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PRELIMINARY STUDY OF MAN-MACHINE PROBLEMS IN HOVERING ABOVE A MOVING PLATFORM

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Preliminary Study of Man-Machine Problems In Hovering Above A Moving Platform

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bstract

The manual low-altitude hovering task, above a moving landing deck of small ship, is very demanding in particular in adverse weather and sea onditions. In this paper, the pilot-helicopter interaction, during this overing task is studied. The purpose of this investigation is to obtain nsight into the problem and to determine which parameters should be visually resented to the pilot, by means of display augmentation, in order to improve he task performance.

A simplified one-dimensional manual hovering task above a moving latform is investigated analytically, using a paper pilot model, based on ptimal control theory. The analytical results are validated in a fixed ase simulation program. The results indicate that the one-dimensional task an be performed satisfactorily in case display augmentation providing absolute osition or velocity information, is used.

The results of the one-dimensional hovering task form the basis for ollow-up research in which all vehicle and ship motions are integrated.

. Introduction

The pilot-helicopter interaction in the very low-altitude hovering ask above a moving platform by visual cues only, is at present not clearly nderstood. An example of such a task is the helicopter approach-to-landing n a small moving ship-deck. The task of the pilot, prior to touch-down, is o keep the helicopter inertially stable above the deck, at a desired inertial eight, until the appropriate moment for touch-down arrives [1],[2].

Low altitude hovering missions are usually performed manually, while e pilot's main source of information is the visual scene. The ship-deck otions are a result of the response of the ship to the sea waves. Since the uman Operator's (HO) visual references are relative to the moving deck rather an to inertial space, his observations (or measurements) are incomplete. oreover, the helicopter is subjected to atmospheric disturbances which onstitute a high frequency forcing function, responsible for the main part the pilot's activity. Since the landing area of small ships is relatively nall, hovering above the deck has to be carried out with great accuracy. or these reasons the hovering task above the moving deck of a small ship is ery difficult and in many cases is not performed satisfactorily.

The subject of this research is to investigate the pilot-helicopter nteraction in the low altitude hovering task above a moving platform, by isual cues only, and to indicate which parameters should be presented to e HO by means of display augmentation, in order to improve the task performance.

In this research the following factors have to be considered: elicopter, Pilot, Atmospheric Disturbances, Ship and Wave Motions.

Technical problems, concerning the helicopter landing on ships, are reviewed in Refs. [1]-[4]. Analytical models of man-machine systems are classified in the literature in two groups. The classical control engineering approach (McRuer et al. [5]-[6]) indicates which state variables are required from control theoretical point of view, without studying how these variables are perceived. The modern optimal control engineering approach (Kleinman et al. [7]-[12]), states that the well-trained, well-motivated HO behaves almost like an optimal controller, subject to his inherent limitations and to the control task. In this model the sources of HO remnant are specified as observation and motor noises. Observation noise accounts for the uncertainty in the perception of parameters, while motor noise accounts for the random errors in executing the intended control movements. The Optimal Control Model (OCM) is used both for instrument display control models [12]-[17] and for visual field control models [19]-[22].

In this paper the results of a preliminary study of the problem are presented. This study deals with a simplified one-dimensional hovering task where both helicopter and ship-deck motions are assumed to exist in the vertical axis (heave) only.

The investigation is carried out according to the following steps:

a) Development of an analytical model of the pilot's hovering task incorporated in an optimal control framework. By using this model the adaptive properties of the HO, as a result of system variations, can be studied. The model can then be used for the prediction of the performance of the HO and of the system.

b) A parametric study of the model to determine the sensitivity of the model output and system performance to changes in parameters.

c) Fixed-base laboratory simulations to determine unknown parameters of the analytical model and to validate the model over a broad range of parameters.

d) Investigation of display augmentation to improve the system performance.

2. The One-Dimensional Analytical Model

The one-dimensional hovering situation is described in Fig. 1. The pilot's task is to keep the helicopter at a desired inertial height, such that a sudden collision between the ship and the helicopter is avoided. The task is performed in the presence of vertical atmospheric disturbances and vertical motions of the ship deck. The desired hovering altitude is presented to the pilot by a horizontal bar attached to the mast of the ship. The bar moves together with the ship deck, thus the pilot can predict the deck motions by looking at the bar. If the position of the ship was inertial stationary the position of the bar would be exactly at the desired hovering height. Since the ship is in motion, the momentary position of the bar does not indicate the exact desired height, but in the long term, this height can be derived from the average position of the bar.

In the one-dimensional task, the pilot controls the helicopter by using the collective control only. The atmospheric disturbances are assumed to influence the vertical motion of the helicopter only. Therefore only the heave motion of the helicopter is assumed to be present. The ship deck motion is also assumed to exist in the vertical axis only, without pitch motions.

2.A. The Dynamic Model

The dynamic model of the system consists of the helicopter's linearized equations of motion and equations representing the ship deck motion and the atmospheric disturbances. This model is represented in Laplace form in Fig. 2. The equations of motion of the helicopter represent the motion in the vertical axis in response to the collective control input commands and to the atmospheric disturbances. The atmospheric disturbances are described by a first-order Markov process. The ship deck motion is described by a second order Markov process. Summarizing the above mentioned assumptions the dynamic model can be described by the following state equation:

$$\dot{\underline{x}} = A\underline{x} + \underline{B}u + \underline{w} \tag{1}$$

Equation (1) is elaborated upon in the Appendix by Eq. (A.4). The state vector is defined as $\underline{x} = \mathrm{col}(Z_h, V_h, Z_s, V_s, u_g)$, where:

Z_h - helicopter vertical displacement

V_h - helicopter vertical velocity

Z_h - ship vertical displacement

V_s - ship vertical velocity

u_g - atmospheric disturbances

$\underline{x}$ - state vector

A - state matrix

$\underline{B}$ - control vector

u - collective control input command

$\underline{w}$ - disturbance vector with covariance matrix W

2.B. The Measurement Model

As mentioned previously, the HO's measurements are relative to the moving deck rather than to inertial space. Therefore during the hovering task the pilot perceives visual information of the position of the helicopter and its velocity relative to the horizontal bar. The observation vector can be described as a linear combination of the state vector by the equation:

$$\underline{y}(t) = C\,\underline{x}(t) + \underline{v}_y(t) \tag{2}$$

where:

$\underline{y}(t)$ - vector of observed variables

C - observations matrix

$\underline{v}_y(t)$ - vector of observation noise components which are assumed to be zero-mean Gaussian white noise processes with covariance matrix V.

When using the relative measurements only, the system $[A,C]$ is not observable and the state cannot be reconstructed by the estimator of the optimal model, to be described in Section 2.C. Thus the vector of observed variables has to be augmented with additional inertial information. This additional information should either be absolute vertical position, vertical velocity or vertical acceleration. Since a fixed base simulator is used in the validation experiments, the acceleration is not included in the measurement model. In the absence of absolute positional information, the additional information is assumed to be the absolute velocity of the helicopter which is perceived to some extent from the image of the sea, perceived by the peripheral vision. Summarizing, the measurements which are perceived by the HO are:

V_h - helicopter vertical velocity

$Z_h - Z_s$ - helicopter vertical position relative to the ship

$V_h - V_s$ - helicopter vertical velocity relative to the ship

Thus, the observation vector is given by:

$$\underline{y} = \mathrm{col}(V_h, Z_h - Z_s, V_h - V_s)$$

It is assumed [7] that the HO perceives measurements which are delayed by τ seconds. Thus the noisy and delayed observation vector, y_p, perceived by the HO, is given by:

$$\underline{y}_p(t) = C\,\underline{x}(t-\tau) + \underline{v}_y(t-\tau) \tag{3}$$

Eq. (3) is elaborated upon in the Appendix by Eq. (A.5).

2.C. The Optimal Control Model

A block diagram of the Optimal Control Model is shown in Fig. 3. A basic assumption of the OCM is that the HO has perfect knowledge of the vehicle and the measurement models. The HO action is accomplished in two parts:

1. Optimal reconstruction of the state from the noisy delayed vector of the observed variables. This is done by means of Kalman filtering and optimal prediction.
2. Determination of the control function u, such that in the steady state the following cost function is minimized:

$$J(u) = \lim_{t_f \to \infty} E\{\frac{1}{t_f}\int_o^{t_f} [\underline{x}^T Q \underline{x} + ru^2 + g\dot{u}^2]dt\} \tag{4}$$

Q is the weighting matrix of $\underline{x}$, while r and g are the weighting coefficients of u and $\dot{u}$, respectively.

Since the Kalman filter has a perfect knowledge of the optimal control, u^* a perfect estimate of the state is obtained by simply integrating the state equations. On the other hand, it is clear that in reality the HO's estimate cannot be perfect. Therefore, in order to prevent the Kalman filter from knowing the control perfectly, HO motor noise is added to the control force u_c, commanded to the neuromuscular system. This motor noise accounts for the fact that the HO does not know his control action precisely.

The inclusion of the control rate in the cost function is mathematical equivalent to the first order neuromuscular dynamics in the transfer function of the HO [7].

The solution of the reconstruction and control problem are derived from Kleinman et al. [7,9,12].

3. Parametric Study of the Analytical Model

A parametric study of the one-dimensional analytical model has been conducted in the time domain. In this study the performance of the HO and of the system is computed in the form of covariances of state and control variables. The purpose of this parametric study is to evaluate unknown model parameters by matching the analytical output of the model with the corresponding experimental data.

In this parametric study the covariances of the position Z_h, the velocity V_u of the helicopter and the control force u, have been evaluated for different values of the following variables:

a) Weighting coefficients $[q_1, q_2, q_3, q_4, q_5] \overset{\Delta}{=} \text{diag } Q$, r and g.

b) Time delay τ and neuromuscular time constant τ_N.

c) Covariances of the observation noises V_y and motor noise V_m, which have been shown by Levinson et al. [11], to be proportional to the covariances of $\underline{y}$ and u_c, respectively. Noise levels are defined as follows:

$$V_{y_i}[\text{db}] = 10 \log[V_{y_i}/\pi E\{y_i^2\}] \quad ; \quad i = 1,2,3$$

$$V_m[\text{db}] = 10 \log[V_m/\pi E\{u_c^2\}]$$

For the parametric study the following nominal values have been chosen:

$$q_1 = 1 \; ; \; q_2 = 0 \; ; \; q_3 = 0 \; ; \; q_4 = 0 \; ; \; q_5 = 0$$

$$r = 1 \; ; \; \tau = 0.1 \text{ sec} \; : \; \tau_N = 0.2 \text{ sec} \; ; \; V_m = -25\text{db}$$

$$V_{y_i} = -20 \text{ db} \quad ; \quad i = 1,2,3$$

The various dependencies of the Z_h, V_h and u covariances upon the above parameters are depicted in Figs. 4÷9. Parameter deviations from their nominal values are indicated in the figures.

3.A. The Effect of the Weighting Coefficients Q and r.

Figure 4 shows the influence of the weighting coefficient q_1 (weighting of the position of the helicopter). For values of $q_1 < 2.5$ the position error (cov{Z_h}) decreases. This is a result of increasing the control activity (cov{u}), which results in an increasing velocity error (cov{V_h}). For values of $q_1 > 2.5$, the position and velocity errors are almost constant.

Figure 5 shows the influence of the weighting coefficient q_1 (associated with the velocity of the helicopter). Increasing q_1 causes a decrease in the velocity error and in the control activity. On the other hand, the position error is increased.

It should be noted that it is not possible to reduce the position error by simultaneously increasing or decreasing q_1 and q_2.

Figure 6 shows that r does not affect the position and velocity errors significantly. However, increasing r causes a decrease in the control activity. This is due to the HO's effort to perform the control task, with minimum control input commands.

3.B. Effect of Variations in τ_N and τ

In the OCM solution a relation has been found between the neuromuscul time constant τ_N and the control rate weighting coefficient g. Therefore onl τ_N is varied and g is adjusted accordingly. Fig. 6 shows an increase in the position and velocity errors, as a result of increasing τ_N while at the same time the control activity is decreased.

Figure 7 shows a progressive increase in the covariances of the position velocity and control activity with increasing time delay.

3.C. Effect of Observation and Motor Noises

It can be seen from Figs. 8 and 9 that the system performance deteriorates with increasing observation and motor noise levels. This deterioration is due mainly to the observation noise level of the absolute velocity V_{y_1}, while V_{y_2}, V_{y_3}, i.e. the relative position and velocity noise levels and the motor noise V_m affect the performance much less.

4. Fixed Base Simulation Program

4.A. Experimental Set-Up

A block diagram of the fixed base simulation set-up is shown in Fig. 10. The HO's control commands are translated into analog voltages and are adjusted and scaled by an EAI-580 hybrid-analog computer. The analog voltages are converted to digital signals and imparted to a Data General Eclipse Mini Computer which is programmed to solve the vehicle equations of motion and which performs the necessary graphics calculations to create, in real-time, a perspective image of the ship and of the sea. This image is displayed on a cathode-ray tube (CRT) display and viewed by the HO and thus used to create the control commands which, on their turn, close the control loop.

4.B. The Experimental Program

The experimental program started after a training period of four weeks of the subject. During this training period the subject became familiarized with the CRT image and, in the absence of acceleration feelings, with the motions of the ship and of the helicopter. The subject was instructed to keep the helicopter at a constant inertial height above the ship deck.

Three major experiments were conducted during the experimental programs:

a) The HO perceived the observations from the basic display configuratio described in Fig. 11a. In this configuration only the horizontal bar was shown with no further display aids. The absolute helicopter's velocity was perceived by peripheral vision of the sea image.

b) The basic display configuration was augmented by presenting to the HO absolute vertical velocity information using a vertical velocity bar, positioned in the center of the display (see Fig. 11b). Vertica motion of the velocity bar indicated the vertical helicopter motion. Like in the basic configuration the measurements relative t the ship deck were obtained by looking at the first horizontal bar.

c) A similar augmentation as in the second experiment was used but the displacements of the bar indicated deviations from the desired altitud (position bar).

For each one of these three experiments six runs of 128 seconds duration each were performed and the time histories of state and control variables were recorded.

The time histories of Z_h, Z_s, u and u_s for a typical run of each one of the three experiments are shown in Fig. 12. The differences in system and HO performance between the configurations are mainly noticed in the time histories of positions Z_h and of the control activity u. From Fig. 12a it is obvious that without display augmentation relatively large helicopter displacements from the desired altitude are obtained. Moreover, a weak correlation is apparent between the helicopter and ship deck displacements, indicating the influence of the ship deck motion on the HO observations and consequently on his control performance. In Fig. 12b it can be seen that using velocity display augmentation the HO performs his task much better. Helicopter displacements are much smaller compared to the first experiment and the combined effort is considerably reduced as well. A small downwards drift in the Z_h average is noticed which can be explained by the fact that the HO's attention is devoted most of the time to the velocity bar and less to the helicopter position. In Fig. 12c finally, the time histories of the third experiment are presented. It can be seen that the use of display augmentation of a position bar yields the smallest deviations from the desired hovering altitude. On the other hand the control effort in this case is considerably larger.

5. Matching the Model Outputs to the Experimental Results

In this matching procedure unknown parameters of the analytical model are adjusted such that the model outputs, i.e. the covariances of state and control variables, match with their corresponding experimental values. The unknown parameters are the weightings Q,r, the covariances of observation and motor noises, V_y and V_m, the time delay τ and the neuromuscular time constant τ_N.

The matching procedure was carried out as follows. First the variances of the state and control variables were computed from the recorded experimental data. Then the mean and standard deviations of the variances, for the six runs of each experiment, were computed. Second, the parameters of the analytical model, which was implemented on an IBM-370 digital computer, were adjusted until the covariances of the state and control variables matched with their corresponding experimental values.

The parameters resulting from this matching procedure are given in Table 1 and the model outputs are shown in Fig. 13. From the matching procedure the following can be concluded:

a) A very good match is established between the analytical and experimental results.

b) In the first experiment, in the absence of display augmentation, the observation noise levels were found to be relatively high. The observation noise related to the absolute velocity was found to be high in particular. From this fact it can be concluded that the HO's absolute measurements, using mostly moving visual references, were inaccurate.

c) In the second and third experiments, where display augmentation was used, the observation noise levels were considerably lower and the performance was accordingly much better.

d) In the second experiment the weighting coefficient of the velocit is lower than the weighting coefficient of the helicopter positio $(q_2/q_1 = 5)$. On the other hand, in the third experiment the posi is weighted more than the velocity $(q_1/q_2 = 25)$. Comparing these cases, it is concluded that the well perceived observations are weighted more than the less accurate ones.

6. Conclusions

The feasibility of the analytical man-machine model for studying t one-dimensional, low-altitude hovering above a moving platform has been demonstrated.

The model is shown to have sufficient sensitivity to relevant syste parameters.

A very good agreement between analytical and experimental results has been obtained.

The model matching to experimental results enabled determination of unknown parameters with sufficient accuracy.

Improvement in the task performance has been obtained by using displa augmentation of the absolute velocity or the absolute position of the helicopter.

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Appendix: Mathematical Formulation of the Optimal Control Model

The helicopter dynamics are given by:

$$\dot{z}_h = V_h \tag{A.1.a}$$

$$\dot{V}_h = z_w V_h + z_u (u + u_g) \tag{A.1.b}$$

where z_w and z_u are the stability derivatives relating the vertical force to vertical velocity and control commands, respectively.

The ship deck motion is described by:

$$\dot{Z}_s = V_s \qquad \text{(A.2.a)}$$

$$\dot{V}_s = -\omega_s^2 Z_s - 2\xi_s \omega_s V_s + w_s \qquad \text{(A.2.b)}$$

where ω_s and ξ_s are the frequency and damping coefficient of the ship deck motion. They both depend on the sea condition. The worse the sea condition the smaller ω_s and ξ_s. w_s is assumed to be a Gaussian, zero mean white noise with covariance W_s. In this study $\omega_s = 0.7[\frac{\text{rad}}{\text{sec}}]$ and $\xi_s = 0.05$. The atmospheric disturbances are described by:

$$\dot{u}_g = \alpha u_g + w_g \qquad \text{(A.3)}$$

Where w_g is assumed to be a Gaussian, zero mean white noise with covariance w_g. The break frequency was chosen $\alpha = 2$[rad/sec]. The helicopter stability derivatives are derived from [23] and the covariances of the white noises have been chosen to produce a standard deviation of the helicopter velocity of 3[ft/sec] and ship-deck standard deviation displacement of 2.5[ft]. Summarizing the equations (A.1) to (A.3), the following matrix equation is obtained:

$$\dot{\underline{x}} = \begin{bmatrix} \dot{Z}_h \\ \dot{V}_n \\ \dot{Z}_s \\ \dot{V}_s \\ \dot{u}_g \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & z_w & 0 & 0 & z_u \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -\omega_s^2 & -2\xi_s\omega_s & 0 \\ 0 & 0 & 0 & 0 & \alpha \end{bmatrix} \begin{bmatrix} Z_h \\ V_n \\ Z_s \\ V_s \\ u_g \end{bmatrix} + \begin{bmatrix} 0 \\ z_u \\ 0 \\ 0 \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ 0 \\ w_s \\ w_g \end{bmatrix} \qquad \text{(A.4)}$$

where the noise vector $\underline{w}(t) = \text{col}[0,0,0,w_s,w_g]$, are white noise processes with covariance matrix, $\underline{W}$.

The detailed measurement (observation) equation is given by:

$$\underline{y}_p(t) = \begin{bmatrix} V_h(t) \\ Z_h(t)-Z_s(t) \\ V_h(t)-V_s(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \end{bmatrix} \underline{x}(t-\tau)+\underline{v}_y(t-\tau) \qquad \text{(A.5)}$$

The measurement noise vector $\underline{v}_y = \text{col}[v_{y_1}, v_{y_2}, v_{y_3}]$ is assumed to be Gaussian, zero mean white noise as well with covariance matrix V_y.

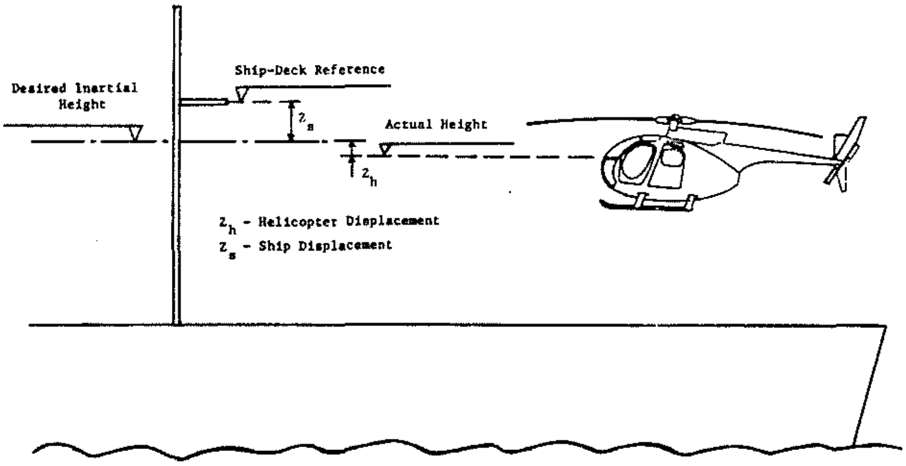

Figure 1: The one-dimensional hovering task.

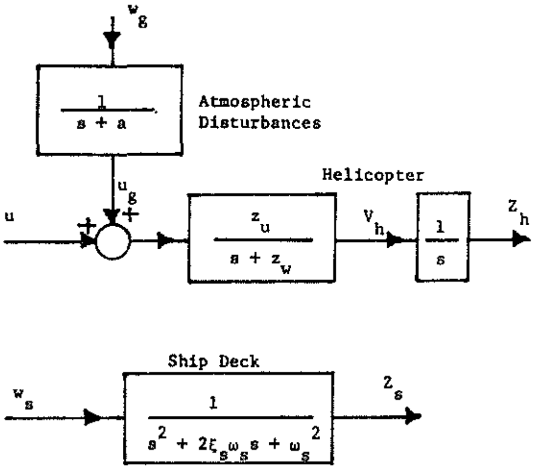

Figure 2: The helicopter atmospheric disturbance model and ship motion model.

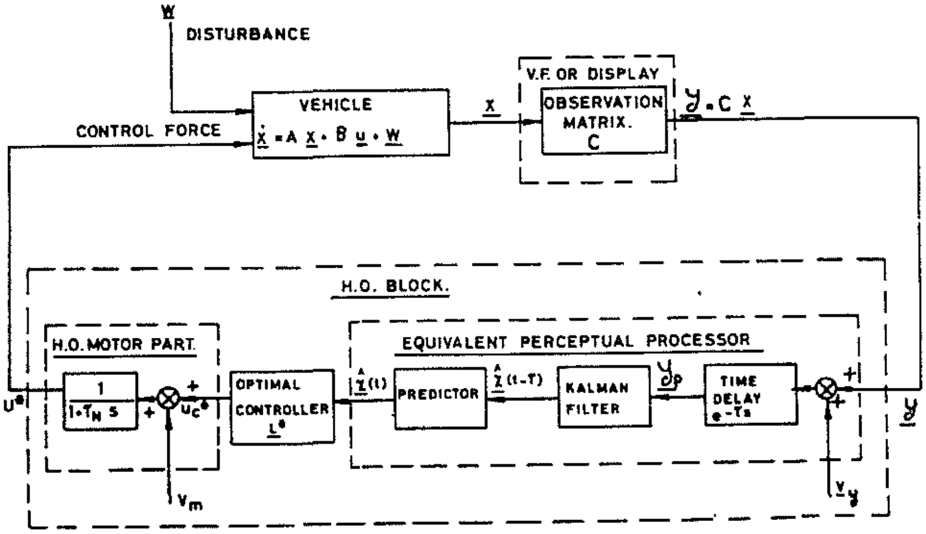

Figure 3: The optimal control model.

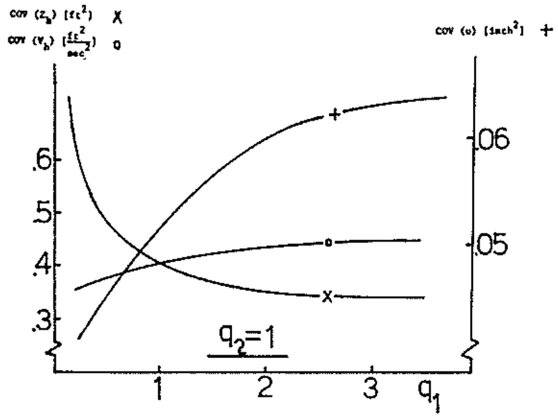

Figure 4: The effect of q_1 on $Cov(Z_h)$, $Cov(V_h)$ and $Cov(u)$.

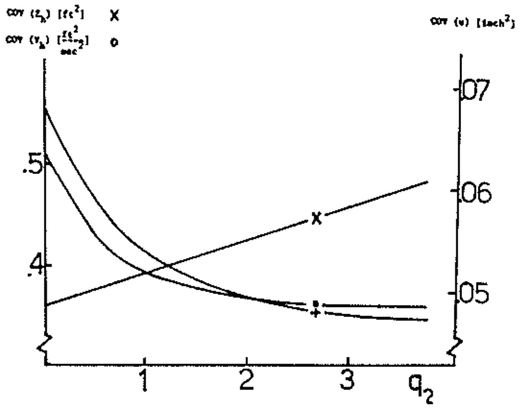

Figure 5: The effect of q_2 on $Cov(Z_h)$, $Cov(V_h)$ and $Cov(u)$.

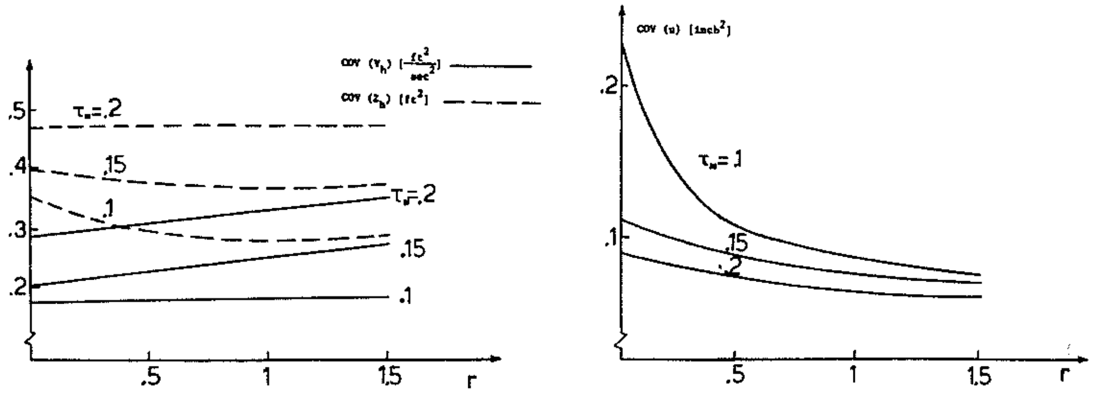

Figure 6: The effect of r and τ_N on $Cov(Z_h)$, $Cov(V_h)$ and $Cov(u)$.

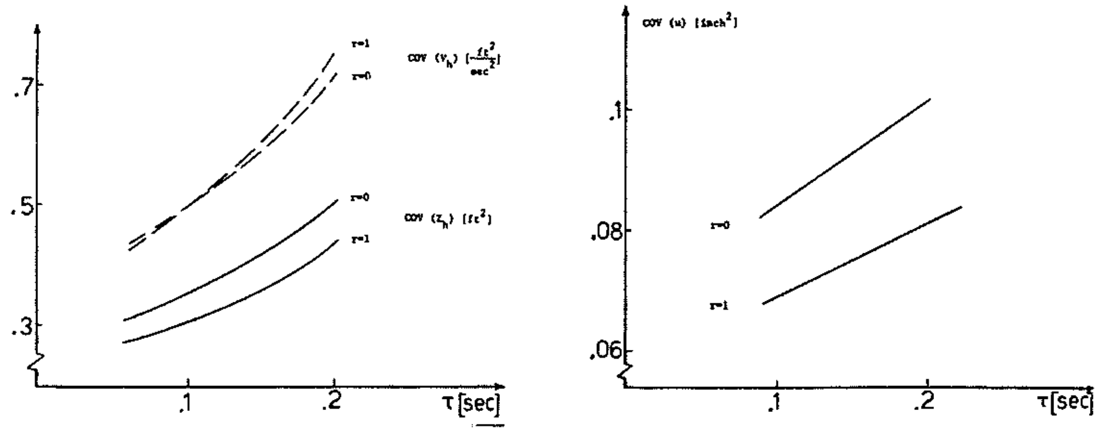

Figure 7: The effect of τ and r on Cov(Z_h), Cov(V_h) and Cov(u).

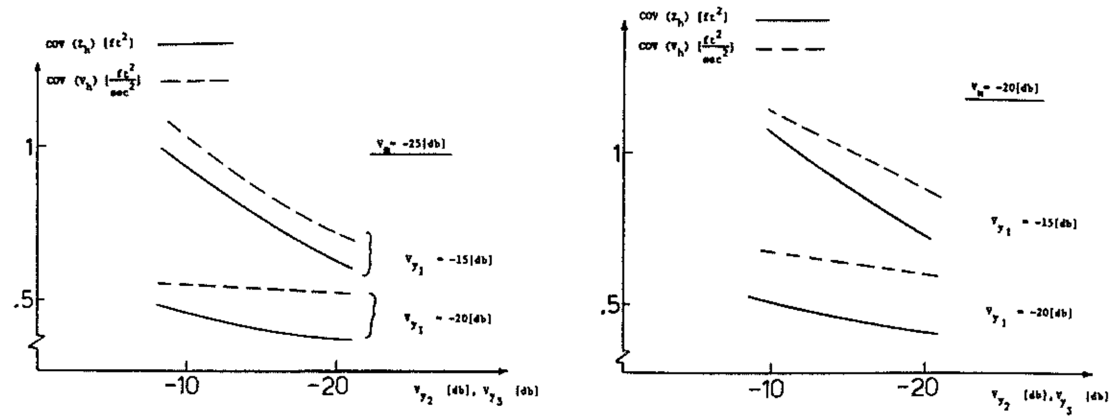

Figure 8: The effect of observation and motor noise on Cov(Z_h) and Cov (V_h).

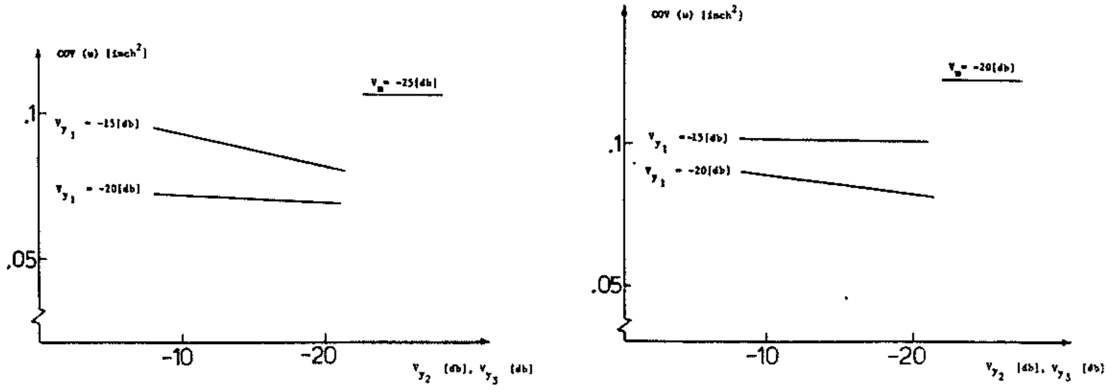

Figure 9: The effect of observation and motor noise on Cov(u).

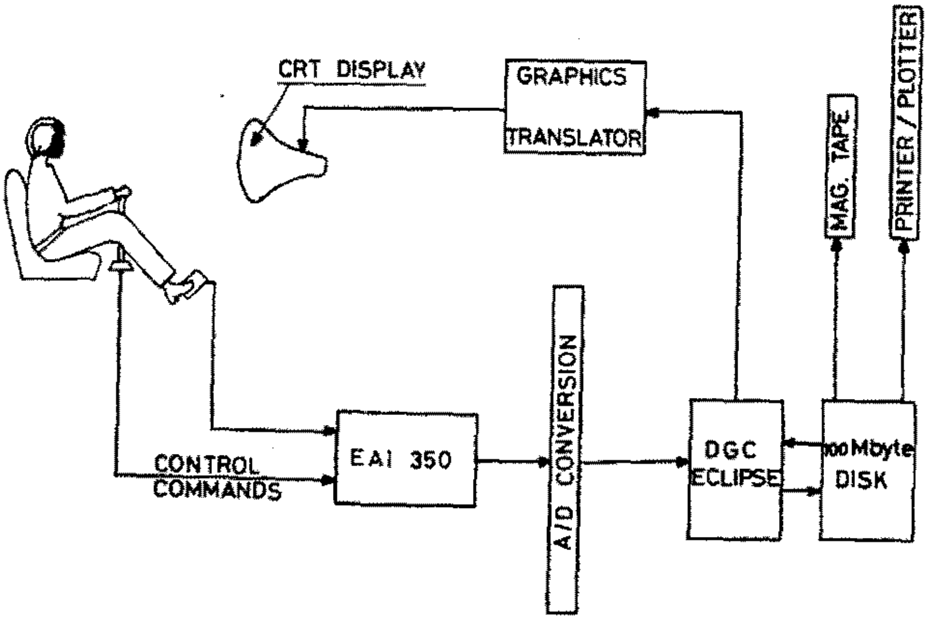

Figure 10: The experimental set-up.

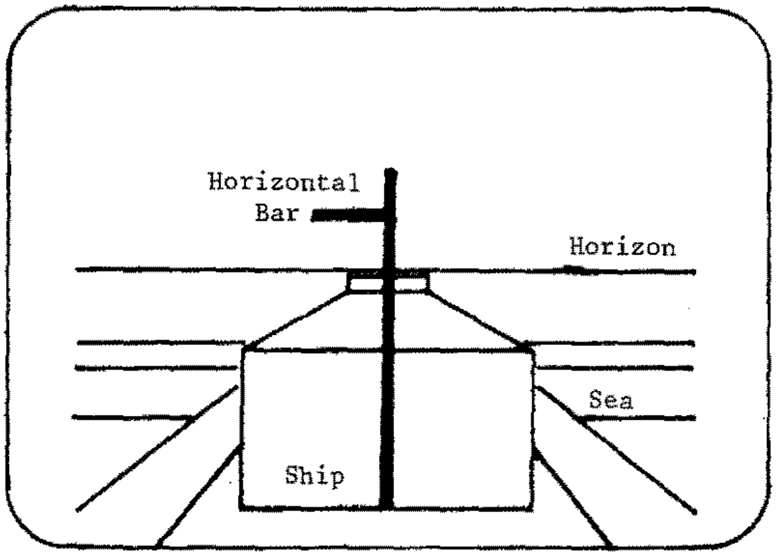

Figure 11a: The basic display configuration.

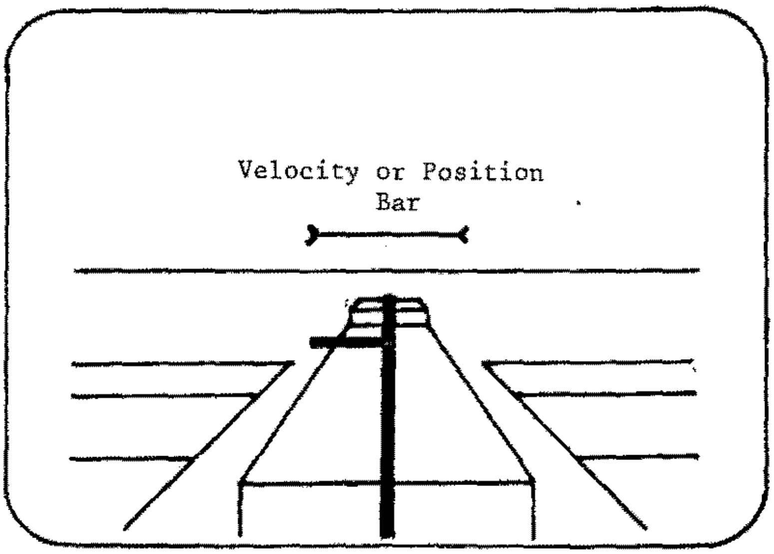

Figure 11b: The augmented display configuration.

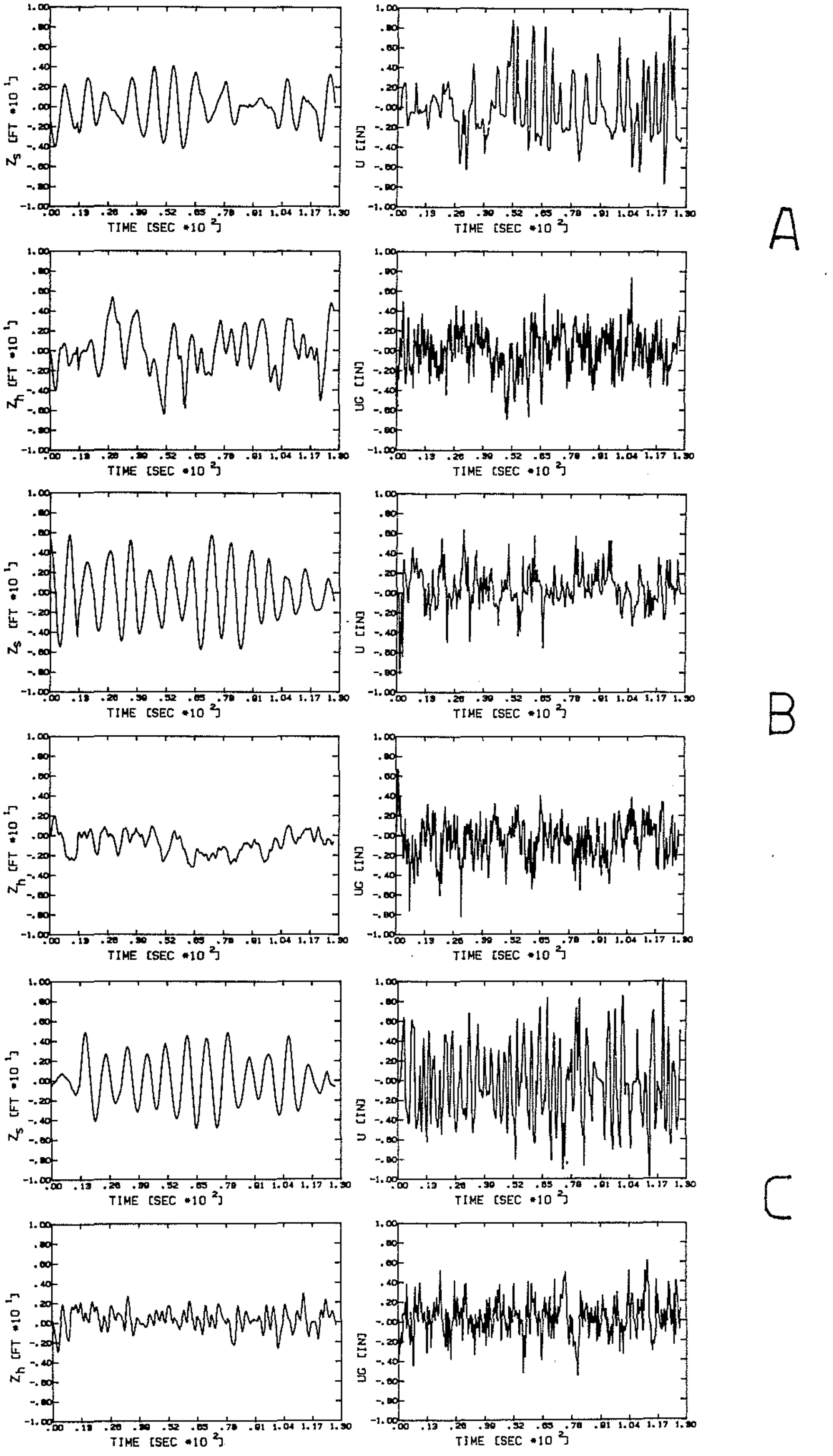

Figure 12: The time histories of Z_h, Z_s, u and u_g.

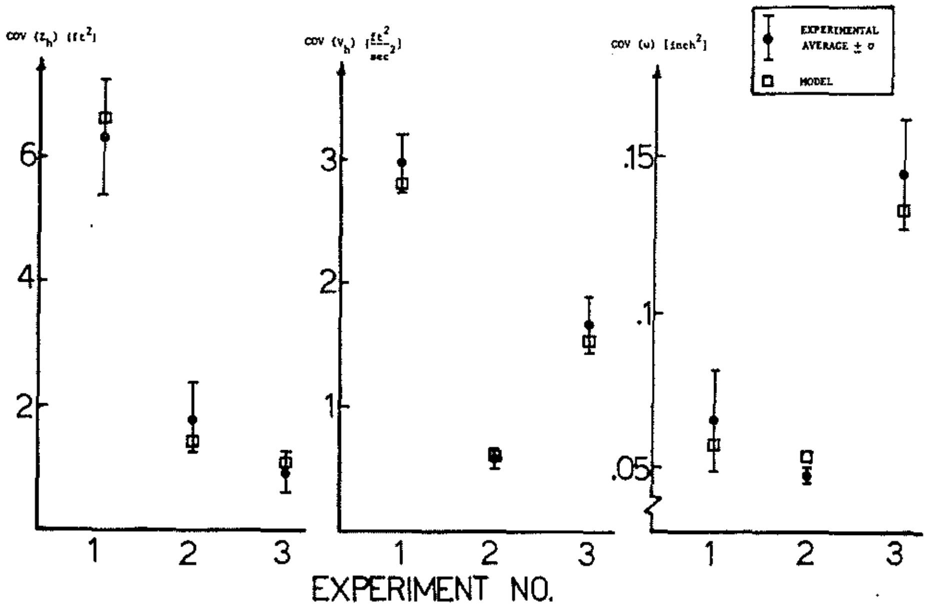

Figure 13: The experimental and analytical matching.

Experiment	Display Augmentation	Measurement Model	V_y[db]	V_m[db]	τ[sec]	τ_N[sec]	q_1	q_2	r
1	None	$y_1 = V_h$	-7	-25	0.1	0.2	0.01	0.	0.5
		$y_2 = Z_h - Z_s$	-10						
		$y_3 = V_h - V_s$	-10						
2	Absolute Velocity	$y_1 = V_h$	-16.5	-25	0.1	0.2	0.04	0.2	1.
		$y_2 = Z_h - Z_s$	-15						
		$y_3 = V_h - V_s$	-15						
3	Absolute Position	$y_1 = Z_h$	-15	-25	0.1	0.2	1.	0.04	0.3
		$y_2 = V_h$	-10						
		$y_3 = Z_h - Z_s$	-13						
		$y_4 = V_h - V_s$	-13						

Table 1: The parameters resulting from the matching procedure.