OPERATIONAL MODAL ANALYSIS OF A LIGHTWEIGHT HELICOPTER TAIL

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Abstract: Modal parameters extraction is considered essential in the development process of a new aircraft. In the case of helicopters where reasonable modal parameters can be obtained in laboratory tests with a classic impact testing approach, it is also important to know what happens to these parameters under operating conditions.
Operating conditions mean, among other things, the possibility to take into account the correct modal contribution of the rotor, that can affect significantly the helicopter dynamics.

In order to do this, before spending time and resources in flights, it was decided by Acoustic & Vibration Department of AgustaWestland to validate the operational modal analysis Balanced Realization algorithm (BR), included in LMS Cada-X Operational Modal Analysis module, with a significant test case. The test case consists in the real structure of a lightweight helicopter tail.

The aim of this paper is to investigate the effectiveness of the operational modal analysis BR algorithm in determining the modal parameters of a helicopter component under different load conditions: from the ideal case (white noise excitation) to real flight data, highly harmonic. Then a comparison with classic impact testing results was performed, showing a very good correlation in terms of frequency, mode shapes and damping.
The next scheduled step will be a operational modal analysis of a helicopter under operating conditions.

1 INTRODUCTION

Frequency response function measures in laboratory conditions have been for several years the only useful tool to build the modal model of a complex structure.
Nowadays many commercial operational modal analysis (OMA) algorithm help engineers to better understand the dynamic behaviour of complex structures under operating conditions.
AgustaWestland Acoustic & Vibration Department, as a first step, decided to perform an experimental campaign to validate the LMS Cada-X BR algorithm on a helicopter component.
In order to validate BR algorithm, OMA was performed on a lightweight helicopter tail and the results compared with a classical experimental modal analysis (EMA) model.

1.1 Description of the structure

The lightweight helicopter tail is composed of aluminum and the dimensions are shown in Figure 1. It was chosen, as a significant test case, for the following reasons:

- it has an important role in the dynamic behaviour of the helicopter;
- this component can be considered approximately as a linear structure;
- the tail has a significant mass (about 40 kg) then the effect of the sensors during tests can be considered negligible;
- it’s easy to move, then many tests with different excitations can be performed.

The tail was in bare condition, without tail rotor and gear box components and also without tail planes.

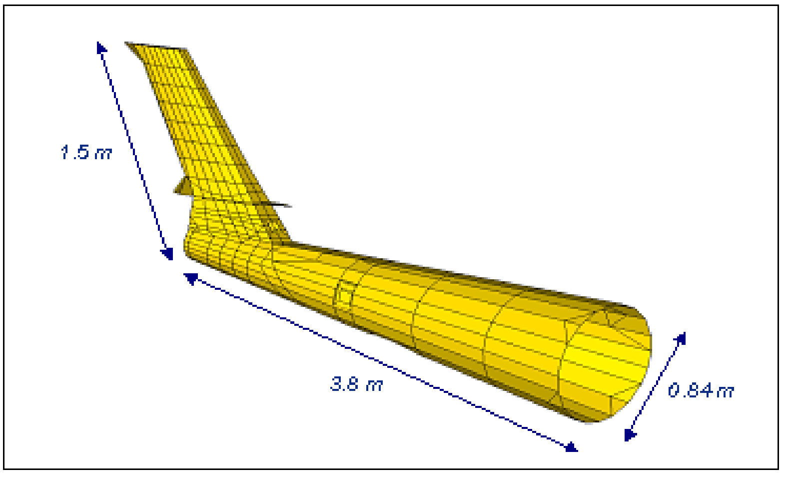

Figure 1: Geometry of the tail

1.2 Geometry

The geometry of the tail was reduced to a wireframe consisting of 30 nodes and 19 connections (see Figure 2).

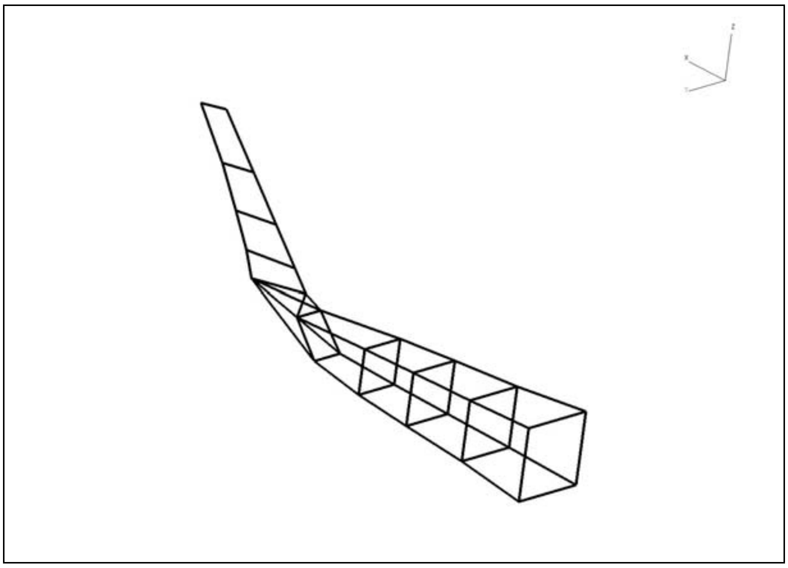

Figure 2: Wireframe

1.3 Experimental modal analysis

A classic experimental modal analysis (Figure 3) was performed in order to extract the modal parameters to compare with operational modal analysis results.

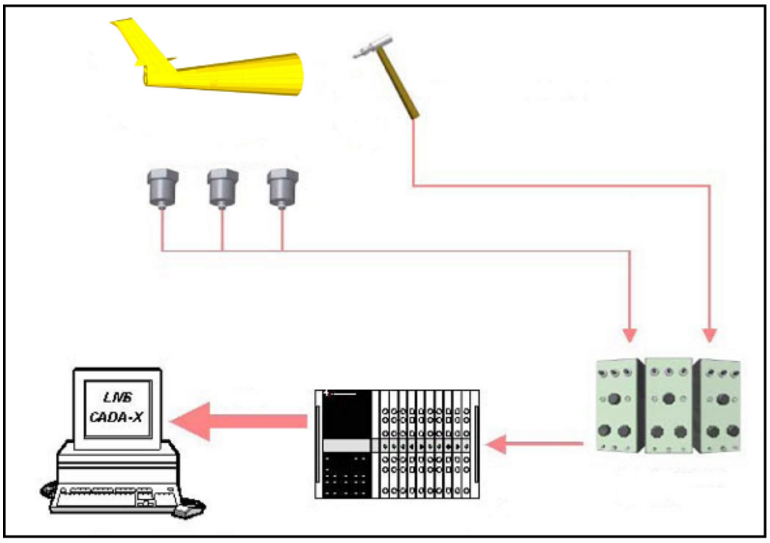

Figure 3: Sketch of experimental modal analysis

The tail was suspended in free-free condition; four driving points (Figure 4) and five sets of accelerometers were used.

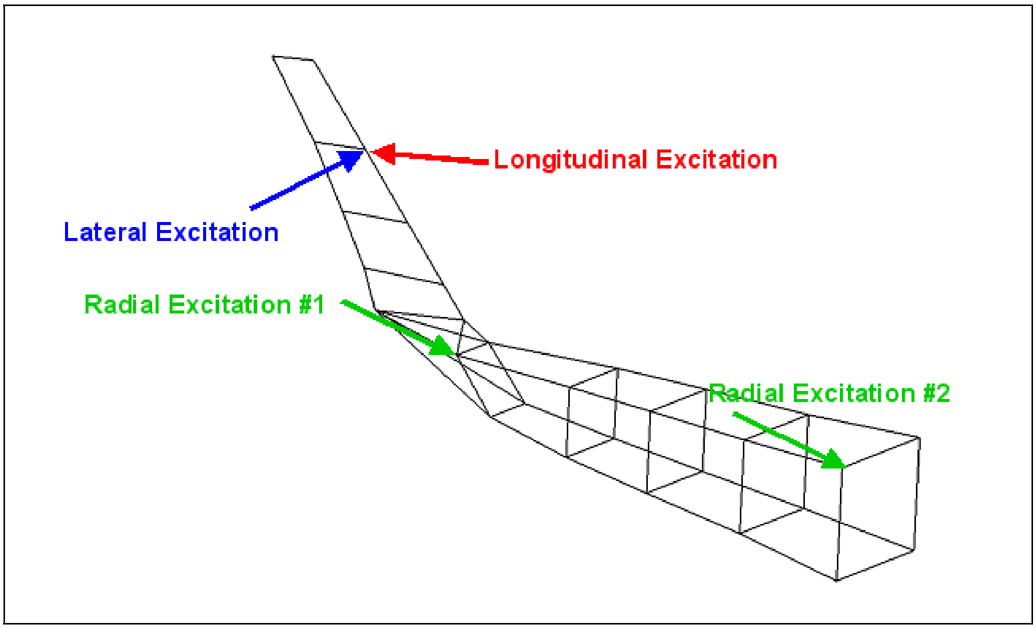

Figure 4: EMA driving points

In Table 1 are listed the first four modes, their frequencies, damping and a brief description of the mode shape.

N	Freq (Hz)	Damp (%)	Shape
1	64.3	1.17	1st lateral fin
2	92.8	0.70	1st vertical tail
3	111.2	0.73	1st lozenge
4	120.7	0.40	2nd lateral fin

Table 1: Experimental Modes

The table above shows a substantial separation in terms of frequency of the four modes extracted: this aspect simplify the extraction procedure in EMA model and will be helpful especially in OMA method.
In figure 5 are shown the shapes of the first two modes, respectively at 64.3 and 92.8 Hz.

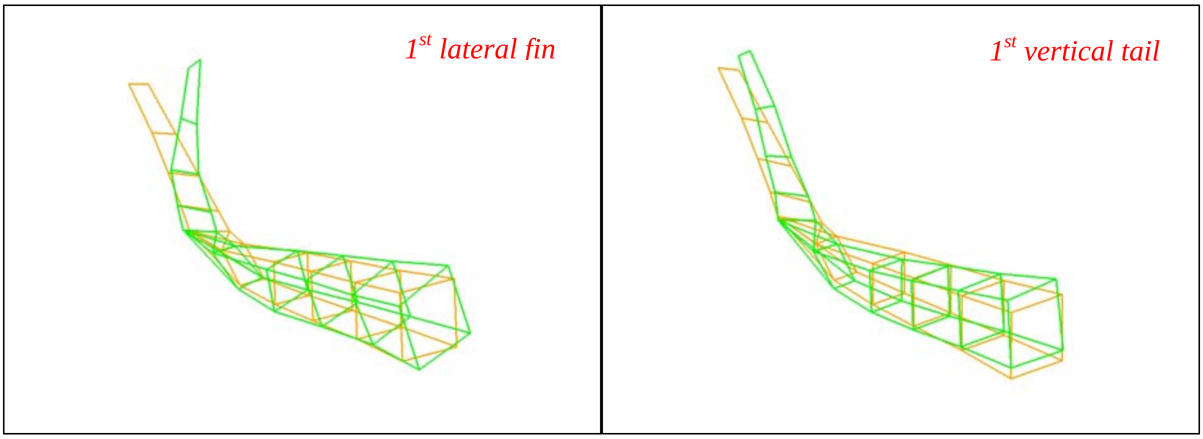

Figure 5: First two modes extracted in EMA procedure

1.4 Operational modal analysis theory

Operational modal analysis, called also output-only analysis, is a dynamic characterization technique employed when the loads applied to the structure are unknown; in fact, in these cases the analysis based on the FRF signals can not be performed.
The Balanced Realization algorithm (BR) belongs to Stochastic subspace methods, that solve the matrix realization problem by exploiting the correlation function in subspace systems. The identification problem and his resolution by means of Stochastic subspace methods can be express in the following way (see Ref. [1]).

Consider the stochastic discrete time state space model below:

$$\begin{aligned} \{x_{k+1}\} &= [A]\{x_k\} + \{w_k\} \\ \{y_k\} &= [C]\{x_k\} + \{v_k\} \end{aligned} \tag{1}$$

where $\{x_k\}$ represents the state vector of dimension n,
$\{y_k\}$ is the output vector of dimension N,
$\{w_k\}$, $\{v_k\}$ are zero-mean, white vector sequences, which represent the process noise and measurement noise respectively,
$[A]$, $[C]$ are respectively the state matrix and the output matrix.

The observability matrix $\lfloor O_p \rfloor$ of order p and the controllability matrix $\lfloor C_q \rfloor$ of order q are defined:

$$\lfloor O_p \rfloor = \begin{bmatrix} [C] \\ [C][A] \\ \vdots \\ [C][A]^{p-1} \end{bmatrix} ; \qquad \lfloor C_q \rfloor = \lfloor [G] \quad [A][G] \quad \cdots \quad [A]^{q-1}[G] \rfloor \tag{2}$$

where $[G] = E\lfloor \{x_{k+1}\}\{y_k\}^T \rfloor$ and E denotes the expectation operator. The matrices $\lfloor O_p \rfloor$ and $\lfloor C_q \rfloor$ are assumed to be of rank $2N_m$, where N_m is the number of system modes.

Given a sequence of correlations:

$$[R_k] = E\lfloor \{\mathrm{y}_{\mathrm{k+m}}\}\{\mathrm{y}_{\mathrm{m}}\}_{ref}^T \rfloor \tag{3}$$

where $\{y_k\}_{ref}^T$ is a vector containing N_{ref} outputs serving as references.
For $p \geq q$, let $\lfloor H_{p,q} \rfloor$ be the following block-Hankel matrix:

$$\lfloor H_{p,q} \rfloor = \begin{bmatrix} [R_1] & [R_2] & \cdots & \lfloor R_q \rfloor \\ [R_2] & [R_3] & \cdots & [R_{q+1}] \\ \vdots & \vdots & \ddots & \vdots \\ [R_p] & [R_{p+1}] & \cdots & [R_{p+q-1}] \end{bmatrix} \tag{4}$$

Direct computations of the $[R_k]$ from the model equations lead to the following factorization property:

$$\left[H_{p,q}\right] = \left[O_p\right]\left[C_q\right] \tag{5}$$

Let $[W_1]$ and $[W_2]$ be two user-defined invertible weighting matrices; pre and post multiplying the Hankel matrix with $[W_1]$ and $[W_2]$ and performing a Singular Values Decomposition (SVD) on the weighted Hankel matrix gives the following:

$$[W_1]\left[H_{p,q}\right][W_2]^T = [[U_1][U_2]]\begin{bmatrix}[S_1] & [0]\\ [0] & [0]\end{bmatrix}\begin{bmatrix}[V_1]^T\\ [V_2]^T\end{bmatrix} = [U_1][S_1][V_1]^T \tag{6}$$

where $[S_1]$ contains non-zero singular values in decreasing order, the *n* columns of $[U_1]$ are the corresponding left singular vectors and the *n* columns of $[V_1]$ are the corresponding right singular vectors.

From equations 5 and 10, it can be easily seen that the observability matrix can be recovered, up to a similarity transformation, as:

$$\left[O_p\right] = [W_1]^{-1}[U_1][S_1]^{1/2} \tag{7}$$

Now the system matrices $[A]$ and $[C]$ can be estimated directly from $\left[O_p\right]$ by means of his own definition (Equation 2).

Balanced Realization algorithm (BR) is obtained from the method above by using the weighting matrices $[W_1]$ and $[W_2]$ equal to identity matrix.

1.5 Operational modal analysis application

Operational modal analysis requires two important hypotheses:

- the perfect linearity of the structure under examination;
- a white noise spectrum as operating condition.

These hypotheses are not completely satisfied in helicopter flight due to helicopter complexity (plays, damping materials applied on the structure) and to the high harmonic content of loads under operating conditions.
The first hypothesis can be considered satisfied for the test case because the tail used was in bare condition.
The tail, as in classic experimental modal analysis, was suspended in free-free condition.

The following loads were applied to the test case by an electromagnetic shaker (Figure 6):

- white noise (which satisfies the second hypothesis of OMA);

- analytical tonal excitation mixed with white noise. The tonal excitation includes the most important tones of the main rotor and tail rotor;

- real flight spectrum.

The driving point, as shown in Figure 6, is located on the fin, near the position employed to acquire the vibration spectrum in flight; during this test 24 accelerometers signals were acquired as time histories, then processed in the LMS Cada-X OMA module.

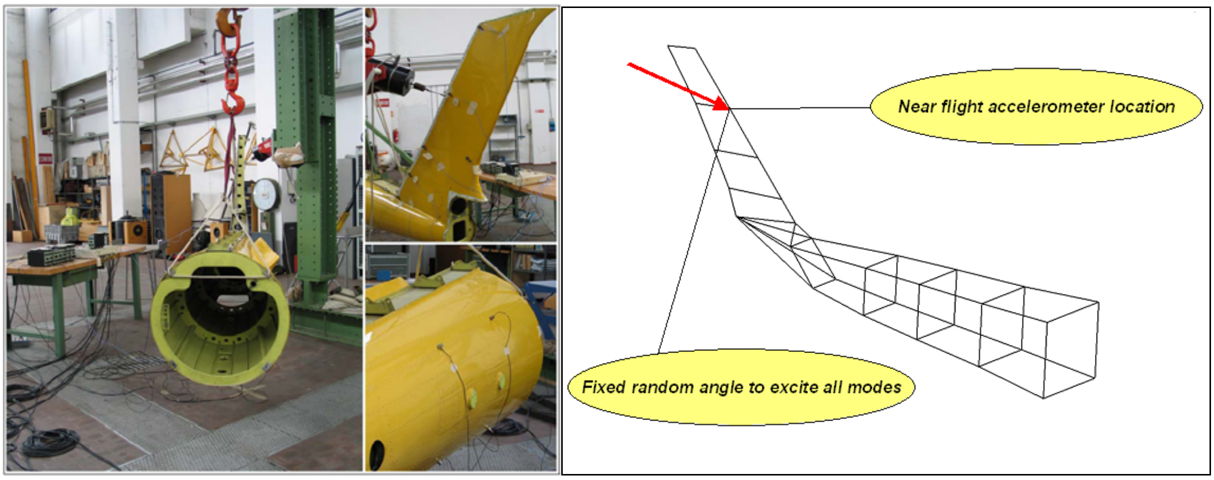

Figure 6: Load application in OMA procedure

1.6 White noise excitation

White noise is the ideal excitation for operational modal analysis. In this condition all modes are well excited in order to be identified in the analysis phase.
A time history with white noise content in WAVE format was used to drive the electromagnetic shaker.
The results are shown in Table 2.

N	Freq (Hz)	Damp (%)	Shape
1	65.5	2.89	1st lateral fin
2	93.9	0.77	1st vertical tail
3	111.9	0.62	1st lozenge
4	121.2	0.44	2nd lateral fin

Table 2: Operational Modes - White noise excitation

Figure 7 shows the MAC matrix between EMA model and OMA model with white noise excitation. All modes are well correlated in terms of frequency, damping and mode shape.

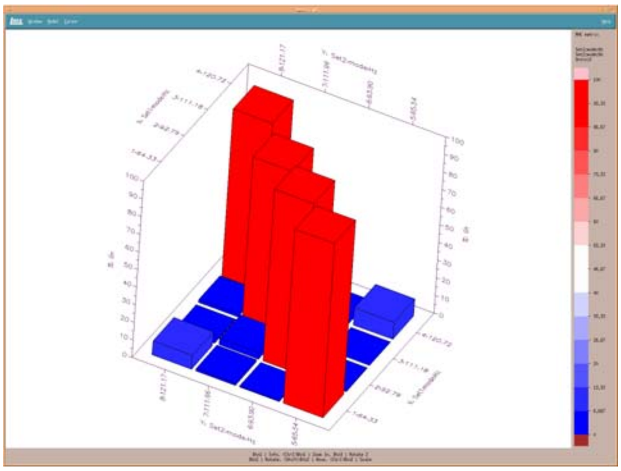

Figure 7: MAC matrix between EMA and OMA - White noise excitation

1.7 Tonal excitation mixed with white noise

Tonal excitation was mixed with white noise in a time history in order to obtain a signal representative of the helicopter flight.
The tones relative to 2,3,4 main rotor revolution and 2,4 tail rotor revolution were added.
The load spectrum is shown in Figure 8 and the results of OMA are shown in Table 3.

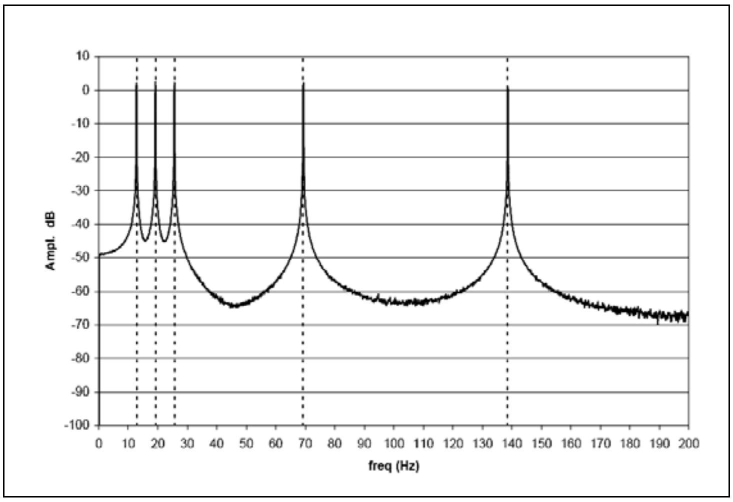

Figure 8: Load spectrum - Tonal excitation

N	Freq (Hz)	Damp (%)	Shape
1	65.0	2.69	1st lateral fin
2	93.4	0.30	1st vertical tail
3	111.7	0.15	1st lozenge
4	122.1	0.90	2nd lateral fin

Table 3: Operational Modes - Tonal excitation

In Figure 9 is shown the MAC matrix between EMA model and OMA model with tonal excitation mixed with white noise. All modes are well correlated in terms of frequency, damping and mode shape.

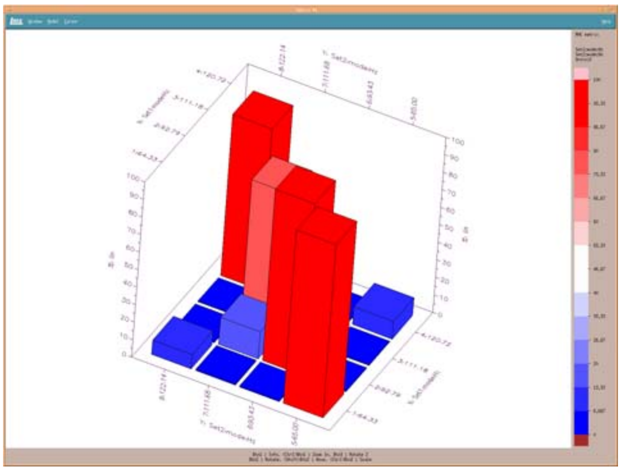

Figure 9: MAC matrix between EMA and OMA - Tonal excitation

1.8 Real flight spectrum

The signal used to drive the shaker is a real accelerometer response (time history) acquired in flight on the fin of the lightweight helicopter.
The real flight spectrum is shown in Figure 10 and the results of OMA in Table 4.

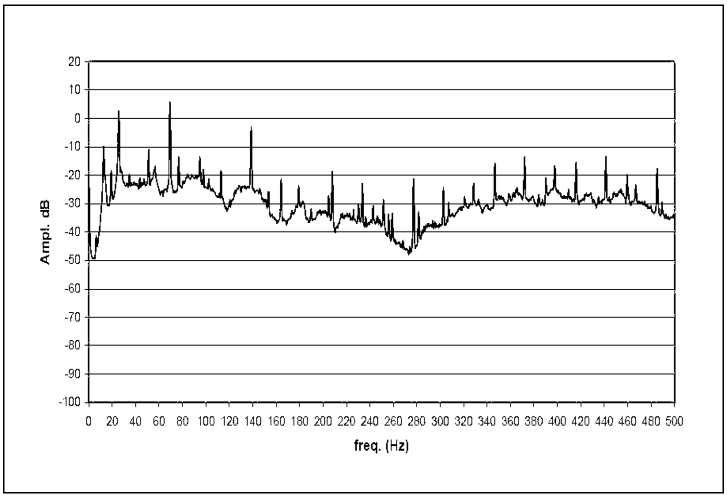

Figure 10: Load spectrum - Flight excitation

N	Freq (Hz)	Damp (%)	Shape
1	64.9	2.12	1st lateral fin
2	91.9	1.13	1st vertical tail
3	112.3	0.70	1st lozenge
4	120.5	0.44	2nd lateral fin

Table 4: Operational Modes - Flight excitation

Figure 11 shows the MAC matrix between EMA model and OMA model with real flight spectrum of excitation. The first four modes are very well correlated.

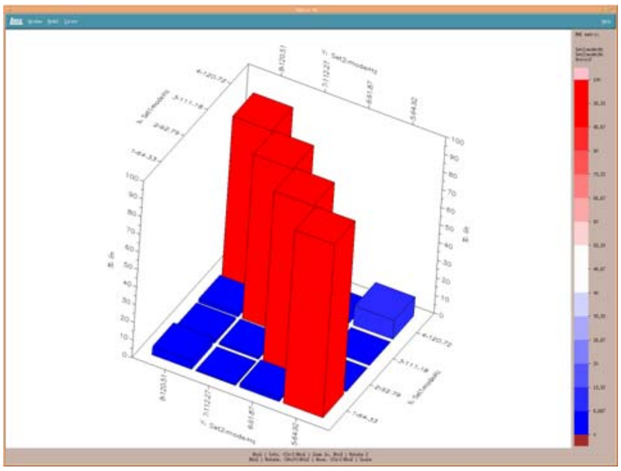

Figure 11: MAC matrix between EMA and OMA - Flight excitation

1.9 Conclusions

In this work the effectiveness of the operational modal analysis BR algorithm in determining the modal parameters of a helicopter component was investigated under different load conditions: from the ideal case (white noise excitation) to real flight data, highly harmonic. A comparison with classic impact testing results was performed, showing a very good correlation in terms of frequency, mode shapes and damping.
The BR algorithm can be considered validated in laboratory tests, then the next scheduled step will be a campaign to validate the algorithm on a complete helicopter in different flight conditions.

2 REFERENCES

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