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TITLE: BIFURCATION ANALYSIS OF A HELICOPTER NON-LINEAR DYNAMICS

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Non-linear dynamics phenomena have become important for various rotorcraft motions. Manoeuvrability of a helicopter in critical flight regimes involves non-linear aerodynamics and inertial coupling. Dynamical systems theory provides a methodology for studying non-linear systems of ordinary differential equations. Bifurcation theory is a part of that theory which is considering changes in the stability lead to qualitatively different responses of the system. These changes are called bifurcations. Several papers can be found in which bifurcation theory has been applied to analyse the equations of aircraft's motion. In this paper a study is presented of the critical flight regimes dynamic of a helicopter. Such dynamics is non-linear and therefore it is evident that bifurcation theory can be used in analysis. The equations of motion used in this investigation assumed a "individual blade" rotorcraft model. Results from dynamical systems theory were used to predict the nature of the instabilities caused by bifurcations and the response of the rotorcraft after a bifurcation was studied.

1. INTRODUCTION

Investigations of controlled flight of a helicopter during extreme flight conditions and breaking through various limits of usage are of great cognitive and practical importance. Such investigations consist of transformations of the rotorcraft through its functional limits. This produces the unique set of information about the rotorcraft behaviour, effects correlation, mutual limits configuration, enabling the rotorcraft to improve safety and widens designed range of usage.

In flight battle areas, military rotorcraft flies close to the ground to utilise the surrounding terrain, vegetation, or manmade objects. The obstacle-avoidance manoeuvres are repeatedly realised in extreme, limiting flight conditions. Such manoeuvres are jointed with a number of singularities, including unexpected rotorcraft motion. As result of them it is possible faulty pilot's action. Therefore it is necessary to investigate rotorcraft flight phenomena in extreme conditions.

In the present paper a non-linear dynamic model of a rotorcraft is considered which enables to determine the helicopter's motion. It is shown [1], [2], [3] that an "individual blade" rotorcraft model, including a correct representation of the rotor-engine drive train, is required to adequately predict rotorcraft response for aggressive manoeuvres. It is assumed that the helicopter fuselage is a rigid body and the motion of rigid blades about flap hinges, lead-lag hinges and axial hinges is considered, while the tail rotor is a linear model using strip/momentum theory with a uniformly distributed inflow. Simplified model of vortex field is applied and spatial structure of tip vortex trajectories is taken into consideration. Unsteady aerodynamics for prediction of rotor blade loads is included, and the ONERA type stall model is used.

Non-linear dynamics of a helicopter motion have become the subject of many works in the literature. The purpose of this approach is to evaluate the risk of helicopter control loses using the continuation methods and bifurcation theory. Continuation methods are numerical techniques for calculating the steady states of systems of ordinary differential equations of motion. Those methods are used to study aircraft roll-coupling instabilities and high angles of attack instabilities [4], [5]. Other works have used continuation methods and bifurcation theory to study the non-linear dynamics of aircraft model that includes unsteady aerodynamics coefficients [6], [7], [8].

In the present paper, after a brief description of the methodology and associated procedures, some flight cases such as "hump witch snatch up" are studied by means of checking the stability characteristics related to unstable equilibria.

Numerical simulations were used to verify the predictions. "Hump witch snatch up" was studying to observe chaos phenomenon in post stall manoeuvres. The realistic non-linear "individual blade" helicopter model my lead to great difficulties for flight analysis when the motion is quasi periodic or chaotic. The results from dynamics systems theory can be used to predict the nature of the instabilities caused by the bifurcations and the response of the helicopter after a bifurcation can be encountered.

2. THEORETICAL BACKGROUND

Dynamical systems theory provides a methodology for studying systems of ordinary differential equations. The first step in analysing a system of non-linear differential equations, in the dynamical system theory approach, is to calculate the steady states of the system and to investigate their stability. Steady states of a system can be found by setting all time derivatives equal zero and solving the resulting set of algebraic equations. The Hartman-Grobman theorem [9] provides that the local stability of a steady state can be determined by linearizing the equations of motion about the steady state and calculating the eigenvalues. A steady state is locally stable if the real parts of all the eigenvalues of the linearized system are negative. If the real part of any eigenvalue of the linearized system is positive, the steady state is locally unstable. In the neighbourhood of a steady state the system will be attracted to the steady state if the steady state is stable and repelled from the steady state if the steady state is unstable.

When the linearized system is non-singular, the implicit function theorem proves that the steady states of the system are continuos function of the parameters of this system [10]. Thus it can be stated, that the steady states of the equations of motion for rotorcraft are continuos functions of the controls deflections. Stability changes can occur as the parameters of the system are varied in such a way that the real parts of one or more eigenvalues of the linearized system change sign. Changes in the stability of a steady state lead to qualitatively different responses for the system and are called bifurcations. Stability boundaries can be determined by searching for steady states, which have one or more eigenvalues with zero real parts.

There are many types of bifurcations and each has different effects on the aircraft response. Qualitative changes in the response of the aircraft can be predicted by determining how many and what types of eigenvalues have zero real parts at the bifurcations point. Bifurcations for which one real eigenvalue is zero lead to the creation or destruction of two or more steady states. Bifurcations for which one pair of complex eigenvalues has zero real parts can lead to the creation or destruction of periodic motion. Bifurcations for which more than one real eigenvalue or more than one pair of complex eigenvalues has zero real parts lead to very complicated behaviour.

2.1. Bifurcation Theory

For steady states of rotorcraft motion, very interesting phenomena appear when even if one negative real eigenvalue crosses the imaginary axis when control vector varies. Two cases can to be considered.

- When the steady state is regular, i.e. when the implicit function theorem works and the equilibrium curve goes through a limit point. It should be noted that a limit point is structurally stable under uncertainties of the differential system studied.
- When the steady state is singular. Several equilibrium curves cross a pitchfork bifurcation point, and bifurcation point is structurally unstable. It breaks in limit points under uncertainties.

If a pair of complex eigenvalues cross the imaginary axis, when control vector varies, Hopf bifurcation appears. Hopf bifurcation is another interesting bifurcation point. After crossing this point, a periodic orbit appears. Depending of the nature of nonlinearities, this bifurcation may be sub-critical or supercritical. In the first case, the stable periodic orbit appears (even for large changes of the control vector). In the second case the amplitude of the orbit grows in portion to the changes of the control vector.

Other domain of interest concerns the behaviour of the system when periodic orbits loose their stability. Three possibilities can to be concerned in this case:

- A real eigenvalue crosses the point +1. It is appeared periodic limit points in this case.
- A real eigenvalue crosses the point −1. It is occurred a period doubling bifurcation in this case. In the vicinity of this point, the stable periodic orbit of period T becomes unstable, and a new stable periodic orbit of period $2T$ appears. This type of stability loss conduct to chaotic motion.
- Two conjugate eigenvalues leave the unit circle. Motion lines on stable or unstable tours surround the unstable orbit after this case of bifurcation.

Gluckenheimer and Holmes [9], Ioos and Joseph [10] and Keller [11] have provided a thorough introduction to the Bifurcation Theory.

2.2. Continuation Methods

Continuation methods are a direct result of the implicit function theorem, which proves that the steady states of a system are continuos functions of the parameters of the system al all steady states except for steady states at which the linearized system is singular. The general technique is to fix all parameters except one and trace the steady states of system as a function of this parameter. If one steady state of the system is known, a new steady state can be approximated by linear extrapolation from the known steady state [8]. The slope of the curve at the steady state can be determined by taking the derivative of the equation given by setting all time derivatives equal to zero. If two steady states are known, a new steady state can be approximated by linear extrapolation through the two known steady states [8]. The stability of each steady state can be determined by calculating the eigenvalues of the linearized system. Any changes in stability from one steady state to the next will signify a bifurcation.

3. NON-LINEAR EQUATIONS OF MOTION

The purpose of this work has been to use bifurcation theory to analyse the equations of motions of a helicopter. This work concentrated on the high angle of attack dynamic of a helicopter. Mathematical model describing the helicopter's flight can be defined in several ways i.e. from classical mechanics or from analytical mechanics [13], [14].

A blade element model was used to determine the main rotor's motion and loads. The coupled equations of motion which involve the main rotor and body degrees of freedom are solved simultaneously, as present in detail in Ref. [14]. The blade element rotor model, in addition to represent non-linear, unsteady aerodynamics, enables correct representation of the flight dynamics of helicopter.

Equations of dynamic equilibrium of forces and moments have been determined in the system co-ordinates fixed with the fuselage and the systems of co-ordinates fixed with rotor blades. Detailed way of determining of these equations can be found in [13] and [14]. Finally it is obtained set of $10+2n$ (n- number of main rotor blades) non-linear differential equations with periodic coefficients which can be presented in the form:

$$\dot{\mathbf{X}} = \mathbf{f}(\mathbf{X}, \mathbf{S}) \qquad (1)$$

Where **X** is the state vector:

$$\mathbf{X} = \big[u, v, w, p, q, r, \dot{\beta}_1 .. \dot{\beta}_n, \dot{\zeta}_1 .. \dot{\zeta}_n, \Omega, \psi_1 .. \psi_n, \beta_1 .. \beta_n, \zeta_1 ... \zeta_n, \Theta_e, \Phi_e, \Psi_e \big]^T \qquad (2)$$

u, v, w are linear velocities of the centre of fuselage mass in the co-ordinate system fixed with the fuselage, p, q, r are angular velocities of the fuselage in the same co-ordinate system; Θ_e, Φ_e, Ψ_e are pitch, roll and yaw angles of the fuselage, β_i - i-th blade flap motion about flap hinge ζ_i - i-th blade lead-lag motion about lead-lag hinge. **S** is the control vector:

$$\mathbf{S} = [\theta_0, \kappa, \eta, \phi_T]^T \quad \text{or} \quad \mathbf{S} = [\theta_0, \theta_1, \theta_2, \phi_T]^T \qquad (3)$$

Where: θ_0 is angle of collective pitch of the main rotor, κ is control angle in the longitudinal motion, η is control angle in the lateral motion and ϕ_T is angle of collective pitch of the tail rotor; θ_1, θ_2 - are angles of cyclic pitches of the main rotor:

$$\theta_1 = \kappa \sin \psi_0 + \eta \cos \psi_0 \qquad (4)$$
$$\theta_2 = \kappa \cos \psi_0 - \eta \sin \psi_0$$

Where ψ_0 is a retardation angle of cyclic pitch control.

4. AERODYNAMIC FORCES AND MOMENTS

Precise describing of aerodynamic forces and moments found in equations of motion is fundamental source of difficulties. In each phase of flight dynamics and aerodynamics influence each other, which disturbs the precise mathematical description of those processes. The requirements for method on aerodynamic load calculations stem both from flow environment and from algorithms used in analysis of helicopter flight. The airframe model consists of the fuselage, horizontal tail, vertical tail, landing gear and wing (if applicable). The fuselage model is based on wind tunnel test data (as function of angle of attack α and slip angle β). The horizontal tail and vertical tail are treated as aerodynamic lifting surfaces with lift and drag coefficients computed from data tables as functions of angle of attack α and slip angle β. The tail rotor is linear model using strip-momentum theory with an uniformly distributed inflow. The effects of rotor wash on the airframe are included in the model. The technique used provides the essential effects of increased interference velocity with increased rotor load and decreased interference as the rotor wake deflects reward with increased forward speed [14].

4.1. Blade Aerodynamics

Aerodynamic data is for a NACA 23012 airfoil in the range +/-23° and the compressibility effects have been included. The data have been blended with suitable low speed data for the remainder of the 360° range to model the reversed flow region and fully stalled retreating blades. Dynamic stall effects have been included.

4.2. Deep Stall Phenomenon

Term „deep stall" means phenomenon of increasing of lift coefficient C_L over the value C_{Lmax} achieved in static airflow conditions. Modelling of airflow on dynamic stall conditions belongs to very involved problems. It is not always possible or profitable to use CFD methods. Therefore dynamic stall phenomenon was a subject of many experimental works. As result of them factors affecting this phenomenon were identificated.

Semi-empirical methods that use differential equations for prediction of unsteady aerodynamic loads are one of efficient methods that predict the unsteady aerodynamic loads. The form and coefficients of these equations are determined by techniques of parameter identification. The basic model was developed by ONERA for loads at rotor blade section in stall conditions. The ONERA model is a semi-empirical, unsteady, non-linear model which uses experimental data to predict aerodynamic forces on an oscillating airfoil which experiences dynamic stall [15], [16]. State variable formulations of aerodynamic loads to allow use existing codes for rotorcraft flight simulation. Non-linear relations describing relations between unsteady lift drag and pitching moment and angle of attack can be establish by following set of differential equations (cf. [17], [18]):

$$C_0 = C_{0_a} + C_{0_b} \tag{5}$$

$$C_{0_a} = s_0 \dot{\alpha} + k_{\gamma_0} \ddot{\theta} + C_{0_\gamma} \tag{6}$$

$$\begin{aligned} \dot{C}_{0_\gamma} + \lambda_0 C_{0_\gamma} &= \\ &= \lambda_0 \left(\alpha_{0_0} \alpha + \sigma_0 \dot{\theta} \right) + \alpha_0 \left(\alpha_{0_0} \dot{\alpha} + \sigma_0 \ddot{\theta} \right) \end{aligned} \tag{7}$$

$$\begin{aligned} \ddot{C}_{0_b} + 2 d_0 w_0 \dot{C}_{0_b} + w_0^2 \left(1 + d_0^2\right) C_{0_b} &= \\ &= -w_0^2 \left(1 + d_0^2\right) \left(\Delta C_0 + e_0 \frac{\partial \Delta C_0}{\partial \alpha} \dot{\alpha} \right) \end{aligned} \tag{8}$$

C_0 represents either the relevant non-dimensional lift force coefficient C_L, drag coefficient C_D or pitch moment coefficient C_m. The coefficients σ_0, λ_0, σ_0, d_0, w_0 and so forth, of these equations must be determined empirically by parameter identification techniques. Narkiewicz [17] published numeric values of those parameters obtained for generic airfoil data. These equations alone, when used in the linear region, provide a full accounting of the unsteady aerodynamic effects including time lag and flow inertia effects. These effects are analogous to the Theodorsen function in two-dimensional oscillatory aerodynamics. Differential equations account for arbitrary airfoil motion and model the history of motion, which is important in unsteady case. The ONERA deep stall model was chosen for adaptation to rotorcraft flight analysis.

5. STEADY STATE FLIGHT CONDITIONS

Bifurcation Theory is a set of mathematical results, which aims at the analysis and explanation of unexpected modifications in the asymptotic behaviour of non-linear differential systems when parameters are slowly varying.

For a fixed control vector S, two types of asymptotic state are commonly encountered. The following relation gives the first:

$$\mathbf{f}(\mathbf{X}, \mathbf{S}) = \mathbf{0} \tag{9}$$

This relation is named steady state. The second relation is given by the equation:

$$\mathbf{X}(T) = \mathbf{X}(0) + \int_0^T \mathbf{f}(\mathbf{X}, \mathbf{S}) dt \tag{10}$$

Starting with an approximation of an asymptotic state, for a given value of parameters, the code determines, by a continuation process, the curve **X(S)** solution of a set of non-linear algebraic equation (11) which computation case dependant.

$$\begin{cases} \text{Equilibrium points} : \mathbf{f}(\mathbf{X}, \mathbf{S}) = \mathbf{0} \\ \text{Limit points} \quad : \mathbf{f}(\mathbf{X}, \mathbf{S}) = \mathbf{0} \\ \qquad \lambda = 0 \\ \text{Hopf points} \quad : \mathbf{f}(\mathbf{X}, \mathbf{S}) = \mathbf{0} \\ \qquad \lambda_{1,2} = \pm 2 i \pi / T \\ \text{Periodic orbits} \quad : \mathbf{X}(T) = \mathbf{X}(0) + \int_0^T \mathbf{f}(\mathbf{X}, \mathbf{S}) dt \end{cases} \tag{11}$$

Continuation process assumes that all functions for (11) are continuity and derivability.

There are several continuation methods algorithms. In the present work the algorithm developed by Doedel and Kernevez [12], which is based on the work of Keller [11] is used.

6. RESULTS

All the results presented in this section refer to a PZL „Sokół" helicopter in forward flight and a gross weight 6500 kg, with the control system turned of (bare airframe configuration). Some results of computation are presented in this paper.

The rotor blade stall affects the limiting condition of operation of the helicopter. Stall on a helicopter blade limits the high-speed possibilities of the helicopter. This is understandable, when one considers that the retreating blade of the helicopter rotor encounters lower velocities as the forward speed is increased. The retreating blade must produce its portion of the lift, therefore, as the velocity decreases with forward speed, the blade angle of attack must be increased. It follows that at some forward speed the retreating blade will stall. In forward flight the angle of attack distribution along the blade is far from uniform, so that it must be expected that some portion of the blade will stall before rest.

Figures 1-10 show the steady states of the PZL "Sokół" as a function of longitudinal swash plate deflection κ for a collective pitch of 18^0 and zero lateral swash plate deflection. Those figures show, that multiple steady states exists for most longitudinal swash plate deflections. For example, a vertical line representing 0 deg of swash plate deflection intersects three steady states. All of them are stable, so the helicopter could exhibit either of these three steady states for 0 longitudinal swash plate deflection.

One stable steady state at 0 longitudinal swash plate deflection represents the trim configuration ($p=q=r=\Psi=\Phi=0$). The other two stable steady states represent humps. The segment of unstable steady states containing the trim conditions between −1,8 and −5,2 deg, because of two saddle-node bifurcation that occur at longitudinal swash plate deflection of −1,8 and −5,2.

For example, if the helicopter is trimmed at an collective pitch 18 deg the steady-state main rotor angle of attack will be given by the angle of attack at 0 longitudinal swash plate deflection contained of in the curve of low-angle-of-attack steady states. If the swash plate deflection is increased slowly enough, the steady state of the helicopter will be given by the curve of stable low-angle-of-attack steady states up to a longitudinal swash plate deflection of −1,8 deg. For longitudinal swash plate deflections smaller then −1,8 deg, the steady states that are at low-angles-of-attach do not exist, so the helicopter jump to a new stable motion. This new motion could be either a stable steady state or some type of time dependent motion. Figures 11-20 show a simulation of the manoeuvre occurred in this unstable region – "hump witch snatch up".

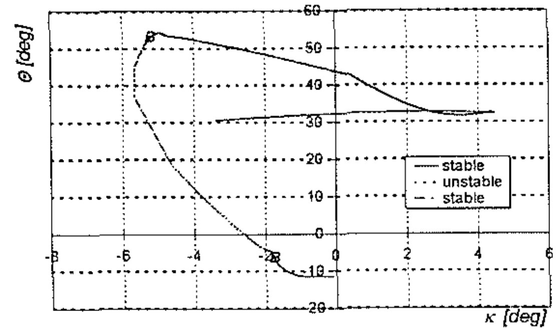

Fig. 1 Steady states for longitudinal manoeuvres – variation Θ(κ), B – saddle-node bifurcation

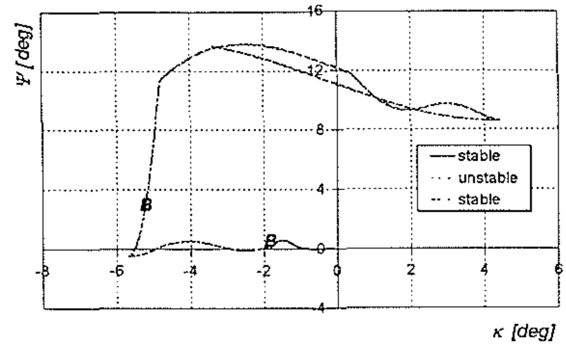

Fig. 2 Steady states for longitudinal manoeuvres – variation Ψ(κ), B – saddle-node bifurcation

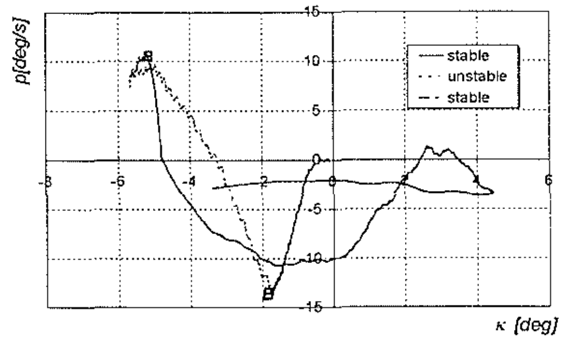

Fig. 3 Steady states for longitudinal manoeuvres – variation p(κ), B – saddle-node bifurcation

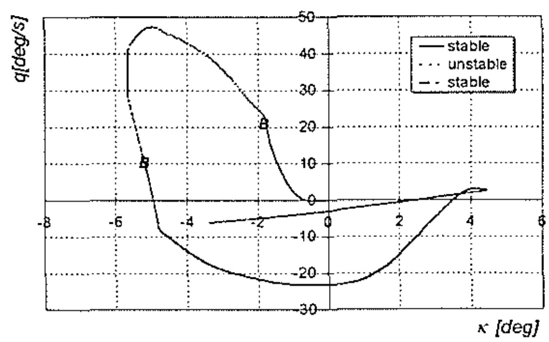

Fig. 4 Steady states for longitudinal manoeuvres – variation q(κ), B – saddle-node bifurcation

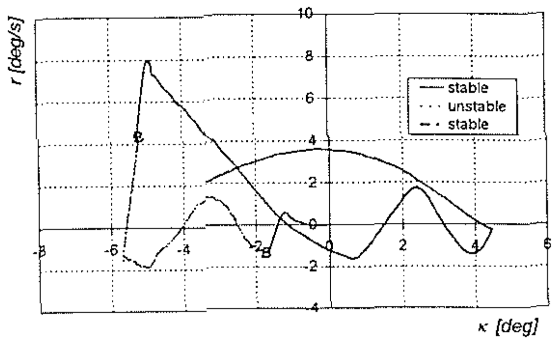

Fig. 5 Steady states for longitudinal manoeuvres-variation r(κ), B – saddle-node bifurcation

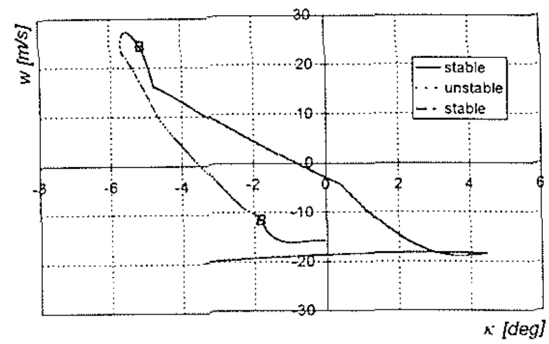

Fig. 6 Steady states for longitudinal manoeuvres – variation w(κ), B – saddle-node bifurcation

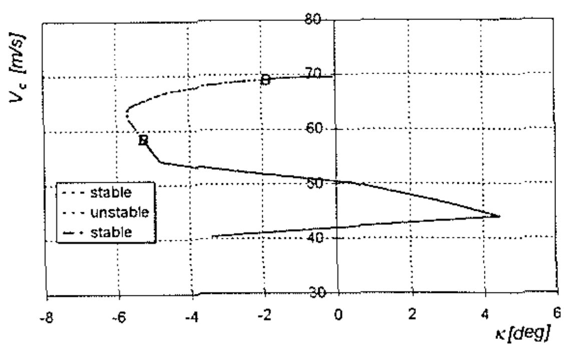

Fig. 7 Steady states for longitudinal manoeuvres – variation $V_c(\kappa)$, B – saddle-node bifurcation

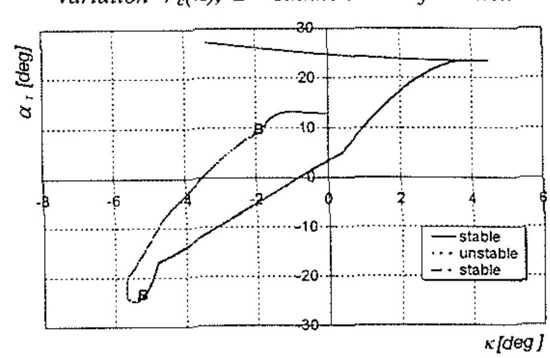

Fig. 8 Steady states for longitudinal manoeuvres – variation $\alpha_T(\kappa)$, B – saddle-node bifurcation

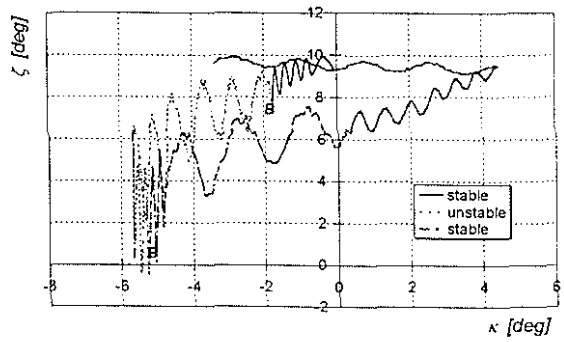

Fig. 9 Steady states for longitudinal manoeuvres – variation ζ(κ), B – saddle-node bifurcation

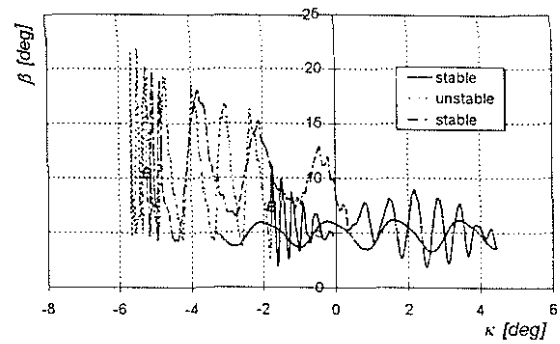

Fig. 10 Steady states for longitudinal manoeuvres – variation ζ(κ), B – saddle-node bifurcation

The hump of a helicopter realised with high entry velocity is characterised by following singularity. When the pilot pulls the stick., helicopter increase angle of attack and normal load factor. After pushing the stick, in consequence of stall of main rotor blades, the normal load factor increased. This phenomenon is called as "hump with snatch up" of the helicopter [3]. Figures 11-21 show the results of numerical simulation of this instability. Fig. 11 shows variation of pitch angle and angular pitching velocity corresponding to pilot's action described above. The pitch angle of the helicopter increased during the first second of motion. This is typical helicopter's reaction. Usually, if the swash plate is deflected backwards, the pitch angles of the helicopter decreased. But in the case of described instability, the pitch angle increased. This is significant singularity of helicopter's hump. That hump becomes unstable because of saddle-node bifurcation.

The phase plots of lagging and flapping motion of main rotor blade are shown in Figs 16 and 18. Those figures show a typical chaotic oscillation (similar to chaotic motion of stalled rotor blade, see Tang and Dowell [18], [19], [20]).

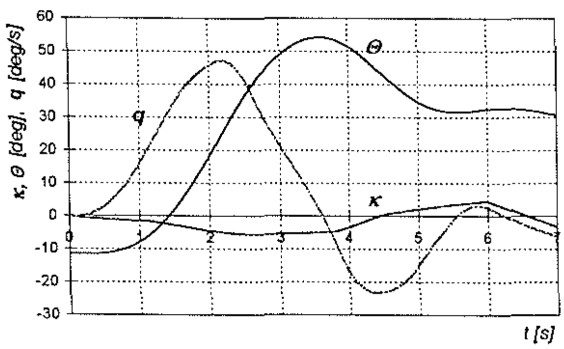

Fig. 11 Variation of pitch angle, pitching rate and longitudinal swash plate deflection

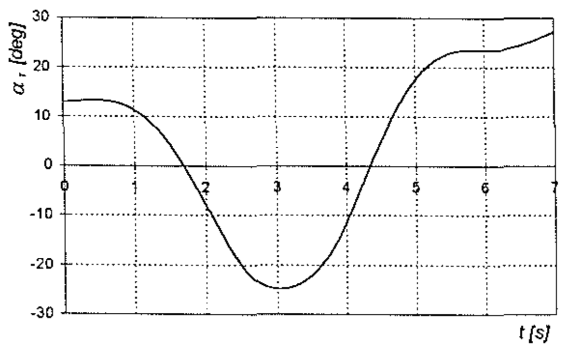

Fig. 12 Variation of main rotor angle-of-attack

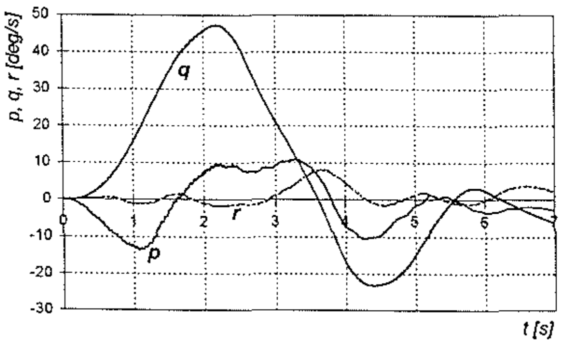

Fig. 13 Variation of pitch yaw and roll rates

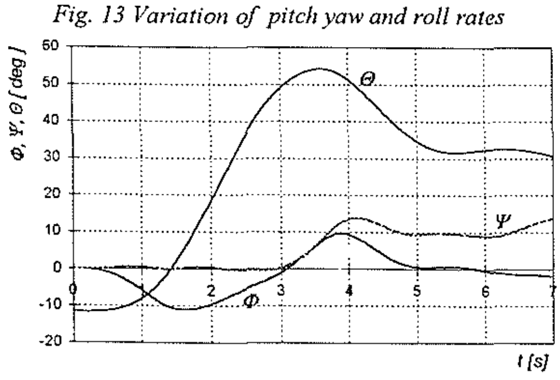

Fig. 14 Variation of pitch yaw and roll angles

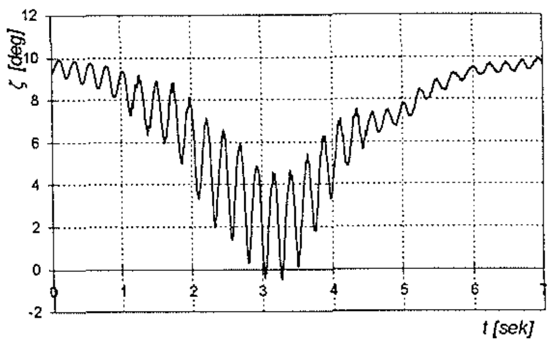

Fig. 15 Variation the lag angle

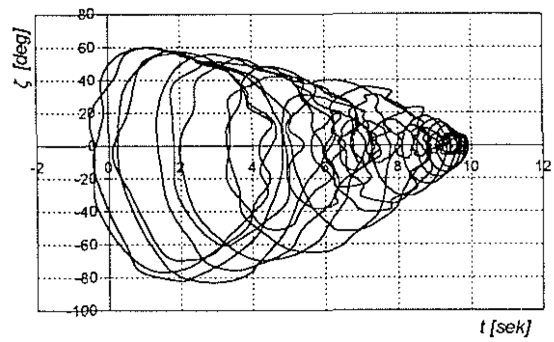

Fig. 16 Phase plane plot of lag motion

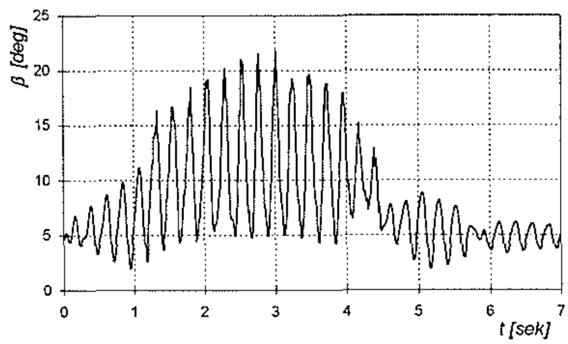

Fig. 17 Variation flap angle

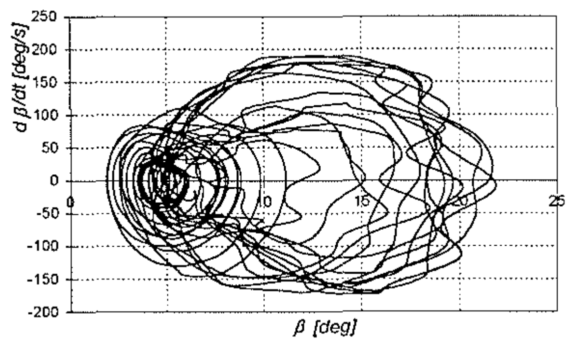

Fig. 18 Pphase plane plot of flapping motion

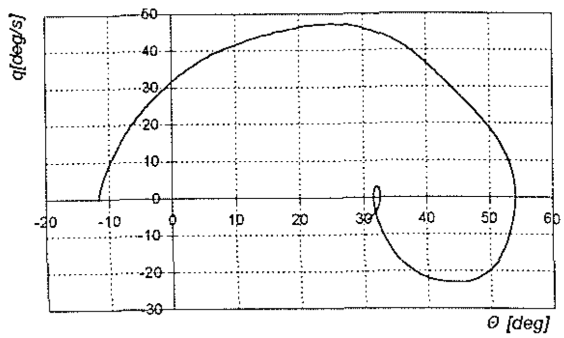

Fig. 19 Phase plane plot of the helicopter pitch motion

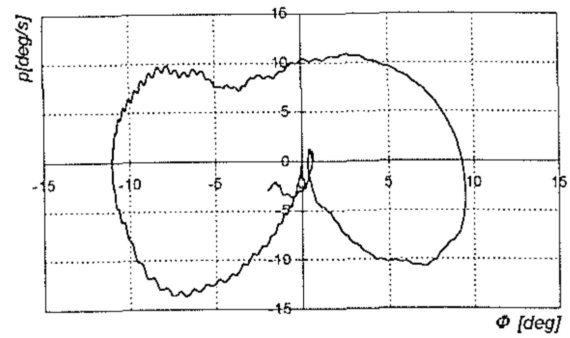

Fig. 20 Phase plane plot of the helicopter roll motion

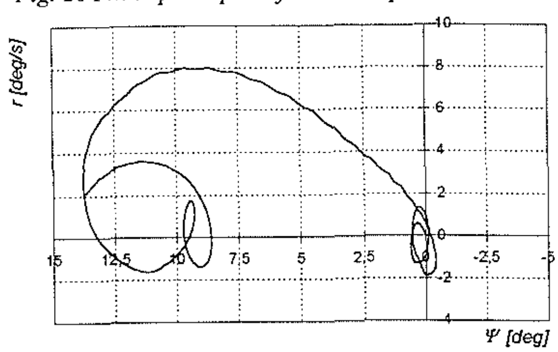

Fig. 21 Phase plane plot of the helicopter yaw motion

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