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HELICOPTER ACTIVE CONTROL WITH BLADE STALL ALLEVIATION MODAL CAPABILITY

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ABSTRACT

An active modal control for high performances helicopters in forward flight is presented in this study. A blade stall alleviation (B.S.A.) control strategy based on the spectral data computed from the flexible blade structural mode of vibration measurements is employed to suppress the blade torsional motion associated with the stall flutter and to reduce the possibility of high angle of attack onset on the retreating blade. The restoring collective pitch commands are derived processing the output data from an electro-optical laser (L.S.U.) sensor by means of a microprocessor performing the power spectral density (P.S.D.) real time computations; these data, obtained implementing a fast fourier trasform (F.F.T.) algorithm and observed within a frequency window centered at the dominant torsional mode frequency are employed as a measure of the actual vibrational level existing on the blade. An optimal control strategy is implemented in order to release the blade loads below the critical limits predicted for the blade stall onset. To reduce the helicopter rigid response sensitivity to the B.S.A. control actuations, its driving signals are applied to the longitudinal pitch decoupling (L.P.D.) unit making the helicopter aptitude and vertical velocity component decoupled. The effectiveness of the B.S.A. control system is investigated by extensive digital simulations and its potential usefulness in widening the helicopter flight envelop is emphasized.

I - INTRODUCTION

An advanced helicopter must operate in severe aerodynamic environments including the atmospheric turbolence and retreating blade stall flutter. In reference [10] a gust alleviation system based on a modal spectral technique has been advanced and in this study this technique is extended to the active control of the blade stall in forward flight to alleviate the violent torsional motion associated to the stall flutter affecting seriously the helicopter flight mission envelop; this is particurarly a problem for the high performances combat helicopter which may encounter, during sharp turn and abrupt pull-up maneuvers involving high blade loads, severe blade stall.

The flutter stall appears, for high blade loading and advance ratio ratios, as a consequence of the high angle of attack on the retreating blade being affected as well by the rapid angle of attack variations experienced by the advancing blade toward the 270° rotor azimuthal angle. This aeroelastic instability involves a coupling of a number of the blade mode of vibration but predominatly developed in proximity of the first torsional natural frequency and implies large variations in the aerodynamic lift and moment coefficients and high torsional moment on the retreating blade. Tipically the stall flutter oscillations are unstable over a part of the blade azimuth covering a critical sector through 270° and damping out rapidly as the blade swings around toward the advancing side; however large amplitude torsional oscillations may occur in the critical azimuthal sector resulting in extreme loads on the blade structure and affecting significantly the helicopter dynamical behaviour. As emphasized in Ref. [2], [3] and [4] the aeroelastic instability associated with the blade stall may be thought as a loss of effecttive damping on the blade dynamics in the time interval in which it is passing through the 270° azimuth causing a growing of any small blade oscillations existing at that time slot and in particular those induced by a low frequency gust inputs; from the last point of view, a blade stall alleviation control implementation associated with a blade ust alleviation control, as that treated in Ref. [10] seems to be the most convenient choice in the design of an integrated modal control system based on the single blade control concept where a number of blade degrees of freedom may be involved.

The individual blade control (I.B.C.) concepts, proposed by various authors

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and experimented by M.I.T. researchers (Ref. 2 and 3) were applied in Ref. 10 in which a new modal control strategy implementing a gust alleviation control (G.A.C.) system was formulated. The Blade stall alleviation (B.S.A.) control system treated in present study can be considered, although involved in different control strategy, as a natural extension of the G.A.C. system sharing with it the Spectral Processor playing a fundamental computing role in both control systems.

An electro-optical laser (L.S.U.) sensor with chordwise and flapwise multireflectors arrangement is employed to measure the blade bending and torsional displacements in respect to a refence datum. The L.S.U. autput data are combined to yield a time function correlated to the effective aerodynamic angle of attack variations on the retreating blade in the time interval in which it swings through the "active sector", extending for 45° before and 75° after the blade 270° azimuth. The power spectral density (P.S.D.) of this time function, computed in a specified frequency window opened in the range of the dominant first torsional natural mode, yields a direct measure of the actual vibrational energy existing on the blade. The actual computed P.S.D. value is continously compared with a predicted critical value giving an indication of the abnormal torsional oscillations associated with the blade stall flutter phenomenon predicted for the particular blade and rotor configuration. If the actual P.S.D. will exceed the critical value, the B.S.A. control system becomesactive actuating the blade collective pitch channel relaxing the excessive airloads supported by the blade for an amount strictly necessary to avoid the stall flutter full development on the blade.The identification phase, the real time spectral process and the actuaction phase take place in the time interval in which the blade is crossing the active angular sector and the B.S.A. process is excluded outside this sector where generally the stall flutter oscillations are vanishing. A new B.S.A. process based on P.S.D. updated value will start at the beginning of the active angular sector for each blade revolution to keep a continous stall flutter control.

An optimal quadratic integral control strategy forcing the blade torsional angular displacement to be reduced to an established reference value in a given number of sampling time, is employed to implement the B.S.A. control system which is designed as an optimal regulator where all the elastic blade state variables are involved.

An important aspect in the B.S.A. control design is the minimization of the effects of B.S.A. control actuations on the helicopter dynamical behaviour. To achieve this goal the B.S.A. control system driving signals are applied to the longitudinal pitch decoupling (L.P.D.) unit described in Ref. [4]; this device is a multi-feedback control system processing all the observable helicopter state variables in such a way that, if a command signal is applied to to its "cyclic pitch channel" input, only the helicopter attitude is varied not affecting the helicopter vertical velocity; instead, the last state variable can be reached, without affecting the helicopter attitude, by a command signal applied to the "collective pitch channel". Serving the the L.P.D. cyclic channel with a reference datum derived by an Inertial Attitude Reference Unit, the B.S.A. control system is made capable, within the validity limit of the linear state variables decoupling theory, to control the stall flutter without affecting the helicopter attitude. Furthermore extending the optimal control strategy to the rigid helicopter state variables, the helicopter dynamical behaviour during the B.S.A. control actuactions, can be forced to follow a specified response model implementing a model following control structure as indicated in Fig. 1.

In the Section 2, the helicopter rotor configuration and the blade geometrical and inertial characteristics assumed as an introductive model for this presentation, are indicated. The results of the modal analysis for the blade lumped mass model are presented in Section 3. The linear mathematical model for the blade structural model describing the coupled flatwise bending and torsional modes is discussed in Section 4.

The spectral modal control concepts and their implications in computing the P.S.D. of the time function derived by the electro-optical laser sensor measurements are treated in Section 5; in that section the P.S.D. computing aspects involving a real time, high speed dedicated F.F.T. microprocessor and some notes on the routine employed to generate a frequency window through which the P.S.D. is evaluated, are discussed. In Section 6 follows the description of the general configuration of the B.S.A. control system including the spectral processor unit. The results of the digital simulations considering the local blade lift coefficient distributions in the active angular sector for the bare and B.S.A. controlled blade subjected to a severe gust excitation effects are presented and discussed in the last concluding section.

2. ROTOR AND BLADE CHARATERISTICS

The geometrical, inertial and aerodynamical characteristics of the blade and rotor configuration and the operating conditions taken into consideration in present study are indicated in Table I. The blade was supposed chordwise balanced, untapered, linearly twisted with uniform mass and stiffness radial distributions

TABLE I - BLADE AND ROTOR CHARACTERISTICS AND KINEMATICS

Datum	Symb	Dimension	Value
Airfoil	-	-	NACA 00112
Chord	c	m.	0.4163
Lift Slope	c_{p_a}	I/rad.	5.73
Blade weight per unit lenghth	w	N/m	100.4374
Hinge offset	e_h	m	0.3048
Aerodynamic center offset	x_a	m	0.0837
Center of gravity offset	x_c	m	0.115
Blade Inertia	I_b	$Kg.\ m^2$	1552.31
Mass moment of inertia per unit length	I_m	Kg. m/m	0.15767
Flapwise Bending stiffness	E.I.	$Kg.m^2$	$8.77\ 10^3$
Torsional Rigidity	G.J.	$Kg.m^2$	$5.8418\ 10^3$
Blade radius	R	m	8.53
Number of blades	N_b	-	4
Rotational freq.	Ω	rad/sec	23.235
Solidity	σ	-	0.0622
Lock Number	γ	-	10.486
Blade twist	β	deg.	-8 (linear)

3. BLADE MODAL ANALYSIS RESULTS

In this study essentially devoted to investigate the vehicle dynamics affected by the stall flutter induced disturbances, only the more involved degrees of freedom of the elastic blade, i.e. the flatwise bending and torsional modes, are considered. To make analysis essential in terms of engineering accuracy requested to analyze the effects of the blade flexibility on the helicopter dynamics, a lumped mass structural model has been assumed for the blade modal analysis. In terms of normal flatwise bending and torsional modes, the solution of the eigenmode problem is given in the following form:

$$z(r,t) = \sum_{k=1}^{\infty} \eta(r) \quad g_k \quad (t) \tag{1}$$

$$\psi(r,t) \quad \sum_{k=1}^{\infty} \xi(r) \qquad \varphi(t) \tag{2}$$

where η_k (r) and ξ_k (r) are respectively the mode shapes for the bending and torsional normal modes referred to the blade section at distance r from the hub hinge and $g_k(t)$ and φ_k (t) are respectively the blade linear and angular displacement time function relative to the k-th bending and torsional mode. The results of the eigenvalue and eigenmodes problem relative to the non rotating blade modelled as 5-masses lumped mass model with zero structural damping are given in Table 2 where only the normalized shapes of the first flatwise bending mode and the first torsional mode, which are essential in the stall flutter effects analysis, are shown.

TABLE 2 - RESULTS OF THE MODAL ANALYSIS FOR THE NON ROTATING 5 LUMPED MASSES NON ROTATING BLADE MODEL

$x = r/R$	First Bending mode ($\omega_b = 31.5$ rad/sec)	First Torsional mode ($\omega_t = 140$ rad/sec)
Q	0	0
0.2	0.059886	0.3093
0.4	0.219028	0.5860
0.6	0.44659	0.8057
0.8	0.71643	0.9493
I.0	I.000	I.000

The modes relative to non rotating blade are assumed as a reasonable approximations for the rotating blade modes. In the next section the mathematical linear model of the rotating elastic blade describing its bending deflection out of the rotational plane and torsional rotation in respect to the elastic axis is described.

4. ELASTIC BLADE DYNAMICAL MODEL

The mathematical model for the rotating elastic blade associated with coupled flatwise bending and torsional modes is formulated on the basis of the theory given in Ref. [2]. The coupled blade bending - torsion equations of motion in terms of the blade fundamental normal modes are expressed by:

$$\ddot{g}(t) = \bar{B}_c \vartheta_c(t) - \bar{B}_I g(t) - \bar{B}_2 g(t) + \bar{B}_3(t)\,\varphi(t) + \bar{B}_4 \dot{\varphi}(t)$$

$$\ddot{\varphi}(t) = \bar{T}_c \vartheta_c(t) - \bar{T}_I \dot{\varphi}(t) - \bar{T}_2(t)\varphi - \bar{T}_3 g(t) + \bar{T}_4 \dot{g}(t) \qquad (3)$$

In Table 3 and 4 the expression of the coefficients appearing in the system equation (3) are given. In Table 4 the modal integrals and constants required as entries in Table 3 are defined.

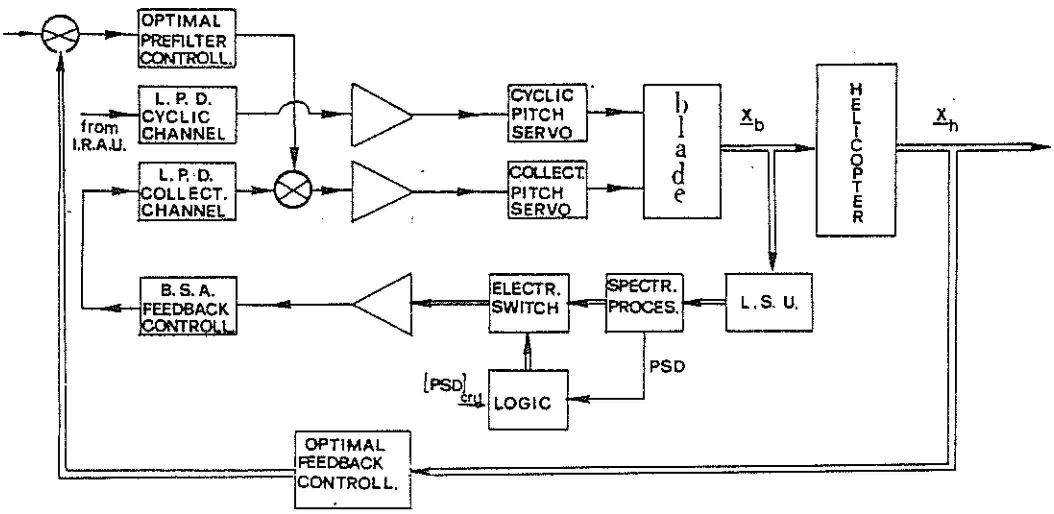

FIG.1 BLADE STALL ALLEVIATION CONTROL SYSTEM

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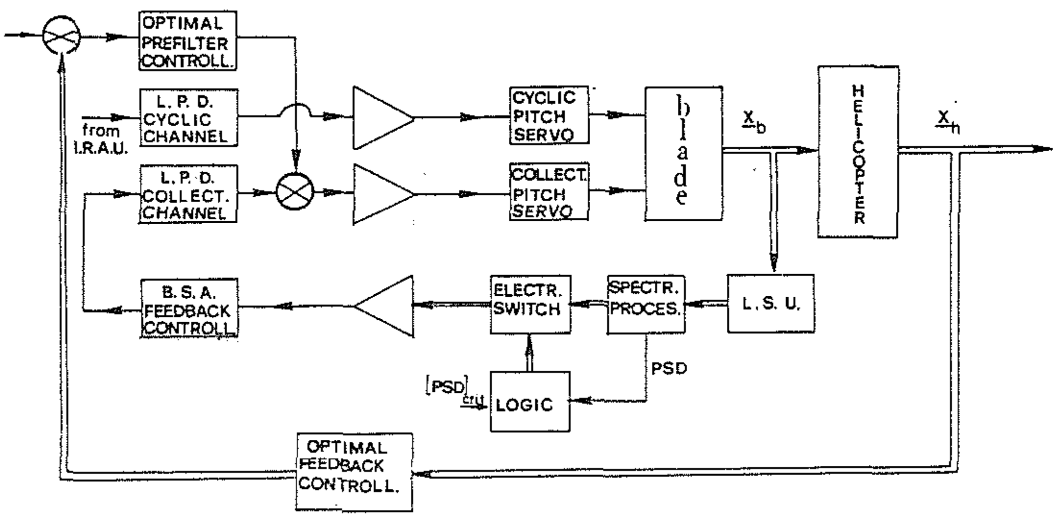

FIG.1 BLADE STALL ALLEVIATION CONTROL SYSTEM

TABLE 3 - COEFFICIENTS IN BLADE BENDING-TORSION DIFFERENTIAL EQUATIONE - UNIFORM BLADE

$B_I = m_D \quad M_R\,\Omega$	$T_I = m_T \quad I_R\,\Omega$
$B_2 = \omega_b^{\,2}$	$T_2 = \omega_t^{\,2}$
$B_3 = m_D\, M_R\,\Omega$	$T_3 = m_D \quad I_R\,\Omega$
$B_4 = M_{R_4}$	$T_4 = \bar{M}_{R_4}$
$\bar{B}_c = m_o \quad M_R \quad \Omega^2$	$T_c = \bar{m}_c \quad I_R \quad \Omega^2$
$\bar{B}_I = \frac{I}{\Delta} \quad B_I$	$\bar{T}_I = \frac{I}{\Delta} \quad T_I$
$\bar{B}_2 = \frac{I}{\Delta}\,(B_2 - B_4\, T_3)$	$\bar{T}_2 = \frac{I}{\Delta}\,(T_2 - T_4\, B_3)$
$\bar{B}_3 = \frac{I}{\Delta}\,(B_4\, T_2 - B_3)$	$\bar{T}_3 = \frac{I}{\Delta}\,(T_4\, B_2 - T_3)$
$\bar{B}_4 = \frac{I}{\Delta}\, B_4\, T_I$	$\bar{T}_4 = \frac{I}{\Delta}\, T_4 \quad B_I$
$\bar{B}_c = \frac{I}{\Delta}\,(B_c - B_4 \quad T_c)$	$\bar{T}_c = \frac{I}{\Delta}\,(T_c - T_4 \quad B_c)$

$$\Delta = I - T_4 \quad B_4$$

TABLE 4 - MODAL INTEGRALS AND CONSTANTS - UNIFORM BLADE

- Generalized mass and inertia

$$M_I = \int_o^I m(x)\,\alpha^2\,(x)\,R^3 dx \qquad M_2 = \int_o^I m(x)\,\beta^2(x)\,x\,dx$$

$$M_3 = \int_o^I m(x)\,\alpha(x)\,\beta(x)\,R^3\,dx \qquad I_2 = \int_0^1 m(x)\,\beta^2\,(x)\,R^3\,x^2\,dx$$

$$I_R = I_I/I_2 \;;\quad M_R = I_I/M_I \;;\quad M_{R_I} = M_3/M_I$$

$$M_{R_2} = M_3/M_I \;;\quad M_{R_3} = x_I\,M_{R_I} \;;\quad \bar{M}_{R_4} = \frac{\varrho^2}{x_I}\,M_{R_I}$$

$$\alpha = \eta(x)/R \;;\qquad \beta = \xi(x)/R$$

- Modal integrals

$$K_\eta = \int_o^I \alpha^2\,(x)\,x\,dx$$

$$K_\xi = \int_o^I \beta^2\,(x)\,x\,dx$$

$$\bar{K}_\eta = \int_o^I \alpha(x)\,x^2\,dx$$

Modal coefficients

$$m_D = \frac{\gamma}{2}\,K_\eta$$

$$\bar{m}_D = x_L\,m_D$$

$$m_T = \frac{\gamma}{2}\,K_\xi$$

$$\bar{m}_T = x_L\,m_T$$

$$m_c = \frac{\gamma}{2}\,\bar{K}_\eta$$

$$\bar{m}_c = x_L\,m_c$$

Defining the state vector:

$$\underline{x}\ (t) = \left[g(t) \quad \dot{g}(t) \quad \varphi(t) \quad \dot{\varphi}(t) \right] \tag{4}$$

The differential equations (3) can be expressed in state variable form:

$$\dot{\underline{x}}\ (t) = A\ \underline{x}\ (t) + B\ \vartheta_c\ (t) \tag{5}$$

where the control variable ϑ_c (t) is defined as the blade collective pitch angle employed as the main effector in the B.S.A. active control.

The state matrix A and the control matrix B have the following form:

$$A = \begin{vmatrix} 0 & I & 0 & 0 \\ -\bar{B}_2 & -\bar{B}_I & \bar{B}_3 & \bar{B}_4 \\ 0 & 0 & 0 & I \\ \bar{T}_3 & \bar{T}_4 & -\bar{T}_2 & -\bar{T}_I \end{vmatrix} \qquad B = \begin{vmatrix} 0 \\ \bar{B}_c \\ 0 \\ \bar{T}_c \end{vmatrix}$$

Neglecting the small unsteady aerodynamic component, the blade incident free velocity including the rotor rotational velocity will be expressed, for a blade section at distance r from the hub, by:

$$U_T = \Omega r + \mu \Omega R \quad \sin \psi \tag{6}$$

and the unsteady flow perpendicular to the blade section assumed as a thin airfoil oscillating in incompressible flow, will be given by:

$$U_P = \eta(r) \quad \dot{g}\ (t) + \mu \Omega R \frac{dz}{dr} \quad \cos \psi \tag{7}$$

The local angle of attack perturbation due to the bending mode, can be approximated by the ratio U_P/U_T . Observing the angle of attack perturbations in proximity of the 270° azimuth at a specific blade section (r_o) where the bending modal shape assumes a known value η_o, and summing up the bending and torsional contributions, it can be expressed in the form:

$$\alpha(t) = K_\alpha \dot{g}\ (t) + \varphi(t)\ ; \quad K_\alpha = \frac{\eta_o}{\Omega r_o - V_o} \tag{8}$$

where the denominator in the K_α expression is the net relative airspeed encountered by the retreating blade passing through the 270° azimuth. Employing Eq. (8), the blade dynamics and associated aerodynamic perturbations in respect to a stationary reference condition, can be obtained integrating simultaneously the Equation (5) with the support of the algebraic output equation:

$$y(t) = C\ \underline{x}(t) = \left[0 \quad K_\alpha \quad I \quad 0 \right] = \alpha(t) \tag{9}$$

Since the all the components of the state vector (4) are measured at discrete time intervals and the function at the input of system (5) is essentially a stepwise constant continuous function, the Eq. 5 and 9 must be discretized becoming:

$$\underline{x}\ (k + I) = A\ (T)\ x\ (k) + B(T)\ \vartheta_c\ (k) \tag{10}$$

$$y\ (k) = C\ \underline{x}\ (k) \tag{11}$$

where A(T) and B (T) are respectively the discrete state and control matrices which are implicitly functions of the sampling time assumed for the data information flow. The equation (10) and (11) will be used as a first approximation for the B.S.A. control system analysis.

5 - THE BLADE STALL ALLEVIATION CONTROL CONCEPTS

As previously introduced, the blade stall alleviation control system proposed in this study has been conceived as an active modal control in which the actual vibrational energy level existing in the retreating blade is continuously observed and when it exceeds some critical value predicted for the blade stall, the blade collective pitch is actuated in order to relax the existing overload supported by the blade avoiding consequentely the full stall development. Since the flutter stall is characterized by a violent torsional oscillations at a frequency almost coincident with the blade first torsional mode, the Power Spectral Density (P.S.D.) of a time function obtained by a direct observation of the torsional angular displacements, is undoubtedly, the more appropiate quantity to be correlated with the blade stall onset. However considering that the torsional vibration are strongly influenced by the bending mode and that the B.S.A. control actuactions must be performed in direct reference to the amount of the angle of attack variations, seems to be more appropiate to assume, as index of the blade vibrational state in relation to the blade stall onset, the P.S.D. of the time function (9) defined in the previous section.

The advantage in performing the spectral analysis of such function which as a matter of fact is the blade of attack perturbation existing on the blade reference section, is represented by the possibility to evaluate the angle of attack magnitude in a frequency slot centered at the first torsional natural frequency where the most stall flutter energy is concentrated. In mathematical terms, defining:

$$A(\omega_t) = \text{D.F.T.}\ [\alpha(t)] = M_A(\omega_t) + j\, I_A(\omega_t)$$

the discrete Fourier transform of the angle of attack perturbation time function defined in (9) evaluated through a triangular (Barlett) frequency window centered at the first natural torsional mode frequency (ω_t), its Power Spectral Density will be expressed by:

$$P_A(\omega_t) = \left(M_A(\omega_t)\right)^2$$

The actual value of the P.S.D. which is considered in the B.S.A. control process will be defined as the mean value of the P.S.D. defined in (13) evaluated in the time interval in which the blade swings through the active angular sector:

$$\bar{P}_A(\omega_t) = E\ [P_A(\omega_t)]$$

The function $\alpha(t)$ is observed in a number of discrete points dictated by the Fourier analysis performed in the Spectral processor and it will be defined on the basis of considerations regarding the frequency and time resolution, the frequency bandwidth to be covered and the maximum time delay expected for the B.S.A. control process. The data measured by an electro-optical sensor are temporarely stored in the Spectral Processor and employed for the P.S.D. real time computation; for the time interval in which this process is being carried out, the measured data are inhibited to enter in the processor R.A.M. area until a new active cycle is initiated. The actuaction stage is empowered to start by the logic unit when the the actual computed P.S.D. exceeds a predicted critical value; a that time the blade collective pitch is decreased to an amount strictly necessary to relax the blade airload below a an established guard value at which the blade stall flutter conditions vanish. This actuaction stage will last a time interval necessary to the blade to complete the active angular sector and the last blade pitch value, resulting at the time in which the blade has left this sector, will be kept invariant until a new active cycle is initiated. The process will be repeated at the sucessive blade revolutions, each performed in reference to an updated P.S.D. value. In the actuation stage the blade collective channel is operated as an optimal regulator with feedback gains adjusted to minimize an integral performance index involving all the blade state variables; the adopted optimal control strategy allows the angle of attack value, measured at the beginning of the active cycle and stored in computer memory, to be reduced below a prescribed value in a number of sampling time interval properly chosen to satisfy the B.S.A. control requirements. As will be treat in next section, the blade dynamics will be regulated by the control law:

$$\vartheta_c(t) = u_{opt}(t) = K_{opt}\ \underline{x}\ (t)$$

where the K_{opt} is the optimal feedback gain vector obtained solving the optimal regulator problem with model response implicit in the state weighting matrix assumed in the integral performance index. Matching properly the aerodynamic and kinematical parameters involved in B.S.A. control design, the actual P.S.D. value will be reduced below the critical value in a time interval depending essentially by the collective pitch servomotor dominant time constant.

To limit the helicopter velocity vector orientation in respect to the terrestrial reference frame, the B.S.A. control command signals are applied to the collective channel input of the longitudinal pitch decoupling (L.P.D.) unit described in Ref. 5, the function of which is to decouple the helicopter state and control variables in two sets of properly chosen decoupled subsystems:

$$S_c = (d_c, \vartheta) \qquad S_{cc} = (d_{cc}, w)$$

The first channel make the helicopter attitude (ϑ) controllable only by the cyclic pitch command (d_c) non affecting the helicopter vertical speed component, while the other channel is empowerd to control, by means of the collective pitch command (d_{cc}), the vertical velocity component (w) independently by the longitudinal attitude. As described in

Ref.5, the L.P.D. unit is a multi-feedback structure, implemented by a feedback and prefilter controllers, involving all the measurable helicopter state variables in the decoupling and regulating processes, the last one based on a specified desidered response model. Driving the cyclic pitch decoupling channel with a reference signal derived by an Inertial Reference Attitude Unit, the B.S.A. actuactions can be developed maintaining the helicopter attitude change within the tollerance required for the flight mission.

5. OPTIMAL ACTIVE CONTROL PROCESS

As indicated in the prevuous sections, the blade stall relaxation is obtained varying the blade collective pitch at the time the computed P.S.D. exceeds a predicted critical value. In the actuaction stage the blade collective pitch is decreased from the value relative to the reference stationary flight condition existing before the blade stall onset to a pitch angle at which the blade aerodynamic load is relaxed to an amount strictly necessary to avoid the full development of the stall along the blades. For this purpose the collective pitch servosystem is made operating as an optimal regulator driving the blades to assume the desidered pitch angle with a specified transient dynamics in a time interval compatible with the servomotor characteristics and chosen on the basis of non stationary aerodynamical considerations involved in blade pitch actuactions.

The optimal control problem is stated in discrete mathematical form which is more appropiate to the B.S.A. control digital implementation. The state equation (10) is taken into consideration in formulating the optimal process in which the performance index:

$$J = \sum_{k=0}^{N-I} 1/2 \; \underline{x}^{T} \cdot Q \cdot x \qquad (12)$$

where Q is a positive semidefinite matrix weghting the state variables involved in the system equation, is minimized in an established number of sampling time starting from an arbitrary initial state:

$$\underline{x}\ (k) = \underline{x}\ (0) \qquad (13)$$

The solution of the optimal control problem yields the in an discrete control law:

$$u_{opt}(k) = \vartheta_c\ (k) = K\ \underline{x}\ (t) \qquad k = 0,1,\ldots N-I \qquad (14)$$

In Eq. 14, K is a time variant feedback matrix found with the application of the linear Riccati transformation which can be found in advanced optimal control texts:

$$K\ (k) = -R^{-I} \left[p\ (k) - Q \right] \qquad (15)$$

where P (k) is the discrete Riccati matrix obtained solving the recursive system equations:

$$P\ (k) = Q + A^{T} P\ (k+I) + W^{-I}(k+I)\ A$$
$$W\ (k) = I + B\ R^{-I} B^{T}\ P\ (k+I) \qquad (16)$$

by a backward in time process starting from:

$$P\ (\ N\ ) = 0$$

to obtain the stationary feedback gain matrix K (0) solving the minimization problem in a given number (N) of sampling time intervals. The system transient behaviour is forced to obey a model following strategy with a response model implicit in the augmented discrete state matrix A(T). Defining the augmented state vector:

$$\underline{x}_a\ (k) = \left[x_I(k), x_2(k), x_3(k), x_4(k), x_5(k) \right] = \left[g\ (k),\ \dot{g}(k),\ \varphi(k),\ \dot{\varphi}\ (k),\ \alpha_M(k) \right] \qquad (17)$$

where $\underline{x}_5$ (k) is the local angle of attack perturbation modelled as a first order discrete dynamics:

$$\dot{\underline{x}}_5\ (k) = K\ T\ \underline{x}_5\ (k) \qquad (18)$$

The constant K in (18) is defined as the inverse of the time constant established for the desidered exponential angle of attack variation starting from a stationaty value. Reformulating the performance index (12) in the following form:

$$J = q_{II}\, x_I^2 + \bar{q}\,(K_g\, x_2^2(k) + x_3^2(k)) + q_{44}\, x^4 - \bar{q}\, x_5^2\,(k) =$$

$$= \bar{q}\,(\alpha_M^2(k) - \alpha^2(k)) + q_{44}\,\dot{\varphi}^2\,(k) + q_{II}\, g\,(k)$$

the performance index J appears to be is involved, beside the bending deflection and the angular torsional rate, by the error in the local angle of attack perturbation in respect to a proposed model. By an appropriate choice of the state weghting matrix elements, the optimization process implemented with the feedback gain matrix indicated in (15) will force the system to minimize the local angle of attack error, or equivalently the local blade loads, in respect to the established model. This strategy results very usefull in regulate the blade loading transients when the the collective pitch is changed to relax the stall onset on the blade. In the actual S.B.A. Control implementation a number of sets of initial conditions with the correspondent computed optimal feedback vectors are stored in the spectral processor. When the B.S.A. Control becomes active the actual set of initial conditions observed and memorized at the beginning of the B.S.A. acquisition phase is brought into coincidence with the stored values making the correspondent feedback vector available for the automatic process allingning the optimal controller gains with the computed reference values; as this time the feedback controller is ready for the collective actuaction driving the blade pitch to the desidered value.

6 - THE B.S.A. CONTROL SYSTEM STRUCTURE

The costituent parts of the proposed Blade Stall Alleviation control system which are shared with the Gust Alleviation control system presented in Ref. 10 , are the electro-optical cal Laser Sensor Unit (L.S.U.), the Spectral Processor and the Longitudinal Pitch Decoupling (L.P.D.) unit. The two active control systems differ from each other essentially for the feedback controllers and their electrical connections to the blade pitch servosystem units. In the G.A.C. control system the spectral processor output is applied to a function generator producing an harmonic signal driving, through the L.P.D. cyclic channel, the blade cyclic pitch servomotor to an amount proportional to the actual computed P.S.D. value. In the Stall Alleviation control system the spectral processor output is applied to a logic unit which empowers the data relative to the difference between the P.S.D. values computed in two subsequent active cycles to be traslated, through the L.P.D. collective pitch channel, to the collective pitch servomotor input for the stall alleviation purposes. The overall topology of the B.S.A. control system for the part relative to the blade feedback optimal controller treated in this study is depicted in Fig. 2, while the complete B.S.A. control system including the feedback helicopter control; which shall be presented in the next future paper, is sketched in Fig. I. The common consituent parts of the B.S.A. and G.A.C. control systems were discussed in Ref. 10 ; in the following further technical informations relative to the B.S.A. control system are given.

- The L.S.U. Pakage

  To measure the blade bending and torsional displacements described in the state equation (5), respectively by the state variables $g(t)$ and $\varphi(t)$, an electro-optical Laser sensor, which is a particular application of the Laser Position Encoder employed for shape measurements in the Large structure in space (Ref. 6,7) and presently in development stage, is proposed as a structural sensor for the B.S.A. and G.A.C. active control systems. This device uses a coaxial trasmitter-receiver pulsed diode diode laser employing a scanning mirror to direct the emitted laser beam to the blade supported reflectors within a fixed angular range; in the same scanning range, the reflected light beam from the reflectors are observed by the laser head and then detected by a photosensitive device.The operation of this system consists of initiating a pulse from the laser emitter which is pointed a the scanning mirror; the emitted pulse strikes the scan mirror and it is sent to one of the reflector targets located on the fence wall fixed on an established blade section. Upon reflection from the target, the pulse returns to the scanning mirror to be observed in a photosensitiv detector via an electronis circuity and fiber optic unit. The detected pulse is amplified and used to trigger another emitted pulse and the process becoms repetitive with a repetition rate uniquely determined by the distance travelled to the target and back; a measure of the repetition rate thus created provides the means required for determining the range from the scanning mirror to the blade reflector. The reflected light beam spot coordinates imaged on the photosensitive array are resolved by a microprocessor using an algorithm computing the numerical value of the imaged spot coordinates which are convertend in a linear and angular displacements of the blade reflector spot from an established reference datum. One set of the reflector points is located on the supporting fence in position aligned with the blade section chordwise elastic axis while another of reflectors set is located in the same fance but forward the elastic axis in order to make possible to compute in one scanning cycle, the linear displacement due to bending at the elastic axis and the angular displacement in respect to the elastic axis due to the blade twist.

Applying the individual blade control technique, the Laser sensor unit head is solidal with the rotating master blade frame and the position of the reflectors supporting fence on the blade, which depends upon the rotor configuration and blade elastic characteristics, has been chosen, for the case treated, at a distance of 0,5 R from the hub. In Fig. 3 a descriptive sketch of the basic L.S.U. principles is given.

The resolution expected from a 0.82 m diode laser operating with a pulsewidth of 28 psec, is in the order of 0.2 mm. and 0.I arc sec. respectively for the blade bending and twist measurements.

- Spectral processor

The same Spectral Processor proposed for the Gust Allevation Control System treated in Ref. 10 may be shared, with an appropiate interrupt microprogram provision, with the Blade Stall Alleviation Control System. It is essentially implemented with an high speed F.F.T. dedicated microprocessor computing, in each blade revolution and in the time slot indicated in Table 5 as the "time interval" in the "computational stage", the averaged convoluted value of the actual Power Spetral Density value employed as the basic information in the stall alleviation process. In Table 5 and Fig. 4 the angular and time bounds for the constituents stages relative to the active operational B.S.A. cycle, as it has been proposed in the system digital simulation, are shown.

In Table 6 and 7 the basic data specifications for the spectral process carried out in the Spectral Processor and some of the characteristic of the microprocessor employed in the laboratory experiments are indicated.

TABLE 5 - ACTIVE B.S.A. CONTROL - PROCESS SPECIFICATIONS

Blade Revolution period	sec.	0.270282
Active Control sector:		
Rotor azimuth coverage (ψ)	deg.	225-345
time interval	sec.	0.09074
Acquisition stage:		
Rotor azimuth coverage (ψ)	deg.	225-284
time interval	sec.	0.044613
Computational stage:		
Rotor azimuth coverage (ψ)	deg.	284-313
Time interval	sec.	0.021929
Actuaction stage:		
Rotor azimuth coverage (ψ)	deg.	313-345
Time interval	sec.	0.024197

TABLE 6 - ACTIVE B.S.A. CONTROL - SPECTRAL PROCESS SPECIFICATIONS

Signal bandwidth	Hz	22.2929
F.F.T. frequency bandwidth	"	44.5829
Number of F.F.T. points	"	32
Sampling time	sec.	0.014178
Frequency resolution	Hz.	I.39321
Barlett window width	"	II.14568

TABLE 7 - F.F.T. DEDICATED MICROPROCESSOR CHARACTERISTICS

Data word lenght	bits	8
Machine time cycle	10^{-6} sec.	I
Number of computing cycle	N.D.	6
Number of butterfly blocks	N.D.	80
Time for F.F.T. batch	sec.	0.005
B.S.A. operational cycles	N.D.	3
Total computing time per B.S.A. control op.	sec.	0.015

7 - THE BLASE STALL ALLEVIATION PERFORMANCES EVALUATION

To evaluate the rotor dynamical behaviour in proximity of the blade stall onset due to local angle of attack increase induced by the blade bending and torsional elastic deformations and the effectiveness of the proposed B.S.A. active control system in relaxing the blade stall effects, extensive digital simulations trials have been carried out; an integrated digital computer program based on the blade linear model described in the preceding sections, including the P.S.D. computational algorithms and logic, as they are implemented in the proposed B.S.A. configuration, was prepared and applied to the specific practical application in order to obtain a significant physical insight on the blade dynamics when subjected to a severe environmental disturbance considered as an energetic source in the blade stall generation process. To simulate the stall flutter instability causing the magnified torsional oscillations, a negative damping factor was switched into the blade bending-torsional coupled equations when the local blade loading at the blade reference station, observed by the L.S.U. sensor, was falling inside a critical specified range in which the blade stall is expected to initiate. However since in this critical range the computed critical P.S.D. of the actual vibration level existing on the blade is also expected to be exceeded, the B.S.A. active control is supposed to be, at that time, operative and if the B.S.A. control system is properly designed, only a low or moderate stall flutter instability effects may appear in the simulation results. Assuming the blade and rotor characteristics in Table I and its modal featuree given Table 2, the state matrix A (T), the control matrix B(T) and the output matrix C (T) appearing in the blade discrete equation (10) referred to the sampling time value (T = 0.0014178 sec.) assumed for the spectral computations, are numerically expressed:

$$A(T) = \begin{vmatrix} 0.999 & I.40077\ 10^{-3} & I.1402\ 10^{-3} & 2.633\ 10^{-9} \\ -I.40543 & 0.99895 & I.6084 & I.14398\ 10^{3} \\ 4.8557\ 10^{-4} & I.80914\ 10^{-8} & 0.979745 & 0.0014178 \\ 0.684952 & 5.11096\ 10^{-4} & -28.573 & 0.979738 \end{vmatrix}$$

$$B(T) \quad \begin{vmatrix} 4.57108\ 10^{-3} \\ I.7469\ \ 10^{-6} \\ 2.46II\ 10^{-3} \end{vmatrix} \qquad C(T) = \begin{bmatrix} 0 & 0.057128 & 0 & 5.73 \end{bmatrix}$$

In order to analyze the elastic behaviour in proximity of the stall onset and, in particular, the local increase in blade loading due to the elastic effects when the blade is passing through the identification sector, a discrete (I-cosine) gust function has been considered as the environmental disturbance; the period T_w and the magnitude K_w of the this gust function expressed by:

$$w(t) = \frac{K_w}{T_w} \left(I - \frac{2\pi}{T_w} t \right)$$

were chosen to contain the amplitude and frequency characteristics typical of the a real atmospheric condition where a gust gradient, with a gradual build up through the rotor disk, is experienced.

Assuming a mean gust velocity corresponding to a severe turbolence (8 m/sec) and a minimum wave length (100 m.) in the range of the predicted value of the gust gradient in relation to the rotor diameter, the gust frequency of 5.8075 rad/sec. corresponding to the spatial frequency of 0.08065 rad/m. with a relative wind velocity of 72 m/sec. has been chosen. This gust function will be resolved in 4 blade revolutions at the assumed revolution period (0.270470 sec.). The solution of the optimal control problem with implicit model following strategy yields, for the flight condition considered in the digital simulation, the feedback gain matrix indicated in Table 8

TABLE 8 - OPTIMAL FEEDBACK MATRIX

Collective Pitch control - B.S.A. Actuaction phase

V_0 = 64 m/sec - 8 m/sec (I-cos) gust function

State Initial conditions at = 225°

Gain	Variable	Sensor	Value
h_{II}	$g(k)$	L.S.U.	8.89545
h_{12}	$\dot{g}$ (k)	"	- 2.545
h_{13}	φ (k)		- 9.58276
h_{14}	$\dot{\varphi}$ (k)	"	- 0.02106

8 - SIMULATION RESULTS

Some of the simulation results are presented in the following diagrams. In Fig. 5 the local lift distribution on the rotor disk azimuthal plane relative to the undisturbed reference flight condition is presented. In Fig. (6-b) the blade load variations, expressed in terms of the local lift coefficients in the blade reference section due to (I-cosine) disturbances plotted in Fig. (6-a) are shown. In Figure (6-c) the P.S.D. levels computed by the Spectral Processor at the end of each active cycle and kept constant until it becomes updated with a new value pertaining to the next cycle,are indicated as they are progressing through the subsequent four cycles; the critical P.S.D. value appears to be exceeded at the second rotor revolution and, at that time, the B.S.A. active stage is initiated. The time behaviour of the local blade load due to the B.S.A. collective pitch actuation governed through the optimal regulator process is given in Fig. 7. In Fig. 8 the local azimuthal lift distribution for the rigid and elastic blade, both subjected to the same (I-cosine) disturbance, are shown for comparison purpose.

9 - CONCLUSION

A Blade Stall Alleviation Control system compatible with the a Gust Alleviation Control presented in Ref. 10 , both based on a new active modal spectral technique treated in previous papers (Ref. 4, 6, 7 and 8) has been investigated in the present work. In this preliminary study in which various overlapping research areas are involved, the main authors aim is to obtain a phisical insight upon the dynamical blade behaviour related to the control problems arising from the particular implementation of the modal spectral process requiring advanced modal measurement techniques and computing capability. From this point of view, the linear model assumed for the the blade in the a individual blade control system has been considered acceptable for a preliminary investigation oriented to obtain the approximate order of magnitude of the stall flutter effects in order to prove the feasibility of a control process based on a spectral process, as it has been proposed in this study. From the simulation results presented in this paper and others not shown for reason of its length, the B.S.A. appears a feasible process the characteristics of which are suitable for further significant improvements dependent on the structural sensor resolution and on the real time F.F.T. dedicated microprocessor features. Considering the significant improvements in the helicopter flight envelope obtainable with a compound use of the Gust Alleviation and Blade Stall Alleviation control systems which may be integrated in a conventional autopilot without substantial helicopter and blades structural modifications, the proposed active modal strategy appears very promising for application on high performances helicopters.

LIST OF SYMBOLS/ACRONYSMS

A State matrix
B Control matrix
C Output matrix
d_c Cyclic pitch command
d_{cc} Collective pitch command
EI Blade bending
E(-) Expected mean value
F_k Generalized force for K-th elastic mode
g Blade bending displacement
GJ Torsional stiffness
I_m Blade section second moment of area, flatwise bending
I_b Blade mass moment of inertia respect to flapping hinge
M_k Generalized mass for the k-th elastic mode
N Number of discrete observations in F.F.T. computations
N_f Number of points in the Barlett window width
K_f Feedback gain vector or matrix
M (-) Magnitude of complex function
P Power spectral density
r Blade section distance from rotor hub
R_e Rotor radius
S_c Cyclic decoupled subsystem
S_{cc} Collective decoupled subsystem
T_s Blade rotational period
T_w gust function period
T_c Sampling time
u Long. velocity component in body axes
w Vert. velocity component in body axes
/Blade weight per unit length
r/R ratio
x_a Blade section aerodynamic center offset from elastic axis
x_c Blade section center of gravity offset from elastic axis
$\underline{X}_b$ Blade state vector
$\underline{X}_h$ Helicopter state vector
z Blade bending displacement
β Blade twist
γ Lock number
ϑ_C Collective pitch angle
η Bending mode shape
φ blade bending angular displacement
ψ Blade angular torsional deflection
ξ Torsional mode shape
σ Rotor solidity
ϱ radius of gyration
ω Rotor rotational frequency
ω_b first bending mode frequency
ω_t first torsional mode frequency

B.S.A. Blade Stall Alleviation
G.A.C. Gust Alleviation Control
F.F.T. Fast Fourier Transform
I.F.F.T. Inverse Fast Fourier Transform

I.B.C. Individual Blade control
I.R.A.U. Inertial Reference Attitude Unit
L.S.U. Laser Sensor unit
P.S.D. Power Spectral Density

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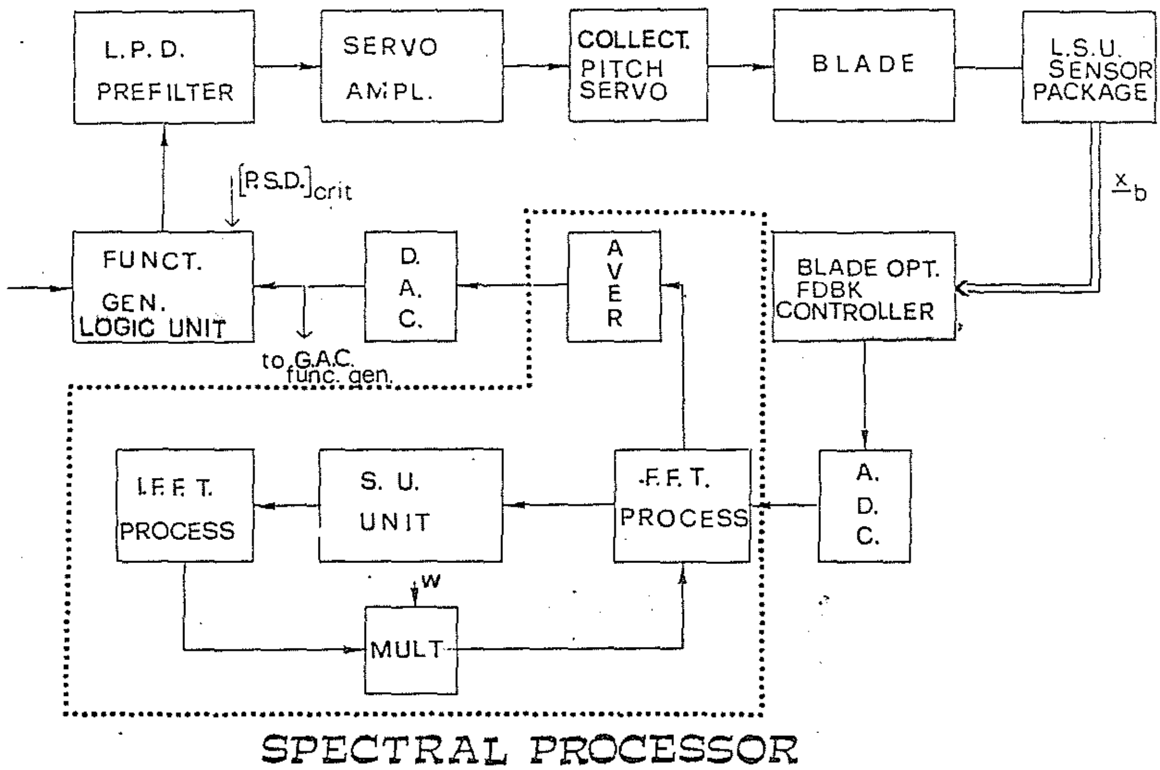

L.P.D.
PREFILTER
SERVO
AMPL.
COLLECT.
PITCH
SERVO
BLADE
L.S.U.
SENSOR
PACKAGE
[P.S.D.]crit
x b
FUNCT.
GEN.
LOGIC UNIT
D.
A.
C.
AVER
BLADE OPT.
FDBK
CONTROLLER
to G.A.C.
func. gen.
I.F.F.T.
PROCESS
S. U.
UNIT
F.F.T.
PROCESS
A.
D.
C.
W
MULT
SPECTRAL PROCESSOR

FIG. 2

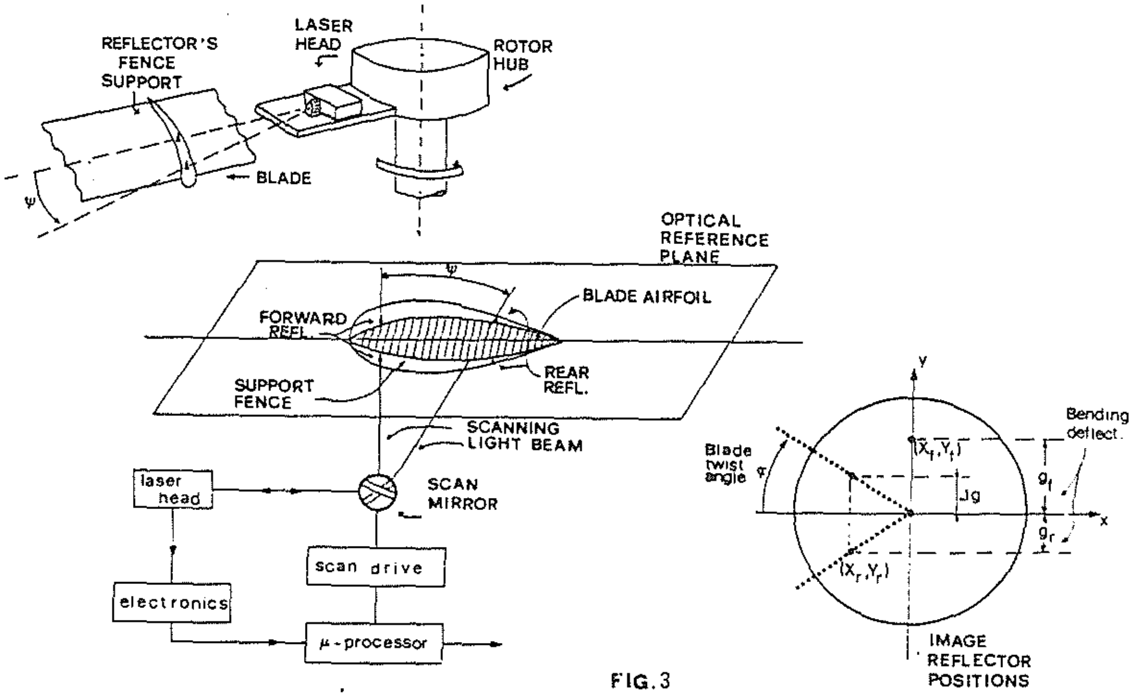

LASER
HEAD
REFLECTOR'S
FENCE
SUPPORT
ROTOR
HUB
BLADE
ψ
OPTICAL
REFERENCE
PLANE
BLADE AIRFOIL
FORWARD
REFL.
SUPPORT
FENCE
REAR
REFL.
SCANNING
LIGHT BEAM
laser
head
SCAN
MIRROR
scan drive
electronics
μ-processor
y
x
Bending
deflect.
Blade
twist
angle
(Xf,Yf)
(Xr,Yr)
gf
gr
IMAGE
REFLECTOR
POSITIONS

FIG. 3

	$\Delta\psi$ (deg)	Δt (sec)
1	225-284	.044613
2	284 - 313	.021929
3	313 - 345	.024197

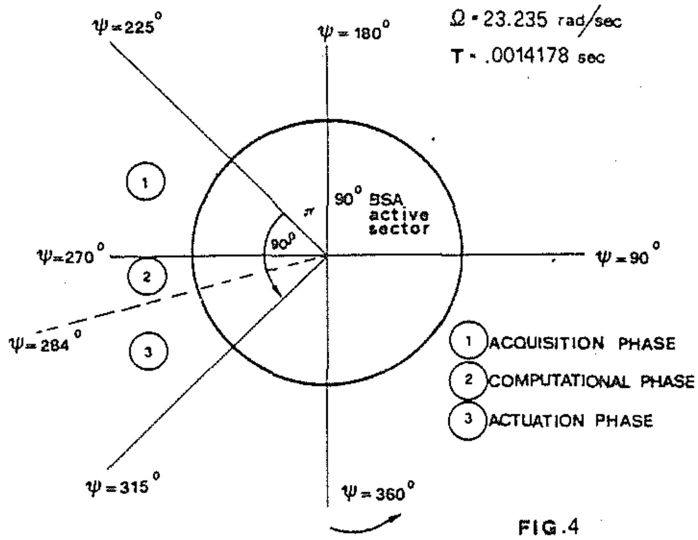


FIG.4

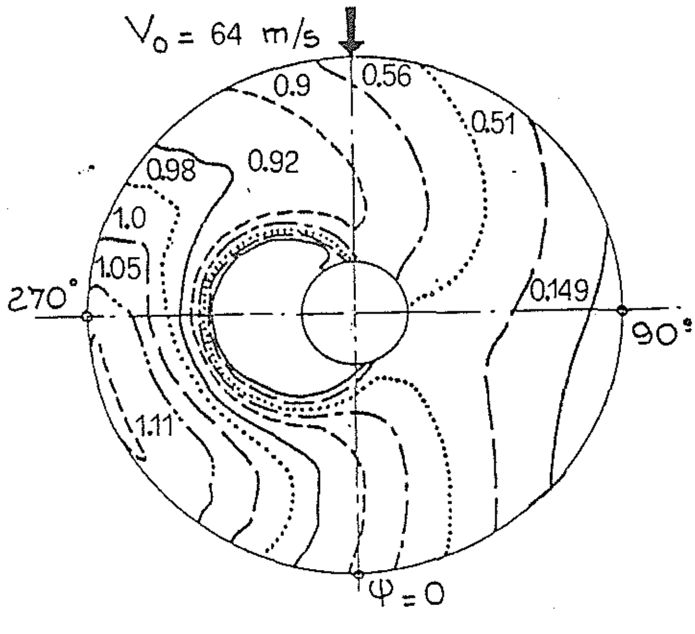


FIG. 5

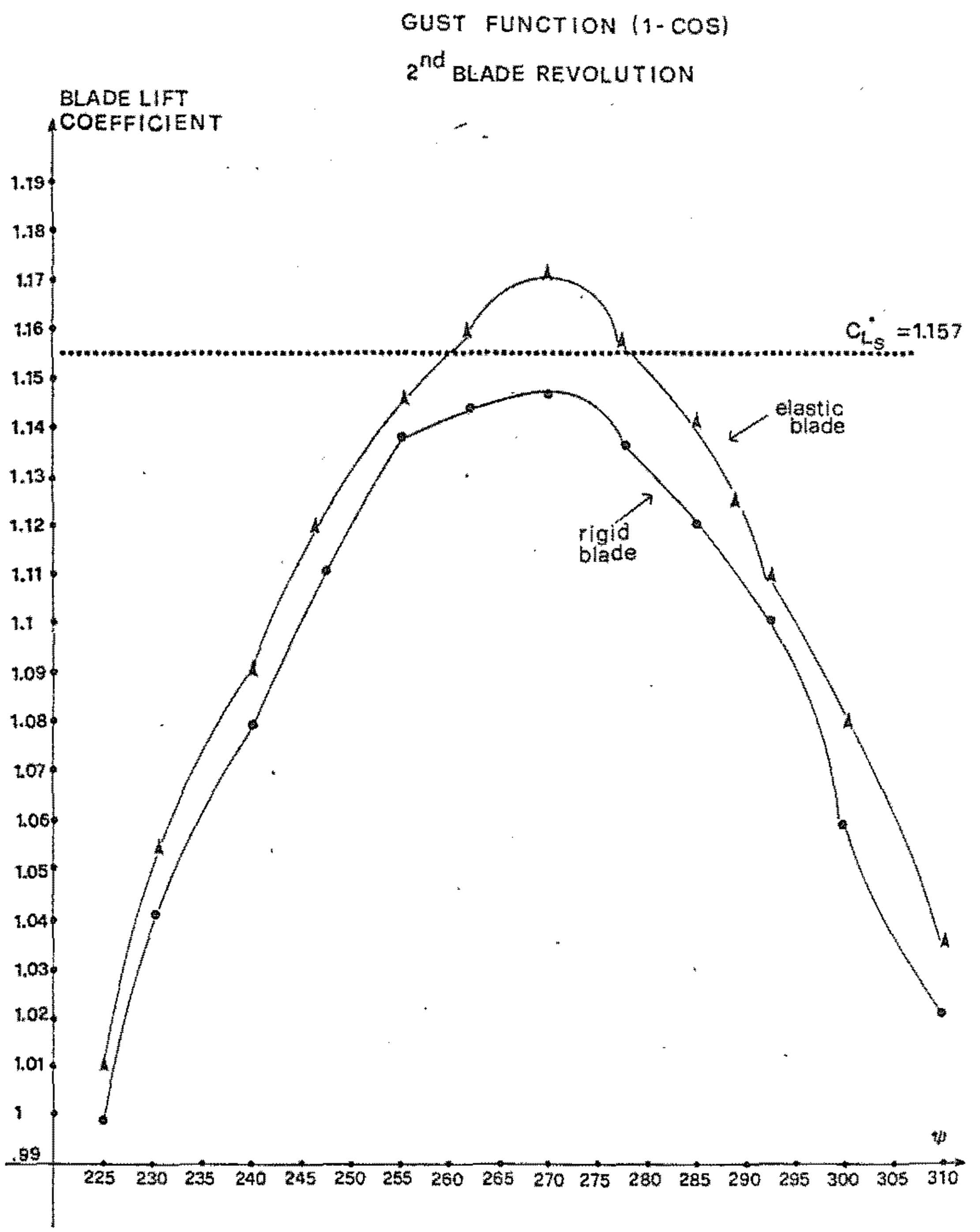
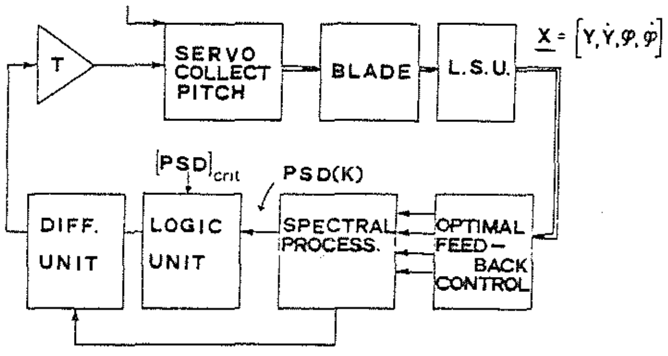

GUST FUNCTION (1-COS)
2nd BLADE REVOLUTION
BLADE LIFT COEFFICIENT
1.19
1.18
1.17
1.16
1.15
1.14
1.13
1.12
1.11
1.1
1.09
1.08
1.07
1.06
1.05
1.04
1.03
1.02
1.01
1
.99
$C^*_{L_S}$ =1.157
elastic blade
rigid blade
225 230 235 240 245 250 255 260 265 270 275 280 285 290 295 300 305 310
ψ

FIG. 8


SERVO COLLECT PITCH
BLADE
L.S.U.
X = [Y, Ẏ, φ, φ̇]
T
[PSD]crit
PSD(K)
DIFF. UNIT
LOGIC UNIT
SPECTRAL PROCESS.
OPTIMAL FEED-BACK CONTROL
PSD(K-1)

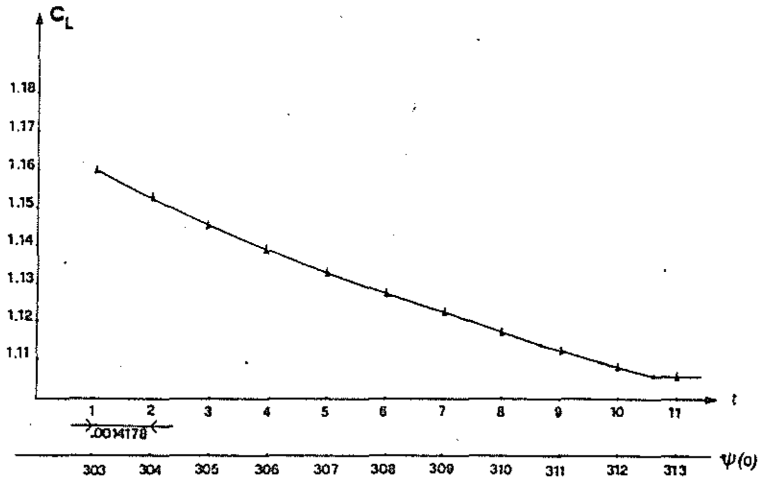

C_L
t
.0014178
ψ(o)

FIG. 7

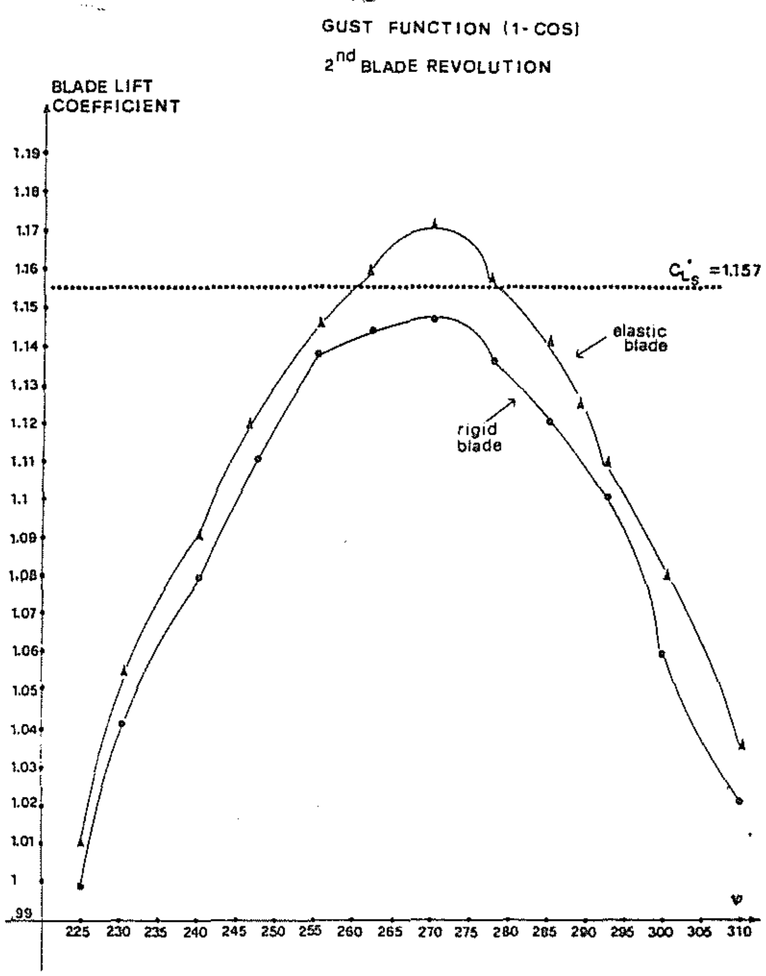

GUST FUNCTION (1-COS)
2nd BLADE REVOLUTION
BLADE LIFT COEFFICIENT
$C^*_{L_S}$ = 1.157
elastic blade
rigid blade
ψ

FIG. 8