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NONLINEAR HELICOPTER

STABILITY

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ABSTRACT

Until now for stability analysis of helicopter almost without any exception, the linear conception (linear equations of motion) were applied. This method is normally used for rigid airplanes. Thus it is possible to state very quickly and relatively easy particulars concerning stability of the airplane under study. This method is however valid only for stability aspects in the vicinity of the equilibrium state since the equations of motion are linearized around this state. In the case of the high nonlinear equations of motion of a helicopter this assumptions can be made only with great neglections. Therefore it has to be achieved to study nonlinear equations of motion without linearization.

For the study of stability of nonlinear systems only a very few methods are reliable. Most of these methods are based on the second method of Lyapunov. A relatively simple and however effective method for determining stability behaviour of nonlinear systems is described by the American J. Roskam in his thesis. This method in modified form will be described.

The study was carried out at the Institute for Flight Mechanics and Flight Control at the Technical University of Munich by order of the Federal Ministery for Research and Technology.

LIST OF SYMBOLS

$\underline{A}^*$	$\underline{P}^{-1}\underline{A}$ system dynamics matrix	v_{io}	mean rotor induced velocity
A	start of airfoil with regard to radius	$v_{ic,is}$	rotor induced velocity coefficients of harmonic terms
a	flap hinge offset	v_i	rotor induced velocity
$\underline{B}^*$	$\underline{P}^{-1}\underline{B}$ control distribution matrix	V'	helicopter velocity with respect to the air
$B_{xd,yd,zd}$	components of angular momentum	X	helicopter longitudinal force
B	tip losses factor	$\underline{x}$	state vector
FH	height	x_g, y_g, z_g	helicopter coordinates in geodatic axis system
F	stability parameter	Y	helicopter side force
F_{AX}	force in x-direction	Z	helicopter vertical force
F_{AY}	force in y-direction	$\beta_o, \beta_c, \beta_s$	flapping angles
F_{AZ}	force in z-direction	δ	stability parameter
g	gravitational acceleration	θ_o	collective pitch angle
$I_{xx,yy,zz}$	moments of inertia	θ_c	lateral cyclic pitch angle
$I_{xz,yz,xy}$	" " "	θ_s	longitudinal cyclic pitch angle
L	helicopter roll moment	θ_H	tail rotor pitch angle
M	helicopter pitch moment	θ_1	linear twist rate
m	mass of helicopter	Φ	helicopter roll angle
M_{GA}	mass moment	Θ	helicopter pitch angle
N	yaw moment	τ	variable of time integration
p	aircraft roll rate	Ψ	helicopter yaw angle
q	aircraft pitch rate		
r	aircraft yaw rate		
T	kinetic energy		
t, t_e	time, final time		
u,v,w	components of velocity of aircraft in body axis		
$\underline{u}$	control vector		
u_g, v_g, w_g	velocity components of aircraft in geodetic axis system		

1 INTRODUCTION

A rigid body in flight possesses in case of steady control six possibilities of motion (-degrees of freedom). It can execute three translational and three rotational motions. In case of a helicopter, the degrees of freedom of flapping, lagging and variable rotor speed are added. Further degrees of freedom result from the case of free controls, from the application of regulators and from respective elastic deformation of various helicopter parts. The sum total of all these degrees of freedom influences all the acting forces and moments on the particular aircraft parts henceforth resulting the dynamics of a helicopter and also its stability properties.

Stability or instability is a characteristic of an equilibrium state. The equilibrium is stable if the system upon a slight disturbance in any of its degrees of freedom returns finally to its initial state.

According to the definition of stability a helicopter is referred to as stable if after a minor deviation of a stationary flight condition without any interfering action of the pilot it will return into this former position. The initial flight position can be a hovering state or a stationary advance flight. Disturbing effects can be gusts of wind or temporary steering deflections. The return into the initial position can occur in the form of oscillations or in aperiodic motion procedure. The following quantities can suffer disturbances during a helicopter flight: height, velocity, inclination angle of flight path, position of helicopter, rotor rotational speed etc. On behalf of various reasons it is wished for that a helicopter indicates stability, that is it does not show too much instability. The pilot is thus greatly relieved. It has been indicated that most helicopters do not fulfill in a strict sense the conditions of stability. If the handling qualities are good the pilots do not have too many objections against a slight instability. The coherence of stability characteristics and handling qualities influences essentially the classification of the flying qualities of a helicopter by the pilot. From mathematical analysis result exact criteria for the stability of an airplane.

The investigation in the stability behaviour can be listed under the following seperate headings.

- Ascertainment of force - and momentum coefficients of wind channel tests, flight tests or theoretically
- Formulation of the equations of motion
- Calculation of the respective stability values

2 LINEAR STABILITY ANALYSIS

For the linear stability analysis of a helicopter the linearized equations of motion of a helicopter are used with the assumption of small disturbances. Every variable x is seperated in a constant part x_o, and in a variable part Δx. Δx is a small quantity and now the variable function. The constant part x_o describes the stationary initial state. The aerodynamic forces and moments are expanded with reference to the vicinity of stationary state in Taylor-series. The equations of motion in this manner simplified and summarized result in a system of linear differental quations with constant coefficients. This differential equation system is shown in figure 1 and discribes the motion of a helicopter with auxiliary linear dynamics. With the assistance of the derivatives expressed in the matrices $\underline{A}^*$ und $\underline{B}^*$ statements concerning static stability of helicopter can be made [7,9]. The dynamic characteristic behaviour of the helicopter is determined by the position of the poles of the characteristic equation [7,9]. The system shown in figure 1 can be written in simplified form thus.

$$\underline{P}\,\dot{\underline{x}} = \underline{A}\,\underline{x} + \underline{B}\,\underline{u} \tag{2.1}$$

$$\underline{x} = [u,v,w,p,q,r,\Phi,\Theta]^T$$

$$\underline{u} = [\theta_o,\theta_c,\theta_s,\theta_H]$$

or

$$\dot{\underline{x}} = \underline{P}^{-1}\underline{A}\,\underline{x} + \underline{P}^{-1}\underline{B}\,\underline{u} \tag{2.2}$$

with

$$\underline{A}^* = \underline{P}^{-1}\underline{A}$$

$$\underline{B}^* = \underline{P}^{-1}\underline{B}$$

it finally results that

$$\dot{\underline{x}} = \underline{A}^*\underline{x} + \underline{B}^*\underline{u} \tag{2.3}$$

The poles of the characteristic equation can be obtained if one determines the eigen values of the matrix $\underline{A}^*$. For the flight case u_g = 27.8 m/s, altitude FH = 1500 m the eigen values of the matrix $\underline{A}^*$ were determined for the model combination MOD1 (see also chapter 4.1) and were entered in the complex number plane (figure 2). As an example helicopter the BO-105 of MBB company served for this and the following investigations.

Modern helicopters with hingeless rotors without stabilizer have according to theroretical investigations generally an instable trajectory oscillation (phugoid), a slightly damped tumbling (dutch roll) and two aperiodic forms of motion (pitch mode and spiral mode) (see figure 2). All eigen values change with air speed, altitude, gross weight and the location of center of gravity and also other system quantities such as e.g. rotor rotational speed, blade mass etc.

3 NONLINEAR STABILITY ANALYSIS

The helicopter forms a nonlinear system with nonconservative forces. The dissipative forces here can also add energy to the system. For these reasons it has not been found possible to apply conventional energy methods or to construct a Lyapunov function [1,2,3,4].

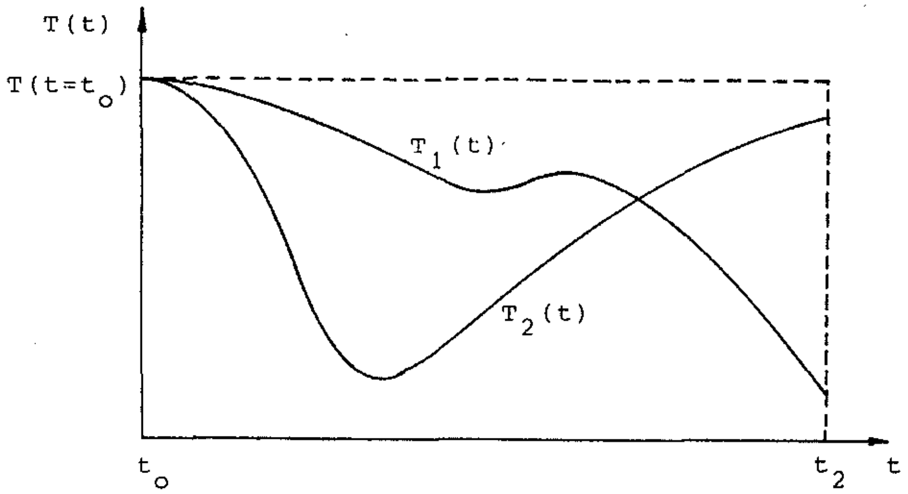
Asymptotic stability of the undisturbed motion implies that all disturbed values vanish after some time. Weak stability implies that all phase variables remain inside some region around the origin. From a handling qualities viewpoint, it is desirable to have those phase variables designated as velocities vanish such that:

$$\lim_{t \to \infty} T(t) = 0 \tag{3.1}$$

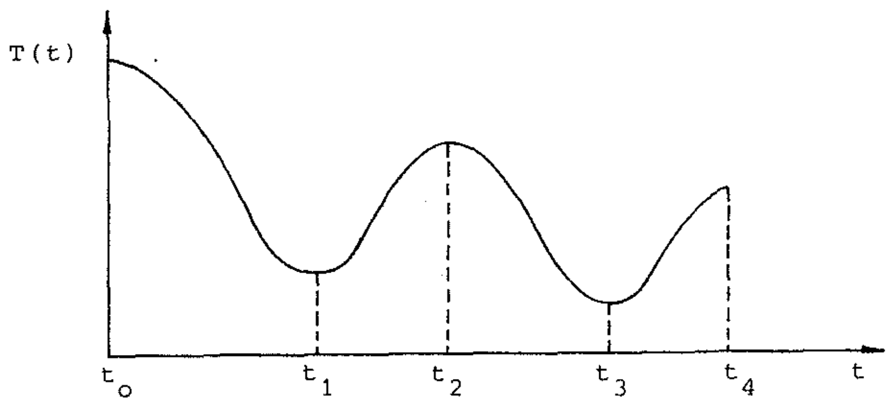
where T is the kinetic energy of the disturbed motion variables. Naturally in most cases it is not interesting to regard the numerically obtained solution over an infinite time interval as it is the case with the conventional stability definitions. It is however necessary to determine the stability by observation of the motion during a limited time interval. Using the definition of stability in a limited time interval due to Lebedev it follows by interpreting T as a positive definite function that in the time interval $t_o < t < t_2$ the condition for stability is:

$$T(t) < T(t_o) \tag{3.2}$$

A consequence of (3.2) is that both motions of the following figure must be called stable. This conclusion is acceptable for $T_1(t)$ but not always for $T_2(t)$.



This unwanted fact can be eliminated if one adds to the mentional inequation (3.2) a condition based upon a time integral of kinetic energy.
Suppose the following energy process:



$$F = \frac{\int_{t_1}^{t_2} T(\tau)\,d\tau + \int_{t_3}^{t_4} T(\tau)\,d\tau}{\int_{t_o}^{t_1} T(\tau)\,d\tau + \int_{t_2}^{t_3} T(\tau)\,d\tau} < 1 \qquad (3.3)$$

$$\text{or} \quad F = \frac{\text{increasing kinetic energy}}{\text{decreasing kinetic energy}}$$

Condition F is a practical stability criteria for the nonlinear equations of motion of a helicopter; especially in cases where stability is to be viewed inside a limited time interval. A motion is called stable inside a time interval $t_o < t < t_e$, if the following conditions are fulfilled:

$$T(t) < T(t_o) \qquad (3.4)$$

$$F \leq \delta < 1 \qquad (3.5)$$

t_o must be chosen such that T has a maximum. If this is not the case the forementioned stability criteria are no longer valid. The reason for this is that for arbitrary initial disturbances $\dot{T}(t=0) > 0$ is possible. This depends entirely on the character of the "kinetic energy generating terms" in the equations of motion.

In the case $t_e \to \infty$ both conditions (3.4, 3.5) are necessary but not sufficient equivalents of the Lyapunov definition for stability.

Since the nonlinear equations of motion are integrated numerically in the program "HESISTAP" and for every integration step the state variables and their derivatives are thus known, it is simple to calculate kinetic energy as a function of time.

Therefore, it is possible to keep track of both conditions, 3.4 and 3.5 and obtain a continous history of the stability character of motion.

In this manner, a numerical procedure for the practical determination of stability of nonlinear equations of motion is obtained.

3.1 APPLICATION OF MODIFIED ENERGY-METHOD WITH NONLINEAR EQUATIONS OF MOTION OF A HELICOPTER

The energy contributions can be found if one multiplies each equation of motion by its characteristic velocity and by a subsequent integration.

The following equations result; with the exception: $I_{xy} = I_{yz} = B_{xd} = B_{yd} = B_{zd} = 0$.

$$
\begin{aligned}
\int_{t_o}^{t_1} F_{AX} u \, d\tau &= \int_{t_o}^{t_1} (m \dot{u} u + m w q u - m v r u) d\tau \\
\int_{t_o}^{t_1} F_{AY} v \, d\tau &= \int_{t_o}^{t_1} (m \dot{v} v + m r u v - m p v w) d\tau \\
\int_{t_o}^{t_1} F_{AZ} w \, d\tau &= \int_{t_o}^{t_1} (m \dot{w} w + m p v w - m q u w) d\tau
\end{aligned}
\tag{3.6}
$$

$$
\begin{aligned}
\int_{t_o}^{t_1} L \, p \, d\tau &= \int_{t_o}^{t_1} (I_{xx}\dot{p} p - I_{xz}\dot{r} p + q r (I_{zz} - I_{yy}) p - I_{xz} p^2 q) d\tau \\
\int_{t_o}^{t_1} M \, q \, d\tau &= \int_{t_o}^{t_1} (I_{yy}\dot{q} q + r p (I_{xx} - I_{zz}) q + I_{xz}(p^2 - r^2) q) d\tau \\
\int_{t_o}^{t_1} N \, r \, d\tau &= \int_{t_o}^{t_1} (-I_{xz}\dot{p} r + I_{zz}\dot{r} r + p q (I_{yy} - I_{xx}) r + I_{xz} q r^2) d\tau
\end{aligned}
\tag{3.7}
$$

where

$$
\begin{aligned}
F_{AX} &= X - m g \sin\Theta \\
F_{AY} &= Y + m g \cos\Theta \sin\Phi \\
F_{AZ} &= Z + m g \cos\Theta \cos\Phi
\end{aligned}
\tag{3.8}
$$

After completing the integrations, adding the equations and rearranging, it is not surprising that a statement of energy balance is recovered:

$$
\begin{aligned}
&\left[\frac{1}{2} m u^2 + \frac{1}{2} m v^2 + \frac{1}{2} m w^2 + \frac{1}{2} I_{yy} q^2 + \frac{1}{2} I_{xx} p^2 + \frac{1}{2} I_{zz} r^2 - I_{xz} p r\right] \Big|_{t_o}^{t_1} = \\
&\int_{t_o}^{t_1} (F_{AZ} w + M q + F_{AY} v + L p + F_{AX} u + N r) d\tau - m \int_{t_o}^{t_1} (p v w - q u w) d\tau - (I_{xx} - I_{zz}) \\
&\int_{t_o}^{t_1} r p q \, d\tau + I_{xz} \int_{t_o}^{t_1} p^2 q \, d\tau - I_{xz} \int_{t_o}^{t_1} (p^2 - r^2) q \, d\tau - m \int_{t_o}^{t_1} (r u v - p w v) d\tau - (I_{zz} - I_{yy}) \\
&\int_{t_o}^{t_1} q r p \, d\tau - I_{xz} \int_{t_o}^{t_1} r^2 q \, d\tau - (I_{yy} - I_{xx}) \int_{t_o}^{t_1} p q r \, d\tau - m \int_{t_o}^{t_1} (w q u - v r u) d\tau
\end{aligned}
\tag{3.9}
$$

The energy-time histories can also be useful in pointing out the effect of individual terms in the equations of motion. Before the energy-time histories of disturbed motion of helicopter are discussed, the program "HESISTAP" should be described briefly. With this program all the investigation studies were carried out.

4 HELICOPTER SIMULATION AND STABILITY ANALYSIS PROGRAM

With the computer program "HESISTAP" the following calculations can be executed (see also figure 3).

- Calculation of trim values (θ_o, θ_c, θ_s, θ_H, Θ, Φ, u, v, w) for an example helicopter with choosable initial velocities u_g, v_g, w_g.
- Calculation of derivatives of an example helicopter for a calculated steady state or for an arbitrary quasistationary state.
- Calculation of eigen values of system matrix $\underline{A}^*$ (linear stability analysis).
- Integration of nonlinear equations of motion of helicopter.
- Calculation of energy-time histories with nonlinear stability analysis thereafter.
- Calculation of optimal control for a desired flight path.

Furthermore for each of the four blade control angles $\theta_O, \theta_C, \theta_S, \theta_H$ three time depandant blade control angles can be chosen:

- constant (trim value)
- doublette
- 3-2-1-1 pulse

Besides it is possible that during a program procedure blade control angles can be read in from a tape. This is especially interesting when a helicopter simulation with measured blade control angles is executed. To adapt the blade control angles to real conditions, the possibility exists to smoothen the chosen step function by application of a filter.

The program "HESISTAP" is also feasible for combining variously complicated mathematical main - and tail rotor models with each other. For this investigation two model configurations (MOD1 and MOD2) were chosen to be described in the following.

4.1 BASIS FOR THE MATHEMATICAL DERIVATION OF FORMULA APPARATUS USED IN THE PROGRAM "HESISTAP"

Deduced from the system of equations of motion for the general case of a (spatial) motion with six degrees of freedom in a body fixed frame, velocity - and acceleration components are calculated existing on a blade element whereas all translational and all rotational motions are considered. After these preparations the flapping motion of blades is calculated. The flapping angle is set such that the sum of correction moment (for consideration of hingeless rotor), massmoment and airloading moment around the (equivalent) flapping hinge disappears. If the sum of these three moments is taken zero, one abtains a system of equations with three equations for the three flapping angles $\beta_o, \beta_c, \beta_s$ to be searched. Next by means of the blade element theory the forces generated by the rotor are calculated. A trapezoidal induced velocity distribution is assumed [8]. Now the moments generated by the rotor around the body fixed axes are calculated. The blade torsional moment is hereby neglected. A rotor shaft angle (in x,z-plane) is considered. Special difficulties always cause the exact appropriation of fuselage aerodynamics. In the program "HESISTAP" therefore a very simplified fuselage model is applied. The following assumptions are made:

- fuselage drag acts in center of gravity
- fuselage drag is effective in direction of resulting velocity of flow from initial direction

with the representation of elevator it is supposed in simplified manner that the direction of elevator lift coincides with z-direction. The elevator drag is supposed to be included in fuselage drag.

The mathematical modeling of tail rotor is based on the see-saw construction mode of rotor. The derivation of tail rotor forces respectively moments succeeds the same as with main rotor; with rotor specific alterations. The formula apparatus resulting from this is called MOD2. MOD1 has with respect to MOD2 the following simplifications:

- No side and no yawing velocity
  $v = r = 0$
- No blade twist; θ_o is thus to be regarded as a mean value of angle of incidence
  $\theta_1 = 0$
- Effective flapping hinge offset zero
  $a = 0$
- The influence of mass moment is neglected
  $M_{GA} = 0$
- The induced downwash and thus the lift are effective on the entire blade length
  $A = 0, B = 0$
- The induced downwash is constant
  $v_i = \text{const.}, v_{ic} = v_{is} = 0$

Furthermore with the tail rotor modeling it is supposed that the tail rotor generates only a force in the y-direction. The influence of the induced downwash on the tail rotor force is considered by a factor F_{v_i} on θ_H. The influence of forward velocity on the tail rotor force is not considered.

(For the exact derivation of mathematical models see [5,6]).

5 APPLICATION

For the procedure described in section 3 a simulation with the model combinations MOD1 and MOD2 was carried out. Based on an ideal hovering in both combinations the forward velocity u was disturbed ($\Delta u = -5$ m/s). Because of this disturbance the resulting time histories of states are represented in the figures 4a)to 4h). The resulting energy time histories are illustrated in the figures 4i) to 4o), whereas in the figures 4i) to 4k) the translational parts of kinetic energy and in the figures 4l) to 4n) the rotational parts of kinetic energy are shown. The figures 4o) and 5 show the time histories of the total kinetic energy composed as such from the (previously mentioned) translational and rotational parts. The blade control angles are shown in the figures 4p) to 4s).

If one compares time histories of the states of the model combinations MOD1 and MOD2, one can notice partly a considerable difference in the amplitudes and on the time histories itself. The reason for this is based on the different modeling (great neglections in MOD1). Furthermore there is an instable tendency of the time histories to be recognized.

If one regards the single energy time histories for this disturbed motion, the forementioned is confirmed. Furthermore one realizes that essentially the translational energy parts supply contributions to the total energy.

Though one would attest the disturbed motion with the aid of the time histories of states an instable character as such, with the assistance of figure 5 and the energy method described in section 3 a range can be found in which "stability within a range" prevails that is, stability criteria 3.4 and 3.5 are fulfilled. For this in figure 5 stability parameter δ in the time interval $0 \leq t \leq 4.5$ s (4.7 s) is represented. δ is in the range $0 \leq \delta \leq .43 (.4)$.

After 4.5 s (4.7 s) furthermore up to 5.7 s (5.3 s) the criteria 3.5 is fulfilled whereas criteria 3.4 is violated. The disturbed motion become quickly instable from this time on; which is also asserted by the known behaviour of examply helicopter BO-105.

6 CONCLUDING REMARKS

The linear stability analysis is in case of the high nonlinear equations of motion of helicopter no more applicable. A procedure was shown with which a nonlinear stability analysis can be performed. With the aid of energy time histories important terms can be identified from the nonlinear equations of motion. The outlined procedure is the beginning of a series of continous investigation possibilities of nonlinear equations of motion of helicopter which are carried out in the Institute for Flight Mechanics and Flight Control at the Technical University Munich. In the future it is relevant to find criteria with the aid of the nonlinear stability theory and thus to be able to make exact statements on the stability behaviour of helicopters.

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$$
\begin{bmatrix}
X_{\dot u} & X_{\dot v} & X_{\dot w} & X_{\dot p} & X_{\dot q} & X_{\dot r} & 0 & 0 \\
Y_{\dot u} & Y_{\dot v} & Y_{\dot w} & Y_{\dot p} & Y_{\dot q} & Y_{\dot r} & 0 & 0 \\
Z_{\dot u} & Z_{\dot v} & Z_{\dot w} & Z_{\dot p} & Z_{\dot q} & Z_{\dot r} & 0 & 0 \\
L_{\dot u} & L_{\dot v} & L_{\dot w} & L_{\dot p} & L_{\dot q} & L_{\dot r} & 0 & 0 \\
M_{\dot u} & M_{\dot v} & M_{\dot w} & M_{\dot p} & M_{\dot q} & M_{\dot r} & 0 & 0 \\
N_{\dot u} & N_{\dot v} & N_{\dot w} & N_{\dot p} & N_{\dot q} & N_{\dot r} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\cdot
\begin{bmatrix} \dot u \\ \dot v \\ \dot w \\ \dot p \\ \dot q \\ \dot r \\ \dot\Phi \\ \dot\Theta \end{bmatrix}
=
\begin{bmatrix}
X_u & X_v & X_w & X_p & X_q & X_r & 0 & K1 \\
Y_u & Y_v & Y_w & Y_p & Y_q & Y_r & K2 & K3 \\
Z_u & Z_v & Z_w & Z_p & Z_q & Z_r & K4 & K5 \\
L_u & L_v & L_w & L_p & L_q & L_r & 0 & 0 \\
M_u & M_v & M_w & M_p & M_q & M_r & 0 & 0 \\
N_u & N_v & N_w & N_p & N_q & N_r & 0 & 0 \\
0 & 0 & 0 & 1 & K6 & K7 & 0 & 0 \\
0 & 0 & 0 & 0 & K8 & K9 & 0 & 0
\end{bmatrix}
\cdot
\begin{bmatrix} u \\ v \\ w \\ p \\ q \\ r \\ \Phi \\ \Theta \end{bmatrix}
+
\begin{bmatrix}
X_{\vartheta_o} & X_{\vartheta_c} & X_{\vartheta_s} & X_{\vartheta_H} \\
Y_{\vartheta_o} & Y_{\vartheta_c} & Y_{\vartheta_s} & Y_{\vartheta_H} \\
Z_{\vartheta_o} & Z_{\vartheta_c} & Z_{\vartheta_s} & Z_{\vartheta_H} \\
L_{\vartheta_o} & L_{\vartheta_c} & L_{\vartheta_s} & L_{\vartheta_H} \\
M_{\vartheta_o} & M_{\vartheta_c} & M_{\vartheta_s} & M_{\vartheta_H} \\
N_{\vartheta_o} & N_{\vartheta_c} & N_{\vartheta_s} & N_{\vartheta_H} \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\cdot
\begin{bmatrix} \vartheta_o \\ \vartheta_c \\ \vartheta_s \\ \vartheta_H \end{bmatrix}
$$

$K1 = -mg \cos\Theta_o$

$K2 = mg \cos\Phi_o \cos\Theta_o$

$K3 = -mg \sin\Phi_o \sin\Theta_o$

$K4 = -mg \sin\Phi_o \cos\Theta_o$

$K5 = -mg \cos\Phi_o \sin\Theta_o$

$K6 = \sin\Phi_o \tan\Theta_o$

$K7 = \cos\Phi_o \tan\Theta_o$

$K8 = \cos\Phi_o$

$K9 = -\sin\Phi_o$

Fig. 1 Linearized equations of motion of helicopter

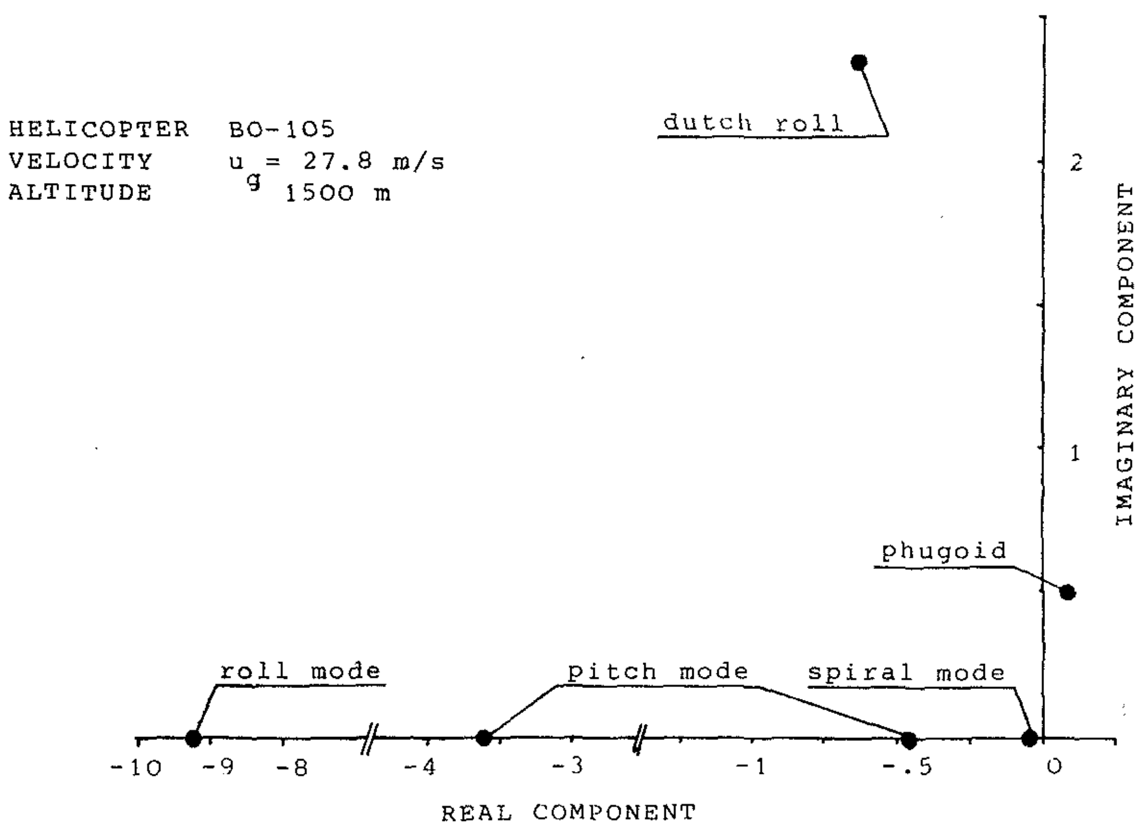

Fig. 2 Complex number plane

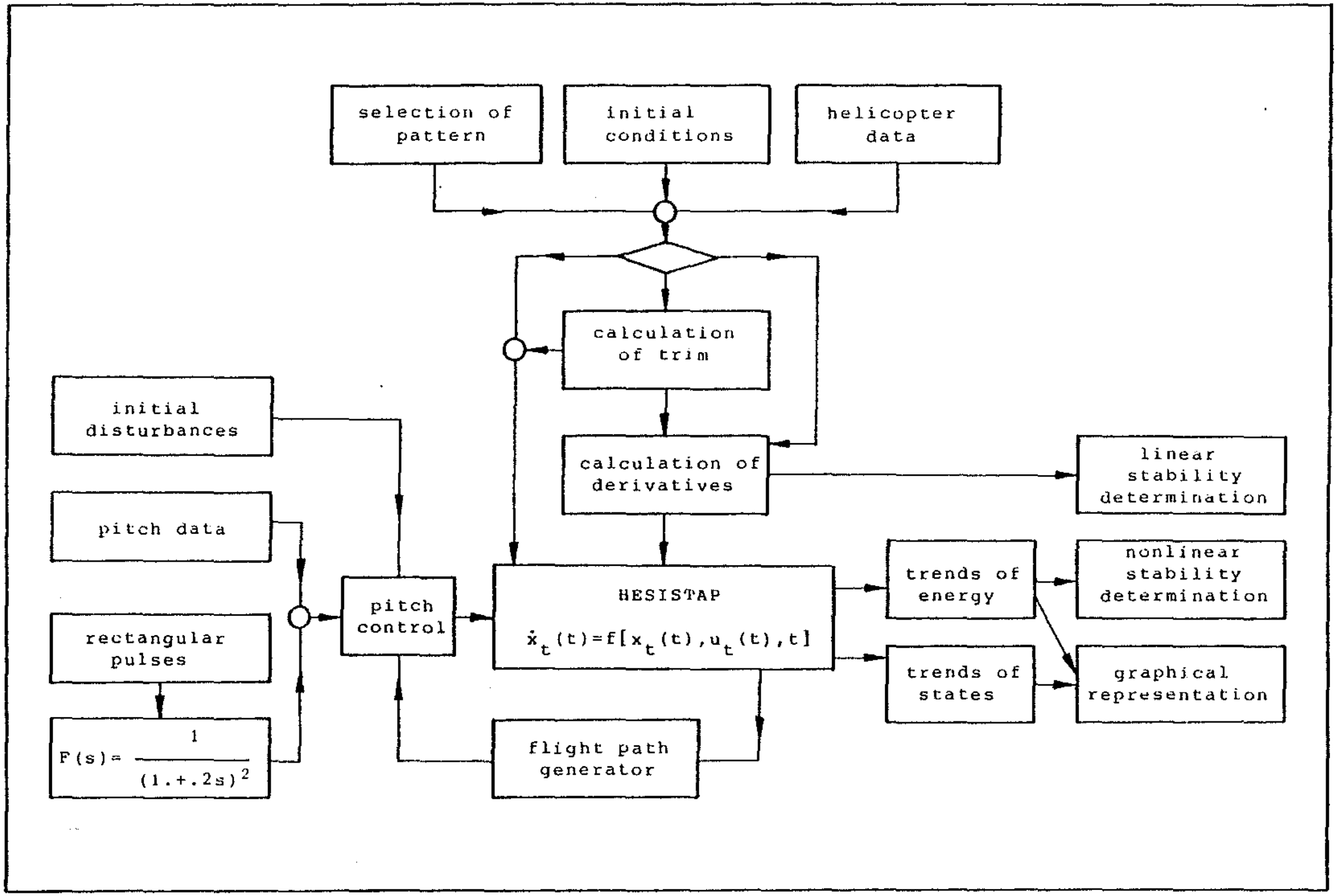

Fig. 3 Block diagram of program "HESISTAP"

HELICOPTER NAME	BO-105
WEIGHT	20600 N
ALTITUDE	1500 m
FLIGHT SPEED	HOVER
MOD1 ———	MOD2 ---------

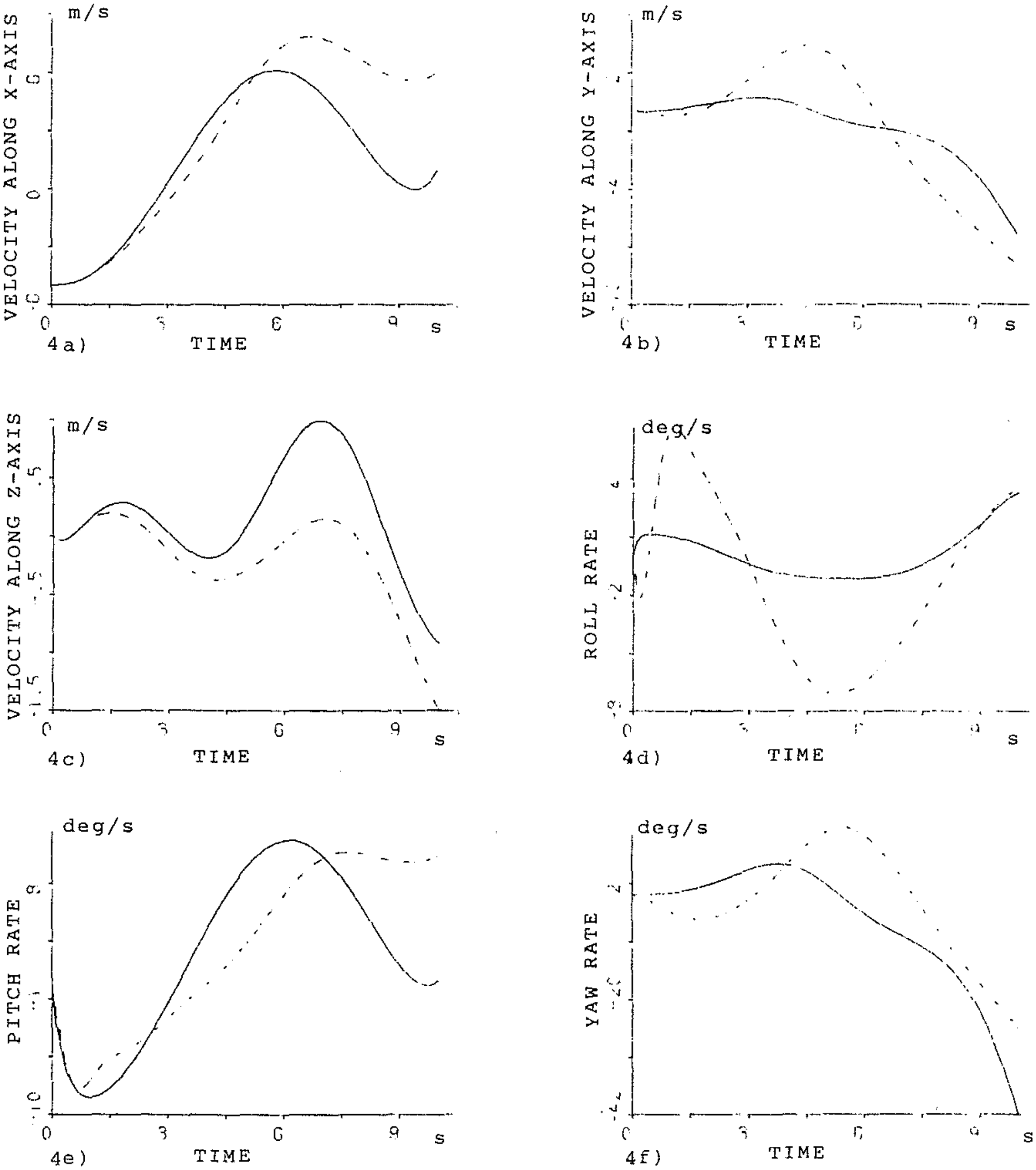

Fig. 4 Initial disturbance response of helicopter (states)

HELICOPTER NAME	BO-105
WEIGHT	20600 N
ALTITUDE	1500 m
FLIGHT SPEED	HOVER
MOD1 ———	MOD2 - - - - - -

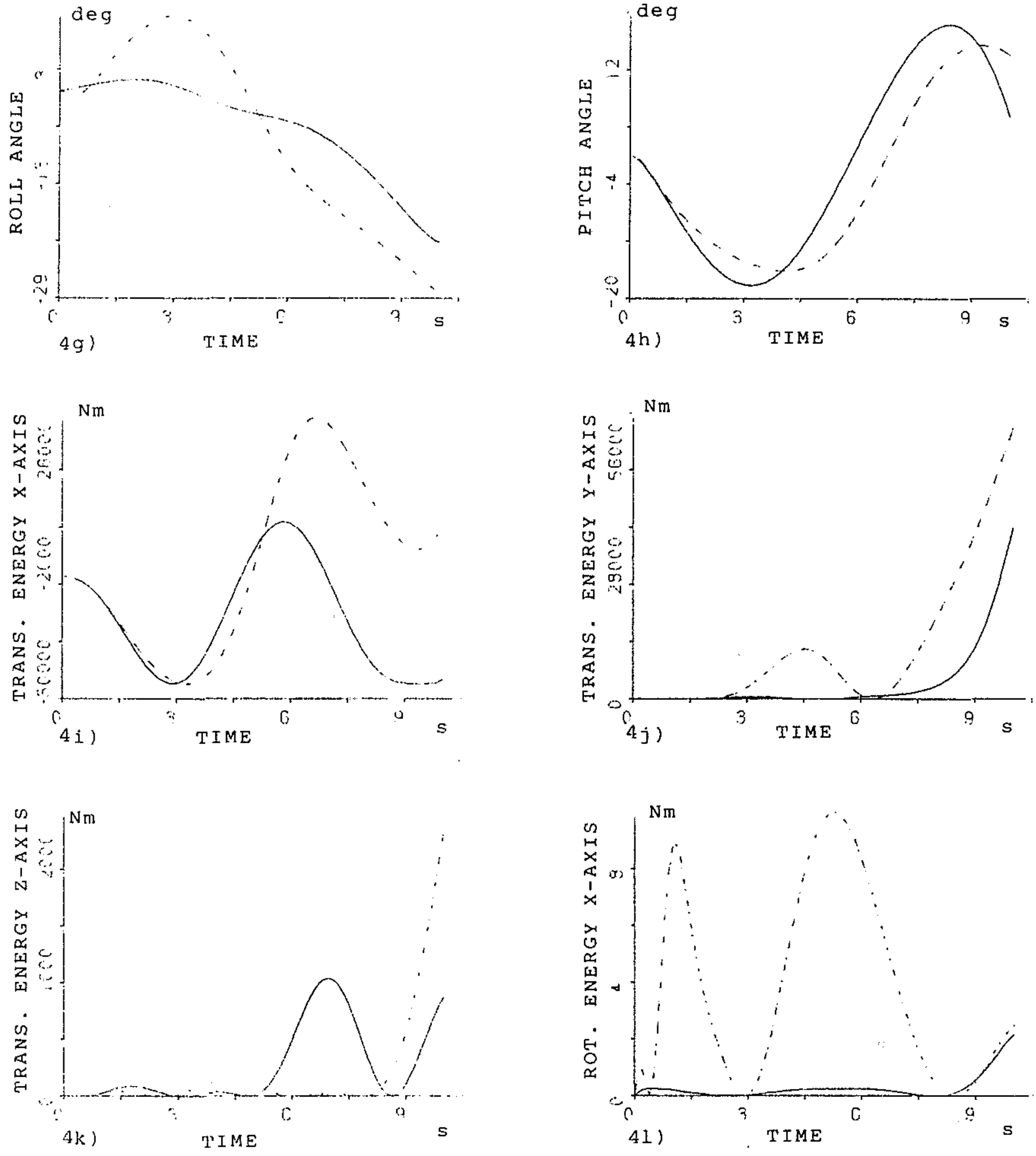

Fig. 4 continued (states, energy)

HELICOPTER NAME	BO-105
WEIGHT	20600 N
ALTITUDE	1500 m
FLIGHT SPEED	HOVER
MOD1 ————	MOD2 ---------

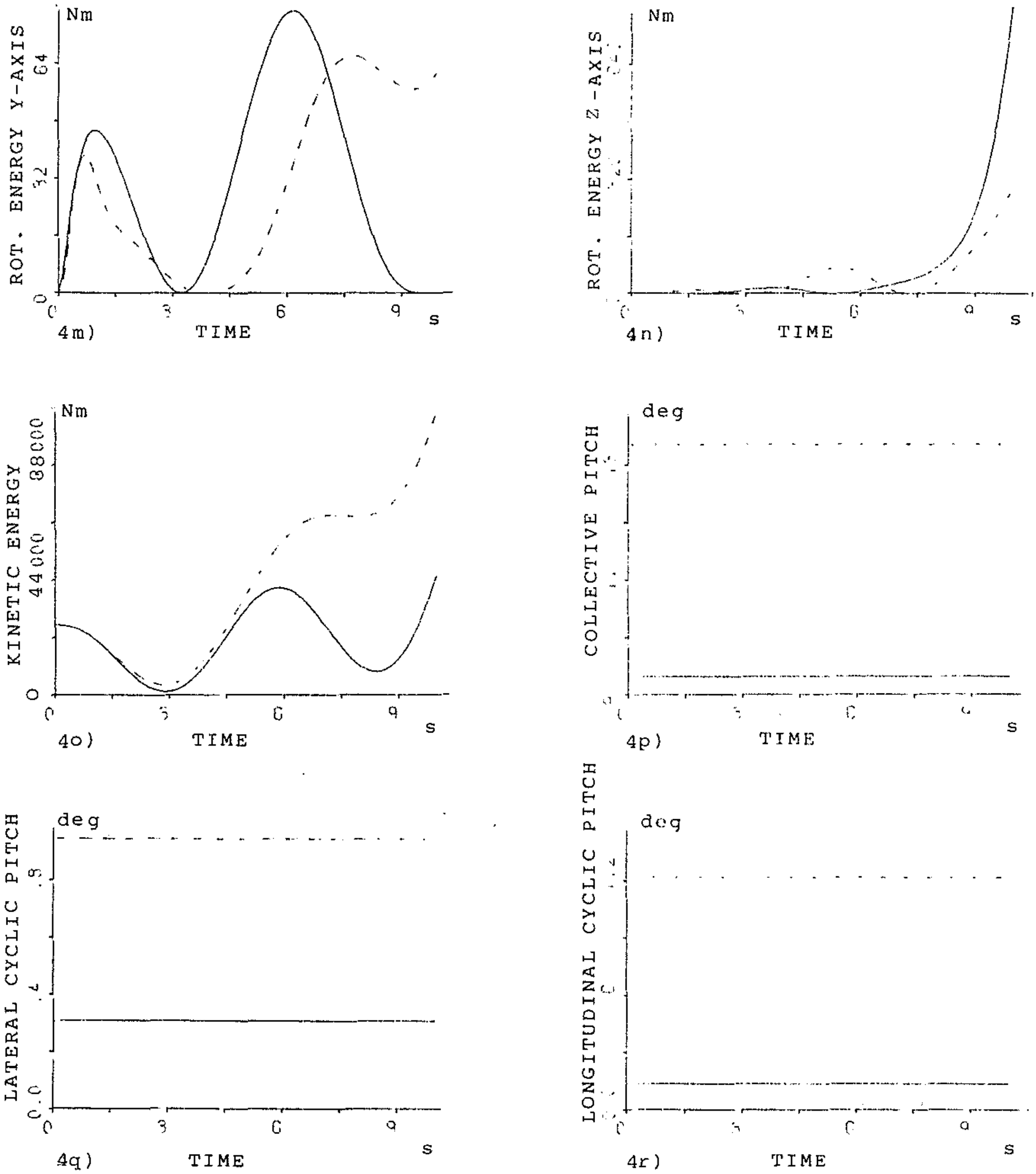

Fig. 4 continued (energy,blade control angles)

HELICOPTER NAME	BO-105
WEIGHT	20600 N
ALTITUDE	1500 m
FLIGHT SPEED	HOVER
MOD1 ————	MOD2 - - - - - - - - -

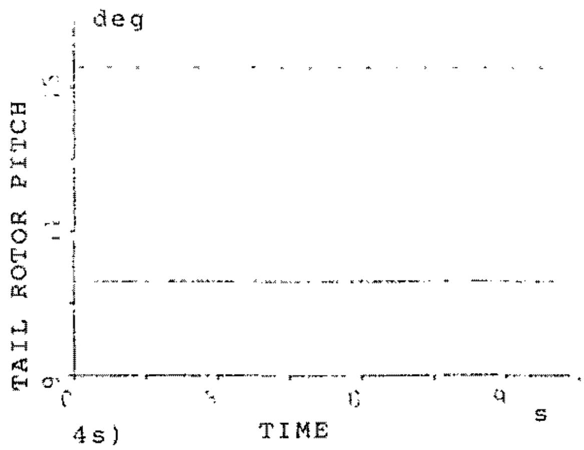

Fig. 4 continued (blade control angle)

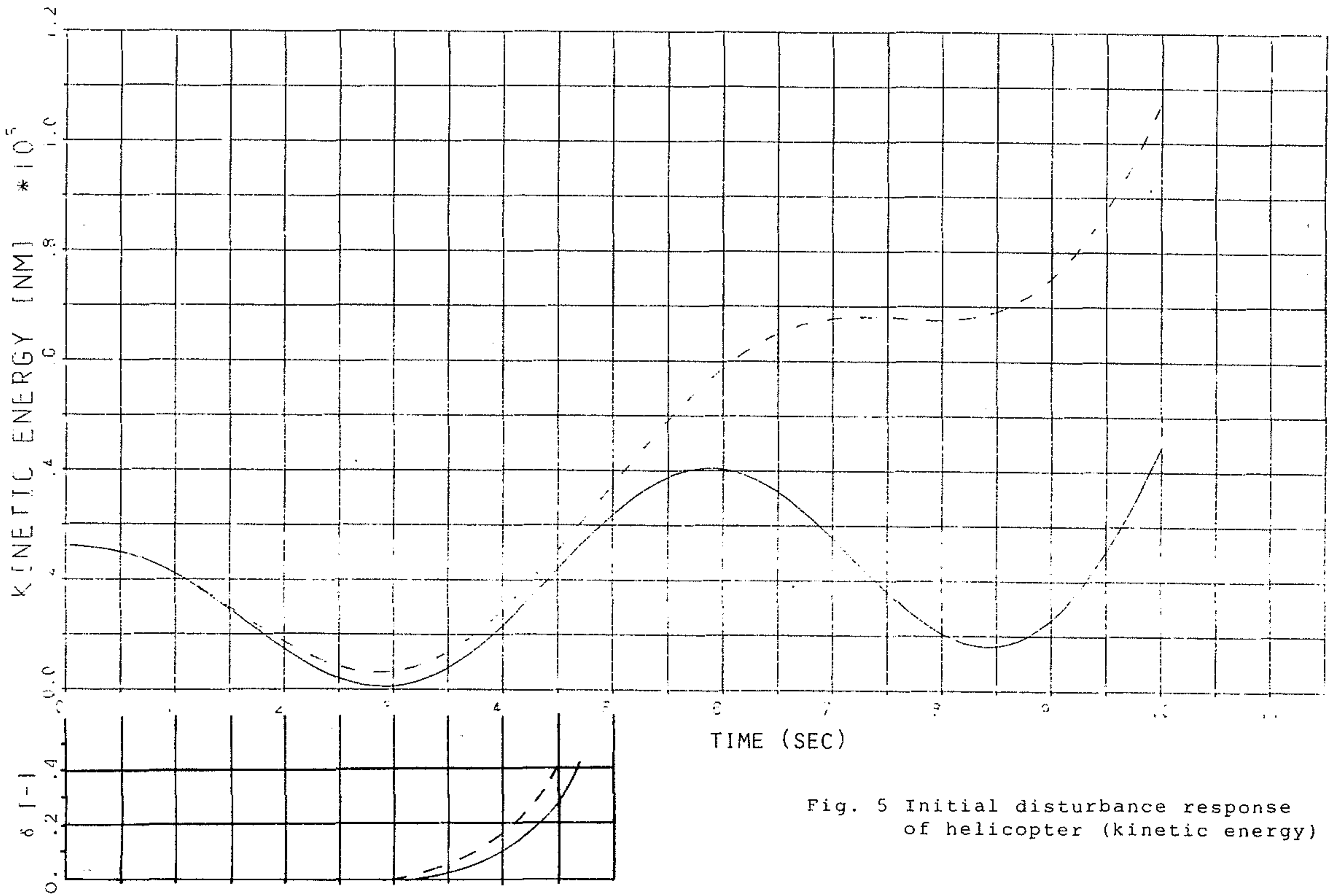

Fig. 5 Initial disturbance response
of helicopter (kinetic energy)