CONTROL DESIGN OF A TILTING MECHANISM FOR THE UK NATIONAL ROTOR TEST RIG FACILITY

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Abstract

This work describes the development of a model and a controller for the tilting mechanism of the UK National Rotor Test Rig Facility (NRTRF). The rig has been designed to operate in two modes: helicopter and tiltrotor. In tiltrotor mode, the regulation of the tilt angle is required between -20° and 90°, measured with respect to the vertical axis. In order to ensure that the tilt angle of the rig is set at the required angular position, a feedback controller is found most appropriate to address disturbances, off-sets and such like. The controller must have access to the tilt angular position and adjust the motor torque (or current) as necessary to achieve the desired position. This work describes the whole process of designing the controller, including the construction of appropriate models from open-loop input-output data. We pursue two control design approaches: classical (PID) and advanced ($\mathscr{H}_\infty$), and provide a comparison of their benefits in terms of robustness and performance via experimental results.

1 INTRODUCTION

The National Rotor Test Rig Facility (NRTRF) is a UK initiative with the aim to provide a state-of-the-art tool for both the national academic and the industrial communities to carry on rotorcraft research and development (Figure 1). The Aircraft Research Association (ARA) is responsible for the commissioning of the NRTRF. The design is based around a grounded rotor able to operate in several UK wind tunnel facilities. The rig is designed to operate in two modes: helicopter and tiltrotor. Operation in tilt rotor mode is the focus of this work and requires the regulation of the tilt angle to a prescribed value, which is in the range between -20° (rearward) and 90° (forward), measured with respect to the vertical axis. The rotor supports four blades attached to the system and can operate up to 4000 RPM.

Figure 1: UK National Rotor Test Rig Facility (courtesy of Aircraft Research Association)

The system concerning the tilt positioning mechanism is comprised of an electric servo actuator connected to a gear box. A simplified schematic of the system is shown in Figure 2, whereby the inner feedback loop of the servo system is not shown and the gear box is represented by the connection of two gears only. Original operation specifications included reaching a demanded tilt position with the rotor spinning and blades off. The operation of the tilt mechanism with the rotor spinning and blades on could introduce dangerous dynamic couplings and hence this is avoided in the first set of tests. Once the tilt position was achieved within 0.1° accuracy, both braking systems (hydraulic and electrical) are engaged in order to maintain the tilt angle at such position.

This work is therefore concerned with the design of the control system for the operation of the tilt angle. The description of the design work is divided into several sections. Section 2 discusses the construction of linear models using system identification techniques and assesses their variation and fidelity. Once linear models are available, Section 3 describes the design of two controllers for the rig system. Given the operation and intended use of the NRTRF, design objectives for the control system are established in terms of measures for tracking, disturbance rejection and stability margins. The two controllers were linear but designed using different techniques: the first was a Proportional-Integral-Derivative (PID) controller using classical methods, which has an appealing but limited structure; the second is an $\mathscr{H}_\infty$ controller which provided better performance but had a more opaque structure. The performance of the controllers is evaluated through simulations using the developed models, and experimentally in the rig (Section 4). The paper concludes with some final remarks in Section 5.

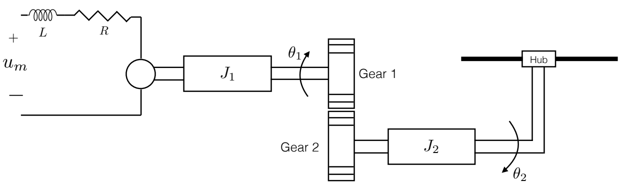

Figure 2: Motor-Gear Tilting Mechanism

2 SYSTEM IDENTIFICATION

The system identification task employs linear time-invariant representations of the tilting mechanism. Such representations are of course an approximation since the physical system contains a few nonlinearities. For instance, the rotor rig tilting system is essentially an inverted pendulum, unstable relative to the vertical equilibrium. The return-to-zero portions of the angular velocity suggest there may be some kind of "jerk" limiting within the black box motor controller. Neither of which can be readily captured in a linear model but this can be compensated for by feedback control.

The models used for controller design of the tilting mechanism were obtained by standard system identification methods [2]. System identification is a data-based process for obtaining, typically, linear models of systems and involves one applying an input to the system over a certain period and time and measuring the output over the same period. If the input signal is sufficiently "rich" in frequency content and the output signal sufficiently noise free, system identification algorithms can provide accurate linear models of systems. The key advantage of system identification is that no *a priori* mathematical model of the system is needed; the disadvantages with the approach are that (i) it can be sensitive to noise, biases, trends and such like, so careful pre-conditioning of the data may be necessary and (ii) typically only linear models are provided, with nonlinearities in the system affecting the fidelity of the linear models produced. For the tilting mechanism, although a rough idea of the model structure was known, there was little information about the model parameters, so system identification, using a number of data sets was used to determine various linear models.

2.1 Linear Model Construction

Various open-loop data sets were gathered from the tilting mechanism using the Rig Control System. The data was gathered without blades attached and without rotation of the hub. Ideally, this data would have been supplemented with that gathered from a spinning and loaded hub, but this was not possible.

The system ID data consists of sets of input and output time series. The input (V) was a waveform generated by the rig control system and can, roughly speaking be thought of as a torque (current) demand to the motor (which it is assumed has its own internal control system). The output data was the tilt angular position, measured from vertical. Velocity estimates were obtained from numerical differentiation of the angular position data. Input-output data was recorded over (typically) 15s intervals and logged every 2ms, potentially providing accurate frequency information up to 250Hz. The input waveforms were typically either doublets of different durations and magnitudes, or sine waves of different frequencies and magnitudes. The different magnitudes enabled some idea of the rig nonlinearity [1] to be obtained: small input signals would only tilt the rig to small angular positions where the rig can be considered approximately linear; large input signals would tilt the rig to large angular positions, where nonlinear effects would be significant. It was also noted that very small inputs (less than 0.25V) provided somewhat spurious output waveforms; this may be due to backlash/hysteresis effects in the motor/gear mechanism which are more dominant for small inputs.

Linear models were constructed from a selection of open-loop data sets. The models were constructed using the `tfest` function in the Matlab System Identification Toolbox. The linear models were estimated as transfer functions from input voltage to tilt angular velocity and then an integrator was added to obtain the transfer function to angular position. The angular velocity measurement was relatively noise-free and un-biased and was, therefore, considered to be useful for system identification purposes. After some experimentation, the identified models representing the transfer function from input to velocity output were stipulated to be of second order and contain one zero, making the overall model including the integrator to be 3rd order. The models obtained in this way demonstrated relatively good agreement with the open-loop data collected.

2.2 Model Fidelity and Variation

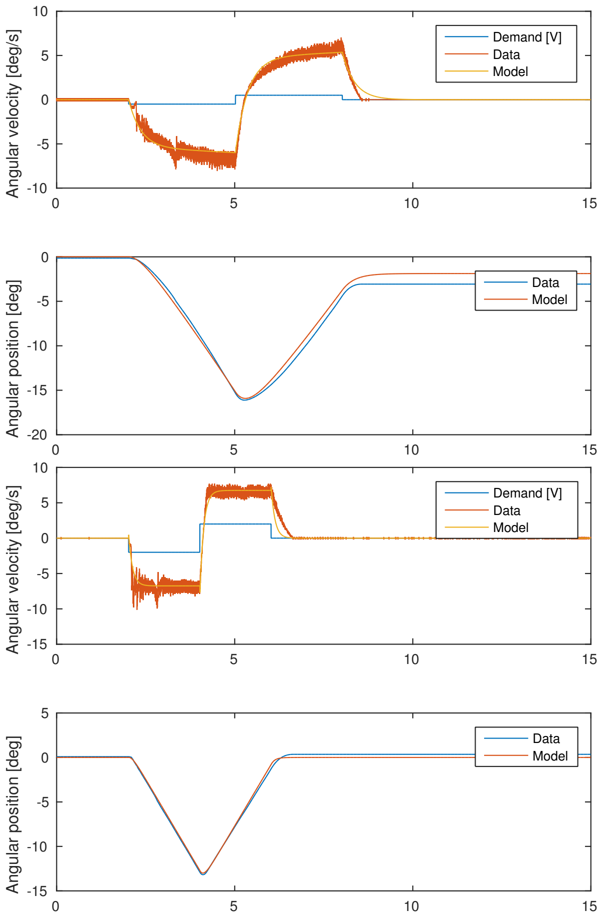

Figure 3: Comparison between data and model constructed using system identification: Top two, 1.5V doublet input of duration 2.5s; Bottom two, 2.0V doublet input of duration 2s

The agreement between the identified model and the data was, for each data point, typically very good, particularly for angular velocity. The angular position was not so well identified because of the (relatively

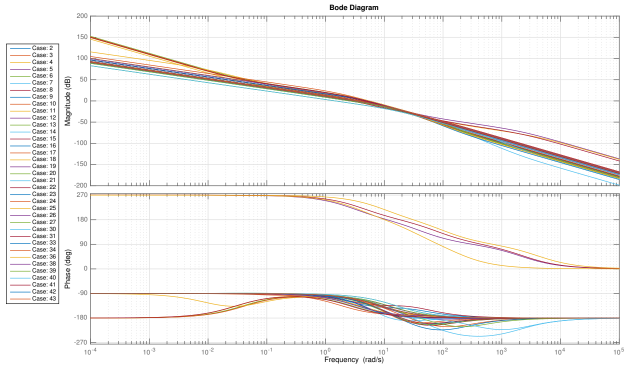

Figure 4: Identified models.

small) offsets due to not all data gathering starting at precisely vertical, but this is mainly an off-set issue, not a problem with the dynamic model. Figure 3 shows a comparison between the identified model using two different data files: the agreement between data and model is deemed to be good.

Each data set led to a different model being constructed and, due to the variation of the input-output data and the nonlinearity of the physical system, differences in the models are expected. Figure 4 shows the frequency response of all the identified models. While there is certainly variation, most models behave similarly and thus it is expected that a linear controller should be sufficient to control the system at all operating points.

3 CONTROL DESIGN

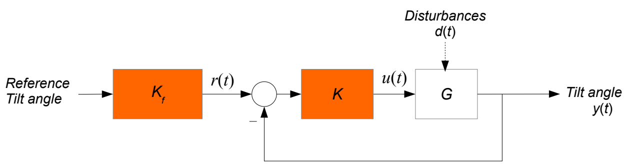

Figure 5: Control architecture.

The main objective of this task was to design a controller for the identified linear models in Section 2. The control structure used was the conventional feedback structure (see Figure 5), where a measurement of the controlled signal, in this case the tilt position angle, is fed back and compared with a filtered reference signal. This signal then enters the linear feedback controller $K(s)$, which produces acceptable control actions that drive the plant G and ensure satisfactory behaviour of the closed-loop in terms of stability, reference tracking, disturbance rejection, control effort and noise attenuation. The feedforward element $K_f(s)$ is designed typically after the feedback design and it is introduced to further improve the tracking capabilities of the system, especially at desired frequencies of interest, and to smooth out control efforts. The design of the tilt positioning system is simpler than that required for trimming of the rotor via swashplate actuators since, in this case, the system is Single-Input Single-Output (SISO). We explore two different controller design methods: classical PID and more advanced mixed sensitivity $\mathscr{H}_\infty$.

The controller design is carried out by assessing four main characteristics of the closed-loop:

- *Robust stability:* This aspect is concerned with the capability of the controller to guarantee stability for the majority of the identified models. For each closed-loop, we consider three metrics which provide an indication of robustness to certain changes in the behaviour of the plant (the tilt mechanism in this case). These metrics are the gain margin (GM), phase margin (PM) and the closest distance to the critical point $\|S\|_\infty^{-1}$, refer to [4] for more details. Typical design requirements for robustness advise that:

$$\begin{aligned} \text{GM} &> 2\ (\approx 6\text{ dB}) \\ \text{PM} &> 30^\circ \\ \|S\|_\infty^{-1} &> 0.5 \end{aligned}$$

- *Tracking:* The ability of the controller to maintain the controlled output as closely as possible to the reference signal when noise and disturbance effects are not significant. Typically, the so-called closed-loop or Cosensitivity transfer function

$$T(s) = \frac{y(s)}{r(s)} = \frac{K(s)G(s)}{1+K(s)G(s)} \qquad (1)$$

  provides comprehensive information about the tracking characteristics in the frequency-domain. Ideal tracking characteristics require $|T(j\omega)|$ to be close to 1 and $\angle T(j\omega)$ to be close to 0, over the frequency region of operation.

- *Disturbance rejection:* The ability of the control system to be insensitive to disturbances entering the system. For this particular application, we are primarily concerned with a disturbance torque originated by rotor imbalances, i.e., when the center of mass of the blades are off the spinning axis. This aspect would be more crucial during helicopter configuration. Disturbance rejection characteristics can be assessed via the Sensitivity transfer function

$$S(s) = \frac{y(s)}{d(s)} = \frac{1}{1+K(s)G(s)} \qquad (2)$$

  Ideal disturbance rejection characteristics require $|S(j\omega)|$ to be 0 over the frequency region of operation.

- *Control effort:* This is also a very important aspect to guarantee operation within acceptable limits of the physical devices and smooth operation. We pay particular attention so control signals are not overly large and they fit within the motor capabilities. The way to improve this aspect in this report was found by the introduction of 5°/s rate limits at the reference signal and input constraints for the signals coming out of the controller restricted as

$$|u(t)| \leq 2.5 \qquad (3)$$

  In the frequency domain and in the absence of measurement noise, the transfer function $K(s)S(s)$

$$u(s) = K(s)S(s)(r(s) - d(s)) \qquad (4)$$

  is considered to assess the control actions efforts. Control efforts require $|K(j\omega)S(j\omega)|$ to be small enough in the frequency region of operation.

- *Steady-state error:* This design specification is related to the step response of the closed-loop system. For the considered application, this design parameter is of high importance and the initial design requirement is that achieved tilt positions should be satisfied within 0.1° accuracy. In terms of the steady-state errors, the relative error is minimum when a tilt demand asks for the largest possible change in tilt position of 110°. This translates in a design requirement of having

$$\text{Steady-State Error} < 9.1 \times 10^{-4}$$

Feedback systems can offer benefits, in terms of tracking and disturbance rejection, over a limited frequency range only. This should cover the expected frequency of operation associated with reference inputs and disturbances. An important metric associated with this frequency of operation is the *bandwidth*, as it denotes the upper limit of such frequency range. This frequency can be related to the so-called gain crossover frequency, which is indicated by ω_c and typically expressed in rad/s. Other measures of bandwidth are associated with the frequencies ω_B, the frequency at which the $|S(j\omega)|$ crosses -3 dB from below and ω_{BT}, the frequency at which $|T(j\omega)|$ crosses -3 dB from above. We will use instead ω_c as our measure of bandwidth since for the majority of practical systems the PM $< 90^\circ$ and therefore

$$\omega_B < \omega_c < \omega_{BT}$$

3.1 PID Controller Design

The PID controller is perhaps the most popular of controller structures and although it is not applicable to all

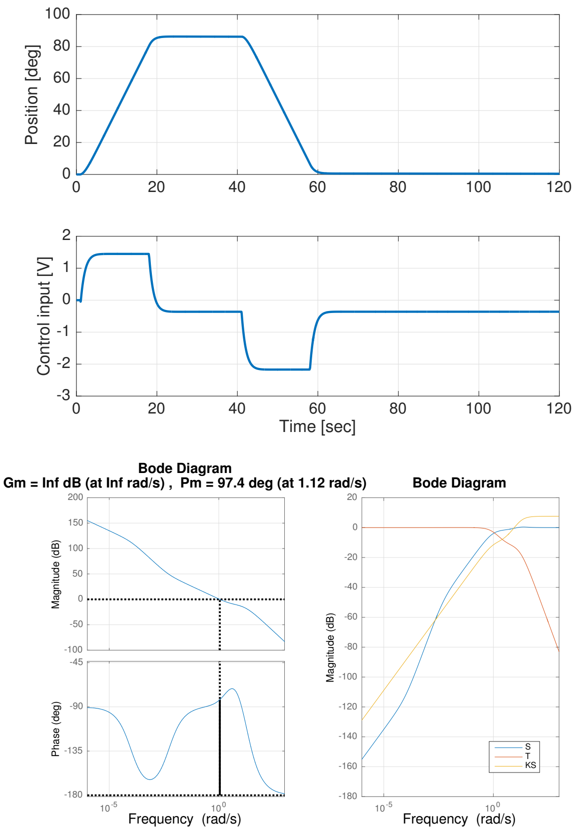

Figure 6: Nominal design results with PD controller.

systems, it was found to be appropriate for the tilt rotor. The PID controller is a three-term controller which has the ideal transfer function

$$K(s) = k_P + \frac{k_I}{s} + k_D s$$

Typically, a low pass filter is added to the derivative part of this controller to avoid amplification of high frequencies. The task of the designer is to tune the three terms so that the controller provides adequate performance and robustness characteristics. Nominal design was performed for a particular identified plant model (so-called 'Case 12'). Some iteration over the controller parameters took place according to a desired shape of the loop transfer function, finally yielding the following feedback controller

$$K(s) = \frac{2.4(s+0.005106)(s+3.329)}{(s+20)(s+0.0001)}$$

The initial design implemented the controller as a Proportional-Derivative only, since the plant was known to have already an integrator component. This integrator is found when the position is obtained from integrating the angular tilting speed. After the first set of experimental tests, this controller was re-tuned and additional integral action was added to further improve the steady-state error. The feedforward compensator was added to smooth out control actions

$$K_f(s) \approx \frac{3.33}{s+3.33}$$

The above filter is also useful to attenuate large control actions that might be obtained from tracking step-like operator commands.

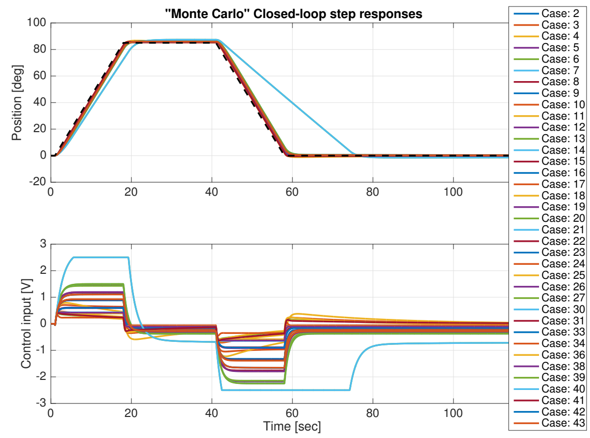

Figure 7: Performance results with PID controller tested with all identified cases.

Nominal design results, both in the frequency and time domain are shown in Figure 6. We observe that the system offer ample stability margins, with infinite GM and PM above 95°. A more comprehensive picture of the controller's capabilities was given by testing the controller using the other identified system models. The responses are shown in Figure 7. We observe tracking of the ramp, as the reference demand goes from 0° to 85° is satisfactory with negligible steady-state error to both, the step and ramp signals (because of the double integrator in the loop). In only one case (so-called 'Case 36'), the performance seems to deteriorate significantly, especially when tilting back to 0° is delayed for few more seconds. Such a delay appears to be caused by the control signals hitting the boundaries at ± 2.5. The sluggishness in the tracking seems to be a consequence of controller *windup* [3]. For most other cases, operation of the control system is not too close to saturation, which is indeed good news in terms of performance and stability.

The closed-loop system is stable for all identified models. Stability margins are very good, with valid measures of GM being larger than 9 dB. In addition, PM $> 72°$ and $\|S\|_\infty^{-1} > 0.77$ for all cases. The performance obtained also via the gain cross-over frequency is also satisfactory. We observe that for valid measures of GM, $\omega_c > 23$ rad/s, which is considered enough for the current application. Finally we mention the steady-state error as this is an important design specification in the control of the tilting mechanism. As expected, the steady-state (SS) error is practically zero for all cases due to the presence of two integrators in the loop.

3.2 Mixed-sensitivity $\mathcal{H}_\infty$ Controller Design

This subsection provide some details about the design of an alternative controller using the mixed sensitivity $\mathcal{H}_\infty$ controller synthesis method [4]. This method consists in the shaping of the sensitivity transfer functions in the frequency domain, and which encapsulates information about robust stability, tracking, disturbance rejection, noise attenuation and control efforts: $S(s)$, $T(s)$ and $K(s)S(s)$. The approach uses so-called performance weights, to express desirable shapes for some or all of these transfer functions. An optimisation algorithm searches among the set of stabilising controllers to provide a controller that satisfies the required loop-shape specifications as much as possible.

The $\mathcal{H}_\infty$ controller was designed using the same linear model ('Case 12') as for the PID design described earlier. The initial weight choice requested $\|S\|_\infty < 1.6$, which is related to the worst-case disturbance rejection scenario and stability margin specification $\|S\|_\infty^{-1} > 0.625$. Additional design requirements included steady-state errors, which were included as 5×10^{-4} and a bandwidth in terms of the sensitivity function $\omega_B >$ 18.85 rad/s. We also included a design specification associated with control signal efforts: $\|KS\|_\infty < 100$.

After running the optimisation routine in the Robust Control System Toolbox in Matlab®, the following feedback controller is obtained

$$K(s) = \frac{3246.7(s+11.4)(s+0.9447)(s+0.001)}{(s+1.018)(s+0.009425)(s^2+109.7s+5585)}$$

In addition, a feedforward compensator was added to smooth out reference signals

$$K_f(s) = \frac{1}{0.5s+1}$$

Nominal design results in the time and frequency domains are shown in Figure 8. Time responses are very good, close tracking of the reference tilt is achieved and control actions are in the allowed operating range. The frequency responses of the key transfer functions, $S(S)$, $T(s)$ and $K(s)S(s)$ show that the initial design specifications are satisfied largely. Control actions are boosted in the frequency region 2-3400 rad/s, achieving a noticeable improvement of tracking and disturbance rejection, in comparison with the PID control design. Such an improvement in performance has an associated cost and, from looking

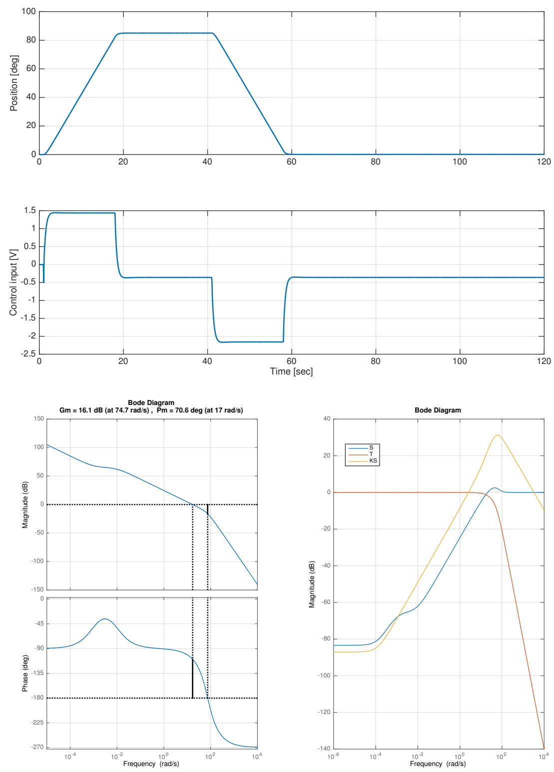

Figure 8: Nominal design results with $\mathcal{H}_\infty$ controller.

at the the peak sensitivity $\|S\|_\infty^{-1} \approx 0.4$, it is clear this controller offers lower robustness properties than the PID design.

More comprehensive tests (see Figure 9) are carried out when implementing the controller also in discrete form (sampling time of 2 ms) for all identified plants. Consistent with the nominal results, the tracking characteristics tested here are excellent, following the reference tilt very closely in the vast majority of cases (except for case 36 again). We observe that for all cases steady-state errors are practically zero, and the original design specifications are met. The bandwidth measured in terms of ω_c is above 26 rad/s, which is above the PID control design by 3 rad/s. The lowest value of $\|S\|_\infty^{-1}$ is around 0.15, which is lower than the original design criterion. However, such a low robustness margin was obtained for very few cases only and, for this reason we expect the $\mathcal{H}_\infty$ controller to be still fairly robust when implemented in practice.

4 EXPERIMENTAL RESULTS

Designed controllers were implemented on the NRTRF real-time Rig Control System. The controllers sat within an overarching state-machine realised within an FPGA, whose operating states - idle, operating, parked etc - were, in turn, controlled by the NRTRF Rig Control System. A number of closed-loop

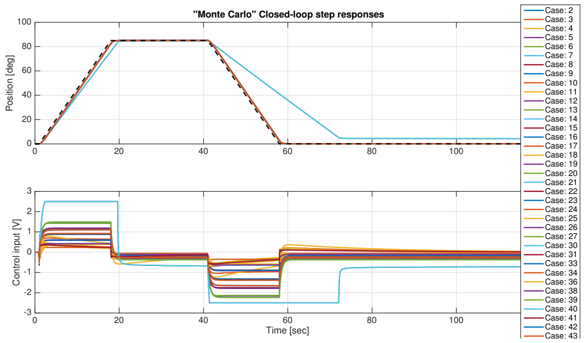

Figure 9: Performance results with $\mathscr{H}_\infty$ controller tested with all identified cases.

tests were performed, with attention primarily given to the tracking characteristics for a set of reference signals. The tests were carried out without the blades on and the rotational speeds of the rotor were 0, 120, 180, 500 and 1000 RPM.

Reference signals include step functions and slow (small frequency) harmonic signals of various amplitudes. Responses shown in Figure 10 are associated with the $\mathscr{H}_\infty$ controller. We observe that obtained steady-state errors are negligible and the achieved tracking speed is about $4°$/s when demanding a step change of $85°$. On the other hand, responses with the PID controllers are shown for the same step amplitudes in Figure 11. The transient is similar in terms of the speed, however, the steady-state errors are a bit larger. The tracking for harmonic signals is also shown to be very good (see Figure 12). Both controllers perform close tracking, with the $\mathscr{H}_\infty$ controller providing a slight improvement over the PID controller.

When the error between demand and achieved angle is less than $0.1°$ then both the solenoid motor brake and the hydraulic brake are engaged. In the case of the PID controller, the steady-state error and settling time were such that the brakes were engaged prior to the controller having finished its control action. This was apparent in the data and could be seen when the control action jumped to zero.

5 CONCLUSIONS

This manuscript has described the controller design process for the rig tilt control system and has discussed implementation results. The main conclusions are:

- The system identification process used to construct linear models of the rotor rig appeared to be successful. This approach could be used again to identify models when the blades are attached to the hub.

- Both PID and $\mathscr{H}_\infty$ controller design techniques produced controllers which performed well when implemented on the rig (blades-off).

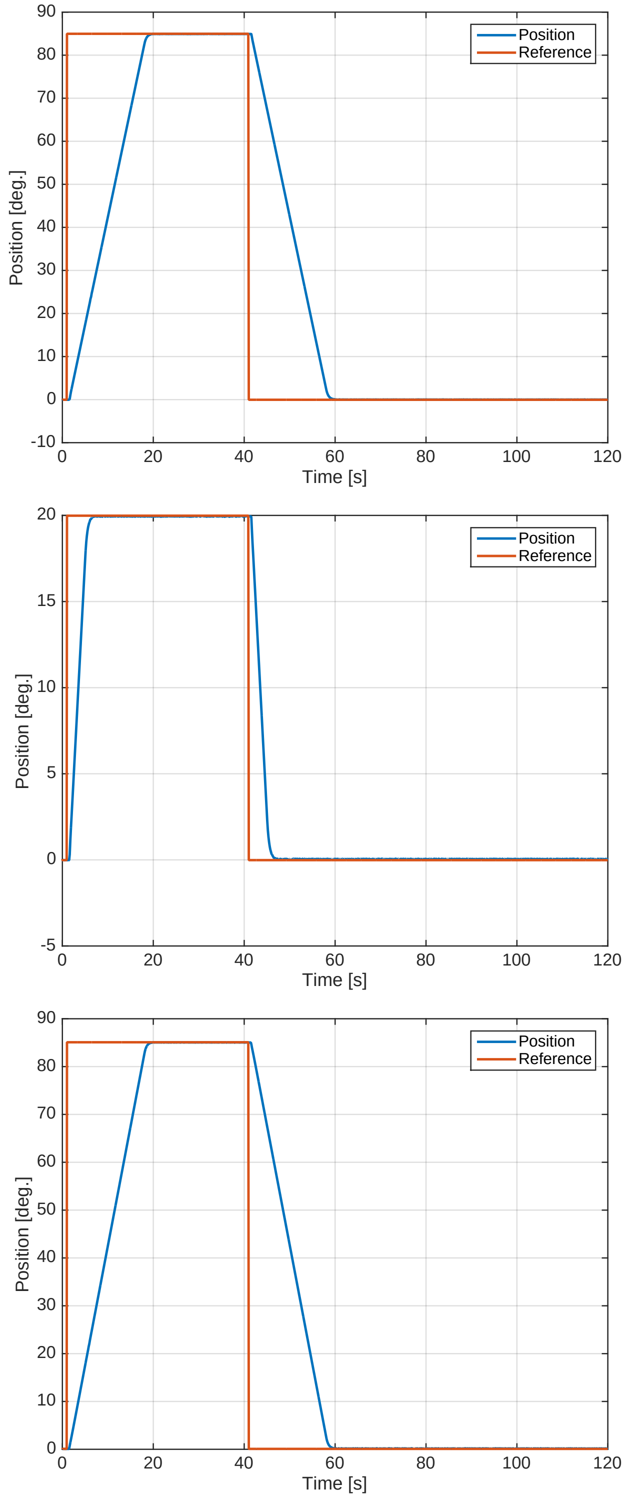

Figure 10: Experimental responses with $\mathscr{H}_\infty$ controller with step references at 500 RPM (top) and 1000 RPM (bottom two).

- The $\mathscr{H}_\infty$ controller had better performance than the PID controller and hence has been adopted

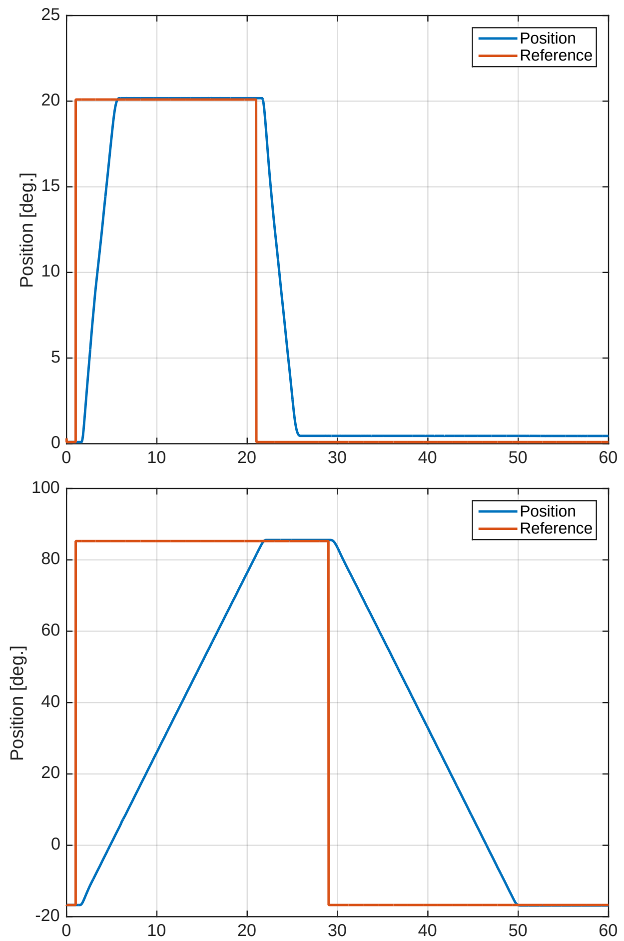

Figure 11: Experimental responses with PID controller with step references at 0 RPM.

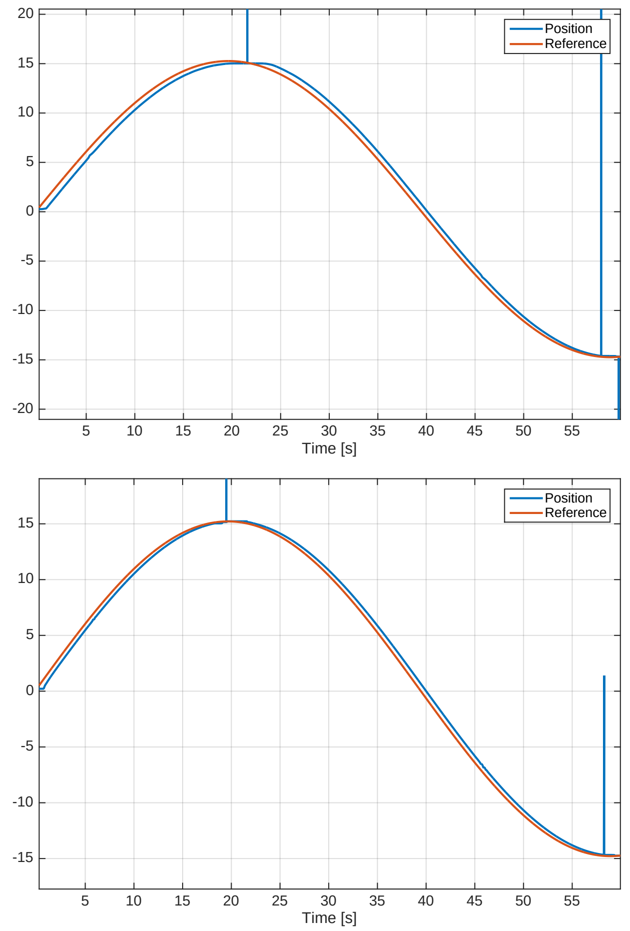

Figure 12: Experimental responses with PID (top) and $\mathscr{H}_\infty$ controllers to sinusoidal reference (Amp. $= 15°$, freq. = 0.08 rad/s.)

as the default tilt controller.

The models used for controller design have been obtained from tests performed in a blades-off configuration. When blades are attached to the rig, it is strongly advisable to re-run the steps mentioned in this manuscript for approval and increased confidence of the control algorithms. More comprehensive results, which include the development and validation of a grey-box nonlinear model to account for the effects of a disturbance torque originated by blade imbalances, will be provided in a future manuscript associated with this work.

Copyright Statement

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