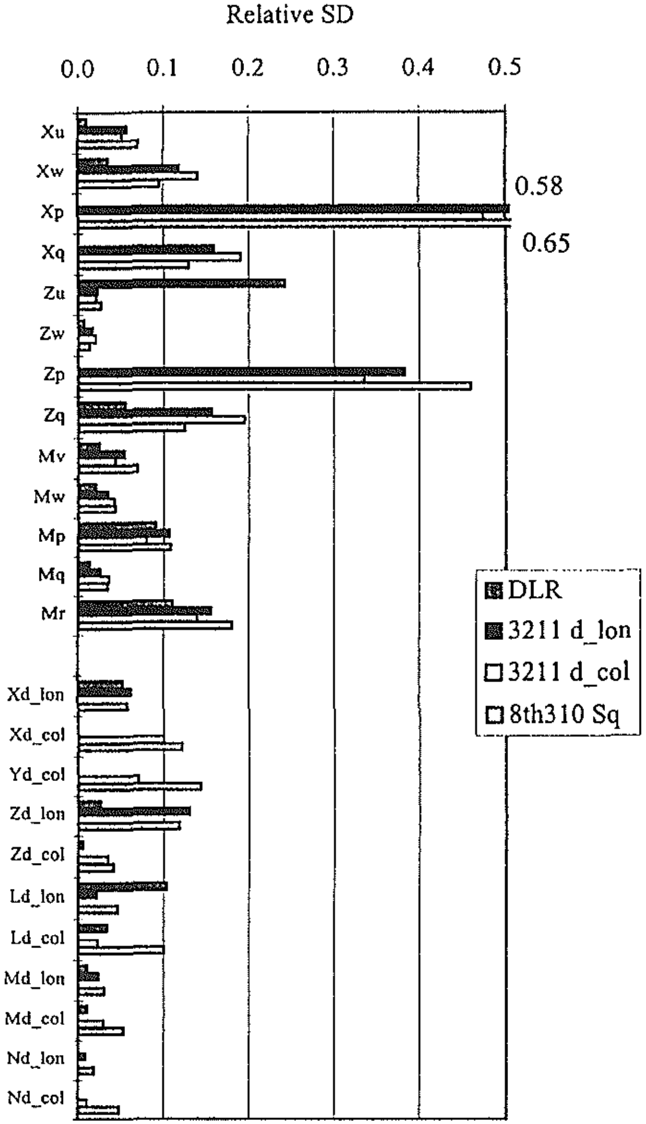


Fig 7 : Force and Moment Parameters 8th Order Model

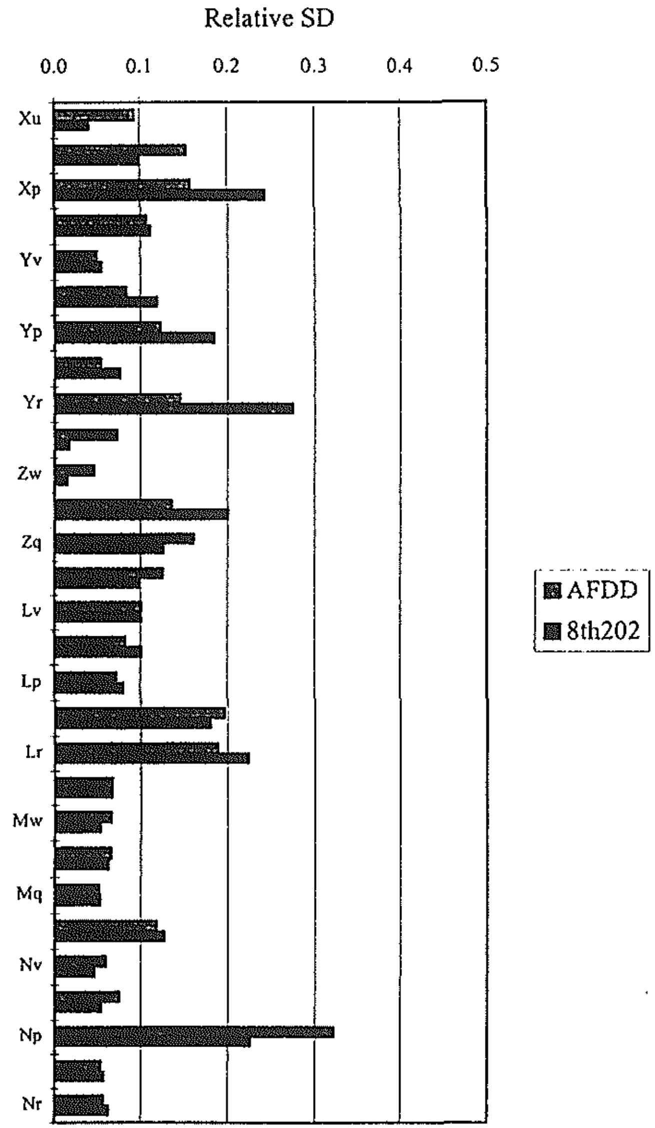
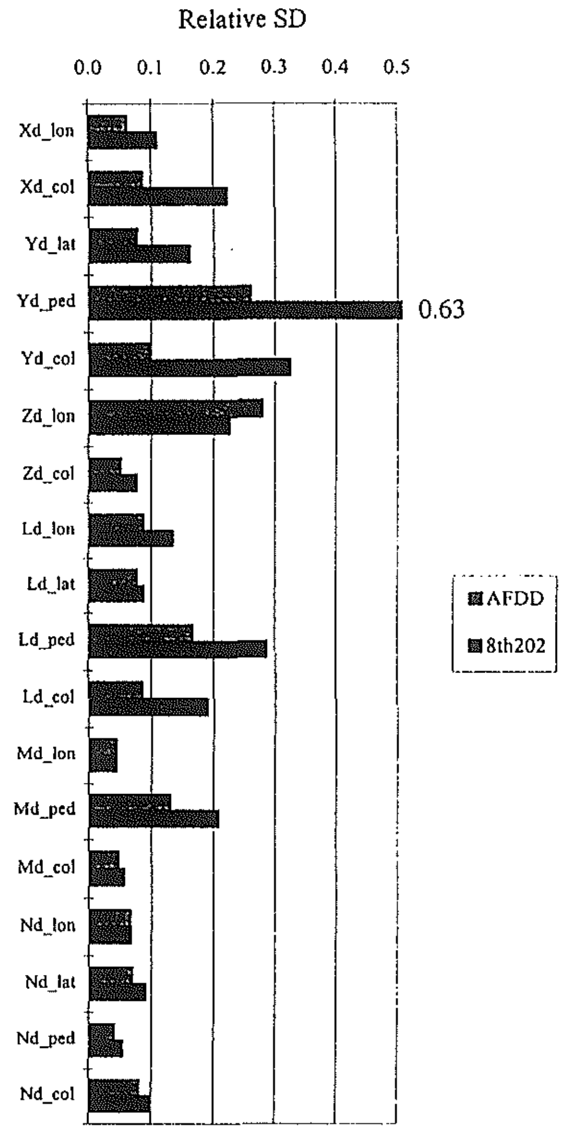
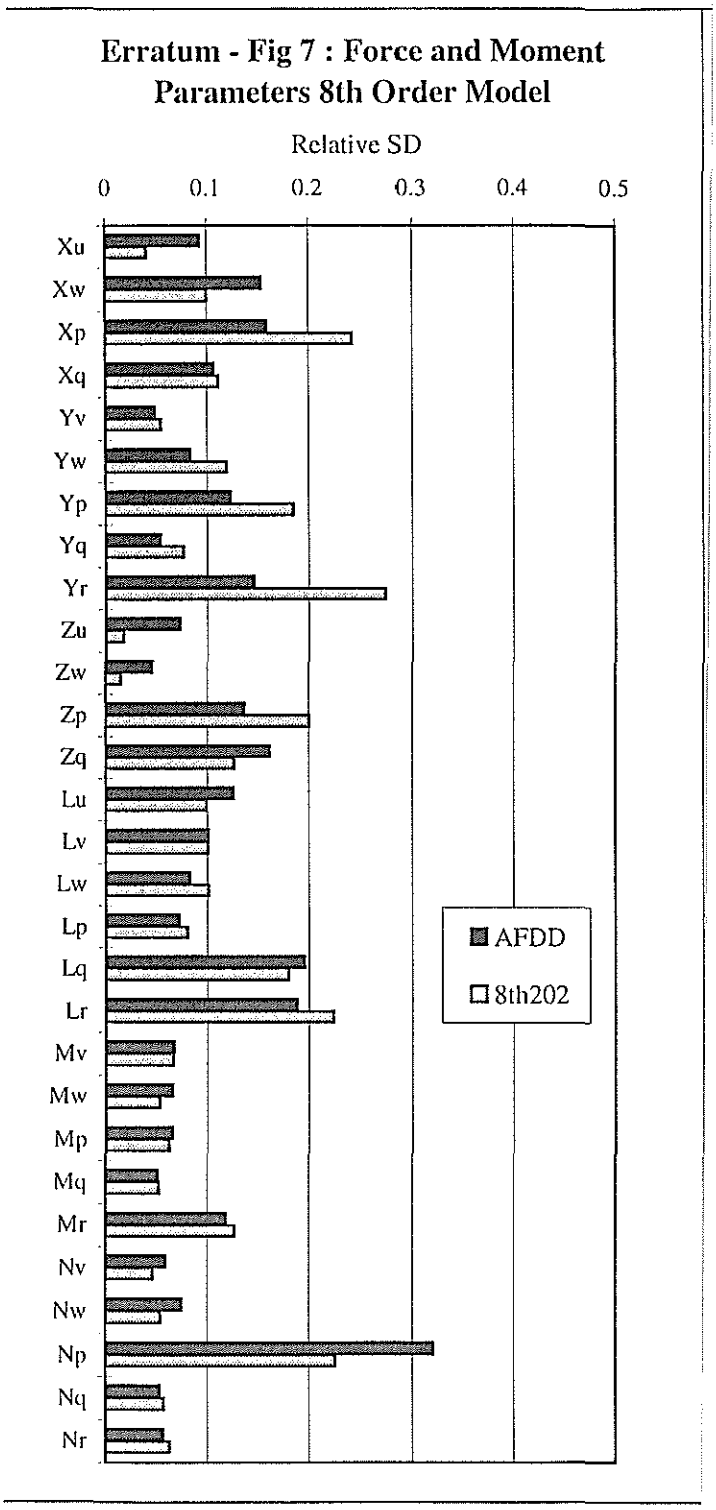


Fig 7 (continued) : Control Parameter Results





Erratum - Fig 7 : Force and Moment Parameters 8th Order Model
Relative SD
0
0.1
0.2
0.3
0.4
0.5
Xu
Xw
Xp
Xq
Yv
Yw
Yp
Yq
Yr
Zu
Zw
Zp
Zq
Lu
Lv
Lw
Lp
Lq
Lr
Mv
Mw
Mp
Mq
Mr
Nv
Nw
Np
Nq
Nr
AFDD
8th202