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THE EFFECTS OF SLIGHT NON-LINEARITIES ON MODAL TESTING OF HELICOPTER-LIKE STRUCTURES

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ABSTRACT

A comprehensive series of modal tests has been made on a specially-built test piece designed to simulate many of the characteristics of helicopter structures. These tests identified the presence of a 'slight' degree of non-linearity and further established that this almost-unnoticed effect causes significant discrepancies in the modal properties deduced from measured data. However, systematic analysis of the measurements may be used to isolate the non-linear effects and to eliminate them from the results.

1 INTRODUCTION

Structural vibration continues to present the helicopter engineer with a major problem and, as a result, much effort is directed towards improving analytical prediction capabilities. However, there remains a continued need for corresponding experimental techniques, not only to check theoretical predictions of vibration modes and frequencies but also to provide information on forced response characteristics, including damping effects. These are of particular concern in helicopter structures and are even more difficult to predict than are the major vibration modes.

There are two methods in use for the modal testing of such structures: the so-called "multi-point excitation method" and the "single-point excitation (mobility or receptance) method." The first provides a means of isolating the structure's basic modes of vibration one at a time using several shakers simultaneously while the second employs a simpler experimental setup but requires a more extensive analysis of the measured data in order to extract similar modal information. The work described in this paper relates primarily to the latter (single-point excitation) method although some of the results have wider implications.

One of the advantages of the mobility method is that it offers the possibility of acquiring redundant data, thus permitting cross checks on the final results. Previous work on helicopter structures in which these cross checks were employed (Ref. 1) indicated inconsistencies in the structure's behaviour: the natural frequency and damping of individual modes varying with excitation point, and mode shape data not satisfying orthogonality criteria. More recently, a method of weighted averaging such duplicated results has been proposed in order to produce a single result (Ref. 2), but it is noted that the discrepancies are often larger than can be accounted for by experimental errors.

The construction of a special test structure for studies of finite element prediction capabilities (Ref. 3) presented an opportunity to examine some of these practical difficulties in more detail than had been possible previously. A comprehensive programme of mobility measurements and modal analyses was carried out and the results are discussed below.

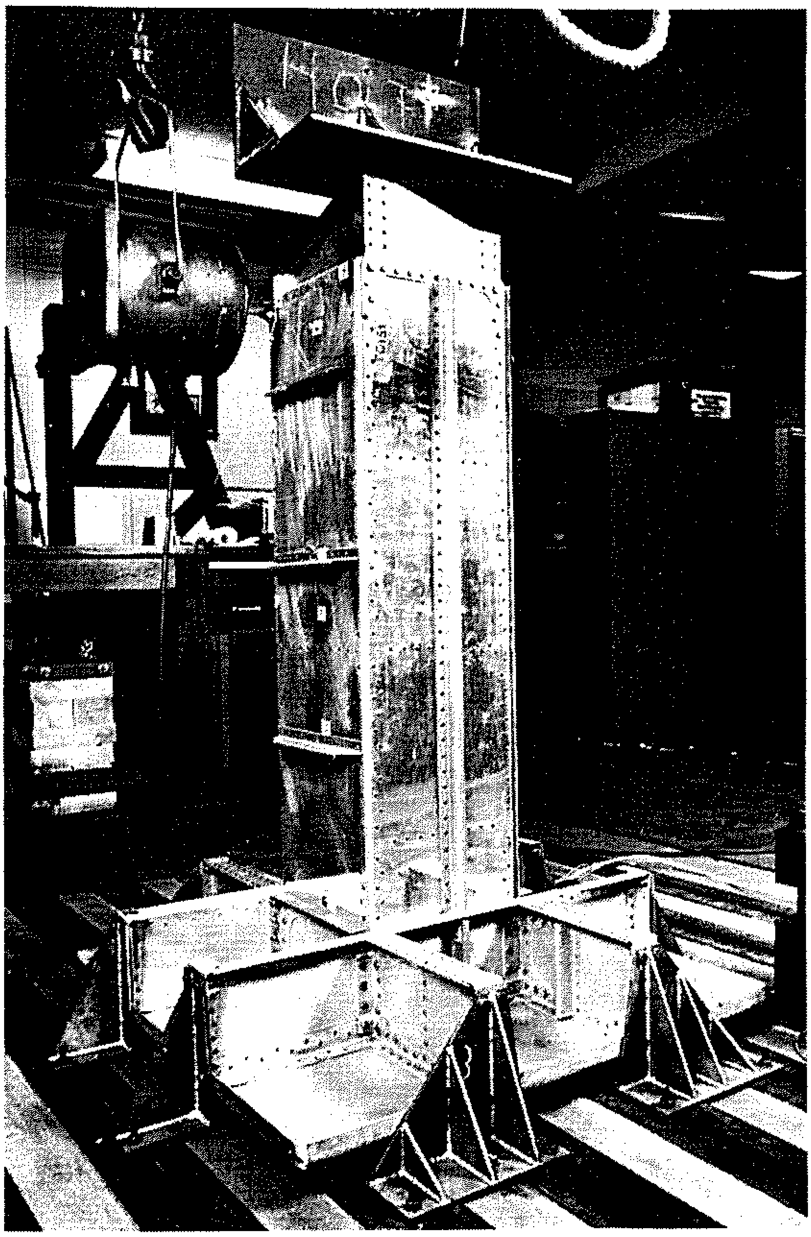

Fig. 1. Test Structure

2 THE STUDY

2.1 Test Structure The structure used in these experiments is shown in Fig. 1. Features of particular interest are: typical lightweight aluminium panels, rivetted together; a composite panel on one side; a concentrated mass mounted on the light panels; total asymmetry. The design and its analysis by finite elements are discussed in detail elsewhere (Ref. 3) and only a very small number of the measurements made on it will be reported here as our interest is in the special nature of its behaviour and this is clearly demonstrated by any of the measured data.

2.2 Test Method It is appropriate to summarise here the modal test method used. The basic principle employs the fact that, given N points of interest on a structure, an N x N matrix of frequency response functions - such as mobilities or receptances - can be constructed and any of these can, generally, be measured. It is theoretically sufficient to measure and to analyse just one row (or column) of this matrix in order to extract the modal properties for all the modes encompassed by the frequency range of the measurements. Thus, N mobility measurements covering M modes, subjected to modal analysis, can yield M natural frequencies and damping factors plus M mode shape vectors each describing the mode's shape in N coordinates.

Even within this minimum set of measured data, there is redundant information as each one of the N mobilities yields values for natural frequency and damping of all the modes. However, only one estimate is obtained for each mode shape for this basic set. A wider cross check becomes available if some of the remaining $(N^2 - N)$ mobilities are measured since in this way duplication of mode shape estimates is obtained.

Methods for the modal analysis of measured mobility data centre around the curve-fitting of a multi-degree of freedom system frequency response function to measured values. Here, we employ a relatively simple application of this method in which just one degree of freedom is considered at a time, while analysing data localised around an individual resonance. This process leads to curve-fits such as that shown by the solid line in Figs. 2 and 3. The modal data thus derived can be refined by using a more general analysis, leading to the results such as those shown by the broken lines in Figs. 2 and 3. These analysis methods are described in detail in Ref. 4.

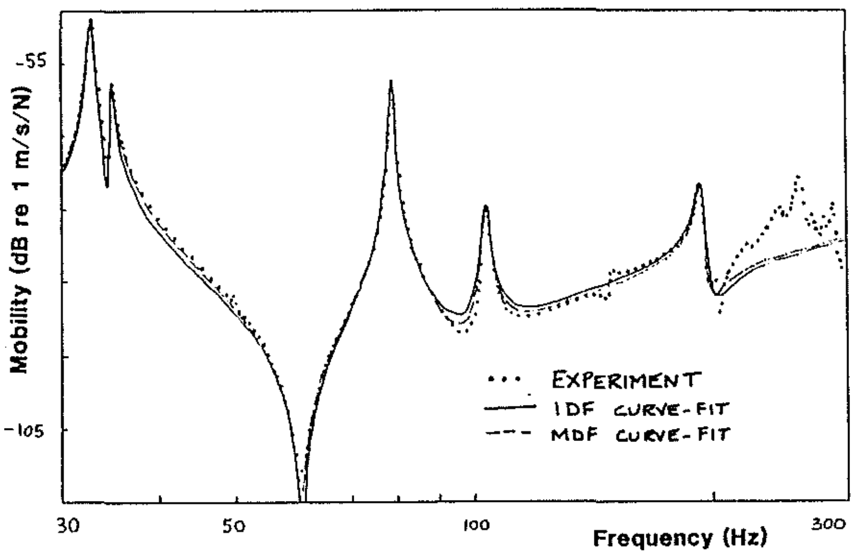

Fig. 2. Typical Curve-Fits to Measured Data

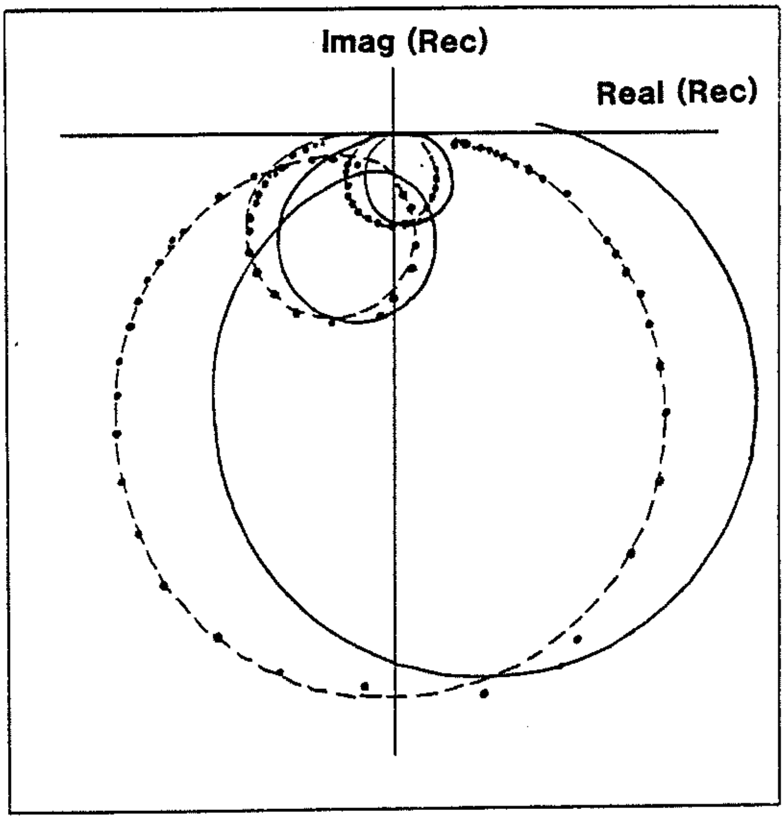

Fig. 3. Typical Curve-Fits to Measured Data

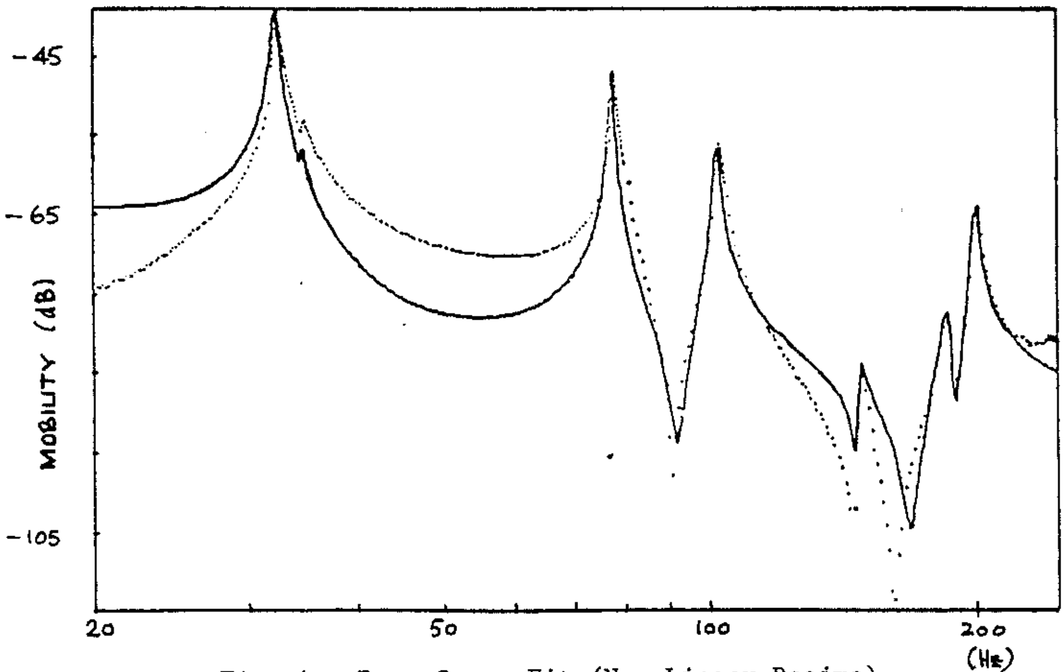

Fig. 4. Poor Curve-Fit (Non-Linear Regime)

3 RESULTS

3.1 Modal Survey Results The results shown in Fig. 2 (and 3) represent some of the better results from the complete modal test programme. They exhibited the by-now familiar trend of small variations in natural frequency (and sometimes more significant variations in damping) from mobility to mobility. However, there were other measurements which were less satisfactory, such as that shown in Fig. 4, in which an acceptable curve-fit could not be obtained. Furthermore, some repeat measurements made after a period of several months produced markedly different modal properties from those obtained in preliminary tests. The problems identified were:

- non-negligible frequency and damping variation;
- significant variations in mode shape amplitudes;
- highly complex modes;
- poor curve-fits and low quality factors.

3.2 Linearity Checks Recognition of these problems instigated some checks on the linearity of the structure since the only known difference between the various tests was the level of the vibration amplitude during measurement. A series of measurements, each with a constant forcing level, were made in the vicinity of one particular mode, and the results are shown in Fig. 5 with the excitation level varying over a range of 20:1. The results from analysing such a set of results (similar ones were found for every mode) are shown in Table 1.

Force Level (Oscillator) Volts)	Nat Freq (Hz)	Loss Factor	Modal Constant	
			Modulus (1/kg)	Phase (°)
0.05	79.165	.00689	5.154 E-3	-148.4
0.1	78.910	.00687	4.579 E-3	-134.1
0.2	78.665	.00631	3.821 E-3	-130.7
0.4	78.335	.00566	3.136 E-3	-122.3
0.6	78.110	.00557	2.932 E-3	-129.8
0.8	77.860	.00528	2.609 E-3	-121.7
1.0	77.615	.00718	3.207 E-3	-120.2

Table 1

The most significant result here is the 2:1 ratio of estimates for the modal constant - the parameter which describes the mode shape - and in the doubling of its degree of complexity (from 32° to 60° away from a 'real' mode shape). This last result is illustrated clearly in Fig. 6 which shows just two of the family of mobilities shown earlier, indicating the circle-fit results and illustrating clearly the marked difference in the apparent complexity of the mode under study (the modal complexity is given by the rotation of the major diameter). For well-separated modes such as these, a high degree of complexity is not expected and it is concluded that the non-linearity of the structure causes the resonant mobility data to suggest - falsely - markedly complex behaviour. It is further concluded that the lowest excitation levels encourage more nearly-linear behaviour in the structure and, to confirm this, a remeasurement and analysis of the poor result shown earlier (Fig. 4), this time at a lower level of vibration, resulted in a much more satisfactory conclusion (Fig. 7).

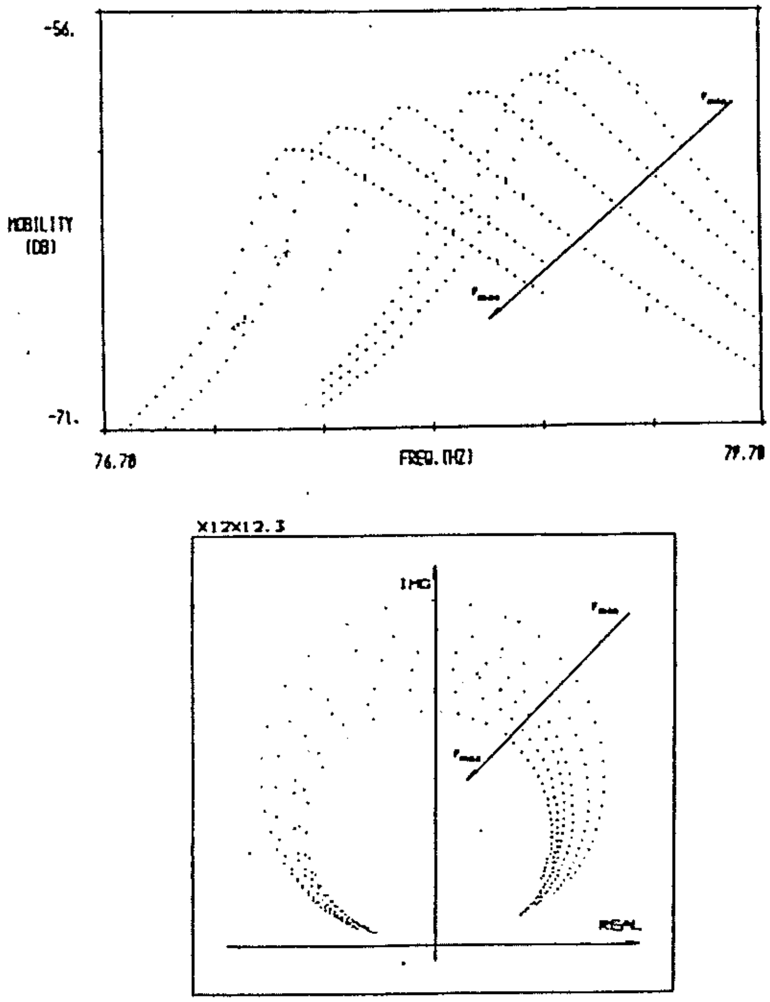

Fig. 5. Linearity Check: Measured Data

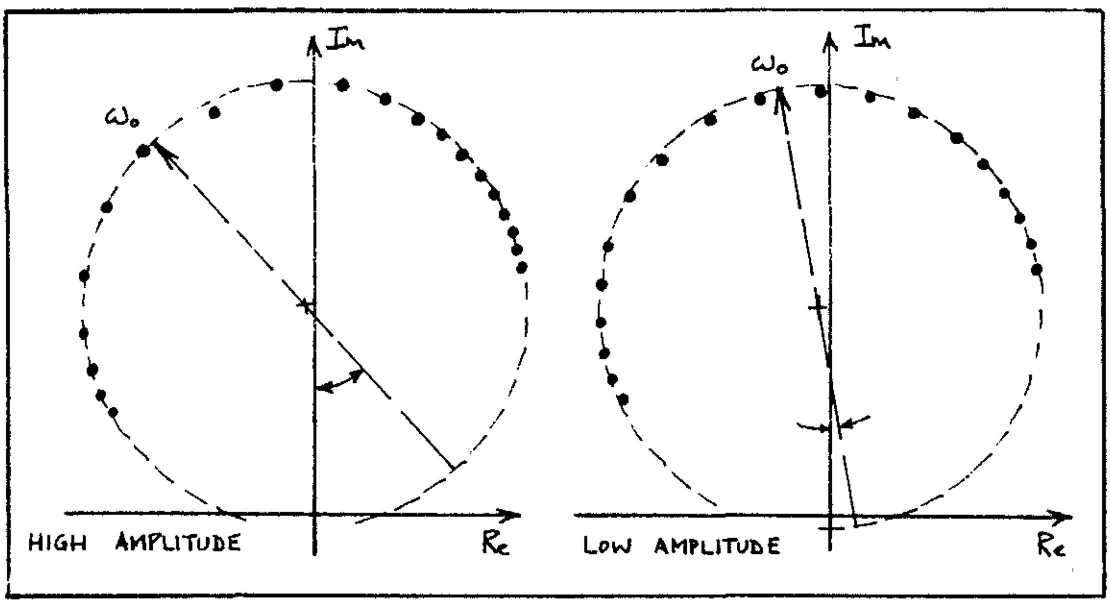

Fig. 6. Modal Analysis of Linearity Check Data

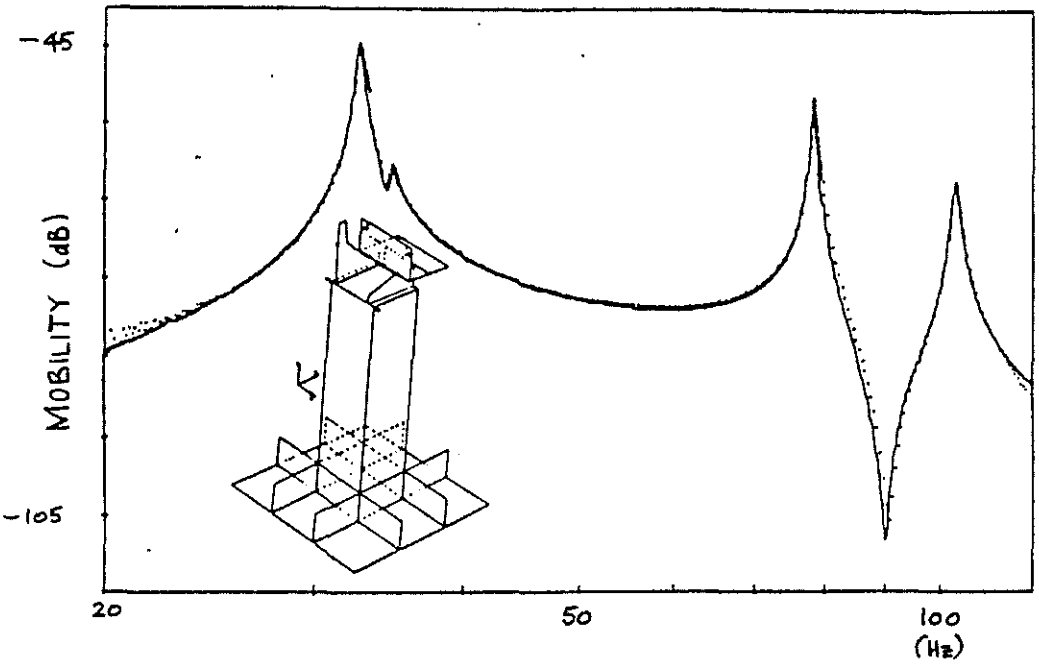

Fig. 7. Improved Curve-Fit (Linear Regime)

4 ANALYSIS FOR NONLINEAR BEHAVIOUR

4.1 Multiple-Level Measurements Clearly, repeating mobility measurements at various levels of excitation provides data suitable for analysis of nonlinear behavior although to be useful, these should strictly be obtained at constant vibration amplitudes (since most forms of non-linearity are amplitude-dependent). The sets of data which result from this approach (such as that shown in Table 1) may be scrutinised for trends, especially asymptotic ones, as these will probably indicate how the structure's analysis should proceed.

4.2 Single Measurement However, the measurement of several curves for each of the many different mobility parameters required is an expensive process and there is considerable incentive to seek some means of identifying non-linear behaviour from a single measurement. Theoretical studies have been made of a single-degree-of-freedom system with various types of non-linearity included, although interest in these studies has been confined to 'slight' non-linearity, where the effect is not immediately apparent in a single mobility curve, such as Fig. 4.

The studies are reported in detail elsewhere (Refs. 5, 6) and we shall describe here two of the cases most relevant to this application, these being: (i) coulomb friction damping, and (ii) cubic stiffness. It is found that the effects of each of these forms of non-linearity, even when present only to a slight extent, has a discernable effect on the Nyquist plot of receptance or mobility. Coulomb friction causes the shape of the receptance plot to distort from its pure circular form although the symmetry about the natural frequency, and the degree of modal complexity, are unaffected. Cubic stiffness, by contrast, does not disturb the circular form at all but it does cause the individual points to be repositioned significantly, as shown in Fig. 8 for a softening spring, in such a way that the resulting plot is almost indistinguishable from that for a linear but highly complex mode. However, a more critical examination of these plots reveals another, detectable, distortion which we can use.

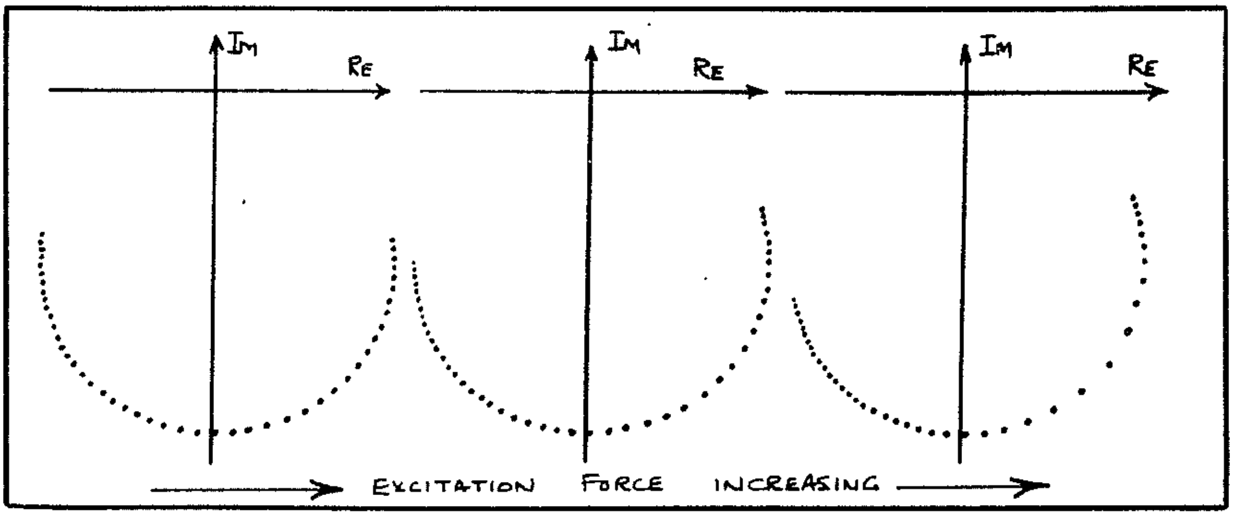

Fig. 8. Theoretical Non-Linear System Response

4.3 <u>Damping Estimates</u> It is common practice to extract from the analysis of resonant data a single value for the damping in that mode, but in fact it is possible to compute several individual estimates, the average of which is the final, single, answer. Close inspection of the individual damping estimates provides some insight into the linearity of the test structure. The method of calculating one such estimate is illustrated in Fig. 9, and uses three frequencies ω_1 , ω_o, and ω_2 and their relative orientation, ϕ_1 and ϕ_2. If we choose ϕ_1 and ϕ_2 to be (approximately) equal, and plot the resulting damping estimate against ($\phi_1 + \phi_2$), we obtain results of the form shown in Fig. 10. For a perfectly linear system, the damping estimate is independent of which points are chosen, whereas for a non-linear system, the estimate depends heavily on the choice, and in a systematic fashion.

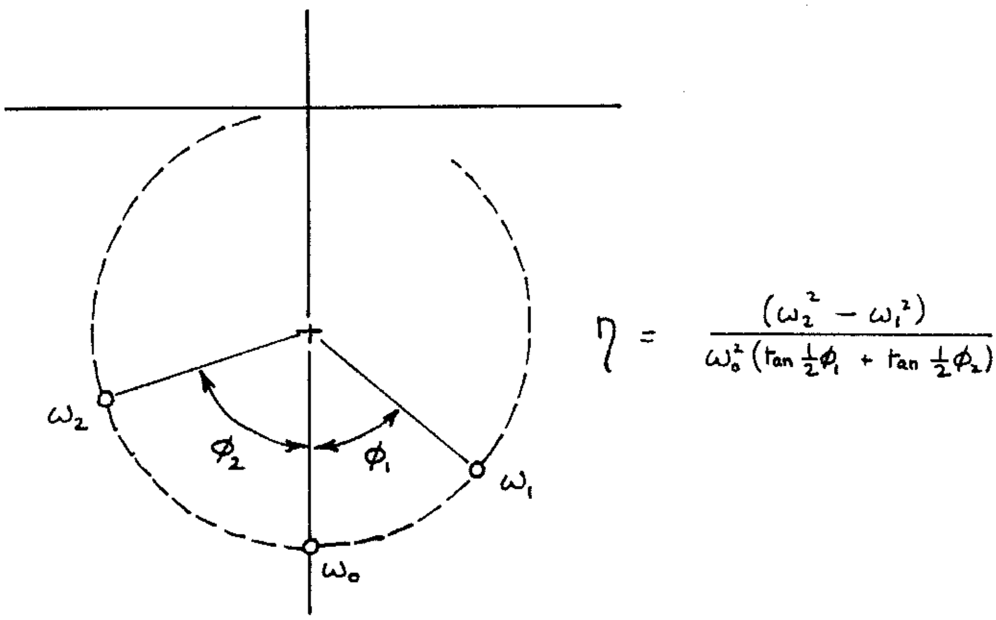

Fig. 9. Damping Estimation Method

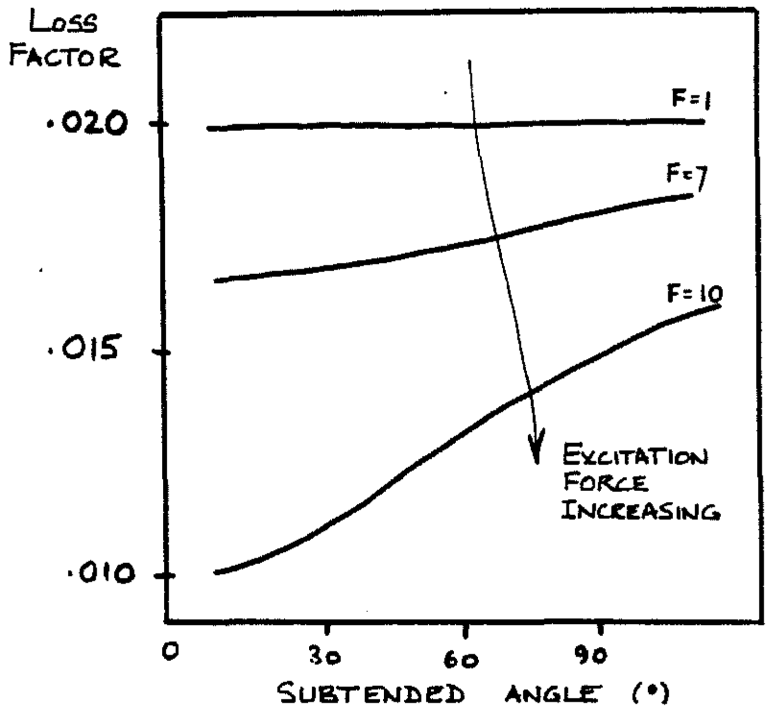

Fig. 10. Damping Estimates for Non-Linear System

4.4 Detailed Analysis of Measured Data The results shown in Figs. 8-10 refer to theoretical single-degree-of-freedom systems. Some results from performing a detailed analysis on data measured on the test structure are shown in Fig. 11. While these results are (understandably) more complicated than those in the previous figure, it is clear that they exhibit a systematic trend which is very similar to that of a softening cubic stiffness spring.

The single "average" damping factor which would result in each case clearly covers a wide range of values and only where the spread of the estimates produced by one measurement is small (represented by a horizontal line in Figs. 10 or 11), can the average value be relied upon, and the system considered to be effectively linear.

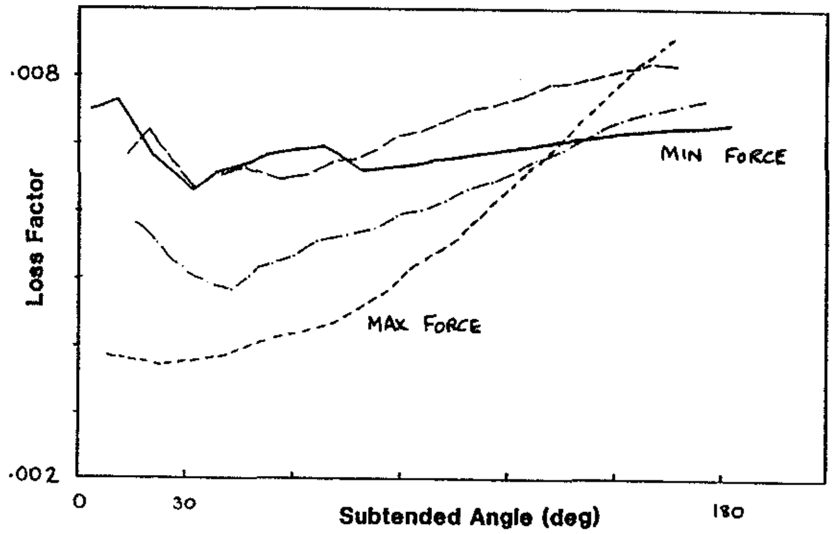

Fig. 11. Damping Estimates for Test Structure

5 CONCLUSIONS

5.1 The application of modal testing techniques to helicopter-like structures often leads to inconsistencies in modal properties which cannot be attributed to experimental inaccuracies.

5.2 Closer inspection of measured data may reveal the presence of slight non-linearity in the structure's behaviour and this can upset the modal analysis process.

5.3 Study of theoretical models with non-linear elements suggests methods of analysing measured data in greater detail to identify and quantify non-linearities.

5.4 Close examination of measured data reveals distinct signs of the trends predicted for theoretical models and confirms that a single mobility curve may be used to detect non-linearity and to distinguish its effect from that of a complex, but linear, mode.

6 ACKNOWLEDGMENTS

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7 REFERENCES

1. D. J. Ewins, J. M. M. Silva — Vibration Analysis of a Helicopter plus an externally-attached Structure. S. Vib. Bull. 50(2), 155-171, 1980

2. J. Kirshenboim — A Method for the Derivation of Consistent Modal Parameters from Several Single-Point Excitation Tests. Imp. Coll. London, Dynamics Grp. Rept. 8010, 1980

3. G. M. Venn, D. J. Boon — A Study of the Techniques of Dynamic Analysis of Helicopter-type Structures. 7th Eur. Rotorcraft & Powered Light Aircraft, 1981

4. D. J. Ewins, J. Kirshenboim — On the Modal Identification of Practical Structures (in preparation)

5. G. R. Tomlinson, J. H. Hibbert — Identification of the Dynamic Characteristics of a Structure with Coulomb Friction. J. Sound Vib., 64 (2), 233-242, 1979

6. J. Kirshenboim — The Effect of Small Non-linearities on the Shape and Modal Analysis of Polar Response Loci. Imp. Coll. London, Dynamics Grp. Rept. 7914, 1979