Adaptive control of dynamic test rigs using complex algebra

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Abstract. Tasks of carrying out dynamic test of helicopter assemblies are resolved using multichannel dynamic test rigs with servocontrol, that make possible test sample load at required directions, with required frequency, statics, amplitude and phase difference between load channels. Dynamic hydraulic actuators are widely used as the sources of force.
At the same time the sample is loaded at close points at the same direction. That yields considerable crosstalk between control channels, up to that, that external unwanted signal, coming to the feedback sensor, have higher amplitude, than required at the concerned control channel.

There are control algorithms, made a good showing, using double feedback loop, in which the first feedback loop is a PID-regulator, maybe some modified, and the second feedback loop is an adaptive loop, keeping up amplitude, statics and phase of the control channel, or maximum and minimum of signal, coming to the task input of the PID-regulator. Such algorithms are well suitable in tasks of control of single-channel and multichannel dynamic test rigs with minor crosstalk between control channels in conditions of unstable environment and sample properties. But such algorithms cannot suppress a considerable crosstalk.

An adaptive algorithm is being introduced, that allows considering test rig as a “black box” with a considerable crosstalk between control channels. Algorithm is based on the introducing a complex coordinate at each channel. That coordinate has information about amplitude and phase of the signal. This allows bringing the object model and the control law to a system of simple algebraic equations with complex coefficients. Proof of convergence is carried out for the case of single channel. The control law for the multichannel case is being introduced.

INTRODUCTION

There are known adaptive control algorithms for dynamic test rigs, which independently keep up amplitude and phase separately for each control channel and for each harmonic [1]. The scheme of the adaptive loop is shown at fig. 1. A_0 and p_0 – are the amplitude and the phase of the load, which should be applied to the test sample according to the official test program.

G_A и G_p – are the gains of the adaptive loop for the amplitude and the phase respectively. No special investigation of the plant (test sample + test rig) is carried out before testing. The official test starts right away. At the start point its assumed G_A:=1, and G_p:=0. The “Identification” block works at each adaptation step – usually a few periods of the signal; e_A and e_p – are hardness coefficients of the adaptive loop for the amplitude and the phase respectively. They are values between 0 and 1. They determine the adaptive loop working speed. The closer they are to unit, the faster the adaptive loop converges: $A \to A_0$, $p \to p_0$. At $e=1$ the adaptive loop may become unstable. There are three adjustable values used in the algorithm: e_A, e_p and duration of the adaptation step. The algorithm could be independently started for distinct harmonics of the polyharmonic load signal.

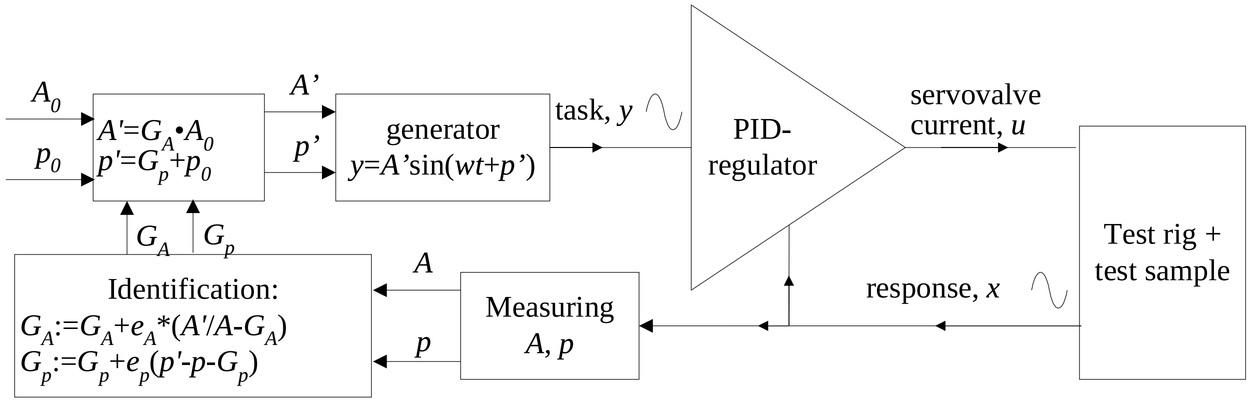

Figure 1. The circuitry of the adaptive loop for maintenance of the amplitude and the phase of a signal, used in [1].

The output of the loop y goes to the input of a PID-regulator as a task. The PID-regulator is the "fast" feedback loop, which controls servovalves of an actuator at the test rig. A force sensor, displace sensor or a strain gage bridge attached to the sample may be used as a feedback sensor. There are also circuitries, where distinct feedback sensors are used for the PID-regulator and the adaptive loop. In such case G_A would not be dimensionless, but would be measured, for example, in $N/(N/mm^2)$, if a force sensor were used for feedback of the PID-regulator and a strain gage bridge – for feedback of the adaptive loop.

The algorithm described above is being used at the laboratory of the dynamic tests of the MHP named after M.L. Mill, and also at the laboratory "Aviatest LNK" (Riga, Latvia) and at the Stupino Plant of the aviation assemblies (Stupino, Moscow Region, Russia). As you can see, the amplitude and the phase are adapted independently. The experience shows, that the algorithm concerned does not provide satisfactory control quality in the presence of a noticeable harmonic noise, which could be caused, for example, by a crosstalk between channels. Furthermore the algorithm fundamentally cannot suppress unwanted signal at some harmonic, i. e. for the case of $A_0=0$.

1. SINGLE CHANNEL CASE

Assume that amplitude at some harmonic is equal to A, and its phase is φ. Then this signal could be presented as a complex number $x=Ae^{i\varphi}$. In this case it isn't important, how the phase is defined: as advance or as backlog from some virtual sinusoidal reference signal.

Assume a signal $y=|y|exp(i\varphi_y)$ comes into the input of the plant, which is aggregate of the test rig, test sample and the PID-regulator. A signal $x=|x|exp(i\varphi_x)$ comes out from the output of the plant. Let present the plant model as quasi-static linear, i. e. while coming throw the plant, the signal changes its amplitude in $|W|$ times, lags on phase in $-\varphi_W$ radians, and an additional harmonic noise at the same frequency with an amplitude $|f|$ and the phase φ_f is added:

$$x=Wy+f \qquad (1)$$

Although the complex values W and f, defining the plant, are unknown at common case, usually the test rig is designed and the PID-regulator is adjusted in such way, in order to y would be reproduced by the x with some accuracy, that can reach –70% in practice. Thus the start estimation can be made: $W_0=1$.

Assume it's needed to get a response equal to $z=|z|exp(i\varphi_z)$. The adaptation algorithm is introduced, composed form a few steps. At each step a certain harmonic task comes into the plant input during a fixed integer number of signal periods, the response is measured at the same time. The amplitude and the phase of the response could be easily calculated from its oscillogram by the integral convolutions, and the complex value *x* could be made up. While coming to the next step, when its needed to send a new task, the task changes smoothly, and after that a new step period counting starts.

Step 1. The task y_1=0 is sent to the plant, the response x_1 is obtained and identified. Then the start estimation of *f* is made:

$$f_0 = x_1. \tag{2}$$

Step 2. Reasoning from assumption that *W*=1, the new task $y_2=z-f_0$ is sent to the plant. Usually such plant as a test rig weaken a few the amplitude of the signal, i. e. usually $|W|<1$. If it isn't so, then it's needed to send $y_2=\kappa(z-f_0)$ to keep the test rig unbroken, where κ~0,01..0,3. At the end of the step, the response x_2 is identified. According to our plant mode, $x_2=Wy_2+f$, then a more accurate estimation of *W* could be made:

$$W_0 := \frac{x_2 - f_0}{y_2} \tag{3}$$

Here it's necessary to check if $|W_0|<<1$. If it is so, then the plant is actually is not controlled by the task *y*, for example as the result of low oil pressure in the test rig hydrosystem, or as the result of cable rupture.

Step k+1>2. The pattern lies in modification of the estimation of *f*. Assume we have values of the task and the response of the previous step: y_k and x_k. As the goal is to converge *x* to *z*, y_k was assigned from

$$y_k=(z-f_k)/W_0, \tag{4}$$

where f_k – is current estimation of *f*. Then we made a new estimation of *f*:

$$f_{k+1} := f_k + \eta(x_k - W_0 y_k - f_k). \tag{5}$$

Here $\eta \in (0;1)$ – is a adaptation hardness coefficient. The closer η to unit, the faster adaptation goes.

Let us find a new task y_{k+1} for the step k+1, using (4) and (5):

$$y_{k+1} = \frac{z - f_{k+1}}{W_0} = \frac{z - f_k}{W_0} - \eta\left(\frac{x_k}{W_0} - y_k - \frac{f_k}{W_0}\right) = \\ = y_k + \eta\frac{z - x_k}{W_0} \tag{6}$$

Thus, f_k was excluded from the adaptation circuitry.

Let us investigate the algorithm concerned for convergence, taking into account the plant model (1).

$$y_{k+1} = y_k + \eta\frac{z - x_k}{W_0} = y_k + \eta\frac{z - Wy_k - f}{W_0} = y_k\left(1 - \eta\frac{W}{W_0}\right) + \eta\frac{z - f}{W_0} \tag{7}$$

Let's introduce designations:

$$A = 1 - \eta \frac{W}{W_0}$$
$$B = \eta \frac{z - f}{W_0} \tag{8}$$

Then, recurrently substituting (7):

$$y_{k+1} = y_{k-1}A^2 + y_{k-1}AB + B = y_1 A^k + B\left(A^{k-1} + A^{k-2} + \ldots + A + 1\right) \tag{9}$$

$$\lim_{k\to\infty} y_k = \frac{B}{1-A} = \frac{\eta(z-f)}{W_0} \cdot \frac{1}{1-\left(1-\frac{\eta W}{W_0}\right)} = \frac{z-f}{W} \tag{10}$$

,

and hence

$$\lim_{k\to\infty} x_k = z \tag{11}$$

i. e. the goal is reached. The sufficient condition for the limit existence is:

$$\left|1 - \frac{\eta W}{W_0}\right| < 1 \tag{12}$$

Actually it means that the error of defining W_0 at the second step should not be very high, and W should not “flow away” with time. In the ideal case, when $W_0=W$, convergence would take place at any $\eta \in (0;1)$.

Modification of the estimation of f is not a sole variant. It’s also possible to adapt, modifying the estimation of W at k+1-th step. In such case the following results are achieved.

$$y_{k+1} = \frac{y_k}{1 - \eta \frac{z - x_k}{z - f_0}} \tag{13}$$

Condition of convergence:

$$\left|1 - \eta \frac{z - f}{z - f_0}\right| < 1 \tag{14}$$

Similarly, this condition means, that error of initial defining of f_0 should not be very high.

It seems that the most attractive variant is serial alternating of steps with the modifying of the estimation of W and steps with modifying of the estimation of f, and the modifying of the estimation of f should be carried out more often.

It should be noticed that both algorithms concerned are insensible for equality or inequality z to zero, thus they could be used for suppression of unwanted harmonics.

The algorithm with modifying the estimation of f is now running at the laboratory “Aviatest LNK” for unwanted harmonic suppression in the challenge of the fatigue test of the fuselage of Mi-26.

2. MULITICHANNEL CASE, POLYHARMONIC SIGNAL

In the case, if the plant have n control channels, they should be considered as vector-columns: $x=[x^1,\ldots,x^n]^T$, $y=[y^1,\ldots,y^n]^T$, $z=[z^1,\ldots,z^n]^T$, each component is a complex value. We will denote a vector component by the upper index, and adaptation step number by the lower index.

There are only plans considered, which have output number equal to input number. If response number is more than task number, then necessarily some problems with controllability would appear, and test rigs with task number more than response number usually are not constructed.

We will consider a linear plant:

$$x=Wy+f \tag{15}$$

Here f is an nx1 vector-column, and W is an nxn matrix.

In order to control additional harmonics, one needs to include a new component to all vectors. Different harmonics behave as independent ones, as far as the plant model is linear. Even though real plant is significantly nonlinear, for example bilinear, then the first harmonic of the input would generate the second harmonic in the output:

$$\text{plant input}: \quad \sin \omega t$$

$$\text{plant output}: \quad \sin^2 \omega t = \frac{1}{2} - \frac{1}{2}\cos 2\omega t$$

Such crosstalk is just included into the W matrix. Thus if we have n_c channels, and we consider as many as n_h harmonics on each one, the x vector would have $n=n_c \cdot n_h$ components.

Consider the algorithm step-by-step.

Step 1 is analogous to single-channel case: send the task $y_1=0$, get the response x_1, and make the initial estimation of f:

$$f_0=x_1. \tag{16}$$

It should be noted, that $f \neq 0$ only in the case if the plant concerned is a part of another large "superplant", at that the influence of other parts of the "superplant" to the plant concerned, can be represented as a slow-changing outer noise.

Step 2. It's necessary to make estimation of the W matrix, i. e. to get n^2 complex values. A few variants are possible here.

Variant 1. Conscientiously measure every component of the W matrix. For that procedure n substeps are required. At each step only one component of the y vector is not equal to zero. At the substep, labeled as (2,k), an equations for k-th column $w^k=[W^{1k},\ldots,W^{nk}]^T$ of the W matrix are obtained:

$$x_{2,k}=w^k \cdot y_{2,k}{}^k+f \tag{17}$$

Hence the estimation of the k-th column:

$$w_0{}^k = (x_{2,k} - f_0)/y_{2,k}{}^k \tag{18}$$

Here $w_0{}^k$, $x_{2,k}$, f_0 are vector-columns, and $y_{2,k}{}^k$ is scalar. Now following question should be answered: how to determine appropriate task for such plant investigation, i. e. what are values $y_{2,k}{}^k$ equal to? Here one should act similarly to the single-channel case: assuming W=1 (unitary matrix): $y_{2,k}^k=\kappa(z^k-f_0^k)$, where κ either is equal to 1, or, if W is supposed to be strongly non-diagonal, appears in the range 0,01…0,3. The rest $y_{2,k}^j=0$ when $k\neq j$.

Variant 2. One can assume W matrix to be diagonal. That means that W can be obtained in one step (or substep). One should send the task $y_2=\kappa(z-f_0)$ and make the estimation of W:

$$W_0^{jj} =(x_2^j-f_0^j)/y_2^j,\ j=1..n,\qquad W_0^{jk} =0,\ j\neq k \tag{19}$$

This method is not suitable, if there are components of the $(z\text{-}f_0)$ vector close to zero.

Variant 3. It's possible to send in one step $y_2=\kappa(z-f_0)$ and assume the W estimation:

$$W_0 =(x_2-f)y^{\mathrm{T}}(yy^{\mathrm{T}})^{-1} \tag{20}$$

If the result of any variant would be singular matrix W_0 (with zero determinant), or close to singular (using the minimal absolute value of eigenvalue), then the plant have some problems with controllability.

Step k+1>2. Here the action is similar to the single-channel case with modification of the f estimation. Following results are obtained:

$$y_{k+1} = y_k + HW_0^{-1}(z - x_k) \tag{21}$$

Here H is a nxn matrix, consisting from adjustable coefficients of adaptation speed. Eigenvalues of this matrix should be less than 1 on absolute value, the matrix must be positively defined. It's recommended to start adjusting from a diagonal matrix, whose elements are the same and belong to the interval (0;1).
Condition of convergence:

$$\left\| 1 - HW_0^{-1}W \right\| < 1 \tag{22}$$

Here ||·|| is the matrix norm – maximal absolute value of eigenvalue. Like the single-channel case, this condition means that the error of the initial estimation W_0 should not be very high.

There is a problem of the algorithm concerned: it cannot trace significant "flow out" of the W matrix. Really the convergence condition stops to hold true, when any eigenvalue of W rotates for more than a quarter a revolve at the complex plane from the initial one.

One cannot re-esitmate W permanently because of insufficient "variability" of the task. Usually z is constant for a very long time in a fatigue test, and any attempt of estimate whole non-diagonal matrix W leads to a system of n^2 linear equations like follows:

$$[x_{k-n+1} \quad \ldots \quad x_k] = W[y_{k-n+1} \quad \ldots \quad y_k] + [f_{k-n+1} \quad \ldots \quad f_k], \tag{23}$$

where W is unknown, f_j are estimates of f at j-th step. Since columns of known matrices are very close to each other, because steps don't differ very much, then solution of this system have very poor accuracy. There are a few tricks to overcome these problems.

Trick 1. Use decreasing adaptation speed matrix H: $H\sim 1/k$ (see the method of recurrent inequalities in [2]). When used in the adaptation law

$$W_{k+1} = W_k + H\left((X - F)Y^{-1} - W_k\right), \tag{24}$$

where $X=[x_{k-n+1},\ldots,x_k]$, $Y=[y_{k-n+1},\ldots,y_k]$, $F=[f_{k-n+1},\ldots,f_k]$, the decreasing *H* matrix makes the system insensitive form close-to-singularity of the *Y* matrix, but actually *W* re-estimation switches off after large amount of steps.

Trick 2. Make re-estimation of *W* only if *x* diverges from *z*. Actually diverging would appear only if an eigenvalue of *W* rotates for more than a quarter a revolve at the complex plane. Diverging of *x* means that *X* and *Y* matrices are not too close to singular ones. The problem is that it's needed not 2, but all *n* different *y* vectors.

Trick 3. Add to *y* small but considerable distortions $s_k \in \mathbf{C}^n$. The matrix $S=[s_{k-n+1},\ldots,s_k]$ should be strongly nonsingular, for example scalar. In such way the *Y* matrix moves off from the singular one. The problem is that distortions make the test process inaccurate. Really the norm of *s* should be lower than the norm of *z* in 20..100 times or more, which can make distortions to become inconsiderable (with no measurable influence to *x*).

Many other tricks and their combinations could be also suggested, such as using of less squares method, direct using of the recurrent inequalities method, nonsensibility zone and so on.

3. WHOLE OR COMPOSITE?

Instead of considering a multichannel plant as a whole, often it's simply to consider it as an aggregate of independent components, an individual component per each harmonic. An external slow-flowing noise comes to each component.

While one component is adapting, the other should be frozen. After modifying the estimation of *f* at *k*-th component and a new task being sent, the external noise at the other channels would change. Then it's required to froze the *k*-th component and to proceed to the *k+1*-th one.

Such a decomposition has following disadvantage: the adaptation time grows linearly with the amount of components. It can result, for example, in that the system would not react in time to hydrosystem oil pressure change, or oil filter fouling and other emergency situations, that proves in the plant characteristics change.

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