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KINEMATIC OBSERVERS FOR ACTIVE CONTROL OF HELICOPTER ROTOR VIBRATION

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Abstract

A simple scheme for estimating the state variables of a helicopter rotor is presented. The method incorporates the use of blade-mounted accelerometers and/or position transducers to reconstruct modal displacements and velocities. The design of the observer structure and feedback gains is simplified by the fact that the method requires only knowledge of basic kinematic relationships between the various modal quantities. The observer structure described is particularly well-suited to control problems where the use of a traditional Kalman Filter approach would be too complex or costly. The technique can be viewed as descreasing the requirements on observer complexity while increasing the need for an enhanced sensor complement.

1. Introduction

Recent efforts to apply active control technology to rotary wings have shown promise in reducing response to atmospheric turbulence, retreating blade stall, vibration suppression, blade-fuselage inteference, and flap-lag modal damping enhancement [1-7]. These applications have all used the method of active pitch control to produce counteracting aerodynamic forces on the rotor blades. The methods for generation of the control actuation, however, can be divided into two fundamentally different approaches, either Higher Harmonic Control (HHC), or Individual Blade Control (IBC). HHC has traditionally been applied almost exclusively to vibration reduction [5,7], where integral multiples of rotor rotational frequency are appropriately scaled and phase shifted so as to generate pitch commands, either open- or closed-loop, that approximately cancel the harmonics of vibration passed down from the rotor to the fuselage. IBC has a larger number of potential applications [1-4], since it involves the use of actuators and sensors on each blade to control the pitch individually in the rotating frame of reference. This latter approach is essentially a "broad band" control of the rotor blade dynamics, as opposed to the HHC limitation of discrete frequency disturbance suppression, and thus is also capable of modifying each blade's aeroelastic stability, modal damping and modal frequencies.

Controller design for the IBC system is most easily done using a state-variable (or "modern" or "optimal") control approach, due to the fact that it can easily handle the many interacting rigid and elastic degrees of freedom present in any rotor system, as well as any periodically time-varying parameters [1]. The consequence of using such a design technique is that one is required to feed back a linear combination of <u>all</u> of the state variables to the control input. This often cannot be accomplished because all of the state variables are rarely available for measurement. Instead, the controls engineer must resort to using estimates of these states produced from an "observer". An observer is a dynamic element that takes the sensor signals as inputs and produces state estimates as outputs. The form of the observer is intimately related to the particular complement of sensors available, and often comprises the most complex part of the controller structure.

The instrumentation used to measure the rotor states and/or responses varies from application to application, but appears to be strongly related to the type of rotor control system employed. In the case of HHC systems, these measurements are often made at several fuselage locations about the aircraft, with the assumption that the vibratory loads vary linearly with changes in harmonic pitch inputs. This approach requires, for most cases, an empirical fit to response data in order to account for the effects of rotor impedence and the complex interactions present in the rotor wake. In the case of IBC systems, however, these measurements are made in the rotating frame of reference, since the feedback loops for this type of control are around each blade individually. This has the advantage of not requiring an accurate representation of the fuselage structure and rotor impedence, and posesses the attractive property of placing the measurement at the source of the disturbance. The increase in potential applications for IBC is accompanied by a more severe estimation task, though, since estimates of the blade's modal displacements and velocities are required for feedback control.

The design of observers for estimating rotor state variables is currently a topic of active research [1,8-12]. Most of these designs use a Kalman filter-type structure, where a mathematical model of the system dynamics being observed is forced by the error between the actual measurements and their predicted values. A full Kalman filter is rarely used, as it requires an a-priori knowledge of the random processes perturbing the rotor system, a knowledge of the structure of the noise corrupting the measurements, and the exact model of the plant dynamics relating the various physical quantities. Given the complex dynamic and aerodynamic environment of most helicopter rotors, this proves to be too great a demand on mathematical modeling ability. Approximations are made in the representation of the plant dynamics or in the assumptions about the signal content of the available sensors.

Recent work on applications of Individual Blade Control to high advance ratio rotor control [1] brought forth a novel and extremely effective technique to estimate the missing state variables of a complex, periodically time-varying system. By incorporating an accelerometer within the observer structure, it was possible to accurately estimate the missing states of the system using a constant-coefficient dynamic element. Also, since the accelerometer signal was used to "force" the observer, an accurate model of the blade dynamics was not necessary. However, this form of observer does require a good description of the sensor dynamics (if present) and modal content of each sensor's output signal over the bandwidth of its response. The significance of the form of the observer is best appreciated after noting the difficulties present in attempting a standard application of Kalman filter theory to the problem.

2. Traditional State Estimation

Consider the linear time-varying state vector representation of the dynamics of an individual rotor blade as:

$$\dot{x}(t) = A(t)\ x(t) + B(t)\ u(t)$$

where:

$$x(t) = \begin{bmatrix} \beta(t) \\ \dot{\beta}(t) \\ g(t) \\ \dot{g}(t) \end{bmatrix}$$

represents the state vector containing the flapping position and velocity ($\beta(t)$ and $\dot{\beta}(t)$), and the first elastic bending mode displacement and velocity ($g(t)$ and $\dot{g}(t)$), and $u(t)$ represents the blade pitch control input ($\theta(t)$). $A(t)$ is a 4x4 matrix of time-varying coefficients from the blade equation of motion, and $B(t)$ is a 4x1 matrix of the time-varying control effectiveness. Observer theory (of which the Kalman filter is a special case) incorporates the concept of negative feedback to force the errors in the state estimates to approach zero exponentially with time. This is done by driving a model of the system with an input proportional to the difference between the actual measurements and the predicted measurements based on the current state vector estimate. If one represents these measurements as:

$$y(t) = C(t)\ x(t) + D(t)\ u(t)$$

then the observer has the form:

$$\dot{\hat{x}}(t) = A(t)\hat{x}(t) + B(t)u(t) + K(t)[\ y(t) - C(t)\hat{x}(t) - D(t)u(t)\ ]$$

or,

$$\dot{\hat{x}}(t) = [\ A(t)-K(t)C(t)\ ]x(t) + [\ B(t)-K(t)D(t)\ ]u(t) + K(t)y(t)$$

where the "hatted" quantity indicates an estimate of the state vector. The choice of the matrix $K(t)$ determines the speed with which the estimation errors are reduced, and depends upon the noise statistics for case of the Kalman filter. Note that the observer has two parts: the first provides a prediction of the rate of change of the state vector by simulating the system equation of motion, and the second provides some corrective action based upon the error between the actual sensor's output and the expected value based on the current state estimate.

The instrumentation proposed for the IBC vibration control system consists of a series of blade mounted accelerometers, with their sensitive axes oriented perpendicular to the blade surface. As shown in figure 1, this particular installation of the accelerometers results in their output being proportional to out-of-plane displacement as well as acceleration, due to their orientation in a centrifugal force field. Since flapping and bending mode acceleration are not state variables but time derivatives of state variables (i.e., time derivatives of modal velocities), one must represent each accelerometer's signal content by incorporating the system dynamics in the observation matrices. Thus, for an accelerometer that senses the combination:

$$\text{accel}(t) = H1\ (t) + H2\ \dot{x}(t)$$

one can reconfigure this to be:

$$\text{accel}(t) = H1\ x(t) + H2\ \{\ A(t)x(t) + B(t)u(t)\ \}$$

or,

$$\text{accel}(t) = \{\ H1 + H2\ A(t)\ \}\ x(t) + \{\ H2\ B(t)\ \}\ u(t)$$

This is indeed an unfortunate situation, for now the representation of the signal content of the sensor depends intimately upon the modeling accuracy of the dynamics. This constraint can make the design of a suitable control law for active helicopter rotor vibration control extremely difficult, due to the complex flowfields and structural nonlinearities often present in such vehicles, as well as the periodically-time-varying nature of the individual blade dynamics in forward flight.

3. Kinematic Observers

Fortunately, a way around this problem is possible by reformulating the equations representing the system dynamics. If one considers the blade dynamics from the previous example, we can reformulate the equations of motion as:

$$\begin{bmatrix} \dot{\beta}(t) \\ \ddot{\beta}(t) \\ \dot{g}(t) \\ \ddot{g}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \beta(t) \\ \dot{\beta}(t) \\ g(t) \\ \dot{g}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \ddot{\beta}(t) \\ \ddot{g}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix}$$

where we have used $w_1(t)$ and $w_2(t)$ to represent ficticious external disturbances. This equation represents nothing more than the knowledge that the position of the blade is the double integral of the acceleration applied to it. If one has knowledge of what this acceleration is (as we do, given that we are using accelerometers for measurement), one can construct an observer for this system that has a form dependent only upon the kinematics of the process being observed. This is accomplished by including the observation equation:

$$y(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \beta(t) \\ \dot{\beta}(t) \\ g(t) \\ \dot{g}(t) \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix}$$

into a standard Kalman-filter type of observer, where we have assumed modal position measurements are also available, and where we are using $v_1(t)$ and $v_2(t)$ to represent measurement noises. By trading off the relative strengths of the process and measurement noise covariances, one can control the bandwidth of the observer dynamics for each mode independently. The net result is simply the double integration of the acceleration information, with the bias errors in the velocity and position estimates driven to zero through feedback of the displacement estimation error.

The significance of this approach needs to be emphasized. By reducing the state estimation problem to a constant coefficient dynamical form, generation of modal rate estimates can be accomplished with relative ease. This allows the use of modern, multi-input/multi-output control law design techniques for rotor control with no penalty on the number or types of state feedback gains required. One does not even need to simultaneously estimate the dynamics of the lower-order modes, since all that is needed is an accurate measurement of the particular modal acceleration and position. This can be achieved by providing two sensors (at least one of which is an accelerometer) for every mode of interest, starting with the lowest mode. Thus, for the two-mode system described above, four accelerometers will provide a unique estimate of each modal position and acceleration for use in the above observer structure. The selection of the bandwidth of each modal observer is made by iterating on the process and measurement noise covariance specifications, such that the particular modal natural frequency is well within the break frequency of the observer. If only the higher frequency modal state variables are required, only one observer need be implemented, but the requirement on the number of sensors remains the same in order to generate a unique measure of the higher mode's acceleration and position.

4. Implementation Issues

A set of computer simulations using this approach were run in support of some experimental work currently in progress at Princeton's Dynamic Model Track. Of specific interest were the various implementation issues associated with the choice of such an observer scheme. Since successful application of Kinematic Observers in closed-loop control tasks depends upon accurate reconstruction of the modal displacements and accelerations from the given sensors, the influences of sensor location, signal nonlinearities, assumed mode shape and choice of observer bandwidth were investigated as they affect estimation error.

The placement of the accelerometers was chosen according to an optimization procedure outlined in [13], where the condition number of the measurement matrix was minimized. That is, the content of the out-of-plane accelerometers can be represented by:

$$\begin{bmatrix} accel_1 \\ accel_2 \\ accel_3 \\ accel_4 \end{bmatrix} = \begin{bmatrix} r_1\Omega^2\partial\eta_1(r_1)/\partial r \; \ldots & \ldots & \ldots \\ \ldots & \eta_1(r_2) & \ldots & \ldots \\ \ldots & \ldots & r_3\Omega^2\partial\eta_2(r_3)/\partial r & \ldots \\ \ldots & \ldots & \ldots & \eta_2(r_4) \end{bmatrix} \begin{bmatrix} \beta(t) \\ \dot\beta(t) \\ g(t) \\ \dot g(t) \end{bmatrix}$$

where η_1 and η_2 represent the rigid flap and first elastic bending modes of the blade. The product of the inverse of the above matrix with the four accelerometer measurements produces a unique measurement of the two out-of-plane modal displacements and accelerations, provided, of course, that the matrix is nonsingular. The larger the condition number for the matrix, the more nearly singular the matrix is, and hence the poorer the measurement of the modal acceleration and displacement become. Various bending mode shapes of increasing order that all satisfied the boundary conditions were selected for the optimization trials, and a plot of the behavior of the four optimum accelerometer locations as a function of mode polynomial order is shown in figure 2. The general trend is that as the curvature of the higher-order polynomials shifts out toward the tip, so also does the set of optimum locations for the blade accelerometers. This result suggested that the observer may exhibit stronger sensitivity to assumed mode shape than originally anticipated. In order to gauge this sensitivity, the condition number of the measurement matrix was plotted as each accelerometer was varied individually away from its optimum location, shown in figure 3. The flatness of the curves indicates that precise sensor placement is not essential, as the condition number does not vary significantly with moderate placement errors.

The second study considered the influence of nonlinearities present in the actual accelerometer's signal on the estimation accuracy of the observer. In order to capture all of the possible nonlinear effects, two out-of-plane modes and one rigid in-plane mode were included in the simulation. A white noise sequence was used as the pitch input to excite the system, providing a particularly challenging tracking task for the observer. The equations of motion used quasisteady aerodynamics with all coriolis coupling terms included in the inertial operators. The flap and lag modes were assumed rigid with a coincident offset hinge, and the out-of-plane bending mode satisfied both natural and geometric boundary conditions at the root and tip, as well as orthogonality with the rigid flap mode.

The non-linear accelerometer signals used in the simulation were:

$$accel(r,t) = \cos\left(\frac{\partial z(r,t)}{\partial r}\right)\ddot z + \sin\left(\frac{\partial z(r,t)}{\partial r}\right)\left[\cos(\zeta)\Omega^2 e + r\Omega^2 - 2r\Omega\dot\zeta + r\dot\zeta^2\right]$$

where:

$$z(r,t) = \eta_1(r)\beta(t) + \eta_2(r)g(t)$$

is the out-of-plane displacement, ζ is the rigid lag angle, e is the offset hinge length, Ω is the rotation speed, and r is the spanwise accelerometer location.

Comparison of the non-linear and linearized accelerometer signals for the farthest outboard accelerometer is presented in figure 4. The two are quite close, and produce almost equivalent estimates for the bending mode displacement and acceleration, indicating that the small angle assumption implicit in the above measurement matrix is indeed valid. These were then used to provide an estimate for the bending mode velocity, and the comparison of the "observed" velocity and the actual velocity generated from the simulation are shown in figure 5. The velocity estimate tracks the actual state almost identically, despite the "unmodelled" white-noise pitch disturbance.

Since the placement of sensors was not found to be overly sensitive to assumed bending mode shape, it was assumed that the coefficients in the measurement matrix would exhibit similar robustness. As a check, the same simulation data was used to produce estimates of the bending mode displacement and acceleration, but with a higher order polynomial used in generating the elements of the sensitivity matrix given above. The results, shown in figure 6, are quite poor, demonstrating that an accurate representation of the blade modal properties is essential for the kinematic observer to prove successful. This requirement can be easily met by performing a few simple modal identification tests using the installed accelerometers prior to initiating any feedback control or estimation tasks.

Finally, as a means of assessing the implications of considering only a limited number of modal displacements, a Kinematic Observer was designed for estimating the displacement and velocity of the rigid blade flapping mode, in the presence of unmodeled higher modal participation. The previous flapping observer bandwidth of 5/rev was used, with only the most outboard and most inboard accelerometers incorporated into the 2x2 measurement matrix. Since the observer bandwidth extends beyond the 3/rev natural frequency of the second out-of-plane mode, it was felt that this would severely limit the observer's performance by introducing significant errors into the reconstructed flap displacement and acceleration data. As can be seen in figure 7, the flap velocity is estimated quite poorly, indicating potential sensitivity to "modal spillover" problems, not unlike conventional observers.

5. Control System Applications

In order for Kinematic Observers to prove useful in state variable control applications, it would be very desirable to be able to design them separately from the feedback control gains. Fortunately, such is the case, due to our favorable choice of system sensors. Since we are driving the "prediction" of the modal state variables by the actual measured acceleration, and since we are using position estimates to correct for any estimation errors, the state estimates may be used with impunity in any state-feedback controller design. Unlike the approaches of [8] and [9], this form of observer makes no approximation in its representation of the equations of motion, and thus the estimation errors are uncorrelated with the system states. Put in other terms, a feedback control system that uses these state estimates will obey the "separation principle" of modern control design, which allows the separate design of a state feedback controller from that for the observer. A simple example for a reduced-order problem will illustrate.

Suppose we have a truncated observer for the first out of plane bending mode of the form:

$$\frac{d}{dt}\widehat{\begin{bmatrix} g(t) \\ \dot{g}(t) \end{bmatrix}} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \widehat{\begin{bmatrix} g(t) \\ \dot{g}(t) \end{bmatrix}} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} \ddot{g}(t) \end{bmatrix} + \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \begin{bmatrix} g(t) - \hat{g}(t) \end{bmatrix}$$

where f_1 and f_2 are the observer gains, and $g(t)$ and $\dot{g}(t)$ are available for measurement. If we wished to utilize these estimated states in a control law of the form:

$$\frac{d}{dt}\begin{bmatrix} g(t) \\ \dot{g}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix} \begin{bmatrix} g(t) \\ \dot{g}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ b_0 \end{bmatrix} \begin{bmatrix} \theta(t) \end{bmatrix}$$

$$\theta(t) = - \begin{bmatrix} k_1 & k_2 \end{bmatrix} \widehat{\begin{bmatrix} g(t) \\ \dot{g}(t) \end{bmatrix}}$$

we could analyze the dynamics of the closed-loop system by first defining the estimation error as:

$$e(t) = \hat{x}(t) - x(t)$$

and thus we get the augmented state dynamics:

$$\frac{d}{dt}\begin{bmatrix} g(t) \\ \dot{g}(t) \\ e_1(t) \\ e_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ (-a_0-b_0k_1) & (-a_1-b_0k_2) & b_0k_1 & b_0k_2 \\ 0 & 0 & -f_1 & 1 \\ 0 & 0 & -f_2 & 0 \end{bmatrix} \begin{bmatrix} g(t) \\ \dot{g}(t) \\ e_1(t) \\ e_2(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ f_1 & 0 \\ f_2 & 1 \end{bmatrix} \begin{bmatrix} v_2(t) \\ w_2(t) \end{bmatrix}$$

where w_2 and v_2 are the process and measurement noise respectively for the bending equation. Of primary interest is the fact that the estimation errors are uncoupled from the system dynamics, and thus the desired closed-loop poles of the state-feedback controller do not move. This uncoupling arises from the fact that by using the actual modal accelerations in our observer structure we are able to <u>exactly</u> predict the time variation in the state variables. Such a result should make control systems using "kinematic observers" more robust than traditional Kalman Filter approaches. An example of such an application was run as part of the above mentioned simulations.

Since it was desired to simulate a closed-loop system using the above observer structure and control law, a disturbance other than simple pitch excitation was used. Instead, a tip vortex encounter was simulated by imposing a spanwise-cubic raised-cosine inflow distribution over a portion of the disk on the advancing side. This "kick" was sufficient to excite the first bending mode of the blade, as shown in the open-loop response plot of the bending acceleration in figure 8. Then, the same disturbance was simulated, with the control system using feedback of the observed bending rate to the blade pitch angle. The closed-loop response, shown in figure 9, shows a diminished bending acceleration, which would translate into a reduce inertial shear load at the hub. Comparison of the open- and closed-loop acceleration spectra are given in figure 10. Even better improvement could be obtained through a methodical feedback control design approach, rather than the heuristic one simulated here.

The ease in which state estimates can be estimated using this technique suggests additional control applications beyond traditional state variable feedback. Since only a kinematic model of the system is necessary for the observer to produce state estimates, and since the modal accelerations are available as a measurement, it becomes possible to solve for the system coefficients describing the differential equation of modal motion. These coefficients can be determined through solution of linear equations or by a least-squares technique. Such a procedure was done for reference [1], and the resulting system identification data was used to design a successful time-varying control law. Were this identification done on-line in a recursive fashion, one may even incorporate the coefficient tracking ability into an adaptive controller, which should exhibit similar robustness as that present in the observer itself.

6. Conclusions

The above method of constructing an observer for rotor state variables presents an alternative to the standard Kalman Filter approach, by utilizing the predictive information content present in an accelerometer. The structure is extremely simple and is not dependent upon the system differential equations, but requires an accurate representation of the kinematic modal content of each sensor. The decision to use such an observer must be based upon the relative costs of implementing an inherently complex Kalman Filter versus adding additional sensors, but for complex rotor vibration control problems, the latter is often the less expensive choice.

The ability of the observer to isolate modal states and accelerations also allows its use as a pre-processor of rotor data in a parameter identification role. Given this wealth of information, such applications should prove valuable in providing more accurate rotor mathematical models to aid the control design process.

7. Acknowledgement

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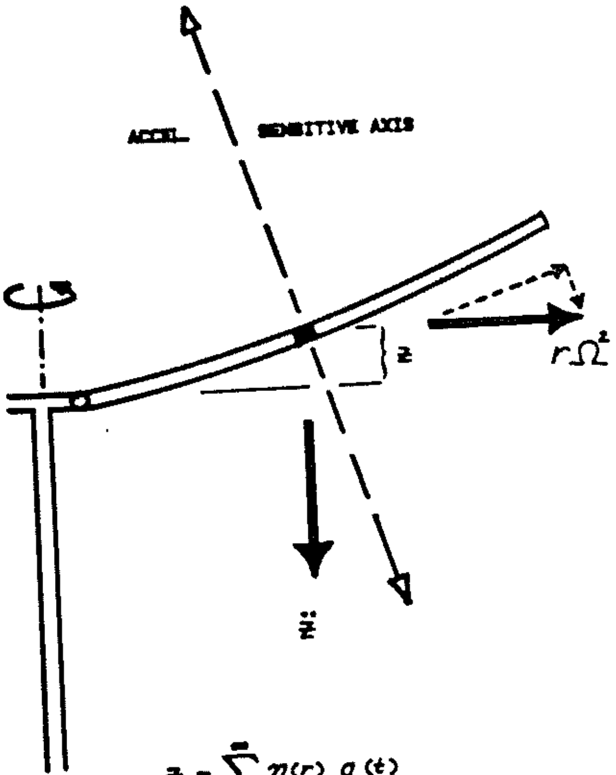

$$z = \sum_{k=1}^{\infty} \eta_k(r)\, g_k(t)$$

$$\text{ACCEL} = \sum_{k=1}^{\infty} \left\{ \eta_k \ddot{g}_k + r\Omega^2 \frac{d}{dr} \eta_k g_k \right\}$$

Fig 1: ACCELEROMETER SENSOR DYNAMICS

Fig 2: OPTIMUM ACCELEROMETER LOCATIONS

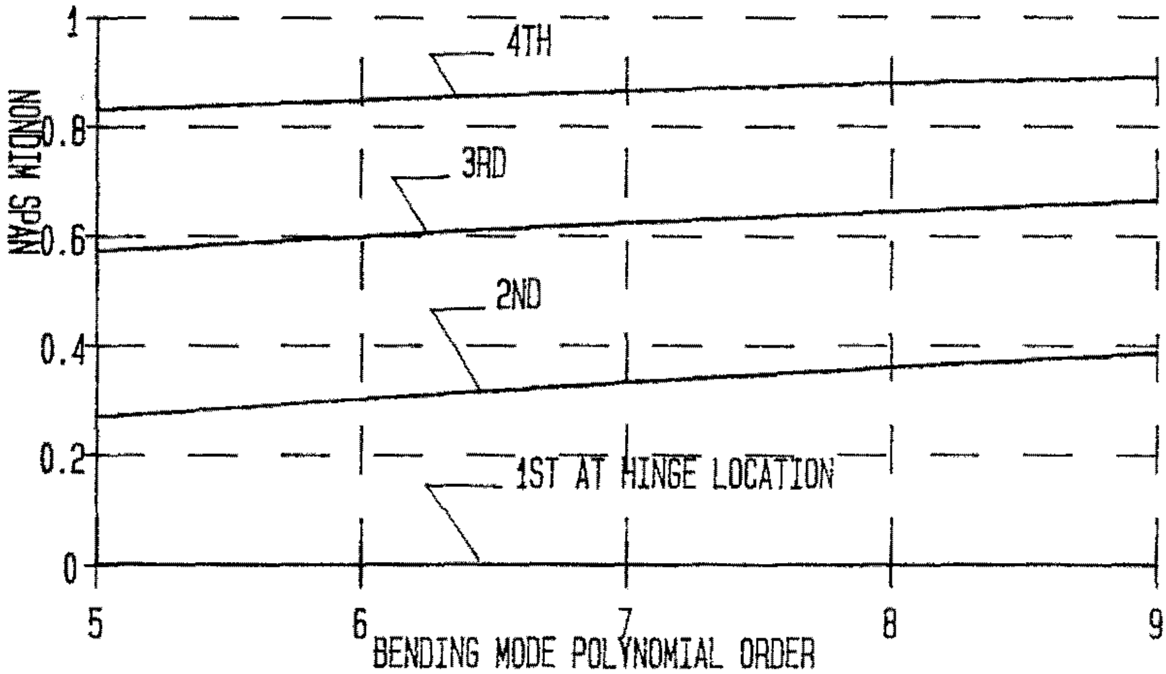

Fig 3: CONDITION NUMBER SENSITIVITY

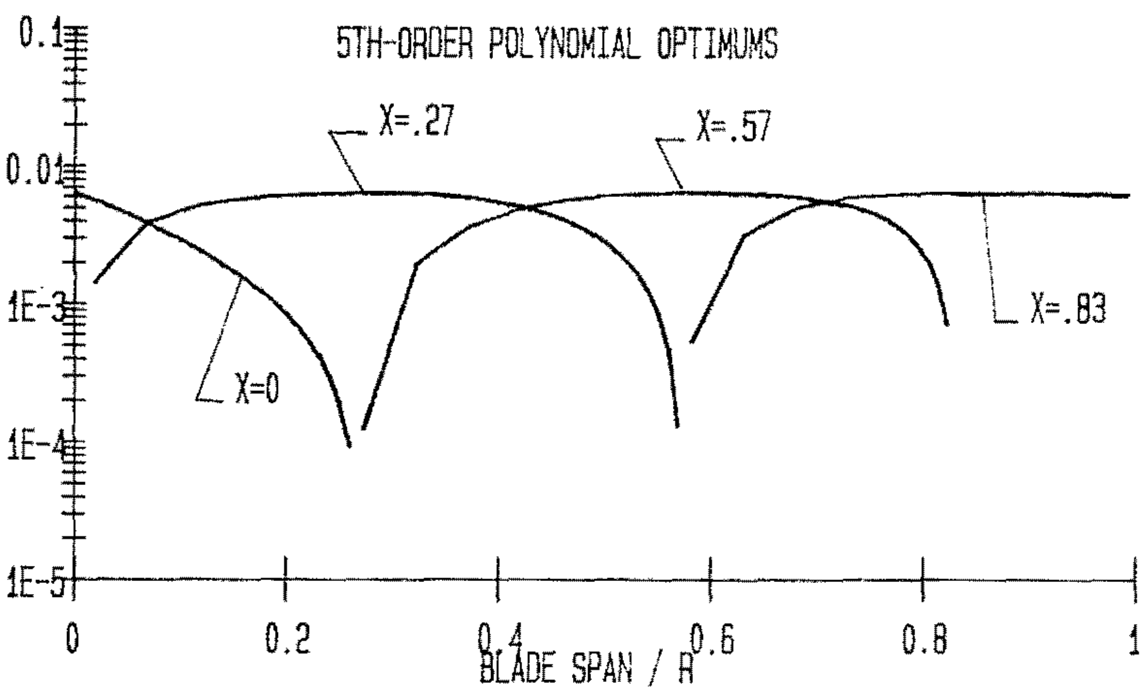

Fig 4: LINEARIZED AND NONLINEAR ACCELEROMETER SIGNAL

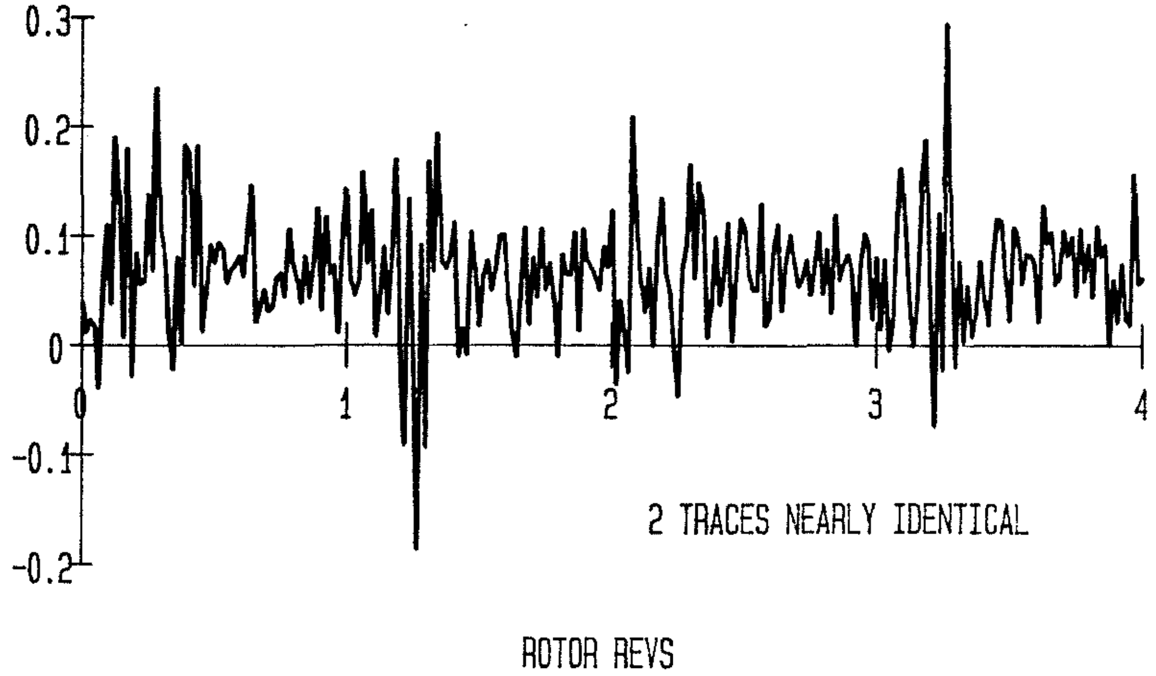

Fig 5: SIMULATED AND OBSERVED BENDING RATE

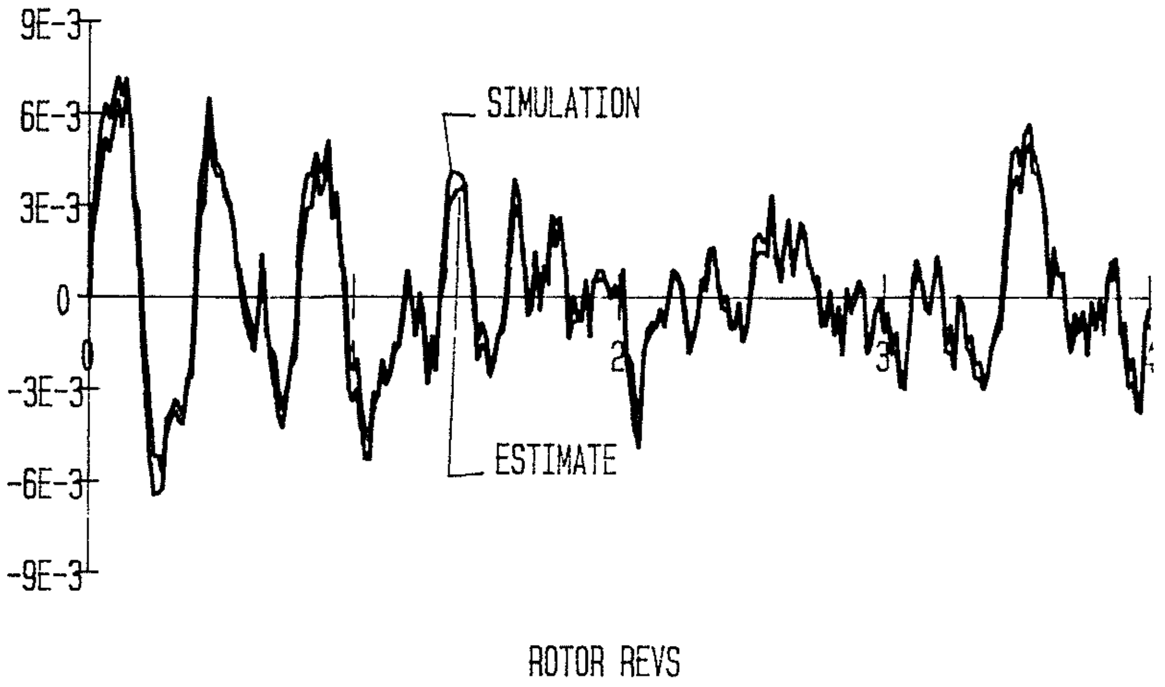

Fig 6: BENDING POSITION ESTIMATE USING WRONG MODE SHAPE

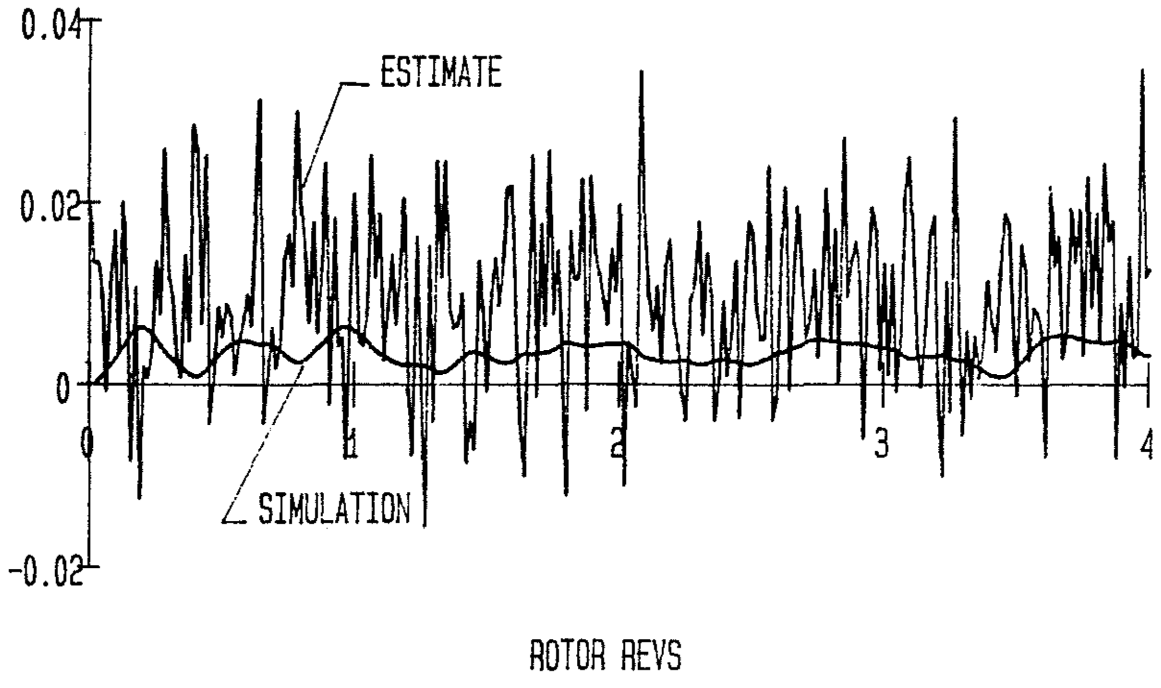

Fig 7: FLAP RATE ESTIMATE NEGLECTING BENDING MODE

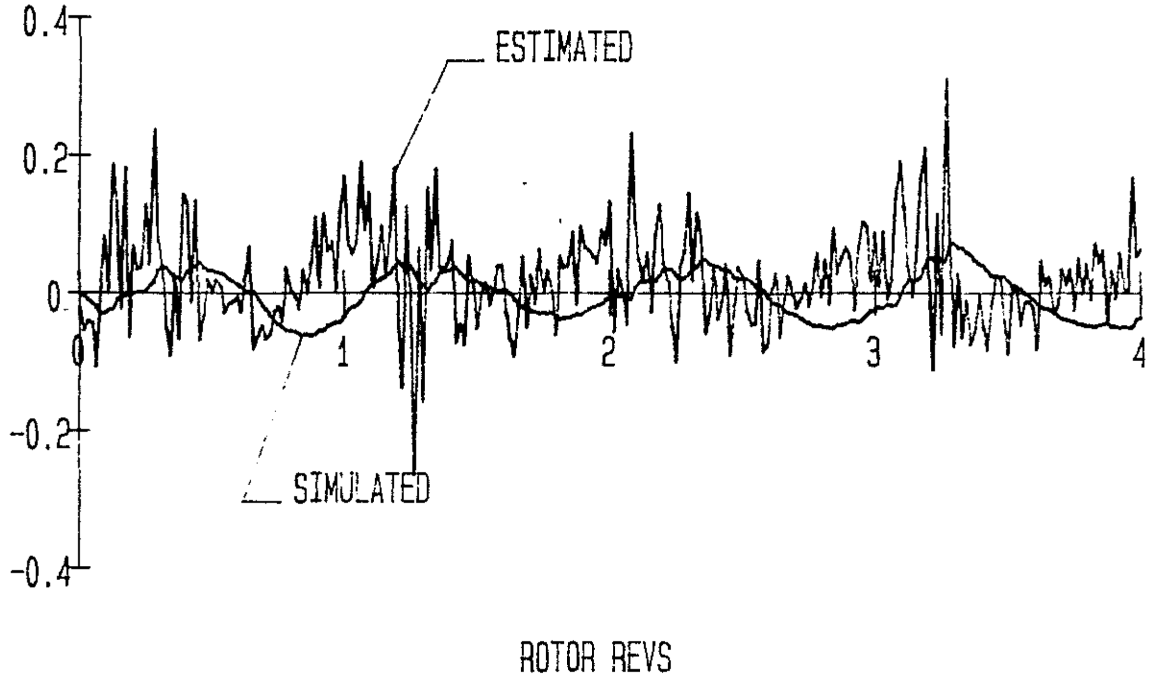

Fig 8: OPEN-LOOP BENDING ACCELERATION

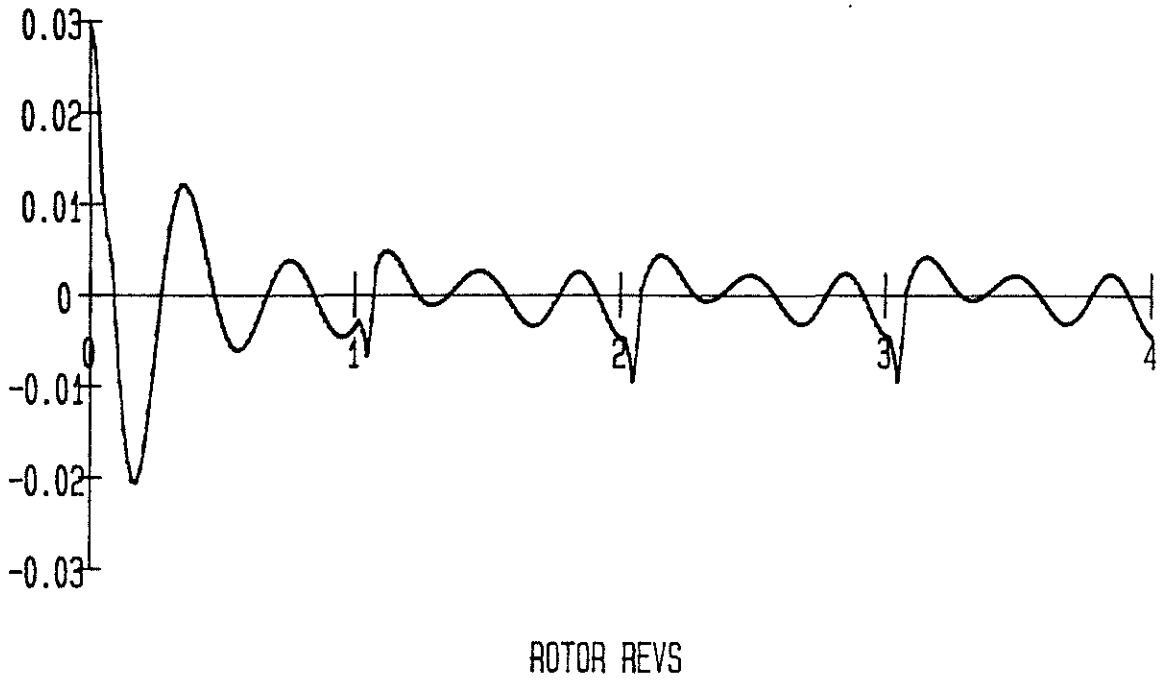

Fig 9: CLOSED-LOOP BENDING ACCELERATION

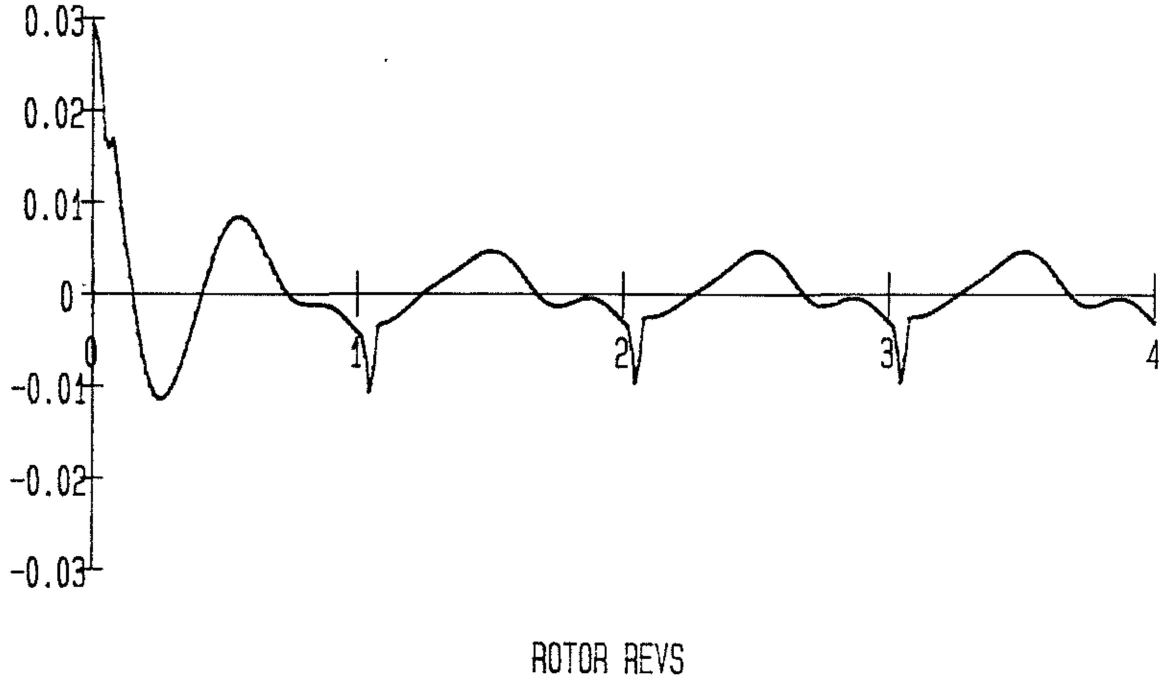

Fig 10: BENDING ACCELERATION SPECTRA, O.L. AND C.L.