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THE PROBLEM OF CALCULATION OF THE FLOW AROUND HELICOPTER ROTOR BLADE TIPS

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Abstract

There are special problems with the theoretical analysis of the flow around helicopter rotor blade tips that are based on the fact that the flow is three-dimensional, unsteady, rotational, and on the advancing blade in fast forward flight also transonic.

Some theories for this problem based upon potential flow theory are presented even though the underlying assumptions of this theory are not strictly fulfilled on the rotating blade. Nevertheless, calculations show that this approach can lead to a sufficient discription of the actual flow.

Additionally, computer programmes for the steady three-dimensional and the unsteady two-dimensional flow past fixed wings have been modified for rotating rotor blades. A few results are shown.

List of Symbols

a	sonic velocity	u	velocity in x-direction
AR	aspect ratio	v	velocity in y-direction
b	span of the wing	w	velocity in z-direction
c	chord length	x, y, z	coordinates
c_D	drag coefficient	α	angle of attack
c_p	pressure coefficient	ε	twist angle
K	reduced frequency	ψ	blade azimuth angle
M_∞	free stream Mach number	μ	advance ratio
M_T	rotor blade tip Mach number	κ	specific heat ratio
p	pressure	ρ	density
r	local radius	τ	thickness ratio
R	radius of the rotor blade	Γ	vortex strength
s	half span of the wing	Λ	aspect ratio
t	time	ϕ	Potential
U_∞	free stream velocity	Ω	angular velocity

Subscripts

t	partial derivative with respect to time
x, y, z	partial derivatives with respect to the indicated coordinate
ϕ	partial derivative with respect to the potential

1. Introduction

Already in steady forward flight there exist fundamental significant differences in the flow around the wing for fixed-wing and rotary wing aircraft. These differences concern Mach number, angle of attack, and yaw angle distributions that depend upon the radial position along the blade as well as upon azimuth angle in the case of the rotor and are therefore unsteady.

The important parameters are the aerodynamic coefficients which change appreciably when going from the subsonic to the transonic regime. This change means also an influence on the helicopter power, vibration, and noise emission.

2. Basic considerations

Most existing theories are developed for fixed-wing aircraft and are based on the potential equation

$$\phi_{xx} + \phi_{yy} + \phi_{zz} = 0 \qquad \text{incompressible}$$

$$(1 - M_\infty^2)\ \phi_{xx} + \phi_{yy} + \phi_{zz} = 0 \qquad \text{subsonic, compressible, linearized}$$

$$[(1 - M_\infty^2)\ \phi_x - \frac{1}{2}(\kappa+1)\ M_\infty^2 \phi_x^2]_x + \phi_{yy} + \phi_{zz} = 0 \quad \text{transonic, linearized}$$

The conditions are

a) isentropic change of state, i.e. heat exchange and friction are negligible ($\Delta s = 0$)

b) irrotational flow, i.e. rot $\bar{v} = 0$:

$$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$$

$$\omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$$

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

The advantage of using irrotational flow is based on the fact that only one unknown ϕ needs to be determined instead of the three unknown u, v, w. This simplifies the numerical treatment.

For treating rotating wings it is necessary to develop a model as is done for fixed wings.

Consider a plane moving through the air with a given velocity. This motion is unsteady with respect to an earth-fixed observer. For calculation purposes it is possible to make the motion steady by fixing the coordinate system of the observer on the plane. Now the air is seen to move past the plane. That transformation applies for measurements in wind tunnels.

Now consider a rotor blade rotating with the angular velocity Ω. The motion again is an unsteady one. When fixing the coordinate system of the observer at the rotor blade the motion is again a steady one (fig. 1) and the flow is rotating around the rotor axis.

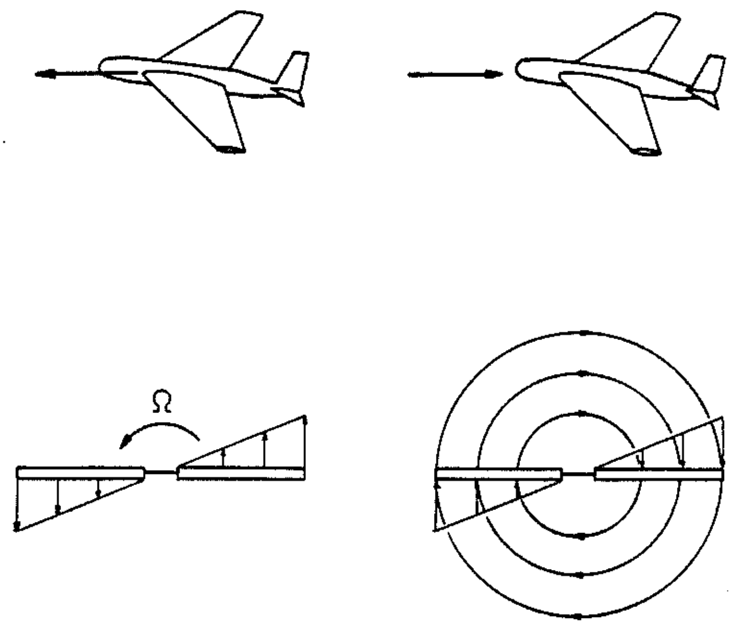


fig. 1 Motion transformation for a fixed wing and a rotor blade

But there is a small physical difference concerning the mass forces not existing for the fixed-wing case. In the unsteady case there are centrifugal forces acting on the blade itself and the adjacent boundary layer. The undisturbed air is at rest and irrotational and therefore there does not exist any gradient in the air.

When considering the steady case the centrifugal forces act on the undisturbed air in the free stream and because of the rotation of the air there exists a pressure gradient as well as a velocity gradient. The boundary layer on the blade itself is not subjected to any centrifugal forces. This motion is called a solid body rotation which is known to be rotational, therefore showing that the motion of a rotor blade is rotational.

The difference of an irrotational and a rotational motion is shown in fig. 2.

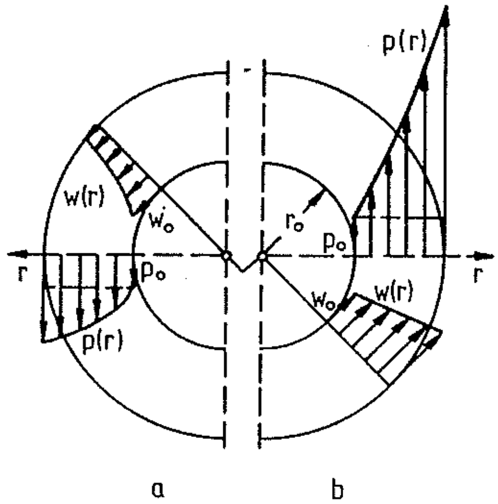


fig. 2 Velocity and pressure gradient of flows on concentric circles [1]
a) w = a/r, irrotational; b) w = a•r, rotational

The problem now is that the theories to be used are irrotational. But there are not any theories known to the author which can treat rotational flow in acceptable computing times.

In the literature most authors use the potential equation for describing the flow around the rotor blade tip. Nevertheless, calculations show that this approach can lead to an acceptable description of the actual flow in the region of the blade tip (fig. 3).

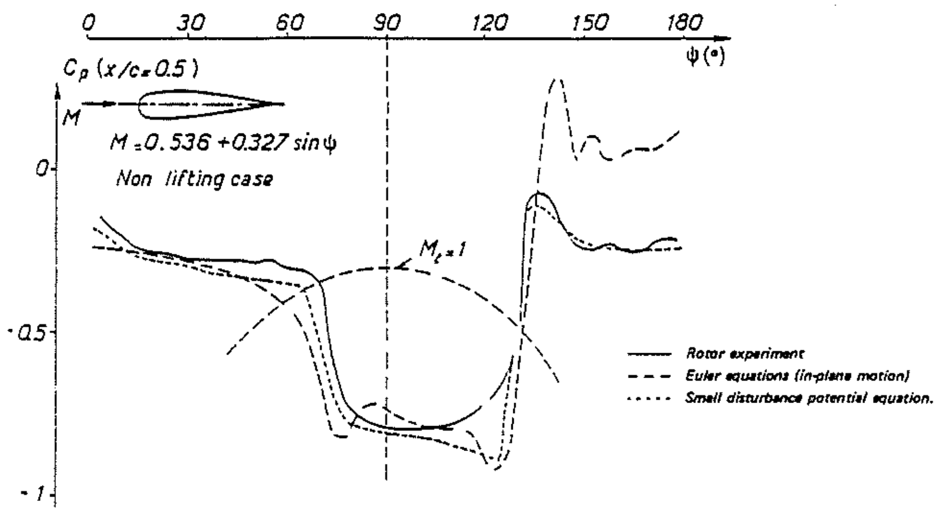


fig. 3 Pressure distribution over a half revolution [2]

3. A Review over some theories for calculating the blade tip effects

3.1 Steady flow calculation for blade tips

This method was developed by Caradonna [3] for calculating the blade tip flow of helicopter rotors in hover. Within the frame of small perturbations the differential equation

$$\left[\frac{1-M_T{}^2y^2}{\tau^{2/3}} - (1-\kappa)\ M_T{}^2 y\ \phi_x\right] \phi_{xx} + \phi_{zz} + \frac{1}{AR^2\tau^{2/3}}\ \phi_{yy} = 0$$

is approximated by means of the finite difference technique.

In fig. 4 this procedure is applied to a rotor blade tip at two different Mach numbers. The shock moves downstream and extends radially with increasing Mach number.

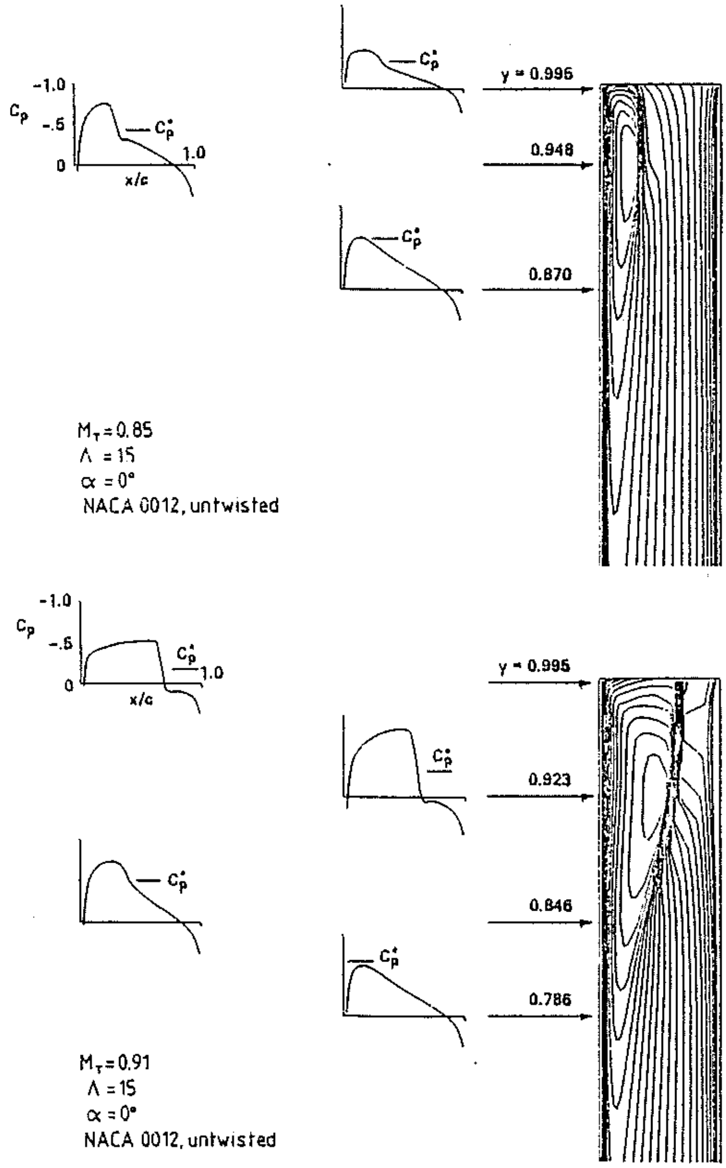


fig. 4 Pressure distributions and isobars on a rotor blade tip

In fig. 5 the drag increase at the blade tip is shown for different Mach numbers.

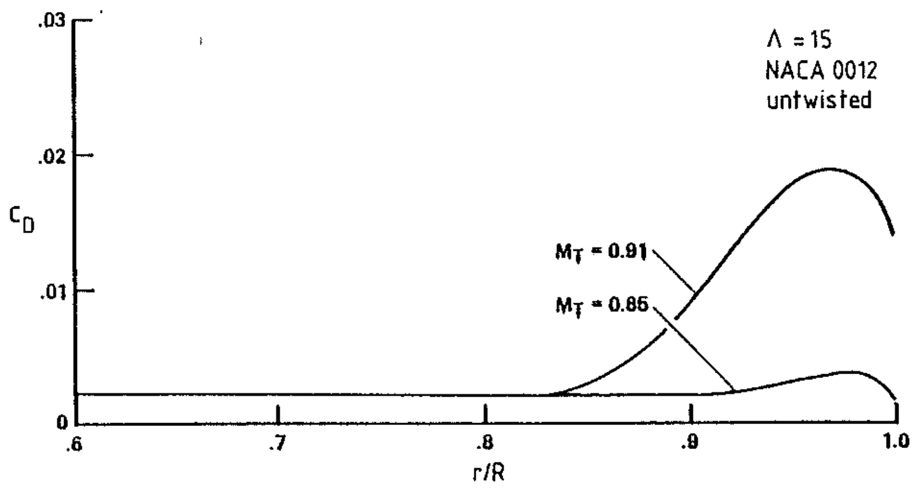


fig. 5 The spanwise drag variation for a non-lifting rotor

The enormous increase of the drag coefficient is seen in the immediate vicinity of the blade tip that is related to the existence of shock waves.

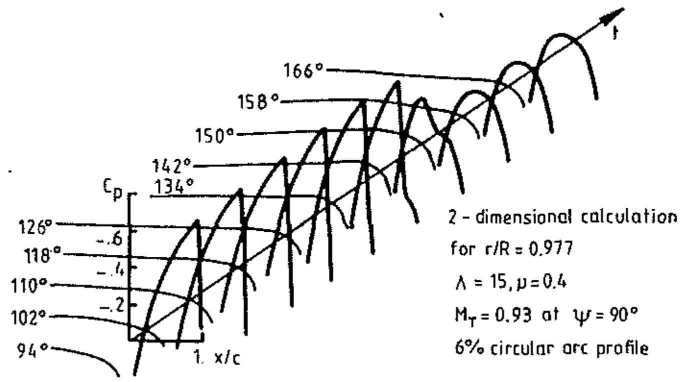
3.2 Unsteady flow calculation for blade tips

In this method the unsteady effects due to forward flight are taken into account. The procedure for unsteady, three-dimensional, non-lifting, non-linear flows for advance ratios $\mu > 0$ has been developed by Caradonna and Isom [4].
The equation

$$\phi_{tt} + \frac{\partial}{\partial t} (\nabla\phi)^2 + \nabla\phi\cdot\nabla \left[\frac{1}{2} (\nabla\phi^2)\right] = a^2\nabla^2\phi$$

defined in the coordinate system at rest is transformed to the blade-fixed coordinate system and solved by the finite difference method.

Fig. 6 shows the influence on the temporal pressure distributions of a blade tip when the calculation is done two-dimensional and three-dimensional.



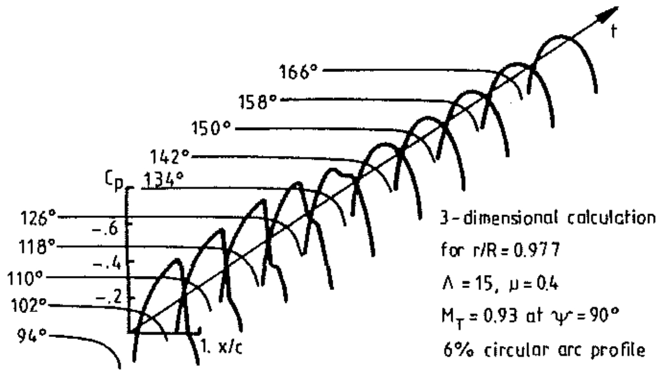


fig. 6 Comparison between two- and three-dimensional calculated pressure distributions

As can be seen the shock waves of the three-dimensional calculation are weaker and decay much faster than in the two-dimensional case.

4. Modifications of computer programmes developed for fixed wings

As already mentioned the choice of a flow model for the rotor creates some difficulties, and also a large computational time. For our problem the programme of Caradonna and Isom would be most suitable. However, because of the computational time programmes of MBB will be used which are developed for fixed-wing aircraft and they are to be modified for rotors.

First of all, the problem will be split up into the steady three-dimensional case and the unsteady two-dimensional case of a fixed wing in § 4.1, 4.2 and then the modifications will be discussed again separately.

4.1 Steady three-dimensional flow past a wing

A programme was developed by Eberle (MBB) [5] for three-dimensional steady flows past fixed-wing aircraft in the transonic flight regime. The Euler equations are transformed to the differential Bernoulli equation by means of the irrotationality condition. By suitable transformations the implicit differential potential equation is obtained which supplies the basis to the variational principle of the aerodynamic potential theory. Introducing the boundary conditions one arrives at the equation

$$\iiint \rho \, (u u_\phi + v v_\phi + w w_\phi) \, dx \, dy \, dz = 0$$

Shocks are generated by means of an artificial viscosity. Because of the condition of small perturbations only weak shocks are admitted. The numerical computation is performed using the finite element method.

In fig. 7 a few chordwise pressure distributions are shown for several spanwise stations near the wing tip at $M_\infty = 0{,}8$ and $\alpha = 0^0$. When decreasing the distance to the tip the shocks become weaker. In this area two-dimensional calculations would provide too high a lift and too strong shocks.

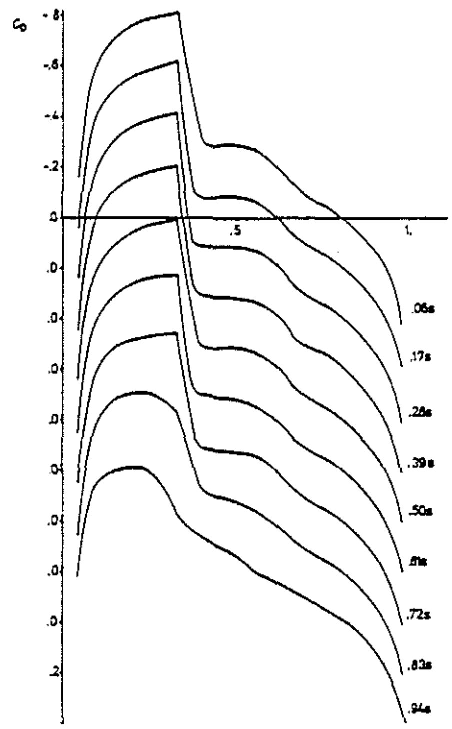


fig. 7 Pressure distributions on a wing for $M_\infty = 0{,}8$ and $\alpha = 0^0$, NACA 0012 profile

4.2 Unsteady two-dimensional procedure

A further programme was developed by Eberle (MBB) [6] for an unsteady flow around a profile with an oscillating flap at the trailing edge. When calculating a steady flow the start of the motion is taken into account.

The transonic potential equation for small perturbations takes the form

$$-M_\infty^2 K^2 \phi_{tt} - 2\, M_\infty^2 K\, \phi_{xt} + [1 - M_\infty^2 - M_\infty^2 (3 + M_\infty^2 (\kappa - 2))\, \phi_x]\quad \phi_{xx} + \phi_{zz} = 0$$

The conservation of mass at sonic lines and shocks is assured by the numerical viscosity. The numerical procedure is the alternating direction implicit method. Again the accuracy can be influenced by the mesh size and also by the number of time steps.

As an example in fig. 8 the starting flow of a steady state case is shown. In three time steps the supersonic pocket builds up and the shock moves into its steady state position.

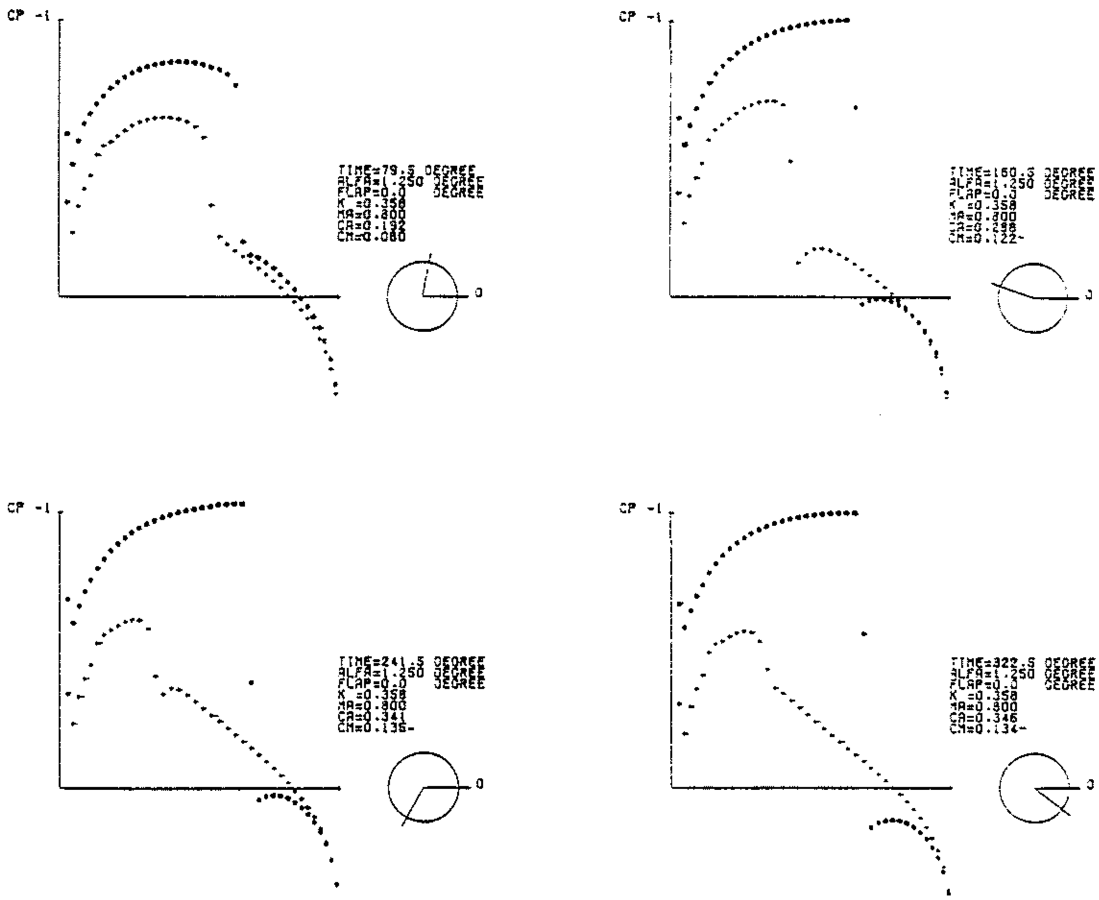


fig. 8 Temporal set up of a steady flow

4.3 The steady flow problem of the rotor

The first step is the investigation of the steady three-dimensional flow problem by a modified programme for the transonic flow past a fixed-wing aircraft.

The following parameters are used as input: the coordinates of the profile of each radial section, the angle of incidence, twist angle, chord length, taper, and Mach number. The data have to be defined for a half wing. The programme generates the whole wing by a reflexion around the x-axis because the geometry and the aerodynamics is symmetrical around this axis.

However, the lift distribution of a rotor is not symmetrical (fig. 9) and so the axis at 75% radius is chosen where the overall lift coefficient C_L of the rotor is assumed to act. As will be seen later the difference between the potential velocity distribution and the actual linear distribution is only acceptably small for a small region of the rotor blade tip. So 0,75 R is chosen as the limit for the application of this programme for the rotor case.

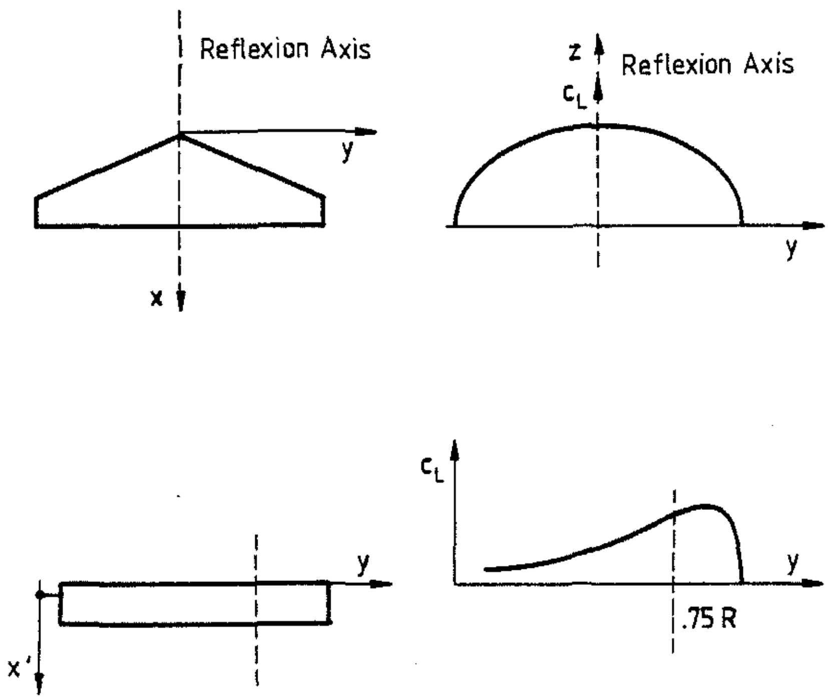


fig. 9 Reflexion of the lift coefficient for a fixed wing and a rotating wing

One possibility is the introduction of a linear velocity gradient into the programme. The resulting pressure distributions are shown in fig. 10. The influence of the finite span can be seen at the blade tip. Farther inboard the influence of the triangular velocity distribution can be observed because of the decreasing shock strength.

However, it must be noted that the condition of irrotationality is not fulfilled in this calculation although it is required of the theory. Consider the component ω_z of rot $\bar{v}$

$$\omega_z = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

the condition had to be

$$\frac{\partial u}{\partial y} = \Omega$$

and it was taken

$$\frac{\partial v}{\partial x} = 0$$

that means

$$\omega_z \neq 0$$

Fulfilling the condition of irrotationality the velocity distribution of the rotor was generated by a superposition of a potential vortex (ϕ_Γ) and a fictitious parallel free stream (ϕ_∞) so that

$$\phi_{rotor} = \phi_\infty - \phi_\Gamma$$

As shown in fig. 11 the actual linear velocity distribution is approximated relatively well. The computational result is shown in fig. 12. Here the same tendencies can be observed as in fig. 10.

The foregoing manner of calculation is necessary because the basic theory of the programmes does not take into account the special conditions of a rotor.

However, all standard methods of determination of aerodynamical coefficients assume irrotational flow. In helicopter aerodynamics the rotor forces and moments are calculated elementwise assuming irrotational flow (two-dimensional blade element theory) and this leads to good results except in the vicinity of the tip. If the two-dimensional calculation is extended to a three-dimensional one the lifting line theory is a possible method to account for the three-dimensional effects of a rotor. Because of simplicity reasons an elliptical wing [7] shall be used.

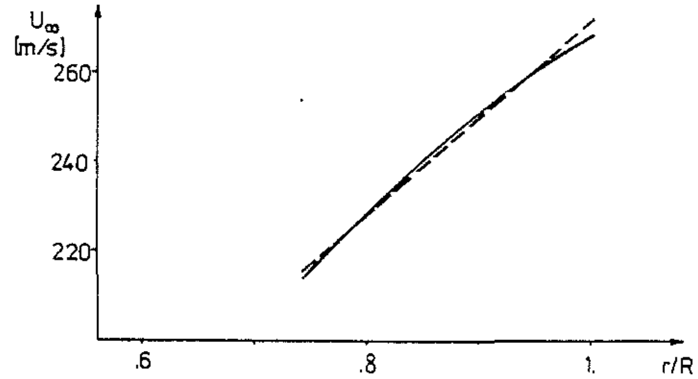


fig. 11 Potential velocity distribution compared to the linear one of the rotor

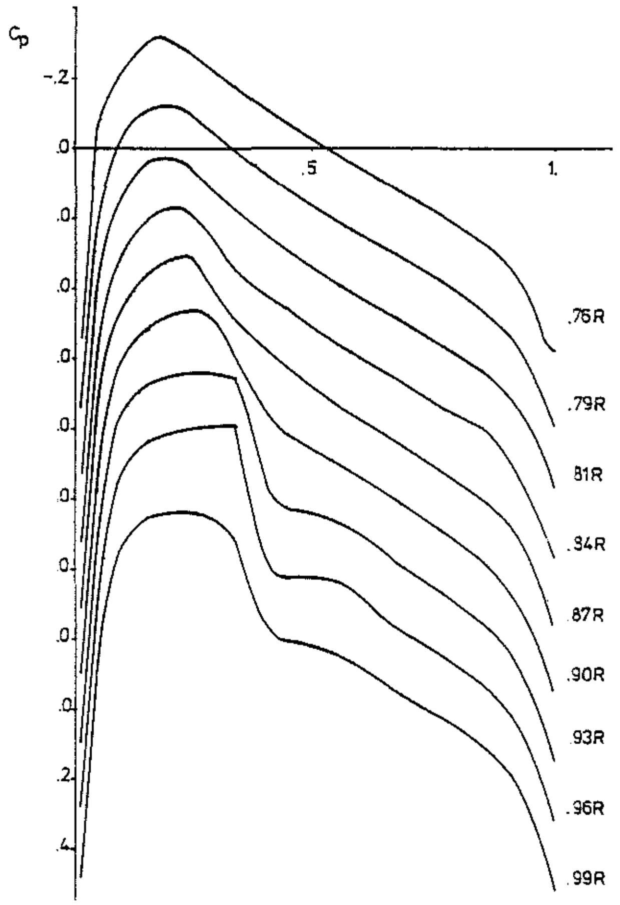


fig. 10 Chordwise pressure distributions for M_T = 0,85, μ = 0,32, α = 0^o, NACA 0012 profile, linear velocity distribution

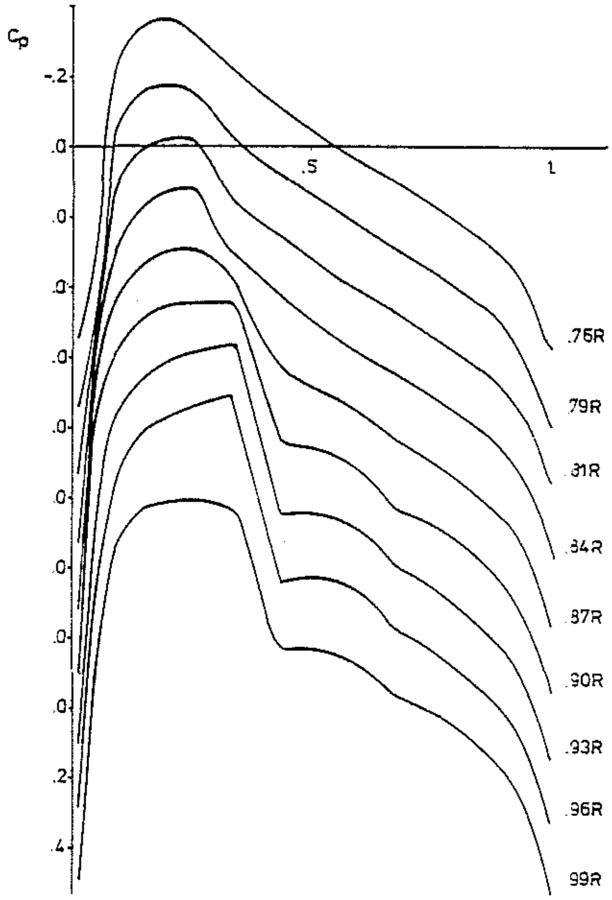


fig. 12 Chordwise pressure distributions for M_T = 0.85, μ = 0,32, α = 0^o, NACA 0012 profile, potential velocity distribution

The lift is obtained by spanwise integrating the vortices Γ (y)

$$\Gamma(y) = \Gamma_o \sqrt{1 - (2y/b)^2}$$

where Γ_o is the vortex strength in the middle of the wing. For the first approximation Γ_o could be determined by the two-dimensional airfoil theory.

The basic idea for calculating the $\Gamma(r)$ distribution of a rotor is that for each position r the Γ_o is determined from the local velocity and from there $\Gamma(r) = f(\Gamma_o(r))$. So one arrives at a $\Gamma(r)$ distribution as shown in fig. 13

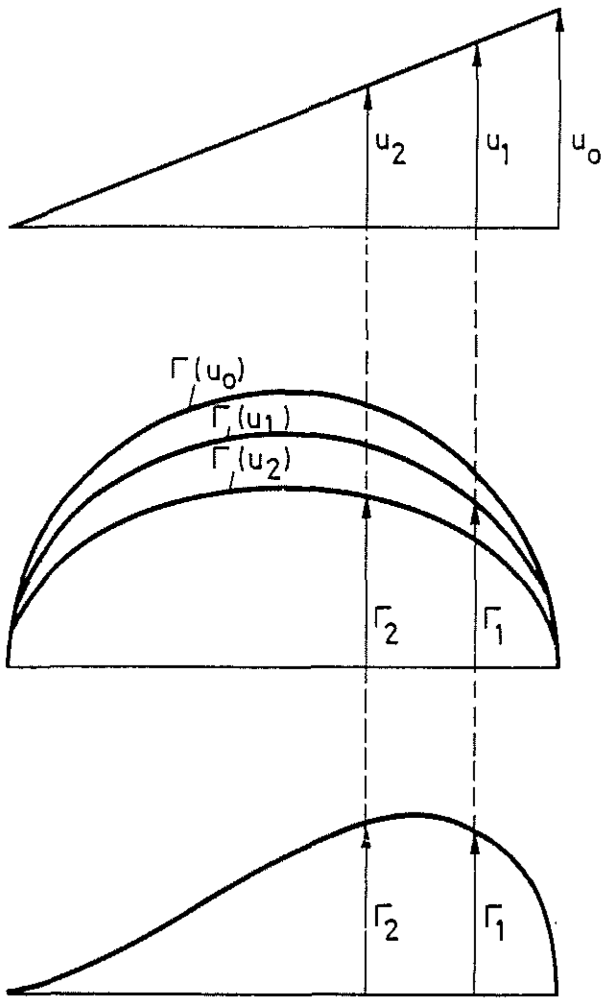


fig. 13 Construction of the vortex distribution on an elliptical rotating wing, $\mu = 0$, zero-thickness

However, it must be remarked that compressibility and thickness effects are not included but the idea is applicable to arbitrary planform shapes and blade twist.

In order to be a true alternative method it would have to be extended to the transonic flow regime.

4.4 The unsteady flow problem of a rotor

The second step is to consider the two-dimensional unsteady flow. Most known programmes are developed for two-dimensional unsteady flow and rarely for three-dimensional unsteady flow because of computation time.

The used programme of interest is written for a profile with an oscillating flap at the trailing edge. In the case of the rotor there is not any flap but the free stream and the angle of incidence are oscillating approximately with the angular velocity Ω for advance ratios $\mu > 0$.

For simplifying the numerical problem the oscillation of the angle of attack is set to zero, firstly. In the potential equation, the Mach number is introduced as a function of time.

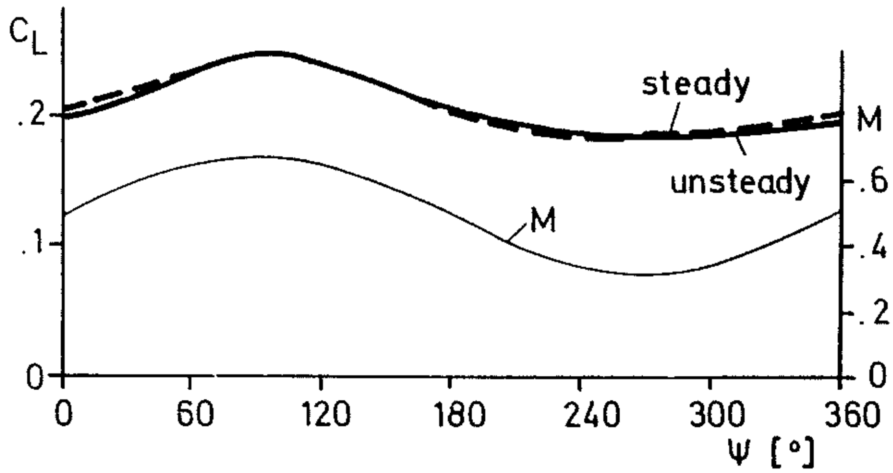


fig. 14 Lift coefficient over one period, M_o = 0,5, α = 2^o, μ = .36, NACA 0012

For comparison results for steady flow are shown, too. As can be seen the influence on the lift coefficient of the Mach number variation during one period is rather small, and the difference between steady and unsteady flow is almost negligible.

In a second step the Mach number is assumed to be constant and the angle of attack will oscillate. In fig. 15 the result is shown for a medium angle of attack of -.11° and an amplitude of 3,78° at a Mach number of .6 over one period. As can be seen the lift minimum position is shifted from ψ = 90° to 100° because of the unsteadiness not corresponding to the minimum setting of the geometric angle of incidence.

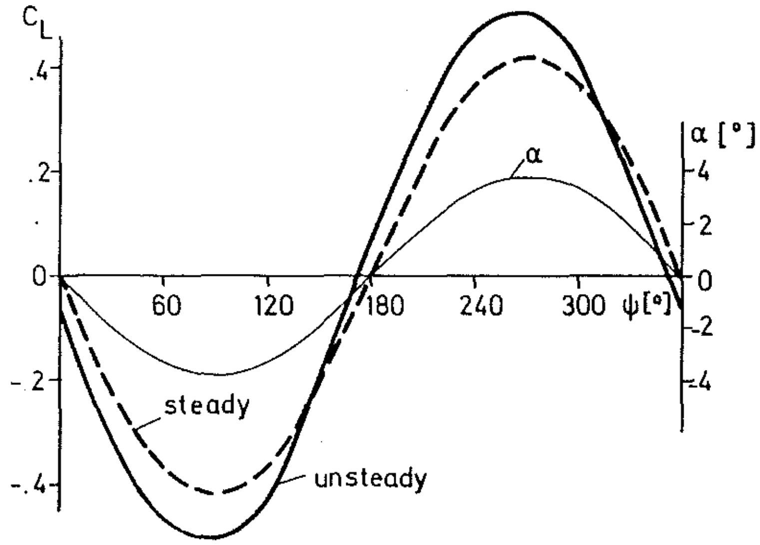


fig. 15 Lift coefficient versus azimuth angle, M_∞ = 0,6, α = -.11°, $\Delta\alpha$ = 3,78°, pitch axis at 0,25 c

The next step is the combination of the two motions. In fig. 6 a combination of the two effects is shown over one period. As it is seen the influence of the oscillation of the angle of attack is dominating the Mach number oscillation.

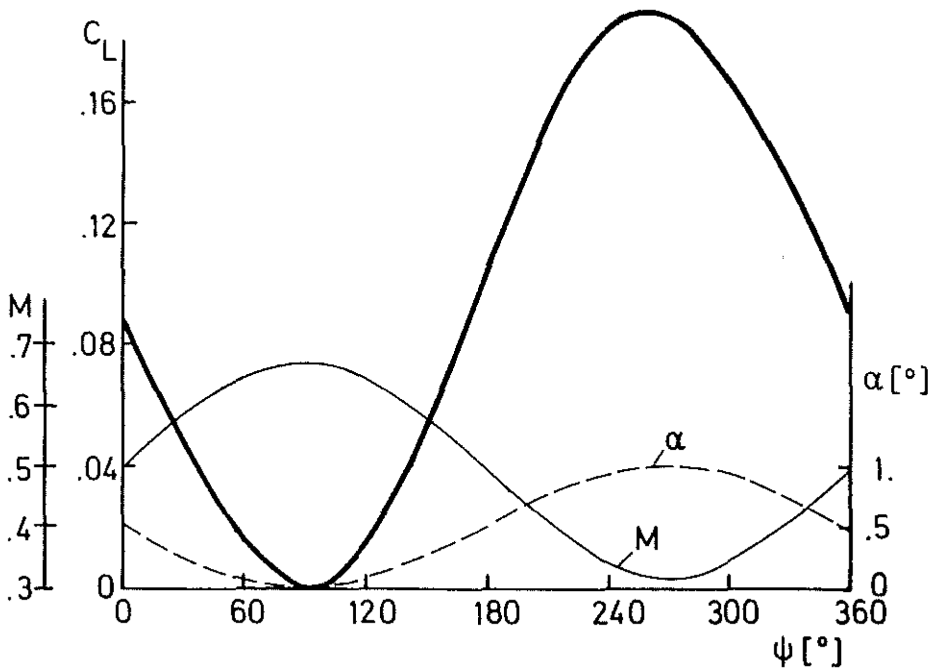


fig. 16 Lift coefficient versus azimuth angle for M_o = .5, α = 1°, $\Delta\alpha$ = 1°, pitch axis at 0,25 c

5. Conclusion

The application of three-dimensional computation methods in rotor aerodynamics has become necessary because the requirements for extending the helicopter flight envelope have increased in recent years, thus necessitating a more deep understanding of the flow around the rotor blades. By aerodynamically optimizing the blade tip an improvement is possible. However, the two-dimensional computation methods are no longer sufficient especially near the tip and thus three-dimensional computer programmes are required. From the foregoing discussion the following conclusions can be drawn:

- The inversion from the rotation of the rotor to the rotation of the fluid for achieving a steady state flow results in a physically unrealistic flow because of creation of actually non-existing pressure gradients in the undisturbed flow. When using potential flow theory these pressure gradients are much smaller.

- The problem of the flow in the vicinity of the tip of a rotor blade has already been investigated in the literature using the potential theory. The agreement between theory and experiment is acceptable. However, the procedures are limited either to hover or to non-lifting flow. Also, the computation time is inacceptably high for industrial purposes.

- A programme for steady three-dimensional flow over a fixed wing has been modified for a rotating blade by taking into account the irrotationality condition of the applied theory. Calculations with this programme provided results as expected on a rotor blade tip.

- The suggestion of modifying the two-dimensional blade element theory into a three-dimensional theory has been made. However, in the present state compressibility effects and thickness effects are not included. Further development had to be done to extend this method to transonic flow problems.

- A programme for unsteady two-dimensional flow has been modified for an oscillating free stream Mach number and angle of attack. The variation of the angle of incidence dominates clearly the influence of the Mach number variation.

- In order to couple three-dimensional effects with the unsteady two-dimensional ones further investigations have to be undertaken.

6. Literature

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