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DETERMINATION OF THE STRUCTURAL PROPERTIES OF HELICOPTER ROTOR BLADES
BY THEORETICAL AND EXPERIMENTAL METHODS

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ABSTRACT

The use of a theoretical method for determining equivalent beam exten-
sional, bending and torsional stiffnesses, for helicopter rotor blades, fabri-
cated from anisotropic material with arbitrary cross-sectional geometry is
discussed and results presented. The method, which uses the NASTRAN finite
element package, also determines all stiffness coupling terms and predicts the
position of the elastic axis and shear centre. Experimental data, derived from
simple section beams and model rotor blades manufactured from composite
materials, are used for comparison with the theoretically derived stiffnesses.

An experimental method used for determining the beam bending stiffness,
torsional stiffness and shear centre is presented. This technique uses a number
of ±1g servo accelerometers mounted on the blade and used as inclinometers to
measure angular deflections for known loads. This method enables shear centre
positions to be determined to an accuracy better than 1% of the blade chord and
bending and torsional stiffnesses to within ±2%.

1   INTRODUCTION

The increasing use of composite materials in the design of helicopter
rotor blades has necessitated a reappraisal of the methods used to determine the
blade section properties required for modal calculations, eg shear centre, flap
and lag bending stiffnesses and inertias. Current rotor modes programs in use
at RAE and Westland Helicopters[1], consider the blade to be composed of a number
of straight line segments, following the locus of the local section shear
centres, with the section stiffness properties expressed in terms of the local
EI and GJ together with a tension centre and an extensional stiffness EA. In
the past programs based on simple thin-walled tube theory have been used to
calculate the section properties, but recent experience has shown up deficiencies
in this approach, particularly in the prediction of the section shear centre.

Alternative methods[2,3] based on finite element methods, have been devised.
The former does not take advantage of the 2-dimensional nature of the problem
and so results in a computationally large task whilst the latter uses a
specially written finite element approach. However, the method of Bartholomew
and Mercer[4] used in this paper, takes full advantage of the 2-dimensional form
of the problem and uses the widely available finite element program NASTRAN to
model the blade section. Thus it is possible to model the rotor blade section
in great detail without incurring the penalties mentioned above. A typical
full-scale rotor blade finite element model is shown in Fig 1.

The technique also calculates the structural coupling terms, which are essential for any work involving aeroelastic tailoring. The final equivalent beam properties are represented by a matrix of order 4 with EA, EI_{flap}, EI_{lag}, and GJ as the diagonal elements and the remaining terms are the associated coupling coefficients. From this structural matrix the tension centre can be derived and from an appropriate shear loading case the shear centre can be found.

This paper discusses the basic method of the analysis and presents the results of work done on simple beam models and more complex rotor blade models using the NASTRAN finite element analysis package.

The second part of this paper details an experimental method used to measure the equivalent beam properties of a rotor blade. The method measures the slope of the blade section when subjected to known loads and deduces the blade properties using the Engineers Theory of Bending. A similar approach is used to determine the position of the shear centre. The experimental method uses dc servo-accelerometers with a range of ±1 g and an output which is proportional to acceleration in its sensitive axis. They can be used as inclinometers rather than accelerometers, thus when they are tilted over the output is proportional to the sine of the angle between the base of the device and the horizontal axis. The linearity of the device is typically 0.02% of full-scale deflection and it can measure angular changes of a few seconds of arc with a range of ±90 degrees, making it ideal for measuring the widely differing angular changes associated with flap and lag stiffness measurements.

2   ANISOTROPIC BEAM ANALYSIS

The method used is that of Bartholomew and Mercer[4] and the reader is referred to that report for a full exposition of the theoretical basis of this paper.

The segment of the rotor blade to be analysed is assumed to be straight, untwisted and to have constant cross-sectional geometry and material composition along its length. The blade is defined with respect to a set of right-handed axes (x,y,z) with the z-axis in the spanwise direction and an x-y plane defining a blade cross-section, Fig 2.

The segment of the blade is defined by two identical cross-sections, P and Q, a distance d apart which can be loaded by a longitudinal force N, along the z-axis, flap and lag bending moments M_f and M_1 respectively and a torque T about the z-axis. The loads and strains are related by the global stiffness matrix $[Q]$

where

$$\{N\} = [Q]\{E\} \tag{2-1}$$

with

$$\{N\}^T = (N, M_f, M_1, T) \tag{2-2}$$

and

$$\{E\}^T = (e_z, k_f, k_1, \phi) \tag{2-3}$$

where e_z is the longitudinal strain

k_f and k_l are the flap and lag curvatures

ϕ is the twist per unit length

The 4×4 stiffness matrix $[Q]$ defined by equation (2-1) is dependent upon the cross-sectional geometry and material composition of the blade. The stiffness matrix is, in fact, found by inverting the associated flexibility matrix $[S]$ the columns of which are the strain vectors $\{E\}_i$, for $i = 1$ to 4, resulting from the application of four separate unit loads, $\{N\}_i$.

Thus we have

$$[S] = [E_1, E_2, E_3, E_4] \qquad (2\text{-}4)$$

where $\{E\}_1$ is the response to the load vector $(1,0,0,0)^T$.

$\{E\}_2$ is the response to the load vector $(0,1,0,0)^T$ etc.

Ref 4 shows that it is permissible to model a small segment of the blade, Fig 2, provided that the displacements of corresponding points on the two faces of the segment (P and Q) are constrained to have the form specified by the following equations.

$$u(Q) - u(P) = \mu_1 - \omega_3 y$$

$$v(Q) - v(P) = \mu_2 - \omega_3 x$$

$$w(Q) - w(P) = \mu_3 - \omega_2 x + \omega_1 y \qquad (2\text{-}5)$$

$$r_x(Q) - r_x(P) = \omega_1$$

$$r_y(Q) - r_y(P) = \omega_2$$

$$r_z(Q) - r_z(P) = \omega_3$$

where u, v and w are translations in the x, y and z directions and r_x, r_y and r_z are the rotations about the x, y and z axes.

μ_1, μ_2, μ_3 represent the rigid body translations of the x, y and z axes respectively and ω_1, ω_2 and ω_3 the rigid body rotations about these axes.

These equations represent displacement fields which, in any cross-section, only differ by a rigid body translation or rotation. Poisson contraction effects and warping are allowed but are the same for all cross-sections within the segment.

In practice μ_1 and μ_2 are constrained to zero to fix the overall rigid body rotations, and then the remaining four constants μ_3 and ω_1, ω_2 and ω_3 represent the elastic response of the blade. The response to an applied load is given by

$$\{E\}^T = (e, k_f k_1, \phi) = (\mu_3, \omega_1, \omega_2, \omega_3)/d \tag{2-6}$$

where d is the length of the blade segment.

3   NASTRAN IMPLEMENTATION

3.1   Stiffness matrix

The NASTRAN Displacement Approach is used to model a short segment of the rotor blade in the required detail. The model need only be one element deep in the z direction and the length d should be chosen so that the aspect ratios of the elements are appropriate for the types of element used. Multipoint constraint (MPC) equations are used to specify the relationships defined by equations (2-5) and scalar points (SPOINT) are used for the quantities μ_1 to μ_3 and ω_1 to ω_3. Single point constraints (SPC) are used to constrain the scalar freedoms μ_1 and μ_2 to zero, to remove the rigid body rotations r_x and r_y, whilst retaining the ability to recover the forces associated with the constraints.

The MSC/NASTRAN Bulk Data Deck, for the simple one element model shown in Fig 3, is given in the Appendix A and the salient features will now be described.

GRID cards 1 and 2 define the two gridpoints on face P (z=0) of the 'blade' segment whilst GRID cards 501 and 502 define the corresponding points on face Q (z=d). The 'blade' section is modelled by a single QUAD4 element with the element and material properties specified on the PSHELL and MAT1 cards. The scalar freedoms 1001 to 1006 on the SPOINT cards are μ_1 to μ_3 and ω_1 to ω_3 respectively and the SPC cards remove the rigid-body freedoms. For this simple model 10 MPC equations are required to implement equations (2-5). Unit loads for each of the scalar freedoms 1003 to 1006 are specified on the SLOAD cards and are selected in the SUBCASEs 1 to 4 in the Case Control Deck. Subcases 2 and 3 also request that the grid point forces (GPFORCES) be output; these are used in a subsequent NASTRAN run to calculate the position of the shear centre of the section.

The calculated displacements for the scalar points 1003 to 1006, for a single load case, when divided by the length of the segment (equation (2-6)), give a column of the flexibility matrix $[S]$. The output from the four load cases, when combined (equation (2-4)) and inverted, give the required matrix $[Q]$. A pre-processing FORTRAN program is used at RAE to produce the MPC equations and post-processing programs calculate the stiffness matrix and process the grid point forces to produce the loading condition required for the shear centre calculation.

3.2   Elastic centroid

The elastic centroid of an anisotropic beam may be defined as the point in the cross-section at which a longitudinal tension will not produce flexural or lateral bending. The NASTRAN model of the beam is defined relative to an origin in the basic coordinate system in a way convenient for the analyst. If the origin of this coordinate system is not at the elastic centroid of the section then there will be non-zero coupling terms in the first row (and column) of the stiffness matrix $[Q]$.

The x and y coordinates of the position of the elastic centroid are calculated as follows

$$x_c = - \frac{Q_{31}}{Q_{11}}$$ (3-1)

$$y_c = \frac{Q_{21}}{Q_{11}}$$ (3-2)

where the Q_{ij} are the elements of the matrix $[Q]$.

Ref 4 shows how the stiffness matrix $[Q]$ can be transformed to a coordinate system at the elastic centroid.

3.3   Shear centre

The shear centre of a beam is usually defined as that point in the cross-section through which a shear force may be applied without inducing twisting throughout the beam. For anisotropic beams a bending-twisting coupling would produce varying amounts of twist along the beam and, therefore, there is no point in the cross-section at which shear forces may be applied to produce zero twist at every cross-section. However, the present method can be used to apply a shear force, S , without a bending moment by considering an infinitesimal section of the beam close to the shear force. To do this it is necessary to rerun the NASTRAN model (Appendix B) with two shear loadings, S_x and S_y , obtained from the gridpoint forces output in subcases 2 and 3 of the first NASTRAN analysis. Reversing the sense of the moments and forces on the front face of the model and constraining the scalar freedoms μ_1 and μ_2 (representing the transverse displacements of the two faces) to have zero displacement produces the required loading. Then the coordinates of the shear centre (x_s, y_s), relative to the origin of the model coordinate system, are given by

$$x_s = \frac{GJ\omega_3}{S_y}$$ (3-3)

$$y_s = - \frac{GJ\omega_3}{S_x}$$ (3-4)

3.4   Inertia properties

When applying the prediction method, the NASTRAN Bulk Data PARAM GRDPNT card is used to calculate the mass, centre of gravity and moments of inertia of the blade segment. Both material density and non-structural mass may be specified in NASTRAN and the rigid body mass matrices with respect to the reference point, selected by the PARAM card, and the section principal axes are output.

4   PRELIMINARY ASSESSMENT

4.1   Rectangular section beam

In order to familiarise ourselves with the method several simple beam test cases were analysed using MSC/NASTRAN. The first study was that of a rectangular section isotropic beam which, initially, used one QUAD4 element to model the section (Fig 3). Subsequently, the number of elements in the x-direction was increased in steps, up to a maximum of 8, to compensate for the over-stiff representation in lag arising from the lack of an in-plane rotational freedom in the element formulation. The effect, on the lag stiffness results, of increasing the number of elements is shown in Fig 4. As a rule one element was enough to give good agreement with theory for all but the lag stiffness terms. The results are compared with theory in Table 1a. It is interesting to note that the value of GJ predicted by the QUAD4 element differs from the theoretical value[6] since the QUAD4 element bases its predictions on a large breadth to thickness ratio. In this example the ratio is 10 and a value $0.312Gbt^3$ should be used instead of the thin walled assumption of $0.333Gbt^3$. Usually the QUAD4 element is used to model plates with an overall breadth to thickness ratio greater than 10 and this discrepancy does not arise. However, the need to understand the assumptions inherent in the element formulation, if accurate results are to be obtained, cannot be overemphasised.

This analysis was repeated using the MSC/NASTRAN isoparametric HEXA element. A single element was sufficient to give good agreement with theory for all stiffnesses except GJ. Increasing the number of elements in the x-direction improved the correlation and the convergence to the theoretical result is shown in Fig 5. The results for the 8-element model are given in Table 1b.

4.2   Channel section

The shear centre prediction capability of the method was assessed using a MSC/NASTRAN QUAD4 model of the channel section beam shown in Fig 6. The results obtained are presented in Table 1b and show excellent agreement with the exact solution.

5   BASIS OF MEASUREMENT METHOD

5.1   Use of accelerometers as inclinometers

The experimental techniques described in this paper use a number of dc servo accelerometers to measure angular changes as loads are applied to the blade. Dc accelerations are measured in their sensitive axis which is parallel to the base of the device. When tilted there is a component of acceleration due to gravity in the sensitive direction which is proportional to the sine of the angle between the base of the device and the horizontal axis. Thus the accelerometer can be used as an inclinometer giving a voltage output which is proportional to the sine of the angle between the base of the device and the horizontal axis.

5.2   Flap and lag stiffness

The calculation of flap stiffness, EI_f , and lag stiffness, EI_l , is derived from the formula for the angular deflection of a built-in cantilever beam of constant cross-section under a point load.

$$EI = \frac{Pl^2}{2(\theta_0 - \theta_i)}$$

EI is the stiffness of the beam, in Nm^2 ;
P is the applied load, in newtons;
l is the distance between the inboard inclinometer and
   the point where the load is applied, in metres;
θ_0 and θ_i are the change in angle as measured by the
   outboard and inboard inclinometers, in radians.

In the technique described in section 6 the angular deflection of the blade is measured at two points using two inclinometers attached to the blade.

The change in angle as measured by the inclinometers, assuming the angle is small, is calculated by

$$\theta = VK \text{ radians} \tag{5-2}$$

where V is the change in output, in millivolts;
       K is the calibration factor, in radians/millivolt.

5.3 Torsional stiffness

The calculation of torsional stiffness, GJ, is derived from the formula for the twisting of beams with a uniform cross-section subject to torsional loading.

$$GJ = \frac{Pml}{\theta_0 - \theta_i} \tag{5-3}$$

where m is the distance between the forward and rearward loading points, in metres.

In the technique described in section 6 the load is applied at two points on the blade chord and the angular defection measured at two points on the blade.

5.4 Shear centre

The shear centre is essentially a point in the structure where a shear load will produce only bending and no twisting of the structure. For our purposes we are primarily interested in the horizontal position of the shear centre in the blade rather than its vertical position. Thus it is sufficient to say that the shear centre is a point on the chord where a flap load can be applied without causing a change in angle of the blade at that point.

$$x_{sc} = x_{fwd} + (x_{rwd} - x_{fwd}) \frac{(\theta_{fwd} - \theta_{sc})}{(\theta_{fwd} - \theta_{rwd})} \tag{5-4}$$

x_{sc} is the position of the shear centre behind the leading edge, in metres;

x_{fwd} and x_{rwd} are the distances behind the leading edge of the forward and rearward point of load application, in metres;

θ_{sc} is the measured pitch angle without any load, in radians;

θ_{fwd} and θ_{rwd} are the measured pitch angles with the load at the forward and rearward points, in radians;

In the technique described in section 6 of the load is applied at two points on the chord and the pitch angle of the blade measured using either one or two inclinometers.

If there are two inclinometers then the required pitch angles are obtained by subtracting the outboard and inboard values as follows

$$\theta_{fwd} = (\theta_{fwd})_0 - (\theta_{fwd})_i$$

$$\theta_{sc} = (\theta_{sc})_0 - (\theta_{sc})_i \qquad (5-5)$$

$$\theta_{rwd} = (\theta_{rwd})_0 - (\theta_{rwd})_i$$

6   MEASUREMENT TECHNIQUES

This section outlines the techniques[9] used to measure the three stiffnesses and shear centre position of the blade. Much of the procedure is common to all the forms of testing and must be carried out diligently to ensure the accuracy of the measured values.

The blade needs to be held rigidly at one end with the axis of the blade horizontal. There are a number of ways of doing this, as shown in Fig 8, each of which has been tried with equal success. Short test specimens of model scale rotor blades can be held by a block of resin moulded onto one end. This can then be used to clamp the blade to a rigid object such as a jig block. Complete blades can be held using the built in root fitting which would normally transmit the flight loads to the inboard hub components, or alternatively the blade could be held by two wooden clamps which fit around the blade section near to the root end.

The inclinometers need to be mounted firmly on the blade as shown in Fig 9, so that the angular deflection of the blade is measured accurately. This is most conveniently done by using double sided tape, or if the surface is highly cambered a block of plasticine can be used. Whatever method is chosen it is important to ensure that the inclinometer is close to horizontal (the unloaded output should be less than 2 mV), and that the sensitive axis is parallel to the leading edge for the flap and lag stiffness measurements, and perpendicular to the leading edge for the torsional stiffness and shear centre measurements. This is especially important for the shear centre measurements, and it may be necessary to use an alignment jig to ensure that the error due to misalignment is kept to an absolute minimum.

A loading fitting is required (a suitable design is shown in Fig 10) to transfer the applied load to the blade. The applied load can be provided by suitable weights on a hanger that can be suspended from the loading fitting at

the appropriate points (see Figs 11 and 12). A combination of weight and moment arm should be chosen to give adequate output from the inclinometers without losing accuracy due to large couplings between flap and torsion deflections. Ideally the applied loads should produce a deflection of one to two degrees over the measured length. However, with full-scale blades it is often difficult to achieve these deflections (being limited to deflections in the range 0.5 to 1.0 degrees) and this can lead to inaccuracies.

7   COMPARISON BETWEEN PREDICTED AND MEASURED BLADE PROPERTIES

7.1   General

The predicted and measured blade properties obtained using the methods described in this paper, have been compared for a model rotor blade and a full-sized rotor blade. The measurements on the latter were carried out at Westland Helicopters under contract.

7.2   RAE model rotor blade

The finite element method was used to analyse the dynamically scaled model rotor blades used on the RAE Helicopter Division rotor test rig[7]. The blade cross-section and the associated MSC/NASTRAN finite element model are illustrated in Fig 7. The foam core of the blade provides the required aerodynamic shape whilst the composite skin and spars provide the required dynamic properties. The layers of composite material were represented by QUAD4 elements with the details of the composite layers specified on the PCOMP Property Card and MAT8 Material Card. The foam core and leading-edge weight were modelled using HEXA solid elements.

The results obtained for this blade section are compared with measured data in Table 2 and show good correlation with experiment.

7.3   Full-size blade

The analysis method has been applied to a full-size blade using a finite element model developed at WHL for previous work[2]. This NASTRAN model has been converted to the required format using a FORTRAN pre-processing program[8] to generate the MPC equations. Some refinement of the NASTRAN model was required, in particular, reducing the aspect ratios of the solid elements at the blade trailing edge eliminated some off-diagonal stiffness terms that had arisen due to numerical ill-conditioning and resulted in a 3% change in the calculated value of GJ. The calculated stiffnesses and the position of the shear centre are compared with measured values in Table 3. These measurements were made in the early stages of the development of the test method and highlighted the need to improve the technique. As a result of this reappraisal formulae were derived to calculate typical values for the length of the segment of blade to be tested and the magnitude of the loads and couples to be applied. Following these guidelines will ensure that adequate deflections are obtained during the test and consequently provide more reliable measured data. Further tests on the blade are planned and it is hoped that the correlation between measurement and theory will improve.

8   CONCLUSIONS

A method for predicting rotor blade structural properties has been developed which should be capable of giving results to a high degree of accuracy. Comparisons with simple exact solutions show very good agreement.

In order to provide measured data for comparison with predicted values, a new method has been devised for measuring rotor blade structural properties.

Comparisons between the predicted and measured properties of a model rotor blade show very close agreement.

Initial comparisons between the predicted and measured properties of a full-sized rotor blade show fair agreement but highlight the need to produce adequate deflections in the blade to improve the accuracy of the measuring technique.

The prediction method gives sensible values for the full-sized blade properties, but more work is needed to determine the best idealisation of the finite elements forming the blade cross-section.

This work should lead to a much improved capability for evaluating blade properties for use in rotor loads prediction methods.
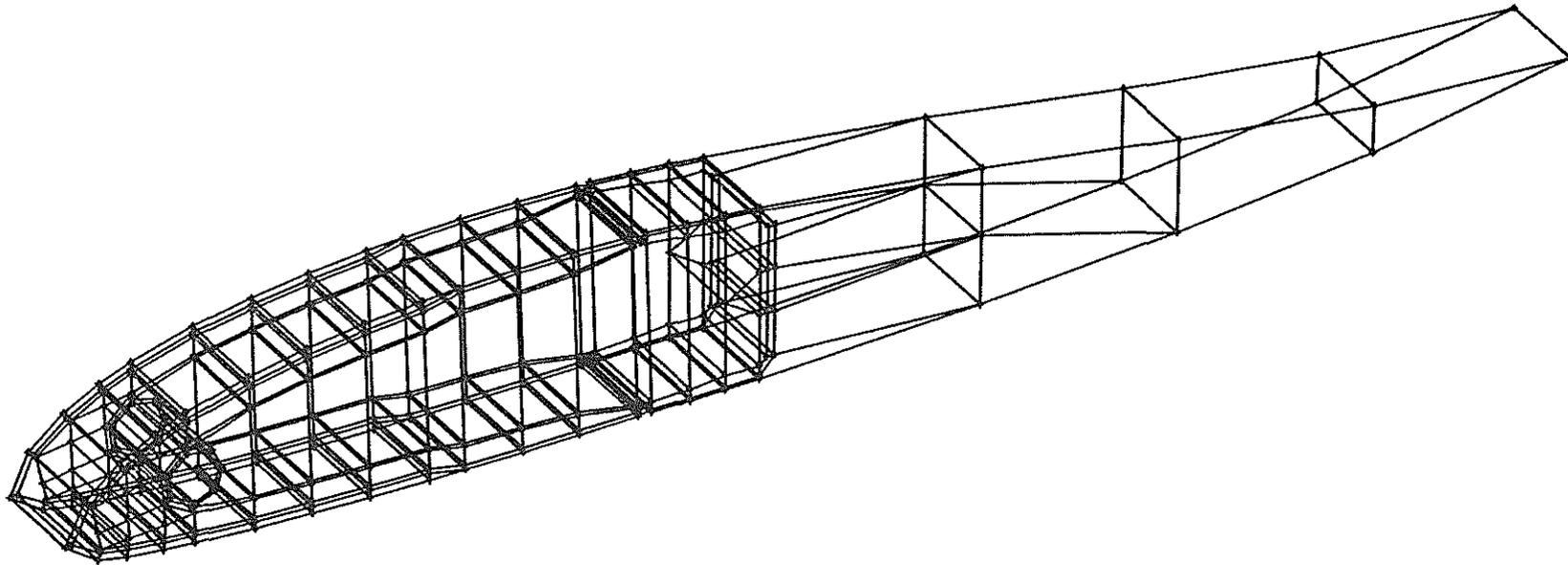
Fig 1   Finite element model of a full scale rotor blade
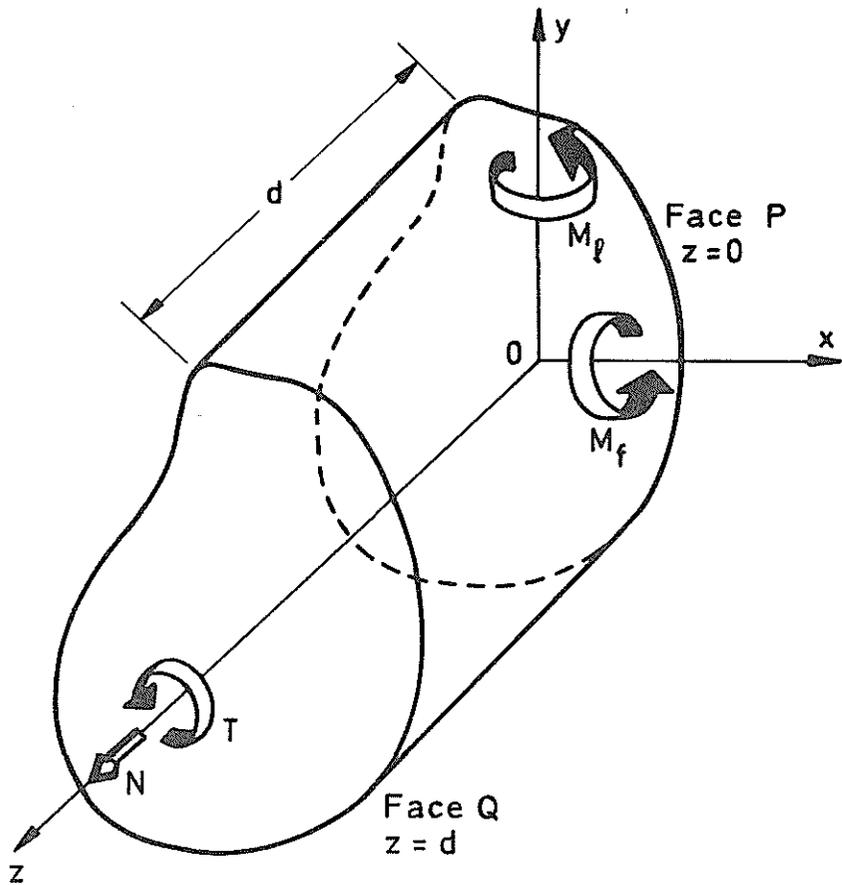
Fig 2   Elemental section of beam
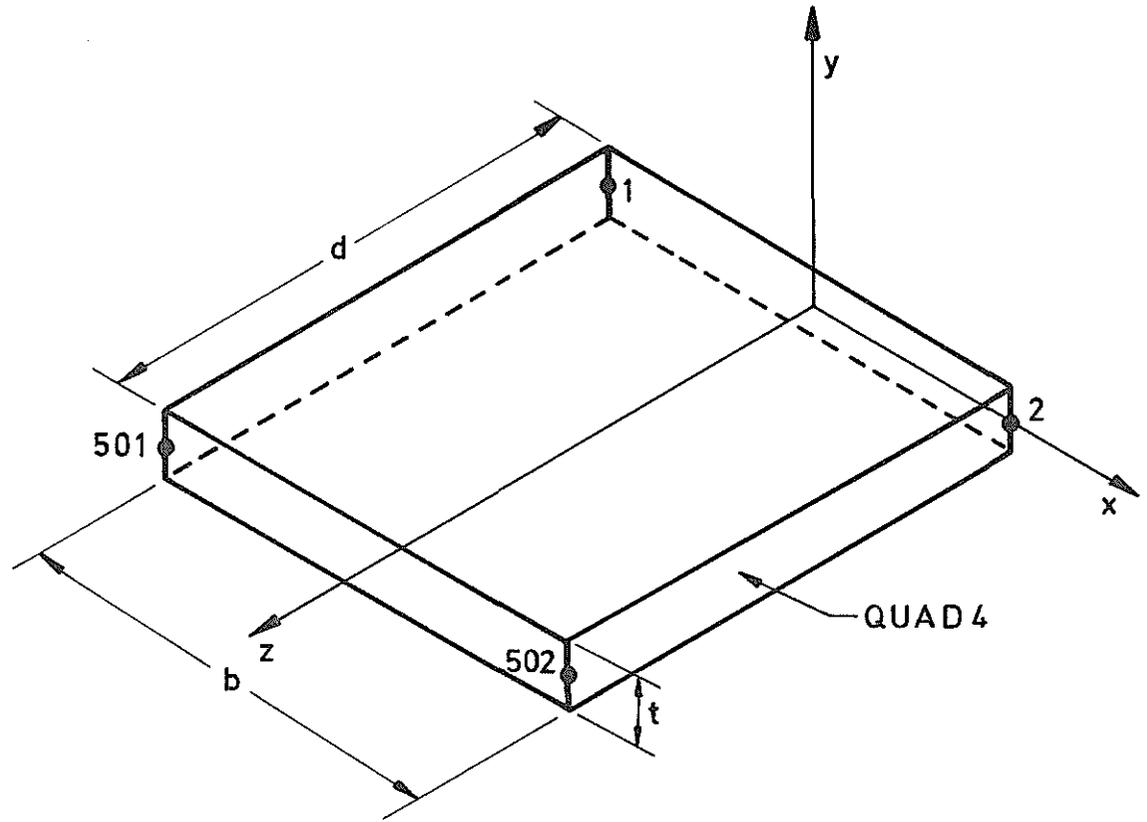
Fig 3   Nastran model of rectangular section (QUAD4 element)
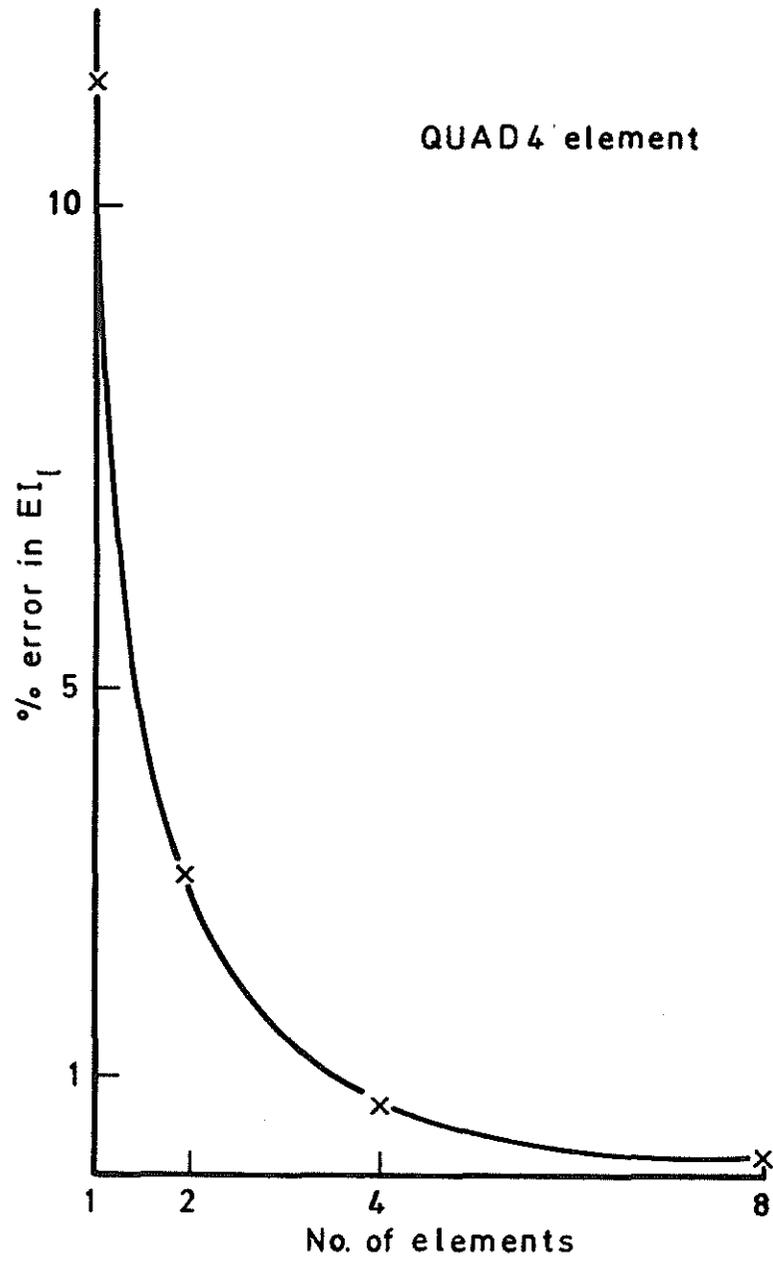
Fig 4   Rectangular section: convergence
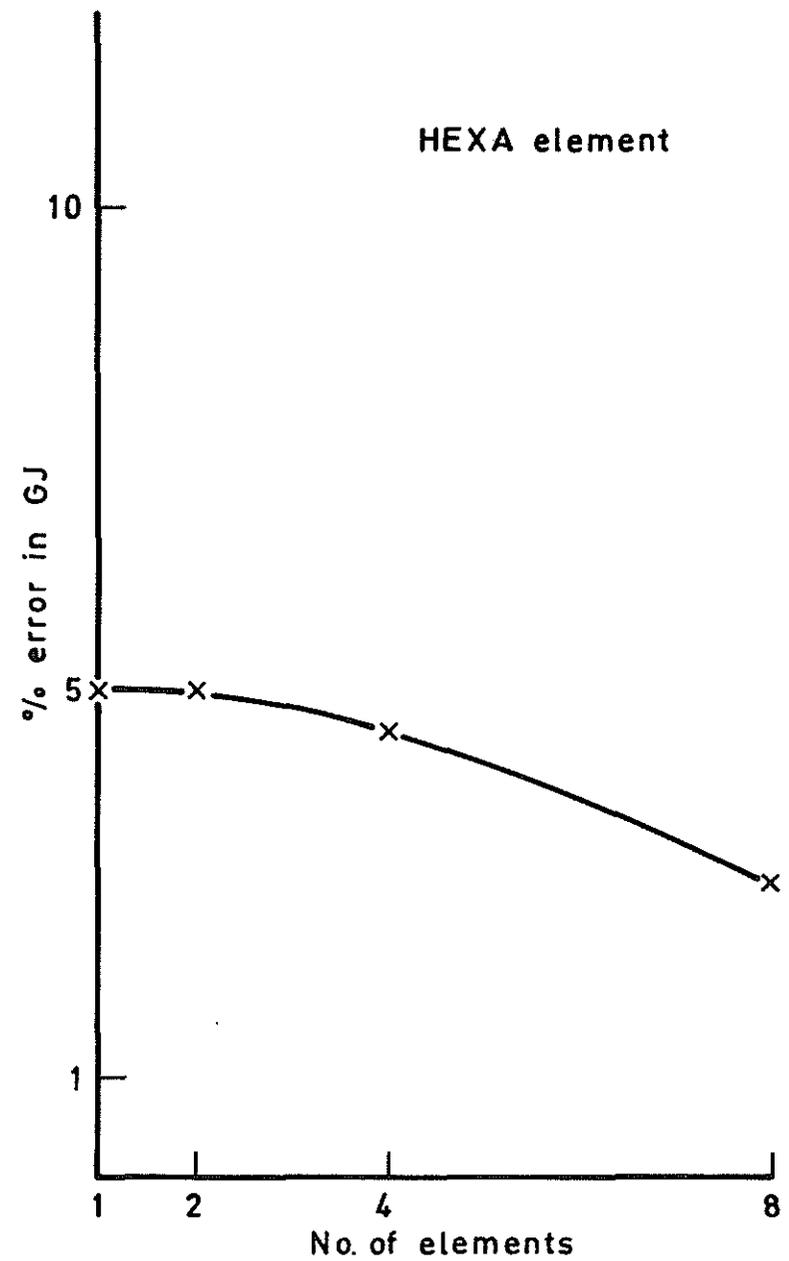
Fig 5   Rectangular section: convergence

Fig 6   Channel section

Lead leading edge weight

CFRP spar

Increasing number of CFRP layers towards leading edge

ROHACELL 110 foam

ROHACELL 51 foam

CFRP skins

Skins and spar → plate elements
Foam → solid elements
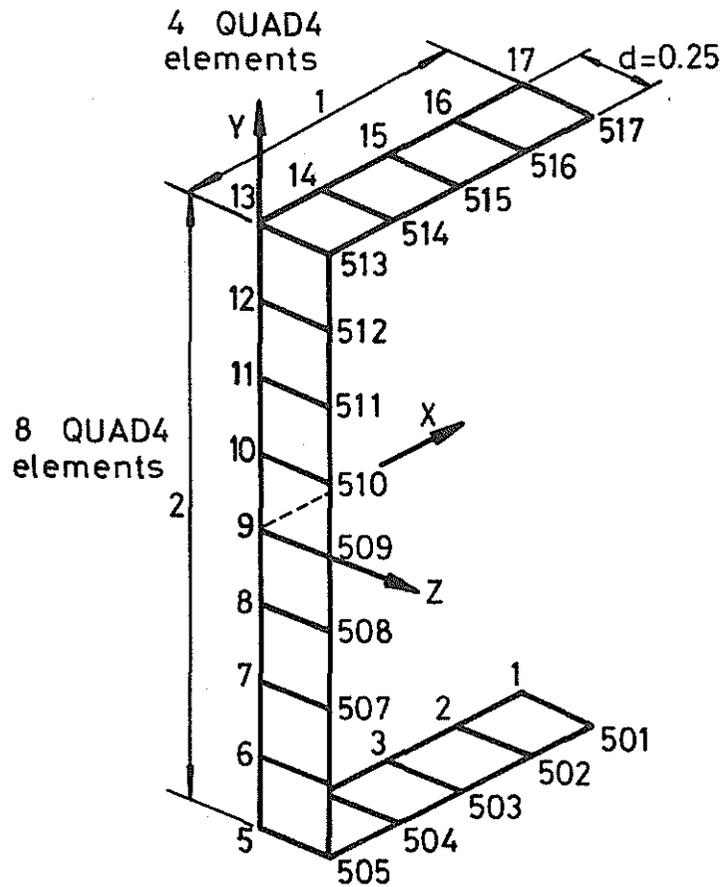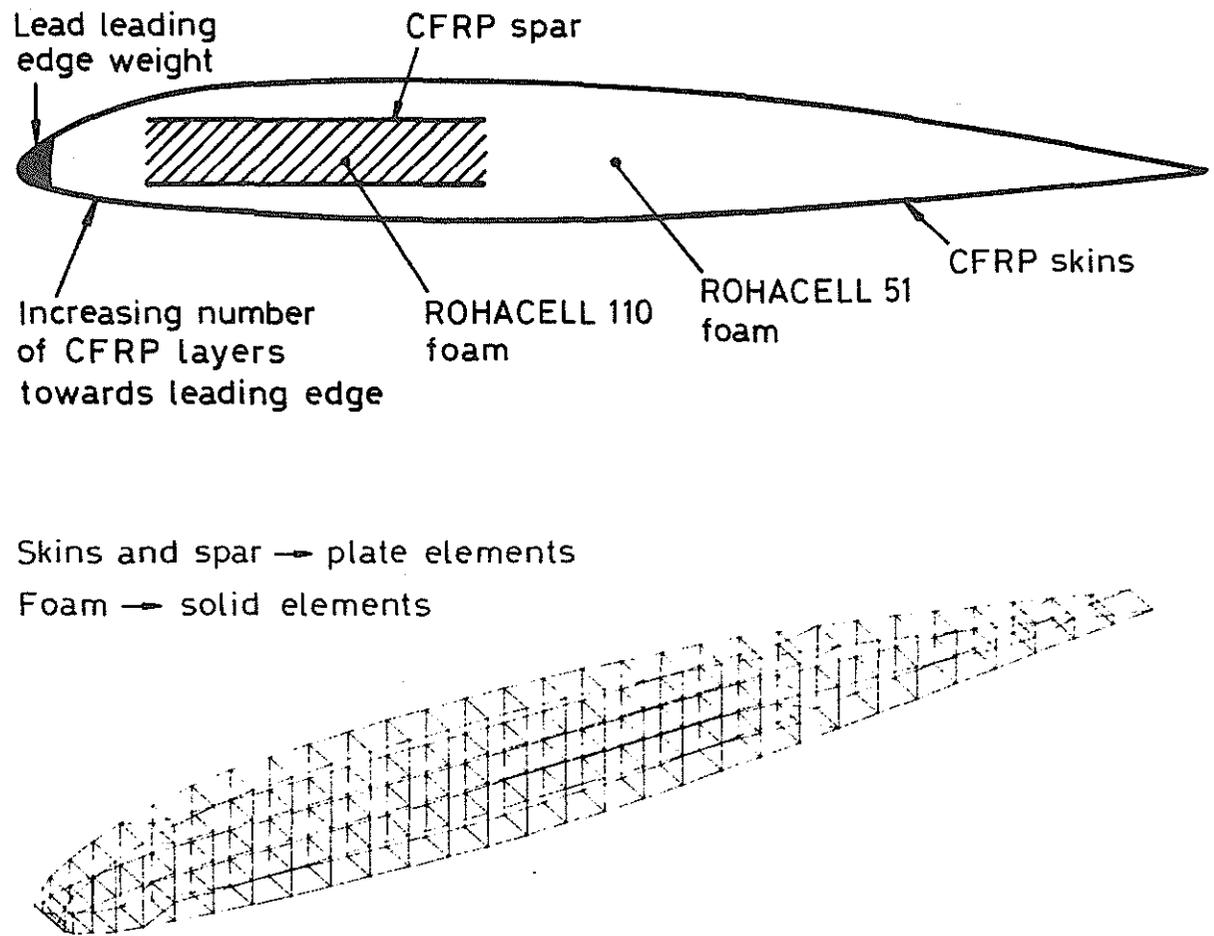
Fig 7   RAE blade and finite element model

Moulded resin block   Test specimen

Built-in root fitting   Blade

Aerofoil section clamps   Blade
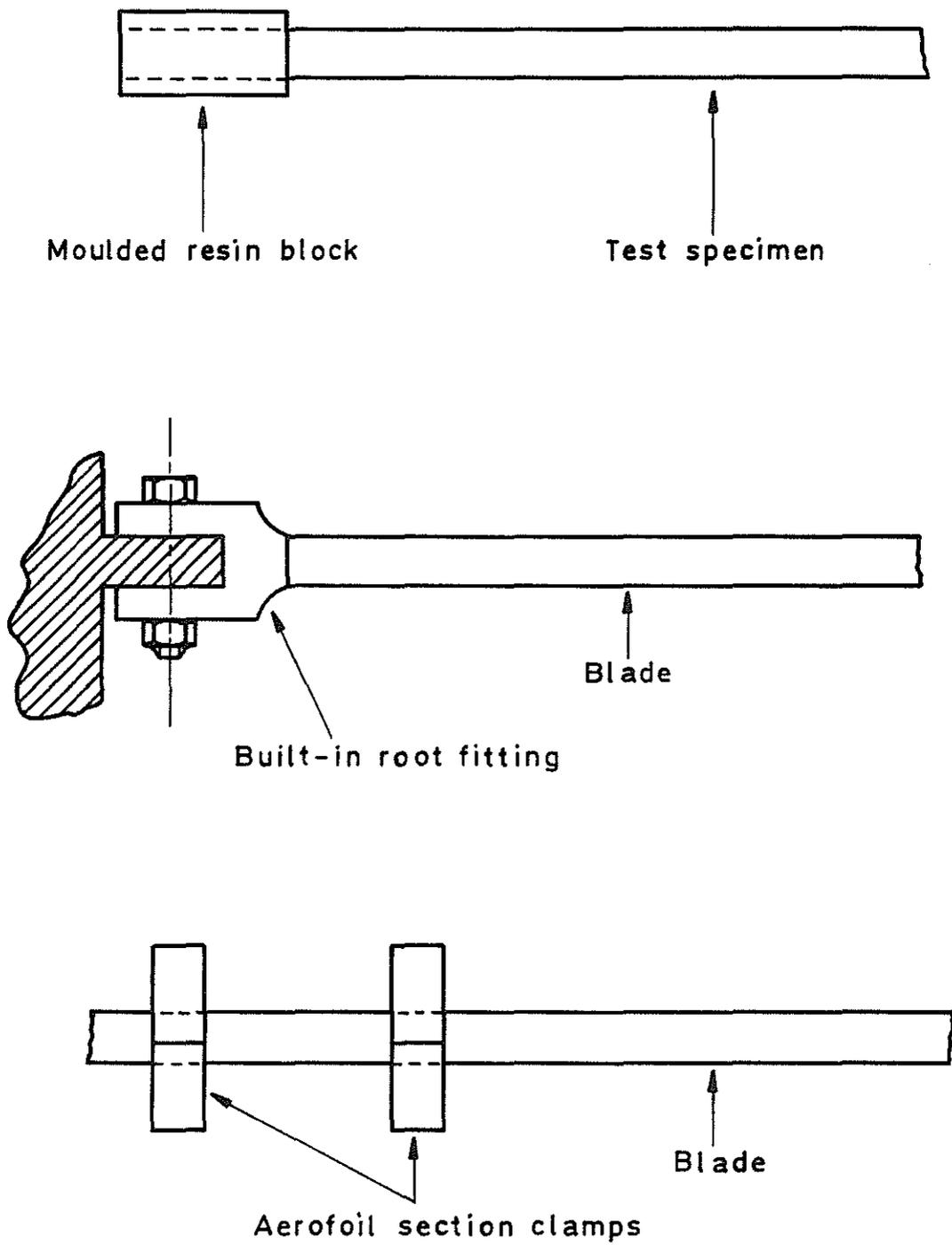
Fig 8   Blade mounting configurations

Double sided tape

Accelerometer

Plasticine
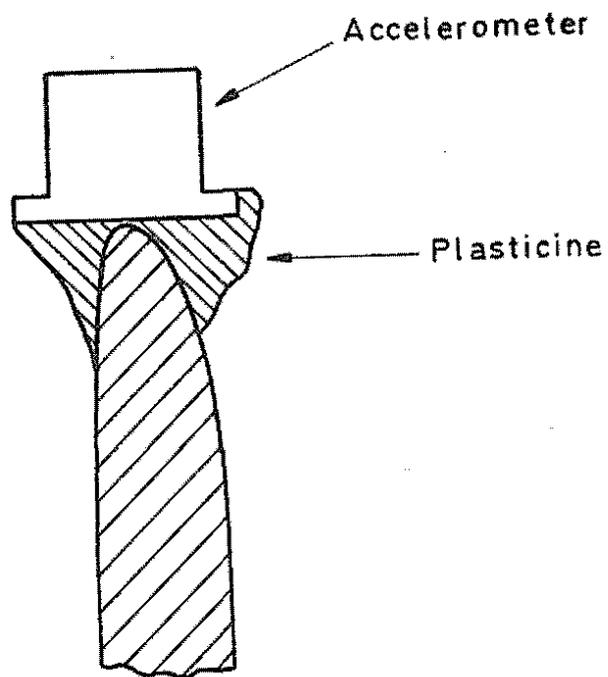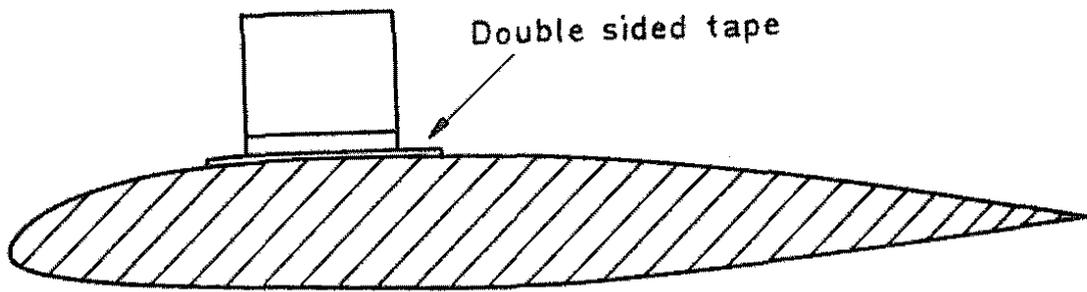
Fig 9　Mounting of inclinometers

Tapped holes — 'A' for flap stiffness measurements

'B' for torsional stiffness measurements

'C' for lag stiffness measurements

Fig 10   Loading fitting

Sensitive
axis

Sensitive
axis

Up to
50mm

End view

Up to
50mm

Fig 11   Lag stiffness measurement

Up to
50 mm

Up to
50 mm

Sensitive
axis

1/2 moment arm

Fig 12   Torsional stiffness measurement