PAPER Nr.: 45

HELICOPTER COUPLED ROTOR-FUSELAGE ANALYSIS THROUGH MODIFIED BLADE ROOT BOUNDARY CONDITIONS

by

W R Walker

Royal Aerospace Establishment
Farnborough, England

FIFTEENTH EUROPEAN ROTORCRAFT FORUM

SEPTEMBER 12 - 15, 1989 AMSTERDAM

HELICOPTER COUPLED ROTOR-FUSELAGE ANALYSIS THROUGH MODIFIED BLADE ROOT BOUNDARY CONDITIONS

by

W R Walker

Royal Aerospace Establishment
Farnborough, England

ABSTRACT

A method is described by which a multi-bladed rotor system mounted on a flexible fuselage may be analysed through modification of the root boundary conditions for a single blade. The method effectively enables hub-fixed equations of motion for the blade to be used directly without modification. If a transfer matrix solution technique is adopted, integrations from the blade tip to root remain identical to their hub-fixed form. It is only when the root boundary conditions are applied that the added complexity of the coupled system is introduced. In this way, the computation is kept to a minimum. The method has application to axial flight eigenvalue analysis only.

1 INTRODUCTION

In a paper presented at the 1987 European Rotorcraft Forum[1], an instability was described that had been encountered on an RAE swept-tip model rotor blade. Comparisons between experimental results and results obtained from a recently developed RAE blade stability program were made and were generally found to be good. However, it was found that the experimentally obtained flutter speeds varied by around 5%, depending on the rotor rig on which the tests were made. One possible cause for the discrepancy was thought to be the differing rig dynamic characteristics, one having a much stiffer tower than the other. This paper describes work recently undertaken at RAE to extend the stability program's capabilities to include the interactions of the rotor mounted on a flexible structure. As well as allowing the influence of hub impedance on individual blade stability to be assessed, this also generalises the program to permit overall vehicle stability in air and ground resonance to be examined.

In extending the analysis to deal with the rotor/fuselage combination, the aim is to do so in a way that is as computationally efficient as possible. The natural way to analyse the problem is to use coordinates that describe the motion of the hub with respect to a fixed point together with coordinates that describe the motion of the blade with respect to the hub[2]. Blade inertia loads then have a contribution from both these sets of coordinates. The root boundary conditions for the blade remain unaffected and are simply the boundary conditions that prevail for the hub-fixed case. The disadvantage of this method is that six extra unknowns, pertaining to the six hub degrees of freedom, are introduced across the blade. With the transfer matrix solution method used in the RAE blade stability program, this has the effect of roughly doubling the transfer matrix size and hence of doubling the computation time.

An alternative approach is to define the blade coordinates not with respect to the shaft axis, but with respect to a spinning axis located at the undisturbed hub position. This spinning axis has a fixed location in space and so has no real physical significance, but it is a useful mathematical artifice to which the problem may be referred. In this frame, the blade equations of motion are unaltered from their hub-fixed form. At the blade root, however, the hub-fixed boundary conditions for the blade coordinates must be replaced to reflect the fact that blade displacements there are now no longer zero. The complexity of the hub motion is thus borne by the root boundary conditions instead of the equations of motion. If integrations over the blade are effected from tip to root, not only is the size of the transfer matrix retained at its hub-fixed level but, by delaying hub motion considerations till the latest possible part of the computation, the minimum amount of re-programming is required.

It is this latter approach that has been adopted in the RAE stability program. In this paper, the appropriate mathematical formulation is described, with due regard being paid to matters such as the Coleman transformation and the inclusion of fuselage hub impedance. Some limited comparison of results with those obtained from other, more conventional rotor/fuselage analyses is made. The initial problem that motivated the work, namely the influence of tower dynamics on RAE model blade stability, has still to be addressed fully, but some preliminary results are presented.

2 THE RAE BLADE STABILITY PROGRAM

The blade equations of motion used in the RAE stability program are derived in Ref 3. They apply to a curved and twisted blade, and take account of the cross-stiffness effects that may be introduced by a fibre composite construction. The equations are adapted slightly in Ref 4 to include the effects of rotatory inertia and a partial representation of transverse shear. The aerodynamics are represented by two dimensional strip theory.

The equations are solved by a transfer matrix technique and are integrated directly from tip to root. Thus no intermediate calculation of rotor blade modes is made. Because the equations are nonlinear, the steady state must first be determined by means of a perturbation procedure. The equations are then linearised about this position and the complex modes and frequencies found.

The integration is effected by means of a simple second order method. Application of the hub boundary conditions yields a frequency equation, the complex roots of which are interpolated by a quadratic fit procedure. The modification of these boundary conditions to accommodate a moving hub is now considered.

3 BLADE ROOT BOUNDARY CONDITIONS

Let axes OXYZ be fixed, with origin at the undisturbed hub position, X pointing aft and Z pointing upwards along the rotor shaft (Fig 1). Let unit vectors $\underline{I},\underline{J},\underline{K}$ be associated with these axes in the usual way. Let $\underline{i},\underline{j},\underline{k}$ be obtained from $\underline{I},\underline{J},\underline{K}$ by a rotation ψ (= Ωt) about $\underline{K}$. A point on the blade flexural axis, initially at location $x\underline{i}$ from O, is displaced to

$$\underline{r} = (x + u)\underline{i} + v\underline{j} + w\underline{k} \quad . \tag{1}$$

As in Ref 3, the displacements u,v,w contain both the built-in and elastic displacements of the blade, denoted by suffixes b,e where necessary. (Thus $u = u_b + u_e$, etc.) The elastic components, however, now embody components due to the motion of the hub. With this notation, neither the elastic nor the inertia terms in the equations of motion require altering from those obtained in a hub-fixed formulation; the reference origin O remains stationary. The root boundary conditions, however, must be modified and are best defined with respect to those that pertain for the standard moving hub formulation.

Let the displacement of the hub $\bar{O}$ from O be given by

$$\underline{R} = R_X\underline{I} + R_Y\underline{J} + R_Z\underline{K} \quad ,$$

and let the rotation of the hub axes due to fuselage motion be given by

$$\underline{\theta} = \theta_X\underline{I} + \theta_Y\underline{J} + \theta_Z\underline{K} \quad .$$

(We may use vectors here since our concern is with eigenvalue analysis and hub motions are thus infinitesimal). Axes located in the rotor are obtained finally by rotating through ψ (= Ωt) about the carried position of $\underline{K}$. New unit vectors associated with these axes are denoted $\bar{\underline{i}},\bar{\underline{j}},\bar{\underline{k}}$, and the displacement of a point on the flexural axis can be written in the alternative form

$$\underline{r} = \underline{R} + (x + \bar{u})\bar{\underline{i}} + \bar{v}\bar{\underline{j}} + \bar{w}\bar{\underline{k}} \quad . \tag{2}$$

Now

$$(\bar{\underline{i}}\ \bar{\underline{j}}\ \bar{\underline{k}}) = (\underline{i}\ \underline{j}\ \underline{k})C \quad ,$$

where the transfer matrix C is given by

$$C = \begin{bmatrix} 1 & -\theta_Z & -\theta_X \sin\psi + \theta_Y \cos\psi \\ \theta_Z & 1 & -\theta_X \cos\psi - \theta_Y \sin\psi \\ \theta_X \sin\psi - \theta_Y \cos\psi & \theta_X \cos\psi + \theta_Y \sin\psi & 1 \end{bmatrix} . \tag{3}$$

On substituting for $\bar{\underline{i}},\bar{\underline{j}},\bar{\underline{k}}$ in equation 2, therefore, and comparing coefficients of $\underline{i},\underline{j},\underline{k}$ with equation 1, we find that

$$u = R_X \cos\psi + R_Y \sin\psi + \bar{u} - \theta_Z\bar{v} - (\theta_X \sin\psi - \theta_Y \cos\psi)\bar{w}$$

$$v = R_Y \cos\psi - R_X \sin\psi + \theta_Z(x + \bar{u}) + \bar{v} - (\theta_X \cos\psi + \theta_Y \sin\psi)\bar{w}$$

$$w = R_Z + (\theta_X \sin\psi - \theta_Y \cos\psi)(x + \bar{u}) + (\theta_X \cos\psi + \theta_Y \sin\psi)\bar{v} + \bar{w} \quad .$$

As in Ref 3, we now let $u = u^N + u^n$ etc, where the superfix N denotes the steady state and n an infinitesimal perturbation about it. Thus $u^N = u_b + u_e^N$. Just as x is common to both formulations under consideration, so $\bar{u}^N = u^N$. Correct to second order, therefore, (assuming u^N is o(2))

$$\bar{u} = u - R_X \cos\psi - R_Y \sin\psi + \theta_Z v^N + (\theta_X \sin\psi - \theta_Y \cos\psi)w^N$$

$$\bar{v} = v + R_X \sin\psi - R_Y \cos\psi - \theta_Z x + (\theta_X \cos\psi + \theta_Y \sin\psi)w^N$$

$$\bar{w} = w - R_Z - (\theta_X \sin\psi - \theta_Y \cos\psi)x - (\theta_X \cos\psi + \theta_Y \sin\psi)v^N \quad .$$

The equivalent relationship for the blade twist variable ϕ is dependent on the twist variable chosen. In Ref 3, the angle of torsion is used. A transfer matrix T results, from $\underline{i},\underline{j},\underline{k}$ to vectors $\hat{\underline{i}},\hat{\underline{j}},\hat{\underline{k}}$ located in the deformed blade, of the form

$$T = \begin{bmatrix} 1 - \tfrac{1}{2}(\chi^2 + \beta^2) & -\chi\cos\phi - \beta\sin\phi & \chi\sin\phi - \beta\cos\phi \\ \chi & \cos\phi(1-\tfrac{1}{2}\chi^2) - \sin\phi\int\chi\beta' & -\sin\phi(1-\tfrac{1}{2}\chi^2) - \cos\phi\int\chi\beta' \\ \beta & \sin\phi(1-\tfrac{1}{2}\beta^2) - \cos\phi(\chi\beta-\int\chi\beta') & \cos\phi(1-\tfrac{1}{2}\beta^2) + \sin\phi(\chi\beta-\int\chi\beta') \end{bmatrix} . \tag{4}$$

This transformation is by no means unique and would be different if, say, the aerodynamic pitch angle were used. The lag and flap angles χ, β equate to v', w' with transverse shear angles removed, whilst the indefinite integral has origin x_o at the point where the torsion boundary condition is applied.

The transformation $\bar{T}$ from $\underline{\bar{i}}, \underline{\bar{j}}, \underline{\bar{k}}$ to $\underline{\hat{i}}, \underline{\hat{j}}, \underline{\hat{k}}$ is equivalent to T but may also be derived from

$$T = C\bar{T} \quad .$$

Comparison of the elements of these matrices, in particular the elements (2,3), (2,1) and (3,1), yields

$$\bar{\phi} = \phi - \theta_X \cos\psi - \theta_Y \sin\psi - \theta_Z \beta_0^N$$

$$\bar{\chi} = \chi - \theta_Z + (\theta_X \cos\psi + \theta_Y \sin\psi)\beta^N$$

$$\bar{\beta} = \beta - \theta_X \sin\psi + \theta_Y \cos\psi - (\theta_X \cos\psi + \theta_Y \sin\psi)\chi^N \quad .$$

We note the presence in the ϕ equation of the root value of flap slope, denoted suffix o. This is evaluated at the point where the torsion boundary condition is applied, ie the origin for the integrals in equation 4.

Equivalent relationships for the blade moments are derived on substituting derivatives of the angular deflections into the load-displacement equations of Ref 3. There result

$$\bar{M}_\xi = M_\xi$$

$$\bar{M}_y = M_y + (\theta_X \cos\psi + \theta_Y \sin\psi)M_z^N$$

$$\bar{M}_z = M_z - (\theta_X \cos\psi + \theta_Y \sin\psi)M_y^N \quad .$$

The force relationships are found from

$$\bar{V}_x\underline{\bar{i}} + \bar{V}_y\underline{\bar{j}} + \bar{V}_z\underline{\bar{k}} = V_x\underline{i} + V_y\underline{j} + V_z\underline{k}$$

and application of equation 3. Thus

$$\bar{V}_x = V_x + \theta_Z V_y^N + (\theta_X \sin\psi - \theta_Y \cos\psi)V_z^N$$

$$\bar{V}_y = V_y - \theta_Z V_x^N + (\theta_X \cos\psi + \theta_Y \sin\psi)V_z^N$$

$$\bar{V}_z = V_z - (\theta_X \sin\psi - \theta_Y \cos\psi)V_x^N - (\theta_X \cos\psi + \theta_Y \sin\psi)V_y^N \quad .$$

Let the perturbed blade displacements and loads be represented by

$$\underline{q}^T = (u \;\; v \;\; w \;\; \phi \;\; \chi \;\; \beta \;\; M_\xi \; M_y \; M_z \; V_x \; V_y \; V_z)^n$$

with a corresponding definition for $\underline{\bar{q}}$, and let

$$\underline{\bar{Q}}^T = (R_X \; R_Y \; R_Z \; \theta_X \; \theta_Y \; \theta_Z \; M_X \; M_Y \; M_Z \; V_X \; V_Y \; V_Z) \tag{5}$$

where $M_X, \ldots V_Z$ denote the net oscillatory rotor loads acting on the fuselage at $\bar{O}$ in the directions $\underline{I}, \underline{J}, \underline{K}$. Thus

$$\underline{\bar{q}} = \underline{q} + (\bar{A} \mid 0)\underline{\bar{Q}} \tag{6}$$

where

$$\bar{A} = \begin{bmatrix} -\cos\psi & -\sin\psi & \cdot & w^N \sin\psi & -w^N \cos\psi & v^N \\ \sin\psi & -\cos\psi & \cdot & w^N \cos\psi & w^N \sin\psi & -x \\ \cdot & \cdot & -1 & -x\sin\psi - v^N\cos\psi & x\cos\psi - v^N \sin\psi & \cdot \\ \cdot & \cdot & \cdot & -\cos\psi & -\sin\psi & -\beta_0^N \\ \cdot & \cdot & \cdot & \beta^N \cos\psi & \beta^N \sin\psi & -1 \\ \cdot & \cdot & \cdot & -\sin\psi - \chi^N\cos\psi & \cos\psi - \chi^N \sin\psi & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & M_z^N \cos\psi & M_z^N \sin\psi & \cdot \\ \cdot & \cdot & \cdot & -M_y^N \cos\psi & -M_y^N \sin\psi & \cdot \\ \cdot & \cdot & \cdot & V_z^N \sin\psi & -V_z^N \cos\psi & V_y^N \\ \cdot & \cdot & \cdot & V_z^N \cos\psi & V_z^N \sin\psi & -V_x^N \\ \cdot & \cdot & \cdot & -V_x^N\sin\psi - V_y^N\cos\psi & V_x^N\cos\psi - V_y^N\sin\psi & \cdot \end{bmatrix} .$$

The root boundary conditions with respect to $\bar{\underline{q}}$ may be written in the form

$$P\bar{\underline{q}} = 0 \quad , \tag{7}$$

for some 6x12 matrix P defined at x_0. Furthermore

$$\underline{q} = S\underline{q}_t \quad , \tag{8}$$

where $\underline{q}_t$ denotes the six unknown tip variables (displacements and rotations for a free end) and S denotes the transfer matrix from tip to root, which will be a function of the complex frequency λ. Combining equations 6, 7 and 8 gives

$$P(S\underline{q}_t + (\bar{A} \mid 0)\bar{\underline{Q}}) = 0 \quad . \tag{9}$$

These then are the six root boundary conditions re-expressed in terms of the six 'hub-fixed' unknowns $\underline{q}_t$. Twelve new unknowns $\bar{\underline{Q}}$ have been introduced, which require further compatibility equations to be identified before a solution can be found. However, consideration will first be given to the Coleman transformation of equation 9.

4 THE COLEMAN TRANSFORMATION

Equation 9 contains periodic coefficients and so an eigenvalue analysis cannot be performed on it directly. The Coleman transformation is applied to eliminate the periodicity. It is only effective provided the rotor blades are identical and number at least three.

We define the azimuth angle for the j th blade to be ψ_j. Application of the Coleman transformation consists of multiplying equation 9 by the coefficients

$$\frac{1}{b}(-1)^j, \quad \frac{2}{b}e^{i\psi_j}, \quad \frac{2}{b}e^{-i\psi_j}, \quad \frac{1}{b} \tag{10}$$

in turn and summing over all b blades. New rotor coordinates result, designated respectively reactionless, progressive, regressive and collective, and are defined with respect to the blade coordinates $\underline{q}_j$ as

$$\underline{q}^r = \frac{1}{b} \sum \underline{q}_j (-1)^j \tag{11}$$

$$\underline{q}^+ = \frac{2}{b} \sum \underline{q}_j \; e^{i\psi_j}$$

$$\underline{q}^- = \frac{2}{b} \sum \underline{q}_j \; e^{-i\psi_j}$$

$$\underline{q}^c = \frac{1}{b} \sum \underline{q}_j \quad .$$

We call these new vectors the Coleman vectors, the titles 'progressive' and 'regressive' implying that the frequency of fixed-frame motion takes the positive sign.

The 6 equations for a single blade, equation 9, thus become 24 equations for the complete rotor. Strictly, the number of types of rotor coordinate defined should equal the number of blades of the rotor. This then ensures that every rotor degree of freedom is represented. The above coordinates are used here for all rotors, however, irrespective of the number of blades. The reactionless coordinate, for instance, may only be formed for rotors incorporating an even number of blades. The means of calculating hub-fixed response is nevertheless retained here for all rotors. Equally, extra coordinates are required if the number of blades exceeds four. These all describe reactionless motions, however, and are unlikely to add to our understanding of a rotor's behaviour.

The Coleman transformation applied to equation 9 modifies matrix $\bar{A}$, removing its periodicity. Further simplification is accomplished, however, if equation 9 is first re-expressed in the form

$$P(S\underline{q}_t + (\bar{A}H \mid 0)\underline{Q}) = 0 \quad , \tag{12}$$

where

$$H = \begin{bmatrix} \bar{G} & & & 0 \\ & \bar{G} & & \\ & & \bar{G} & \\ 0 & & & \bar{G} \end{bmatrix} \quad ,$$

$$G = \begin{bmatrix} ½ & ½ & 0 \\ -½i & ½i & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and

$$\underline{Q} = H^{-1}\underline{\bar{Q}}$$

$$= (R_X + iR_Y, R_X - iR_Y, R_Z, \ldots) \quad .$$

The Coleman transformation may now be considered to act on $\underline{q}_t$ and $\bar{A}H$, producing in the latter four new matrices, given in shorthand form by

$$
A = \begin{array}{c} \\ \end{array}
\overset{\begin{array}{cccccc} + & - & c & + & - & c \end{array}}{}
\begin{bmatrix}
-1 & -1 & . & iw^N & -iw^N & v^N \\
i & -i & . & w^N & w^N & -x \\
. & . & -1 & -ix - v^N & ix - v^N & . \\
. & . & . & -1 & -1 & -\beta_0^N \\
. & . & . & \beta^N & \beta^N & -1 \\
. & . & . & -i - \chi^N & i - \chi^N & . \\
. & . & . & . & . & . \\
. & . & . & M_z^N & M_z^N & . \\
. & . & . & -M_y^N & -M_y^N & . \\
. & . & . & iV_z^N & -iV_z^N & V_y^N \\
. & . & . & V_z^N & V_z^N & -V_x^N \\
. & . & . & -iV_x^N - V_y^N & iV_x^N - V_y^N & .
\end{bmatrix} .
$$

Column superfixes: $+ \quad - \quad c \quad + \quad - \quad c$

Not all of this matrix is used at once. Thus the columns of A that are appropriate to each type of rotor motion are identified by their superfix. Progressive motion, for instance, requires only the first and fourth columns of A, reactionless motion requires none of A. Strictly, the elimination of A in the reactionless case only occurs if the number of blades is even. This is the result that we assume still applies to rotors with an odd number of blades, in order that hub-fixed response can be found in all cases. The use of the expanded form of A in the complete equations of motion is illustrated in Section 7.

Finally, we note that the form of the Coleman transformation used here is as originally defined by Coleman (Ref 5). Often, cyclic rotor motions are defined through application of coefficients cos ψ_i and sin ψ_i, instead of the progressive and regressive coefficients of 10 (see Ref 6, for example). The use of progressive and regressive coordinates, however, decouples the cyclic rotor motions and also permits the frequency of the blade motion in the rotating frame to be specified directly.

5 LOAD COMPATIBILITY RELATIONSHIPS

Load compatibility relationships between the rotor and fuselage reflect the fact that the root loads generated by the rotor are input loads to the fuselage. Such relationships are best derived by invoking reciprocity in the system. If the fuselage loads in $\underline{Q}$ had been defined to act in the fixed frame OXYZ, then load compatibility could be expressed in the form

$$
B\underline{q}_c + (0 \mid E)\underline{Q} = 0 \quad , \tag{13}
$$

where the vector $\underline{q}_c$ denotes a Coleman vector. The matrix B may be written down directly by comparison with its counterpart, matrix A, the moment compatibility components being given first. Thus

$$B = \begin{array}{c} + \\ - \\ c \\ + \\ - \\ c \end{array} \begin{bmatrix} iV_z^N & -V_z^N & V_y^N - iV_x^N & . & M_y^N & M_z^N & -1 & \chi^N - i & \beta^N & -iw^N & w^N & ix - v^N \\ -iV_z^N & -V_z^N & V_y^N + iV_x^N & . & M_y^N & M_z^N & -1 & \chi^N + i & \beta^N & iw^N & w^N & -ix - v^N \\ -V_y^N & V_x^N & . & . & . & . & -\beta_0^N & . & -1 & v^N & -x & . \\ . & . & . & . & . & . & . & . & . & -1 & -i & . \\ . & . & . & . & . & . & . & . & . & -1 & i & . \\ . & . & . & . & . & . & . & . & . & . & . & -1 \end{bmatrix}$$

on recalling that M_y and β act in opposite senses. Again, a shorthand notation is used and only the rows indicated should be used for each type of rotor response.

The matrix E takes the form

$$E = \frac{1}{b} \begin{bmatrix} 2 & & & & & \\ & 2 & & & 0 & \\ & & 1 & & & \\ & & & 2 & & \\ & 0 & & & 2 & \\ & & & & & 1 \end{bmatrix}$$

and reflects the normalising constants of equations 11.

We recall, however, following equation 5, that fuselage loads are defined with respect to $\bar{O}$, with resolutes remaining in the directions $\underline{I},\underline{J},\underline{K}$. The hub moments therefore incorporate a contribution arising from the steady transverse loads and the shift of origin to a non-fixed point. This additional moment is given by

$$\underline{M} = -\sum \underline{R} \wedge (V_x\underline{i} + V_y\underline{j} + V_z\underline{k})$$

$$= b(R_X\underline{J} - R_Y\underline{I})V_z^N \quad .$$

Thus equation 13 must be replaced by

$$B\underline{q} + (D \mid E)\underline{Q} = 0 \quad , \tag{14}$$

where

$$D(1,1) = -D(2,2) = 2iV_z^N$$

and the rest of matrix D is zero. Again the factor 2/b has been incorporated.

6 FUSELAGE LOAD-DISPLACEMENT RELATIONSHIPS

For any given fixed-frame frequency, we assume that fuselage load-displacement relationships are defined at the rotor hub in the form

$$\bar{F}\bar{\underline{Q}} = 0 \quad .$$

This may be transformed to

$$F\underline{Q} = 0 \quad , \tag{15}$$

where

$$F = \bar{F}H \quad .$$

The matrix $\bar{F}$ may be partitioned in any way convenient, allowing the input of either an impedance matrix or a receptance matrix, or some suitable hybrid. In the RAE stability program, $\bar{F}$ is determined from fuselage elastic mode data. An arbitrary number of modes may be input, the data required being the mode shape at the rotor hub, generalised mass, frequency and damping.

7 COUPLED EQUATIONS OF MOTION

The frequency equation for the complete system can now be assembled from the Coleman transformation of equation 12 and from equations 14 and 15. Collectively, these may be expressed in the form

$$\left[\begin{array}{cccc|cccccc|c} PS(\lambda) & & & & & & & & & & \\ & PS(\lambda-i\Omega) & & & PA^+ & & & PA^+ & & & \\ & & PS(\lambda+i\Omega) & & & PA^- & & & PA^- & & \\ & & & PS(\lambda) & & & PA^c & & & PA^c & \\ \hline & B^+S & & & & & & & & & \\ & & B^-S & & & & & & & & \\ & & & B^cS & & & & & & & \\ & B^+S & & & & & D & & & & E \\ & & B^-S & & & & & & & & \\ & & & B^cS & & & & & & & \\ \hline & & & & & & F & & & & \end{array}\right] \begin{bmatrix} \underline{q}_t^r \\ \underline{q}_t^+ \\ \underline{q}_t^- \\ \underline{q}_t^c \\ \\ \underline{Q} \\ \\ \end{bmatrix} = 0 \qquad (16)$$

for a single load path blade. (Blades with multiple beam-type connections to the hub can be treated by suitable summation). It is apparent that the reactionless motion of the rotor is independent of the response in all other degrees of freedom and can be determined separately. Motion in the other rotor degrees of freedom is coupled by virtue of whatever couplings are present in the fuselage motion.

Now the vector $\underline{q}_j$ for the j th blade can be expressed in the component form

$$\underline{q}_j = (-1)^j \underline{q}^r + \tfrac{1}{2} e^{-i\psi_j} \underline{q}^+ + \tfrac{1}{2} e^{i\psi_j} \underline{q}^- + \underline{q}^c \quad .$$

Clearly, application of the Coleman transformation to this equation correctly yields equations 11. If the complex frequency of both the fuselage response and the rotor coordinates is denoted λ, then the complex frequencies associated with the reactionless, progressive, regressive and collective rotor components of $\underline{q}_j$ within the rotating frame are respectively λ, $\lambda - i\Omega$, $\lambda + i\Omega$ and λ. The transfer matrix S must therefore be evaluated at these frequencies as appropriate to each type of rotor motion, (as indicated in equation 16). Complex frequencies λ which satisfy equation 16 may then be found by quadratic interpolation.

Coupled rotor-fuselage modes are more complicated to define than their hub-fixed counterparts and require careful interpretation. Since the fuselage frequency λ is generally different from the rotor speed Ω, the relative position of the rotor and fuselage at the start of each fuselage response cycle changes. Although this makes no difference to the blade response in the collective and reactionless modes, it does in the cyclic modes. In the RAE stability program, blade response in the cyclic modes compatible with a particular fuselage response is derived specifically for a blade at azimuth angle zero.

It should be noted that, if the dominant blade motion is in a regressive lag mode, say, the other blade motions will be at frequencies λ and $\lambda - i\Omega$. Such frequencies are far removed from those for the corresponding collective and progressive lag modes. It is incorrect, therefore, to label such components collective and progressive lag. They are simply the collective and progressive components of the regressive lag mode.

It should be remembered finally that the blade response derived by this method is relative to the undisturbed hub position. Thus, comparisons with mode shapes produced by other methods are unlikely to be straightforward. They are not attempted here.

8 RESULTS

The above equations have been programmed in FORTRAN for a VAX computer. Comparisons of the frequencies produced for a hypothetical ground resonance model have been made with those found using the current Westlands ground resonance computer program. That program uses a modal formulation, and the blades were represented in it by simple, uncoupled, fundamental flap and lag modes. The fuselage was represented by its first five modes, the yaw mode being omitted, with 5% critical damping in each. The modal frequencies for the coupled system derived by the two programs are compared in Table 1, and the agreement is generally good.

It is pointed out in Ref 3 that, for moving hub applications using the approach outlined in this paper, the radial variable u must be treated as o(1) instead of the more usual o(2), and its inertial contribution to the blade tensile equation must be retained. The tensile equation then takes the form

$$V'_x = m[\ddot{u} - 2\Omega(\dot{v} - z_G\dot{\phi}) - \Omega^2(x + u)] \quad ,$$

when radial aerodynamic loading is neglected. The centrifugal term in u is particularly important for it is non-conservative, effectively providing negative stiffness.

To check the importance of the u terms in the tensile equation, a comparison is made of ground resonance predictions with the terms either retained or omitted. The model of the previous example is modified slightly, to include 4° collective pitch and to eliminate fuselage roll mode damping. The variation of regressive lag mode damping with rotor speed is depicted in each case in Figs 2 and 3. No structural damping is included in the blade lag sense and ground resonance occurs, but the way in which it does so is markedly different.

Some consideration has been given to the problem that originally stimulated this work, namely the influence of tower flexibility on model rotor blade stability, but the results have so far been disappointing. With both the RAE rigs, the high generalised mass of the tower modes compared to that of rotor modes of comparable amplitude appears to preclude significant tower interference. Even when the tower frequencies are modified theoretically to stimulate coupling, the effect on blade instability does not appear to be pronounced.

The mode frequencies measured on the more flexible of the RAE towers together with the frequencies predicted with the spinning rotor mounted upon it are given in Table 2. They are largely uninfluenced by the rotor's presence, save for the mode at 15hz which interacts with the first progressive lag mode. These unremarkable results parallel the findings of Ref 7. Changes to the rotor mode frequencies do not merit recording for they are even less pronounced. This is perhaps surprising but is possibly due to there being a greater concentration of rotor modes than fuselage modes. A full investigation, however, awaits the modification of the second of the RAE towers to accommodate more than a single rotor blade. Further theoretical work is also planned to explore more fully the differences in the tip aerodynamics for the one and three bladed rotors, possibly following the lines of Ref 8.

9 CONCLUSIONS

A method has been described which enables coupled rotor-fuselage systems to be analysed through modification of the root boundary conditions for a single blade. The method has application to helicopter stability analysis in axial flight. Its advantage is that it minimises the computation required when a transfer matrix solution technique is used across the blade. Current estimates are that the savings in computer time amount to around 70%.

Important terms in the blade equations of motion are identified, which are normally neglected for hub-fixed applications but which must be retained for the hub-free case when analysed by this approach. Simple validation exercises have been carried out and the method has been found to perform satisfactorily. Preliminary indications are, however, that the flexibility of the support structure has little influence on the flutter speed of the RAE swept-tip model rotor.

Acknowledgment

The provision of the results from Westland Helicopters ground resonance program by Mr P T W Juggins is gratefully acknowledged.

REFERENCES

1	W.R. Walker C. Hatch	Pitch-flap flutter instability of a swept-tip model rotor blade. Paper 6.12, 13th European Rotorcraft Forum, 1987
2	P.T.W. Juggins	A comprehensive approach to coupled rotor-fuselage dynamics. Paper 48, 14th European Rotorcraft Forum, 1988
3	W.R. Walker	Equations of motion for a fibre composite helicopter rotor blade. RAE Technical Report 88053, 1988
4	W.R. Walker	A computer program for determining helicopter rotor blade stability. RAE Technical Report, to be published
5	R.P. Coleman A.M. Feingold	Theory of self-excited mechanical oscillations of helicopter rotors with hinged blades. NACA Report 1351, 1958
6	A.R.S. Bramwell	<u>Helicopter Dynamics</u>, pp34-35. Edward Arnold, 1976
7	G.T.S. Done P.T.W. Juggins M.H. Patel	Further experience with a new approach to helicopter aeroelasticity. Paper 6.11, 13th European Rotorcraft Forum, 1987
8	D.A. Peters Y-W. Chiu	Extension of classical tip loss formulas. J. Am. Hel. Soc., Vol 34, no 2, p68-71, 1989

	WHL frequency predictions, rad/sec		RAE frequency predictions, rad/sec		Mode Type
	imaginary	real	imaginary	real	
Reactionless modes in vacuum	22.108 37.915	0.0 0.0	22.031 38.252	0.0 0.0	Lag Flap
Reactionless modes in air	22.117 34.41	-0.126 -15.885	21.987 34.325	-0.0476 -17.197	Lag Flap
Rotor/fuselage modes	0.0 0.0 6.5089 12.052 13.944 22.117 34.368 56.806 68.566	-13.117 -15.185 -1.7751 -0.1144 -1.0707 -0.126 -15.876 -0.1536 -15.808	0.0 0.0 6.556 12.171 13.783 21.987 34.283 56.722 68.465	-14.977 -16.529 -1.494 -0.0265 -1.0658 -0.0476 -17.181 -0.0741 -17.139	Reg flap Reg flap Fuselage Reg lag Fuselage Coll lag Coll flap Prog lag Prog flap

Table 1 Coupled rotor/fuselage mode frequency predictions from WHL and RAE programs.

Measured tower frequency, Hz	Predicted frequency with rotor added, Hz
2.44	2.36
2.62	2.53
11.27	11.04
15.0	12.42
20.71	20.6
29.64	29.43

Table 2 Predicted changes in tower frequencies due to spinning rotor.

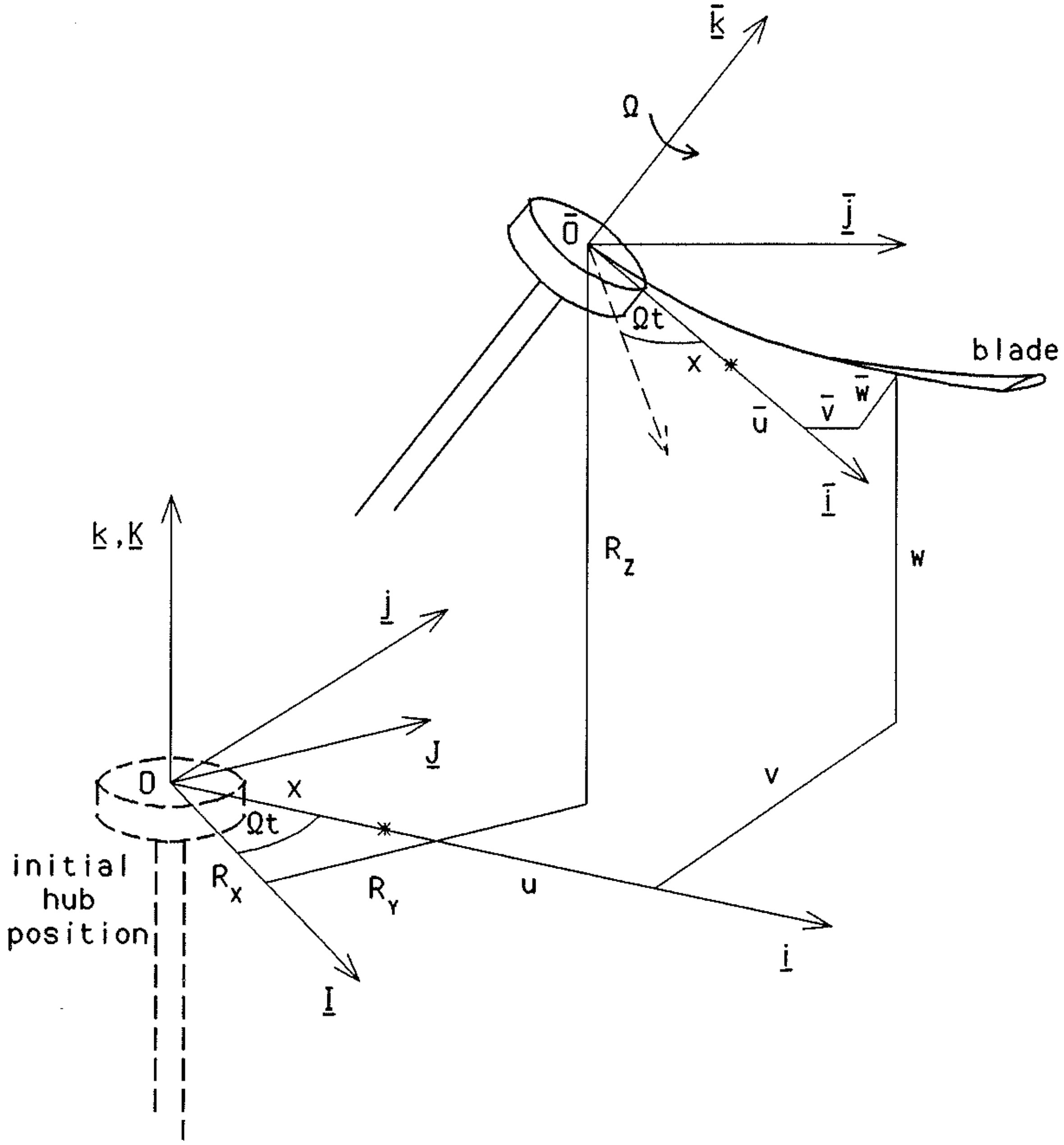

Fig 1   Axes systems with respect to displaced and undisplaced rotor hub

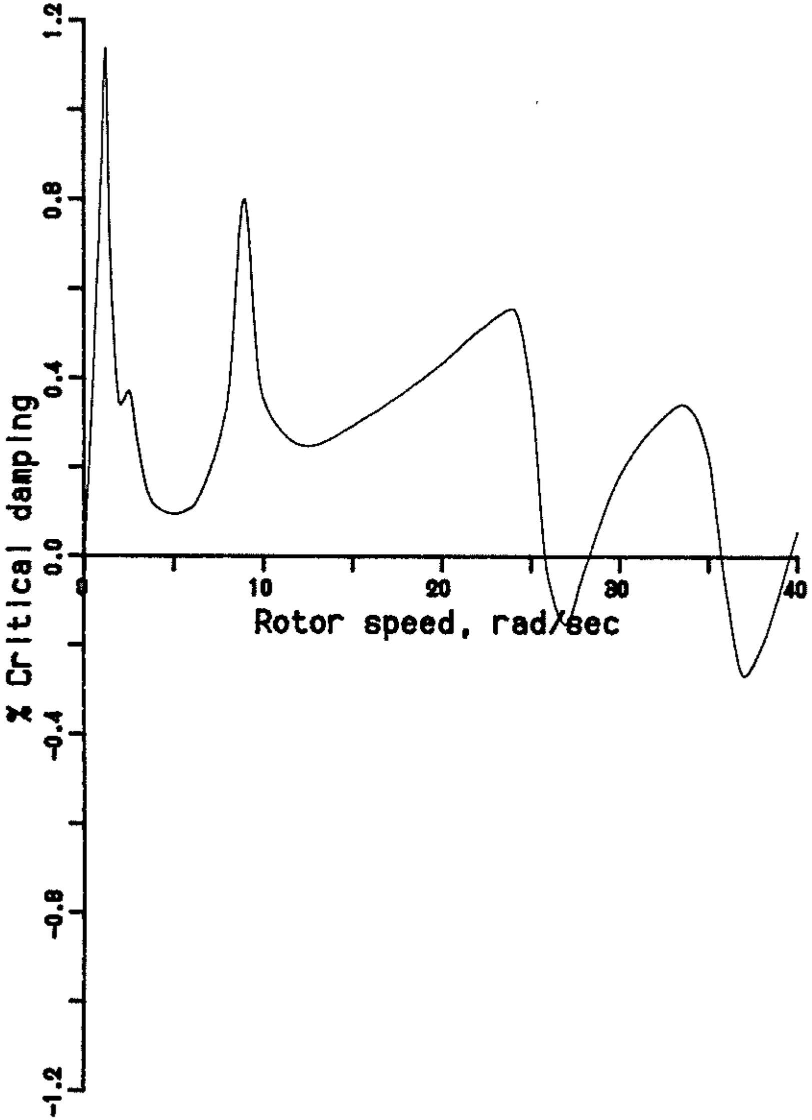

Fig 2 Regressive lag mode damping with u dependence included

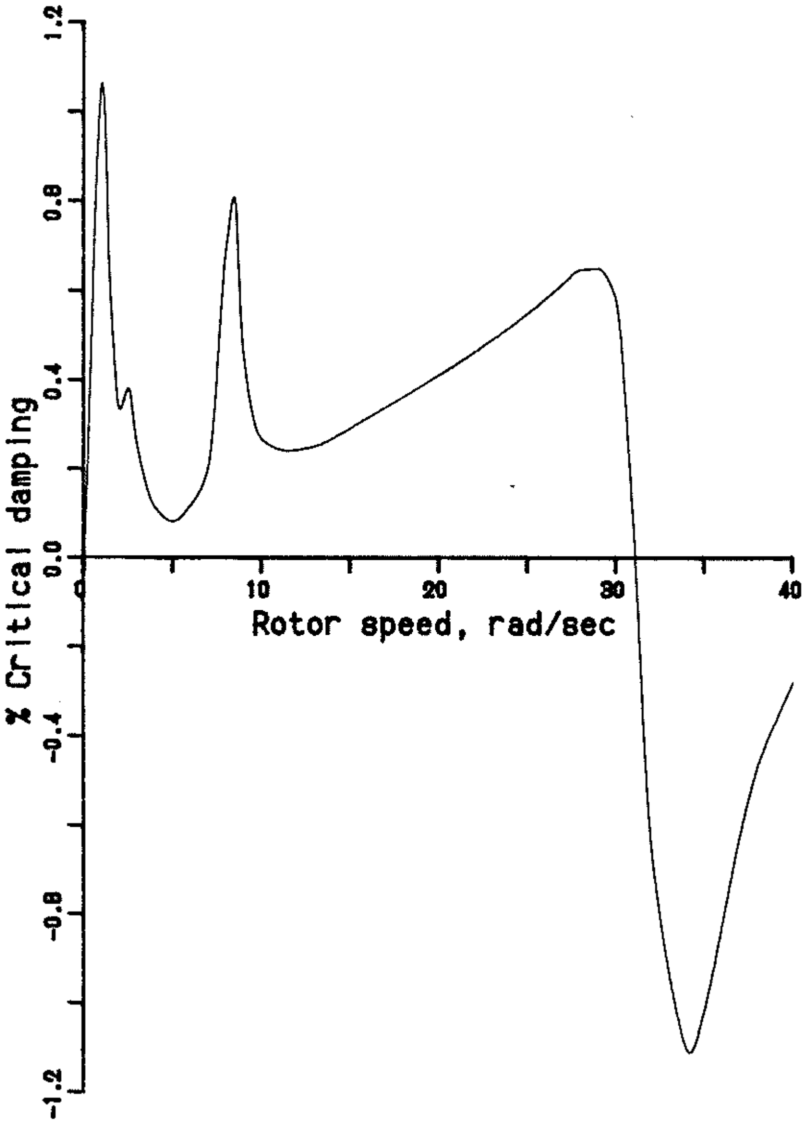

Fig 3 Regressive lag mode damping with u dependence excluded