TWENTYFIFTH EUROPEAN ROTORCRAFT FORUM

Paper n° G22

COAXIAL HELICOPTER ROTOR
DESIGN & AEROMECHANICS

BY

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SEPTEMBER 14 – 16 , 1999
R O M E
ITALY

Introduction

The advanced technology is based on mathematical simulation (software), on a model testing, on a flight tests and on an experience of helicopter development, on the new developed materials.

The Fig.1A & Fig.1B show basic parameters of coaxial KAMOV's helicopters.

1. Coaxial rotor mathematical simulation & physical experiments

The survey of the theoretical and experimental coaxial rotor aerodynamic research published by Coleman, C.P. [4] and by Bourtsev, B.N. [9,10].

The paper [9,10] presents the results of figure of merit analysis for single and coaxial main rotors at hover as well as for a helicopter with a tail rotor and a coaxial helicopter. The analysis has been performed using a simple physical model based on the results of numerical simulation, wind tunnel tests and full scale flight tests.

It is demonstrated [9,10] that a characteristic feature of coaxial main rotor is a high aerodynamic perfection at hover caused by an additional amount of air being sucked in by the lower main rotor (Fig.2). The coaxial rotor at hover demonstrates a 13% larger figure of merit value in comparison with a single rotor unbalanced by torque (Fig.2). Absence of tail rotor power losses provides a 20% larger figure of merit for a coaxial helicopter as a rotorcraft (Fig.2, Fig.4). Fig.3 presents the coaxial rotor figure of merit. These results were obtained by measuring in full scale flight test at hover.

Full-scale flight investigation of a Ka-32 coaxial helicopter tip vortex wake structure was successfully completed [5,6]. A smoke visualization method was applied using blades smoke generators installed. A tip vortex wake was visualized in hover, at low flight speeds and medium flight speeds (Fig.3, Fig.5). A specific dimensionless number has been adopted to determine inductive velocities and flight speeds related to an ideal inductive velocity in hover (Fig.5). The wake form was determined and the tip vortex velocities were measured in hover. The tip vortex vertical velocities turned out to be less then the inductive velocity of the ideal rotor in hover. The measured wake contractions were equal to 0.85R for the upper rotor and 0.91R for the lower rotor (Fig.3).

The tip vortex coaxial rotor wake was visualized in a forward flight. In the rotor front part the free tip vortices were positioned above the tip path planes. That flat part of a free wake could extend along the rotor up to 3/4 of its radius (Fig.5).

2. Basic design solutions and aeroelastic phenomena

It is very important to have a substantiation of aeromechanical phenomena. This is feasible given adequate simulation making possible to explain and forecast:
- natural frequencies of structures;
- loads & deformation;
- aeroelastic stability limits: stall flutter, transsonic flutter, ground resonance;
- helicopter performance & maneuverability.

KAMOV Company has developed software to simulate a coaxial rotor aeroelasticity [1,2,3,8,9,10]. Aeroelastic phenomena to be simulated are shown in the Fig.6 as (1-7) lines in the following way:

(1) - system of equations of coaxial rotor blades motion ;

(2) - elastic model of coaxial rotor control linkage (boundary data);

(3) - model of coaxial rotor vortex wake;

(4,5,6) - steady and unsteady aerodynamic airfoils data;

(7) - elastic / mass / geometry data of the upper / lower rotor blades and of the hubs.

The lines (1-8) in Fig.6 show functional capabilities of the software. Columns (1-5) conform to versions of the software . Both steady flight modes and manoeuvres of the helicopter are simulated .

Based on experience of KAMOV Company new key design approaches regarding the coaxial rotors of the Ka-50 helicopter were developed.

The blade aerofoiles were developed by TsAGI for the Ka-50, Ka-115, Ka-226 helicopter specially (Fig.7). Optimal combination of $C_L, C_D, C_M(\alpha, M)$ characteristics was a necessary condition to achieve:
- high G-load factor & stall limit ;
- acceptable margin of flutter speed ;
- low loads of rotor & linkage ;
- low vibration level ;
- high helicopter performance.

Blade sweep tips developed by KAMOV Company affords for the same purposes.

Usage of all key approaches regarding rotors becomes an sufficient condition to achieve high performance of the rotor system and therefore of the helicopter as whole.

3. KAMOV's Key Technologies

3.1 Fiberglass & fiber graphite rotor blades

In the end - 1950's KAMOV Company designed, built and tested fiberglass rotor blades. In 1965 first production fiberglass rotor blades were successfully flown on the KAMOV Ka-15 helicopter. In 1967 first production fiberglass rotor blades were successfully flown on the KAMOV Ka-26 helicopter. In the end 1970's the graphite & glass fiber rotor blades were successfully produced by KAMOV Company.

The graphite fibers had potential specific stiffness values which six times those of current aircraft materials. This proved to be highly significant in addressing elastic stability and aeroelastic design problems.

The combination of glass and graphite fibers resulted in excellent, benign failure modes and provided aerodynamics, structures and dynamic engineers.

Ka-50 helicopter advanced rotors geometry is:
- special airfoil;
- blade optimal twist;
- blade swept tip (Fig.7).

All KAMOV's blades fitted with electrothermal de-icing system.

3.2 Advanced rotor blade retention technology

All before Ka-50 KAMOV's helicopters had full articulated rotor hubs. Ka-32 helicopter has metal hubs, mechanical & elastomeric bearings & dampers (Fig.8).

Ka-50 helicopter has metal and composite hubs, elastomeric bearing & damper, flex element for pitch, flapping and lag (Fig.8).

3.3 Rotor control linkage

The rotor control linkage parameters determine a flap-lag-pitch motion and a motion stability of rotor blades (Fig.6, Fig.8, Fig.9).

The mathematical model of coaxial rotor control linkage was developed by KAMOV Company [1]. This math model is used for control linkage design and for frequency & stability analysis.

The control linkage elasticity matrix-functions was measured for full scale coaxial helicopters of four types.

The analysis of the experimental results determined to develop a control linkage mathematical model and a adequate formula

for the "approximation-calculation" of the matrix-function elements. With the help of this formula the rigidity characteristics of the control linkage aggregates were determined without their physical measuring for coaxial helicopters of four types. It is demonstrated that the eigenvectors of the elasticity matrix are actually torsional modes of the six blades on the control linkage and the eigenvalues are actually linkage "dynamic elasticities" of the corresponding mode which are usually measured in a different way - that is by the linkage frequency testing. The study results are illustrated on Fig.9 [1].

4. The basic design aeromechanic & aeroelastic problems of coaxial rotor helicopter have been developed and key technologies have been achieved

4.1 KAMOV Company experience concentrated in the Ka-50 attack helicopter

Acceptable margin of flutter speed and stall flutter speed of the Ka-50 helicopter were substantiated by mathematical simulation and validated by flight test results (Fig.10). Flight tests results are shown by (V_{TAS} - ωR) relation in Fig.10. A part of flight test points is given in the frame, namely the following range: from V_{TAS} ~ 300 km/h to V_{NE} = 350 km/h , till V_{TAS} = 390 km/h. Flutter is non-existent in restricted range of calculated curve what is verified by flight test results. Calculated curve presents that there is a flutter speed margin of about 50 km/h (Fig.10).

Vibration level of the coaxial helicopter have been discussed in paper [2]. The alternating forces apply to hubs of the upper/lower rotors and excite airframe vibration. Configuration of the Ka-50 helicopter rotors affords applying of minimal alternating forces to the airframe. In this case vibration level of the airframe is minimal too.

Vibration level is not in excess of 0.01g in main flight modes. Rotor pendulum and anti-resonant isolation system are not used. The example shown on Fig.11 [2].

Special task for the coaxial helicopter is making a provision for the acceptable lower-to-upper rotor blade tips clearances. As a task of aeromechanics it is analogous with a task to provide the blade tips-to-tail boom clearance of the helicopters with the tail rotor.

KAMOV Company applies at analysis both calculation methods and flight tests [3, 8]. The clearances are measured with the help of optic devices at each of 6 crossing points when the upper blades are arranged above lower blades during their relative rotation with doubled angular speed.

The mechanics can be outlined as following (Fig.12). The planes of the upper / lower rotor blade tips are in parallel at hover. Their clearance is even more than a clearance between rotor hubs.

At forward flight variable azimuthal airloads occur in the rotor disk, that results in flapping motion. Because of this , planes of the upper / lower rotor blade tips are inclined to equal angles in flight direction (forward / backward) .

In lateral direction (viewed along flight direction) the planes of blade tips are inclined to each other because of counter-rotation of the rotors (Fig.12).

The upper-to-lower tips clearances on one disk side decreases and increases on the opposite one. In lateral direction an inclination angle of blade tip planes is approximately equal to blade tip flapping angle (to the left / to the right) and depends on flight mode (Fig.12). As known from aeromechanics, there are relations between blade flapping angle and the rotor parameters, especially to Lock number, blade geometrical twist angle and blade/control linkage torsional stiffness.

Calculation and flight test results show the values of coaxial rotor parameters mentioned above which ensure acceptable safety clearance.

Fig.12 demonstrates measured blade tip flapping angles made during flight tests of Ka-50 helicopter and comparison with calculation data .

Ka-50 helicopter generalised measurement results for the forward flight and manoeuvres are presented in the Fig.12, Fig.13.

The acceptable upper-to-lower rotor blade tips clearances were substantiated by mathematical simulation and validated by flight tests results for all approved envelope of manoeuvres.

The acceptable lower rotor blades to tail boom clearances were validated .

4.2 Ka-50 helicopter maneuverability features

Load factor / speed envelope was substantiated and validated by Ka-50 flight test results:
- within operational limitations (pitch, roll, rotor speed, rotor loads,...);
- within special aerobatic limitations.

A part of flight test points is illustrated by the Fig.13 :

at **3.5 > g-factor > 2** & at **g-factor ≈ 0.**

Each point corresponds to one of the performed manoeuvres . The most part of them are shown at the Fig.13 No established limitations have been exceeded .

Fig.13 also shows the test flight results of Tiger's helicopter [13].

The table on the Fig.14 presents parameters of manoeuvres within special limitations for aerobatic flights. It is notable in this case parameters of "flat turn" and pull-out from the skewed loop at **g–factor = 3.5.**

4.3 The means of aerobatic flight monitoring and analysis

The NSTAR software was created to provide processing and analysis of Ka-50 helicopter test flight data. Using records made by aircraft test instrumentation the NSTAR software makes possible to restore the flying path and to calculate flight parameter additional values [14].

NSTAR software is comparable both with test flight record system and with standard record system. NSTAR results are used for the following purposes:
- analysis of actions , assistance in pilot training;
- examination for critical parameter limits;
- as input data for simulation.

Fig.14 presents an example for skewed loop path recovery.

5. Conclusions

5.1 The basic aeromechanic & aeroelastic problems of a coaxial rotor helicopter have been developed and Key Technology have been achieved.

5.2 Key Technology determine the coaxial helicopter performance and manoeuvrability.

7. References

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Basic Parameters
of Coaxial KAMOV'S Helicopters

POWER LOADING

$$W_{MAX}/P, \quad [kg/h.p.]$$

	Ka - 8	Ka - 10	Ka - 15	Ka - 18	Ka - 26	Ka -126	Ka -226	Ka - 25	Ka - 32	Ka - 29	Ka - 50	Ka - 52
	7.1	7.1	5.2	5.3	5	4.5	3.7	4	2.9	2.6	2.5	2.6

MAX SPEED

$$V_{MAX}, \quad [km/h]$$

	Ka - 8	Ka - 10	Ka - 15	Ka - 18	Ka - 26	Ka -126	Ka -226	Ka - 25	Ka - 32	Ka - 29	Ka - 50	Ka - 52
	80	115	150	150	160	190	200	220	250	280	310	300
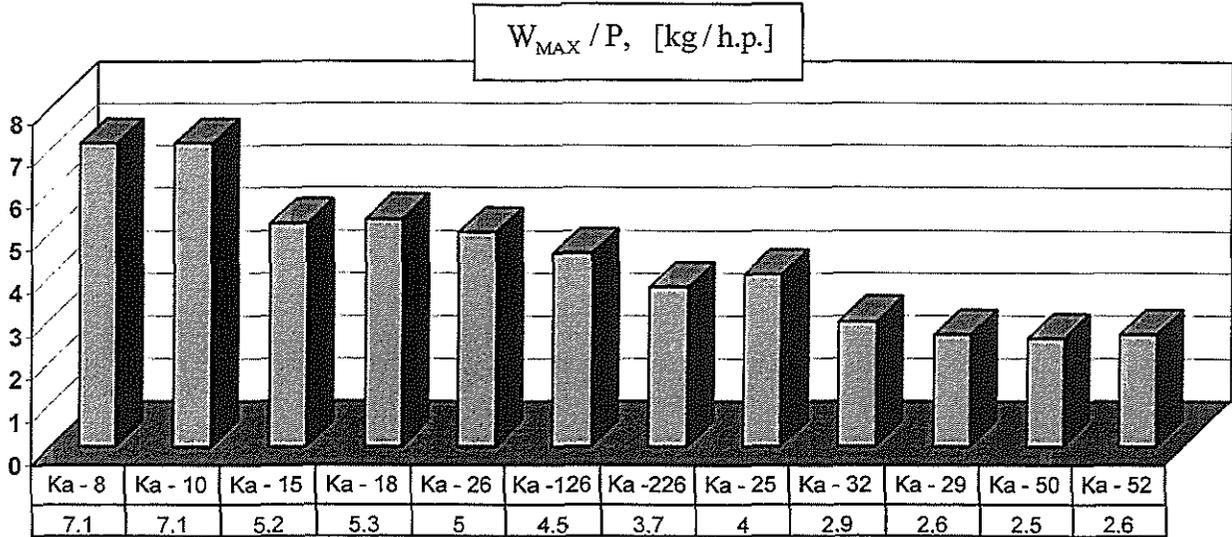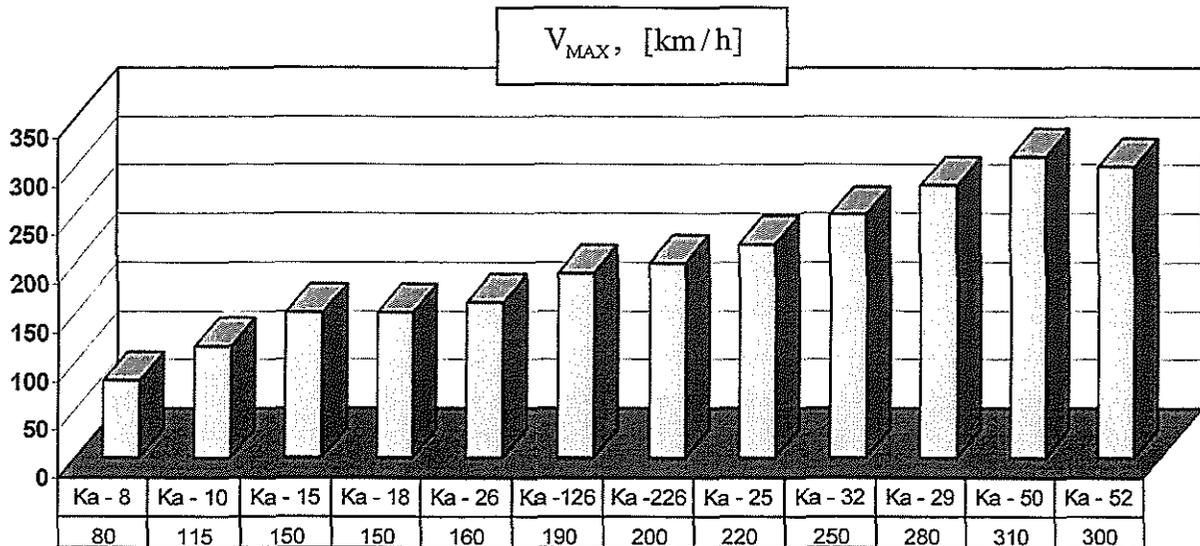
Fig.1A

Basic Parameters
of Coaxial KAMOV'S Helicopters

RELATIVE HUB CLEARANCES

$$H_0 / D$$

	Ka - 8	Ka - 10	Ka - 15	Ka - 18	Ka - 26	Ka -126	Ka -226	Ka - 25	Ka - 32	Ka - 29	Ka - 50	Ka - 52
	0.098	0.1	0.085	0.085	0.09	0.09	0.09	0.095	0.093	0.093	0.097	0.097

DISK LOADING

$$W_{MAX} /(\pi R^2), \ [kg / m^2]$$

	Ka - 8	Ka - 10	Ka - 15	Ka - 18	Ka - 26	Ka -126	Ka -226	Ka - 25	Ka - 32	Ka - 29	Ka - 50	Ka - 52
	13	14.8	18.7	19	24.5	24.5	25.6	37.5	63.5	57.9	65.4	73.1
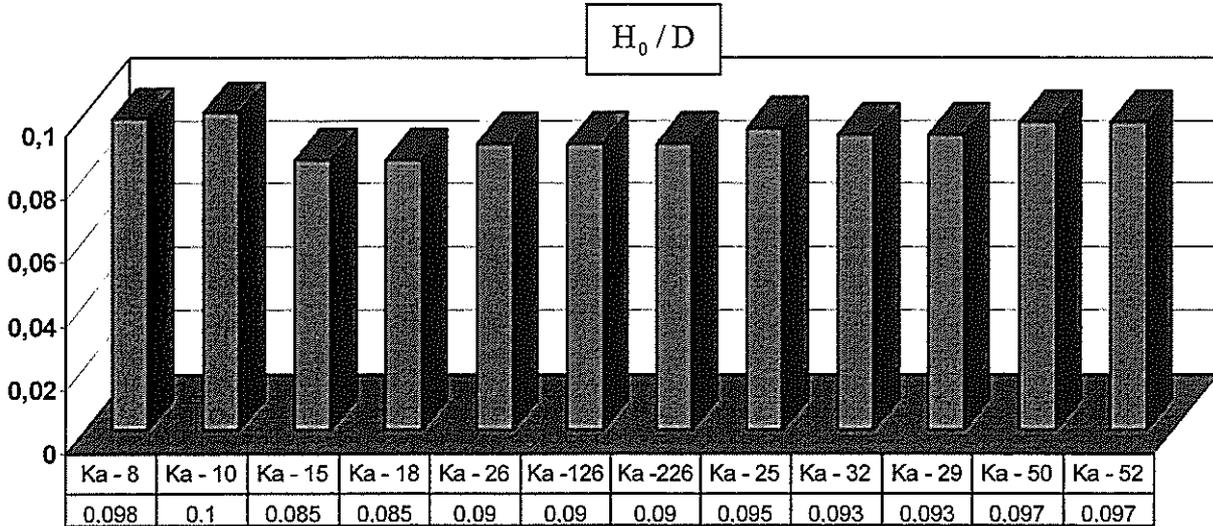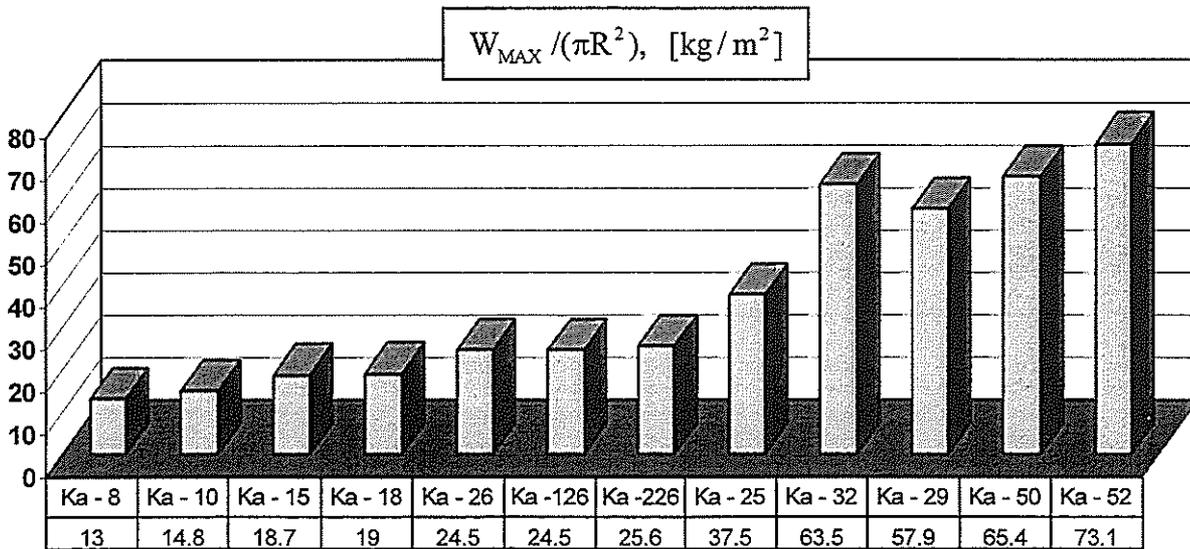
Fig.1B

Single & Coaxial Rotors
active disk areas, effective diameters, power & thrusts at hover

SINGLE ROTOR

COAXIAL ROTOR

$$As = A = \pi D^2/4$$

$$\delta A = 0,28A$$

$$Ac = \pi D^2/4 + \delta A = 1,28\,As$$

$$D_{EFF} = \sqrt{4As/\pi} = 1,13\,D$$

$$Ts = (33,25 \cdot \eta_s \cdot D \cdot P\sqrt{A})^{2/3}$$

$$Tc = (33,25 \cdot \eta_c \cdot D \cdot P\sqrt{A})^{2/3}$$

$$\eta_c = \frac{D_{EFF}}{D} \cdot \eta_s = 1,13\eta_s$$
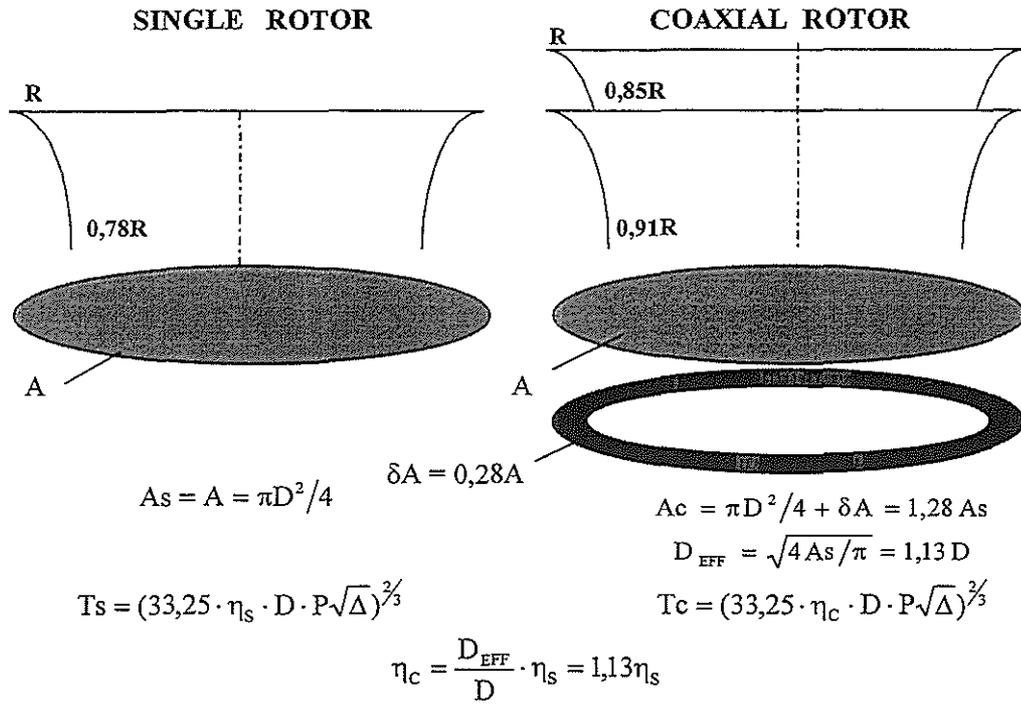
Single & Coaxial Rotor Helicopters
main rotor diameter, power & thrust at hover

HELICOPTER WITH A TAIL ROTOR

COAXIAL HELICOPTER

$$Ts = \left(33,25 \cdot \eta_s \cdot Ds \cdot 0,9P\sqrt{A}\right)^{2/3}$$

$$Tc = \left(33,25 \cdot \eta_c \cdot Dc \cdot P\sqrt{A}\right)^{2/3}$$
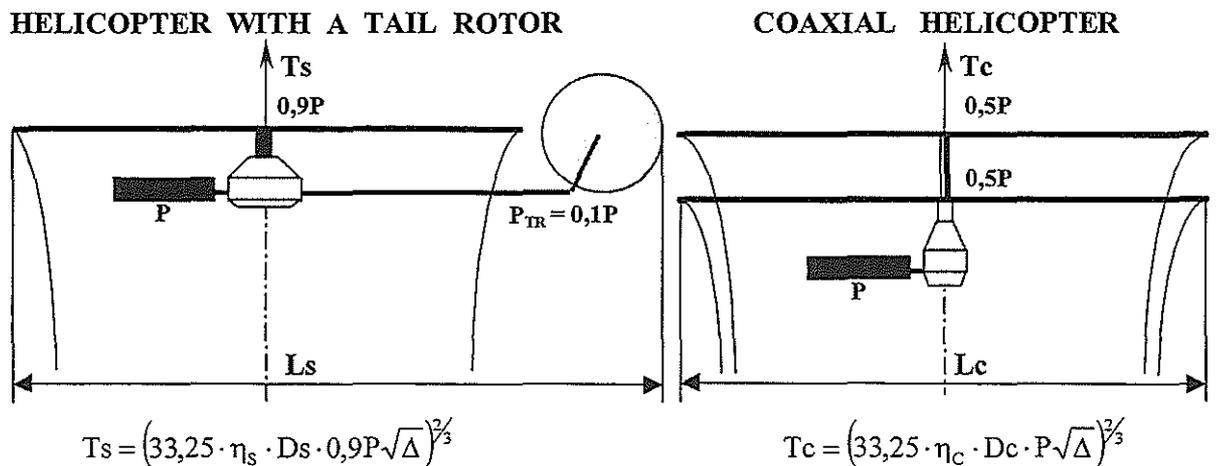
From $\eta_c/\eta_s = 1,13$ and $Pc = Ps = P$ and $P_{TR} = 0,1P$:

1. At $Dc = Ds$ the thrust ratio is $Tc/Ts = (1,13/0,9)^{2/3} = 1,16$;
2. At $Ts = Tc$ the diameter ratio is $Ds/Dc = 1,13/0,9 = 1,26$.

Fig.2

Figure of merit of coaxial rotors
(Flight test measured data)

η_c

Hover
Airframe Thrust & Power Loss
was taken into account

Ka - 25

Ka - 26

Ka - 15

Legend:
- —— Ka - 32 advanced rotor
- ● Ka - 50 prototype
- ■ Ka - 50 production - type
- ▲ Ka - 32 production - type
- ★ MAX measured value

y-axis: 0.50, 0.55, 0.60, 0.65, 0.70, 0.75, 0.80, 0.85, 0.90

x-axis: 0.05, 0.08, 0.10, 0.13, 0.15, 0.18, 0.20, 0.23, 0.25

C_T / σ
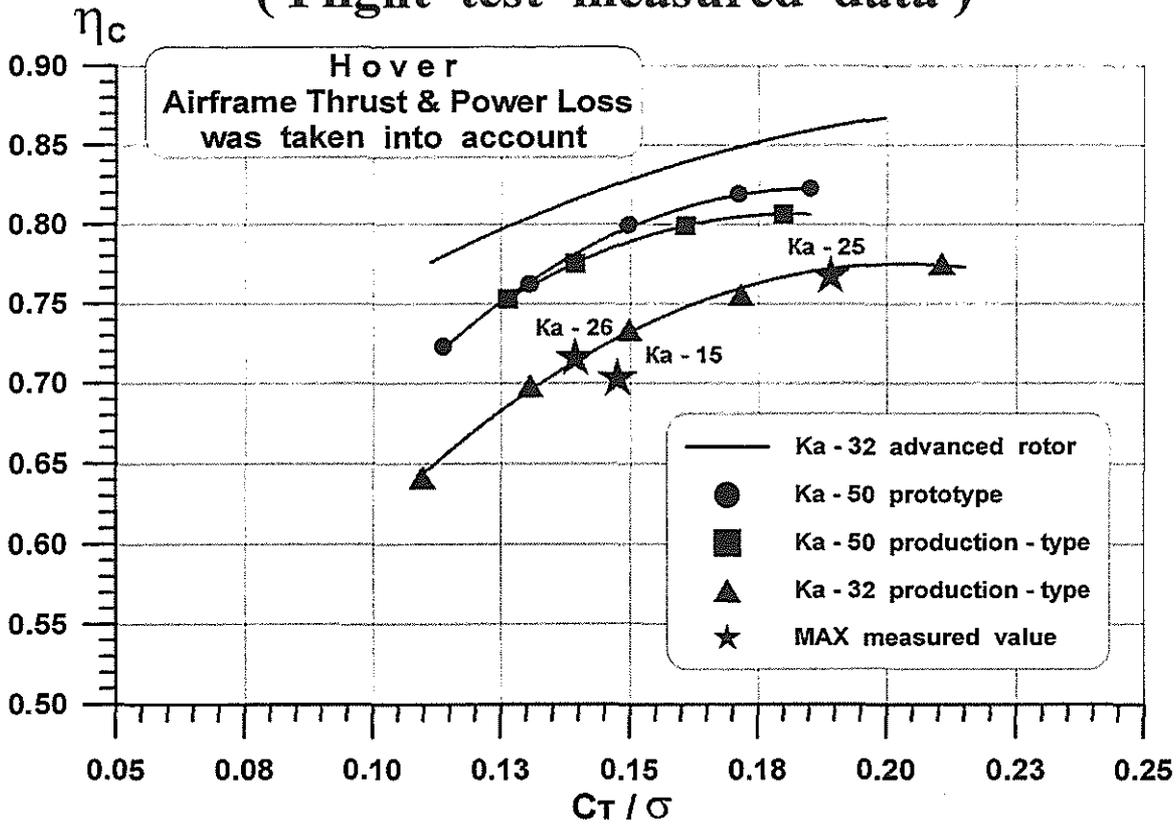
Wake form in hover & its approximation

AXIAL LOCATION, y/R

RADIAL LOCATION, r/R

y/R

K1

K2

$2\pi/3$

0.85

r/R

Ka - 32 COAXIAL ROTOR

WAKE FORM APPROXIMATION :
+ - Upper Rotor : K1 = 0.0371 , K2 = 0.1028
× - Lower Rotor : K1 = 0.0431 , K2 = 0.0854

$$r / R = A + (1 - A) / ch (n \lambda \psi)$$

Upper Rotor : A = 0.82 , n = 2
Lower Rotor : A = 0.91 , n = 4
$A_{LR}^2 / A_{UR}^2 = C_{T_{UR}} / C_{T_{LR}} = 1.23$

WAKE AZIMUTH ANGLE , ψ deg

0, 30, 60, 90, 120, 150, 180, 210, 240, 270, 300, 330, 360
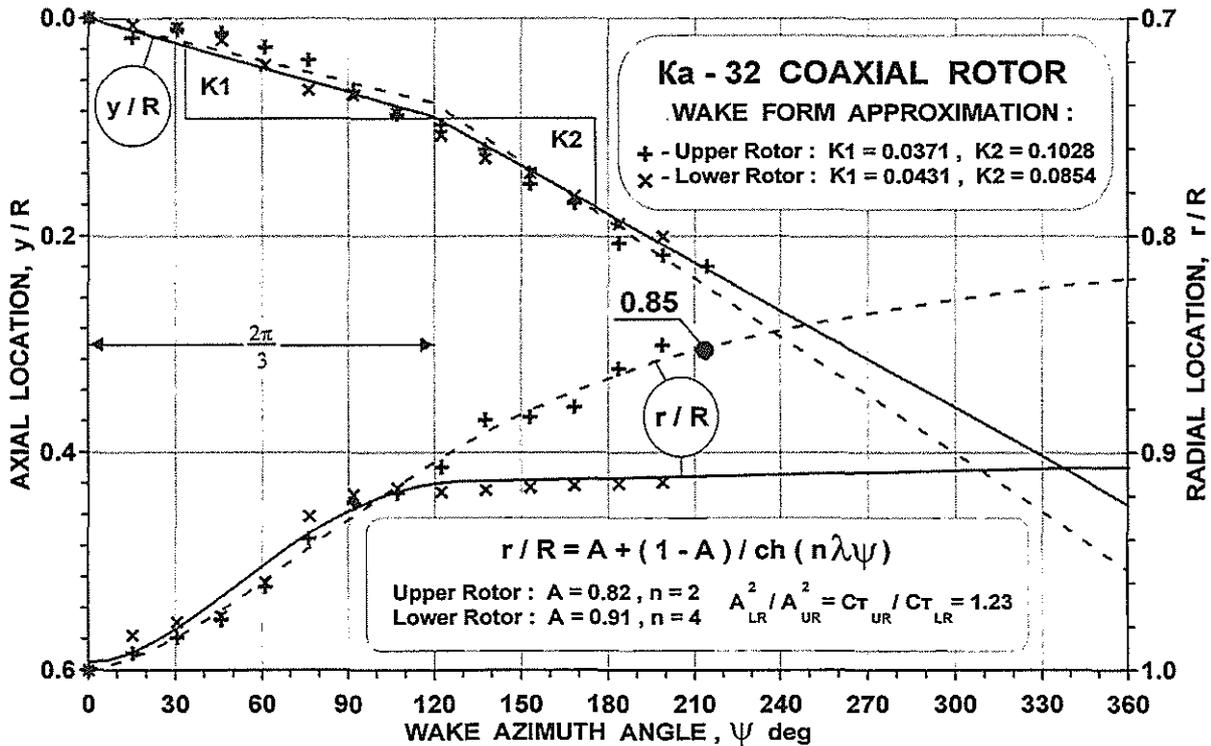
Fig.3

THE STATISTICAL CHART
Power Loadind - Disk Loading - Design Figure of Merit
of Coaxial Helicopters & Helicopters with a Tail Rotor
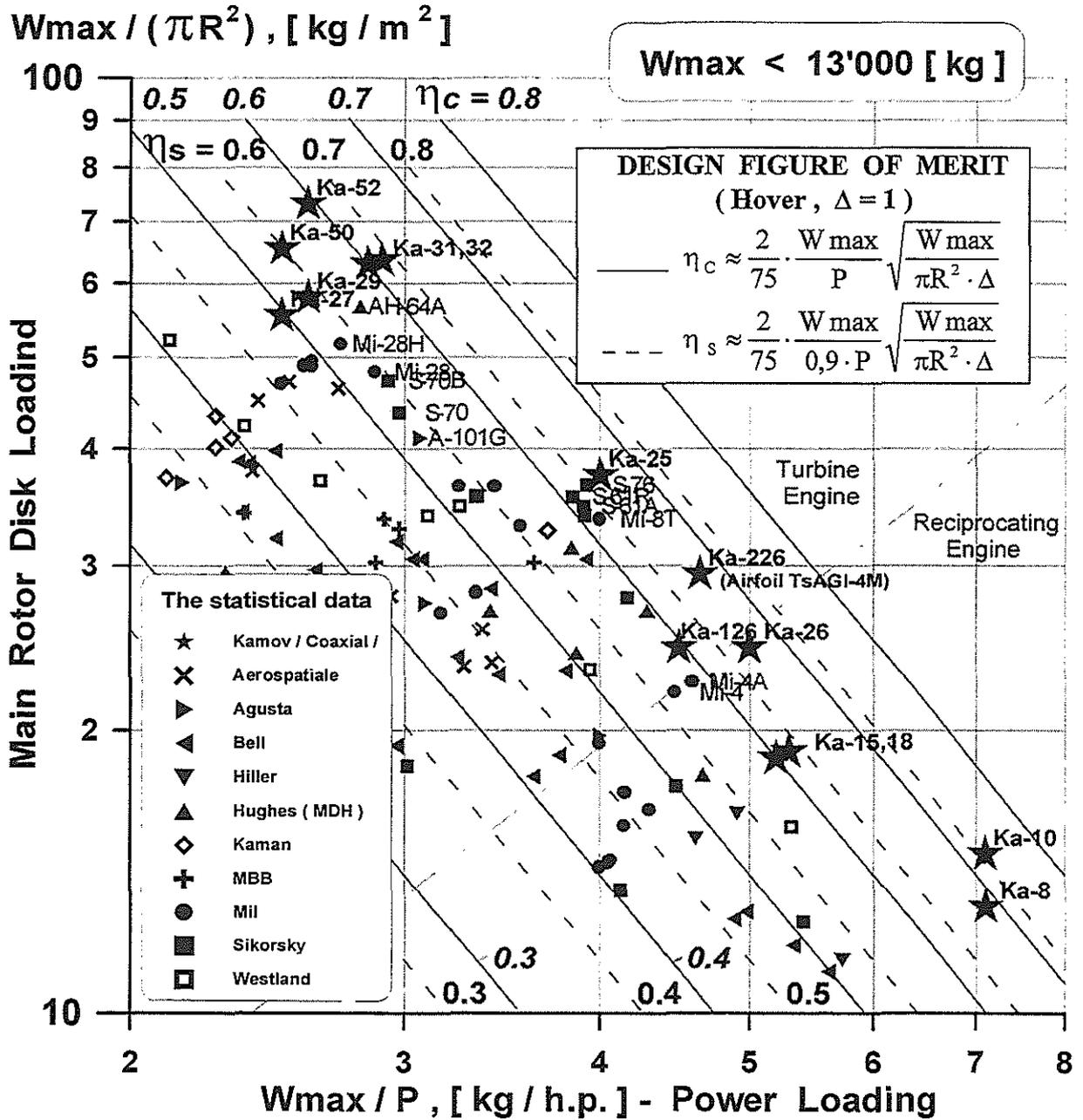
Fig.4

Coaxial Rotor Wake Side Views
for Several Flight Speeds

V_{TAS} = 5 [km/h]

V_{TAS} = 73 [km/h]

V_{TAS} = 138 [km/h]

V_{TAS} = 227 [km/h]
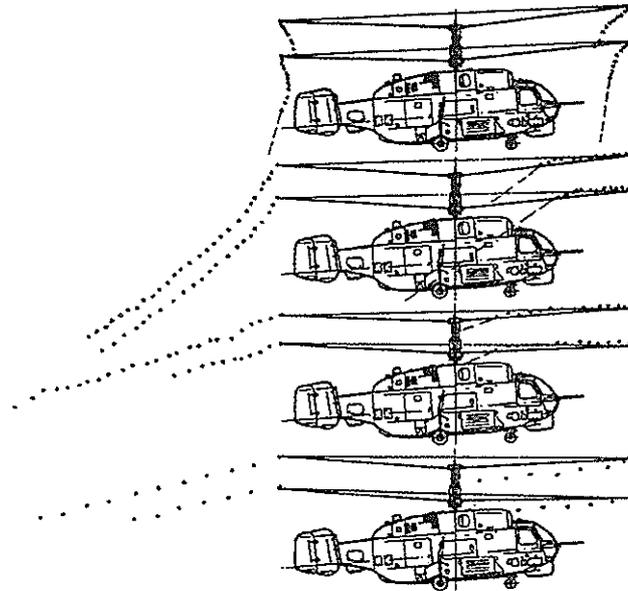
Wake Front Boundary Longitudinal Position
Versus Flight Speed

$V_{TAS} \cdot Cos\alpha$

r/R

r_o/R

$\widetilde{V}$

3.5

r/R

V

$$\begin{cases} r/R = 1 - 2\pi\mu, & \text{for } r/R > r_o/R, \ \widetilde{V} \leqslant 3.5 \\ r/R = r_o/R, & \text{for } r/R \leqslant r_o/R, \ \widetilde{V} \leqslant 3.5 \\ r/R = 1, & \text{for } \widetilde{V} > 3.5 \end{cases}$$

$$\mu = V_{TAS} \cdot Cos\alpha / (\Omega R), \ \widetilde{V} = V_{TAS} / \sqrt{T/(2\rho A)}$$
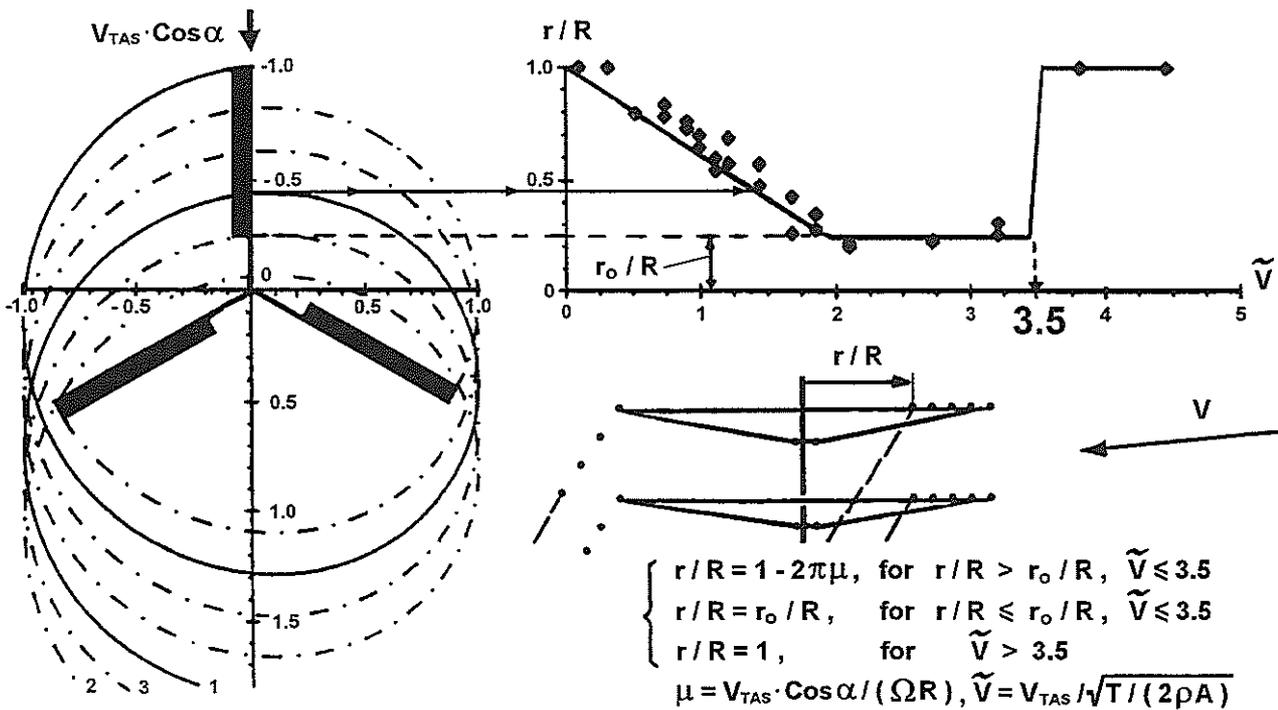
Fig.5

Simulated Aeroelastic Phenomena of Coaxial Rotor

	Simulated Phenomena	SIMULATION VERSION				
		ULISS-6	ULISS-1	ULMFE	FLUT	MFE
1	$EI_x\,(r/R,\omega t)$ $EI_y\,(r/R,\omega t)$ $GI_p\,(r/R,\omega t)$	✓	✓			
2	$\overline{\varphi}_o=\|\upsilon_{ij}\|\times\overline{M}$	✓			✓	✓
3	$V_i\,(r/R,\,\psi)$	✓				
4	$C_L,\,C_D,\,C_M$ $(\alpha,\,\dot{\alpha},\,M,\,\dot{M})$	✓	✓	✓		
5	C_{L_MAX} $(\alpha,\,\dot{\alpha},\,M)$	✓	✓	✓		
6	Airfoil Aeroelastic Deformation	✓	✓	✓	✓	
7	Upper/Lower Rotor Data	✓	✓	✓	✓	✓

Analysis Results of Coaxial Rotor Aeroelastic Simulation

	Analysis Results	SIMULATION VERSION				
		ULISS-6	ULISS-1	ULMFE	FLUT	MFE
1	Stall flutter boundary	Coaxial Rotors	Blade	Blade		
2	Bending moments, Pitch link loads, Actuator loads	Coaxial Rotors	Blade	Blade		
3	Elactic Deformations	Coaxial Rotors	Blade	Blade		
4	Alternate loads on Hubs	Coaxial Rotors				
5	Blade tips Clearances	Coaxial Rotors				
6	Flight test flutter	Coaxial Rotors	Blade	Blade		
7	Ground test flutter	Coaxial Rotors			Blade	
8	Natural frequencies			Blade		Blade

Fig.6

Blade Aerofoil Data

Ka-50 Aerofoil

AEROFOILES

OA313
RAE9645
OA213
VR-12
DM-H4
OA312
Ka-50 Aerofoil
NACA-23012
TsAGI-4
OA212BFM
RAE9648
TsAGI-2
OA309
DM-H3
OA209
SC 1095
RAE9634
NACA-0012
VR-15
OA207
TsAGI-3

Axes: C_{L_MAX} AT (M = 0.4) versus M_{DD} AT (C_L = 0)

Anti Erosion Strip — Skin — Trailing Edge

Balance Bar — Fibercarboglass Spar — Nomex Honey Comb

Heater Blanket

Sweep Tip

Blade Fitting — Aft Section — Trim Tab

Aerodynamic moments of existing TsAGI-2, TsAGI-4 airfoils & advanced TsAGI-4M airfoil

Legend:
- TsAGI - 2 airfoil
- TsAGI - 4 airfoil
- TsAGI - 4M advanced Ka - 226 airfoil

C_L axis: 2.0, 1.5, 1.0, 0.5, 0

M=0.3   0.4   0.5   0.6   0.7   0.8

C_M (0.25 chord)
-0.2   -0.1   0.0   0.0   0.0   0.0   0.0   0.0   0.1
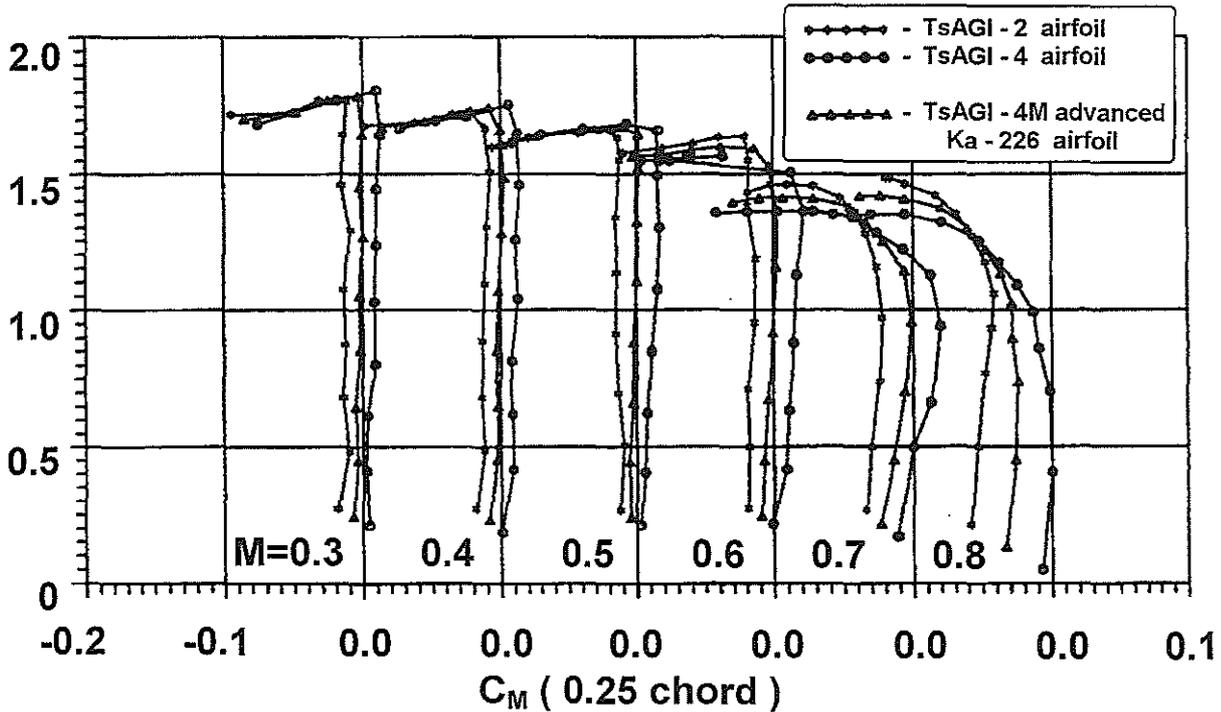
Fig.7

Ka-32 rotors control linkage model
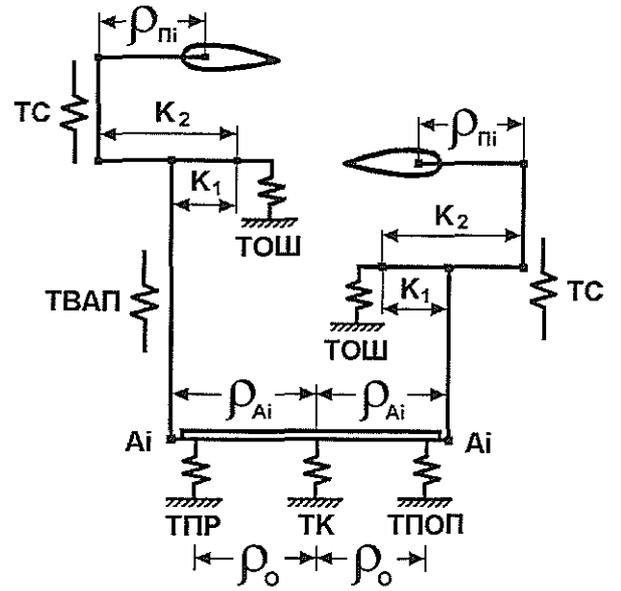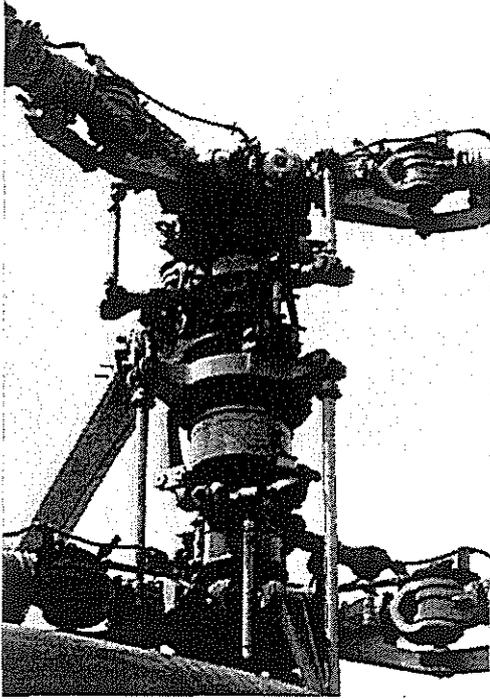
Ka-50 rotors control linkage model
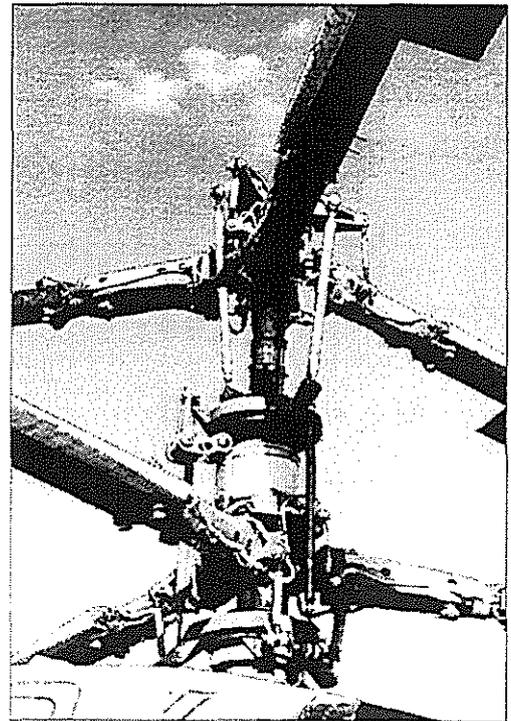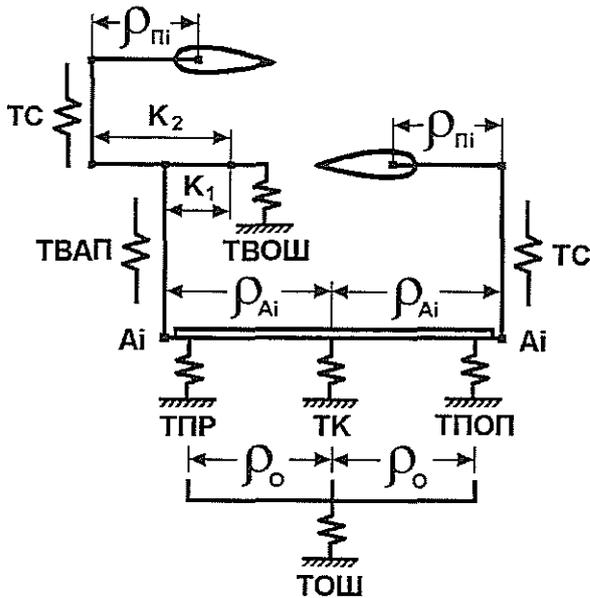
Fig.8

The scheme of experiment on determination of the control linkage elasticity matrix

$$\overline{\varphi} = \left\| \vartheta_{I,J} \right\| \times \overline{M}$$

$$
\begin{Vmatrix} \overline{\varphi}_1 \\ \overline{\varphi}_2 \\ \overline{\varphi}_3 \\ \overline{\varphi}_4 \\ \overline{\varphi}_5 \\ \overline{\varphi}_6 \end{Vmatrix} =
\begin{Vmatrix}
\vartheta_{11} & \vartheta_{12} & \vartheta_{13} & \vartheta_{14} & \vartheta_{15} & \vartheta_{16} \\
\vartheta_{21} & \vartheta_{22} & \vartheta_{23} & \vartheta_{24} & \vartheta_{25} & \vartheta_{26} \\
\vartheta_{31} & \vartheta_{32} & \vartheta_{33} & \vartheta_{34} & \vartheta_{35} & \vartheta_{36} \\
\vartheta_{41} & \vartheta_{42} & \vartheta_{43} & \vartheta_{44} & \vartheta_{45} & \vartheta_{46} \\
\vartheta_{51} & \vartheta_{52} & \vartheta_{53} & \vartheta_{54} & \vartheta_{55} & \vartheta_{56} \\
\vartheta_{61} & \vartheta_{62} & \vartheta_{63} & \vartheta_{64} & \vartheta_{65} & \vartheta_{66}
\end{Vmatrix} \times
\begin{Vmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \\ M_6 \end{Vmatrix}
$$

Elasticity matrix

APPROXIMATION : $\quad \vartheta_{I,J}(\psi_{1LR}) = f(\,TC, TK, T\Pi O\Pi, T\Pi P, TBA\Pi, TOШ, TBOШ, \psi_{1LR})$

CALCULATION :

$$\vartheta_{I,J}(\psi_{1LR}) = \frac{KAЧ_J}{\rho_{\Pi J}} \cdot \frac{KAЧ_I}{\rho_{\Pi I}} \cdot \left\{ TK \cdot \left[KA_I \cdot (Sin\varphi_I + Cos\varphi_I) - 1 \right] \cdot \left[KA_J \cdot (Sin\varphi_J + Cos\varphi_J) - 1 \right] + \right.$$

$$\left. + KA_I \cdot KA_J \cdot (T\Pi O\Pi \cdot Cos\varphi_I \cdot Cos\varphi_J + T\Pi P \cdot Sin\varphi_I \cdot Sin\varphi_J) \right\} + TC + TBA\Pi + \vartheta_{0\,I,J} \,;$$

$$\vartheta_0 = \begin{Vmatrix} TOШ & 2TOШ \\ 2TOШ & 4TOШ + TBOШ \end{Vmatrix} \,;\quad
\varphi_I = \begin{cases} \psi_{1LR} + O\Pi - \dfrac{\pi}{2} - \dfrac{2\pi}{K}(I-1); & I = 1,2,\dots,K \\[2mm] 2\pi - (\psi_{1LR} + O\Pi + DFI) - \dfrac{\pi}{2} - \dfrac{2\pi}{K}(I-K-1); & I = K+1,\dots,2K \end{cases}$$

Main rigidities of the elasticity matrix
## & dynamic rigidities obtained from frequency testing

$$\begin{cases} M\ddot{\varphi} + \vartheta^{-1}\varphi = 0 \\ \varphi = u \cdot e^{ipt} \\ (\vartheta M - E/P_K^2) \cdot u = 0 \\ P_K^2 = 1/(\lambda_K I) \end{cases}$$

MEASURED Ka – 32 ROTOR LINKAGE RIGIDITY $\Rightarrow$

$C_1;\ 1/\lambda_K$, kgm / rad

2 mode — $1/\lambda_2$

1 mode — $1/\lambda_1$

$1/\lambda$ – calculation of eigen value

C_1 – frequency testing (Ka - 32)

Ψ_{1LR}, deg
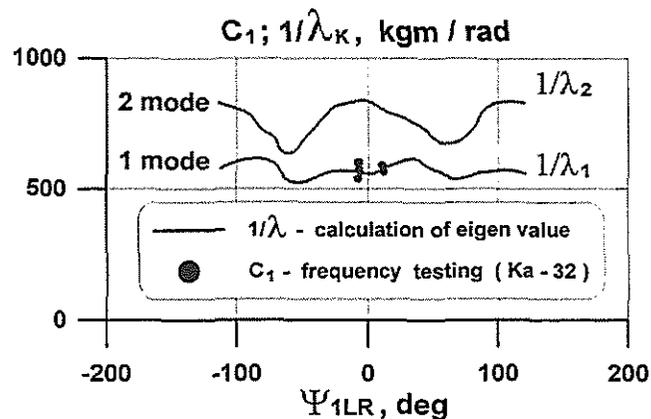
Fig.9

Demonstrated Rotor Speed Range

Blade Tip Mach Number (ISA SLS) :

Rotor Tip Speed, ΩR [m/sec]

max. Transient : 255 [m/sek] (98%)

Calculation : NO FLUTTER

max. Transient : 246 [m/sek] (98%)

Ka-50

● - Ka-50 Flight Tests

Ka-32

min. Transient : 216 [m/sek] (83 %)

min. Transient : 208 [m/sek] (83%)

V_{TAS} [km/h]

Stall Flutter Boundary

C_T / σ

$$C_T / \sigma = \frac{n_z \cdot m \cdot g}{\rho / 2 \cdot A_B \cdot (\Omega R)^2}$$

Stall Limit (Theory)
New Profiles DMH4, DMH3
TIGER, EC135, BO105 KWS

Ka - 50 Flight Tests
Approximation

Blade loading [-]

●	Ka - 50 Flight Tests
◇	TIGER , PAH2 / HAC
✛	TIGER , HAP
✕	EC135 Stress Flights
☐	WG13 Lynx Record
△	Dauphin DGV Test

Advance ratio, true airspeed μ TAS [-]

Fig.10

Ka - 25 Helicopter 3ω Vertical Vibrations

Fig.11

Blade Tip Coefficients
Comparision of Calculations and Flight Test Results

$$Y\_UR(\psi) = A_{0\_UR} + A_{1\_UR} \cdot \cos \psi + B_{1\_UR} \cdot \sin \psi + \ldots$$
$$Y\_LR(\psi) = A_{0\_LR} + A_{1\_LR} \cdot \cos \psi + B_{1\_LR} \cdot \sin \psi + \ldots$$
Clearance: $H(\psi) = H_0 + Y\_UR(\psi) - Y\_LR(\psi)$

The Upper-to-Lower Rotor Blade Tips Clearancies
Versus Level Forvard Speed & Blade Azimuth

Fig.12
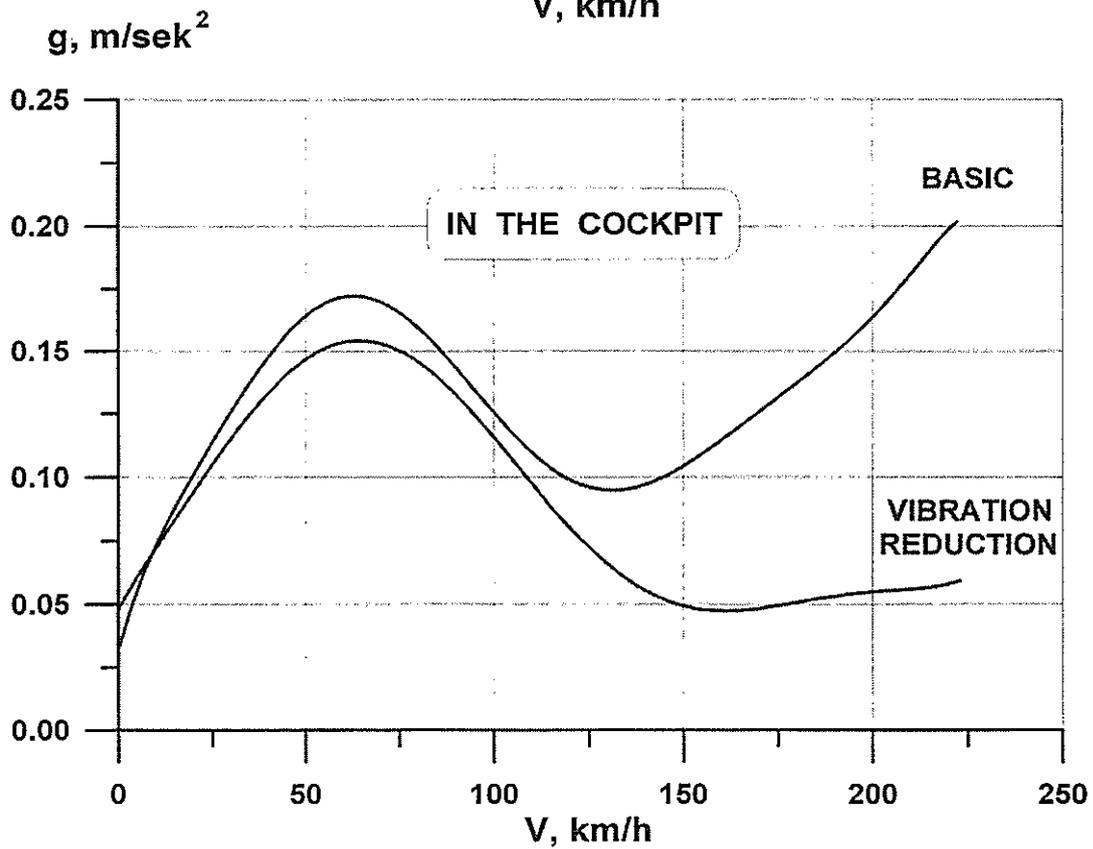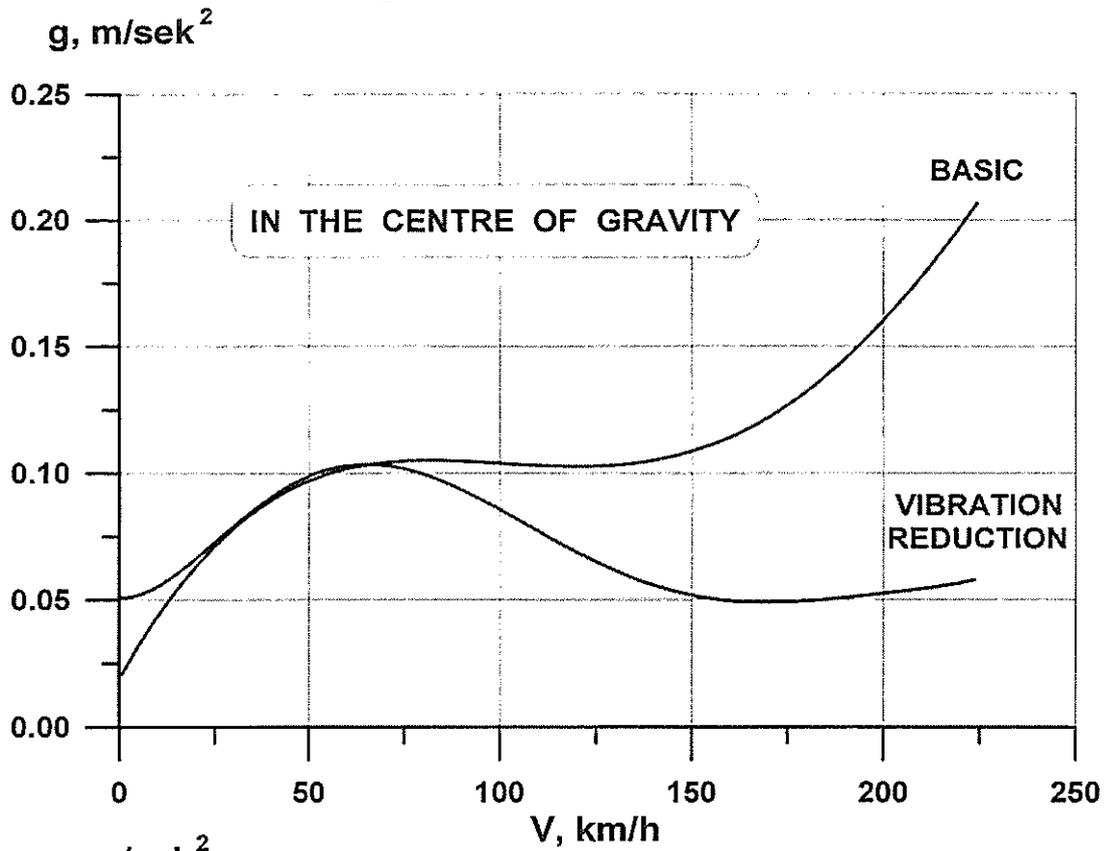
Measured Upper-to-Lower Rotor Blade Tips Clearancies
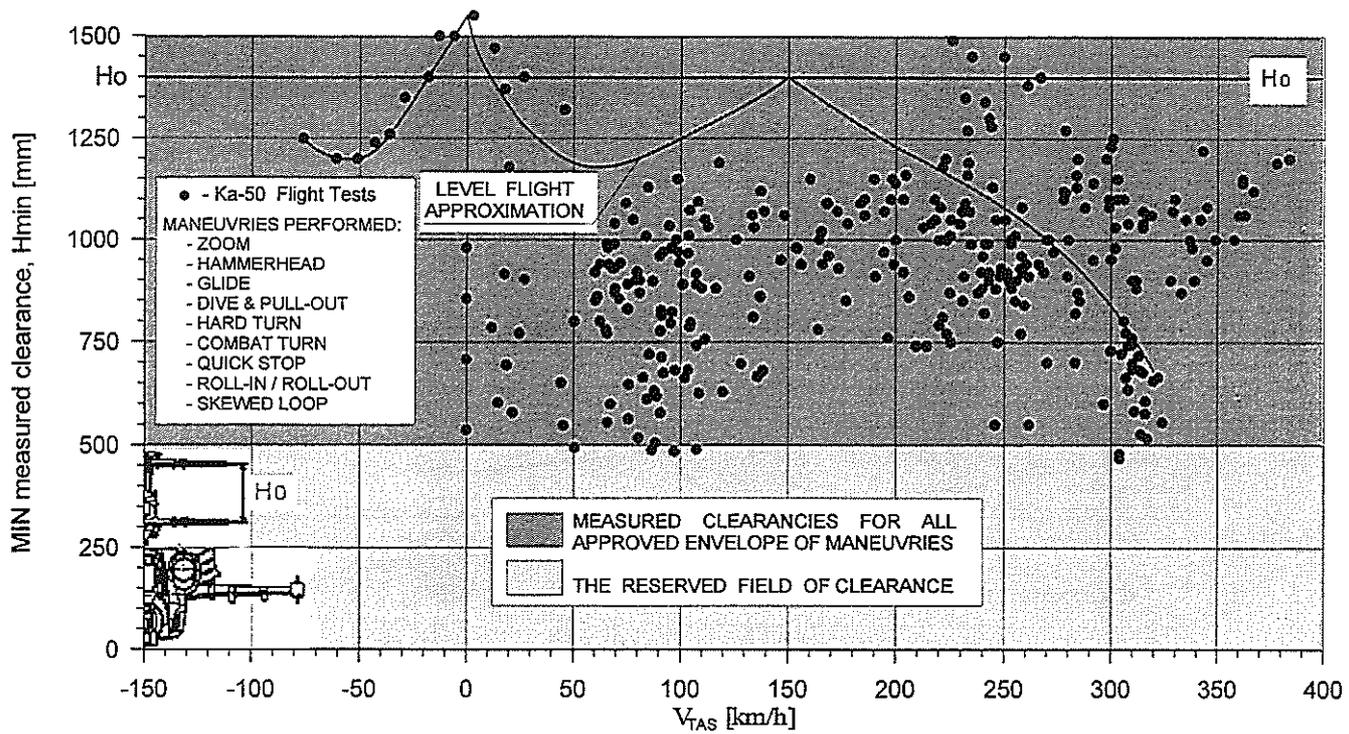
Load Factor / Speed Envelope

(Structural Qualification)
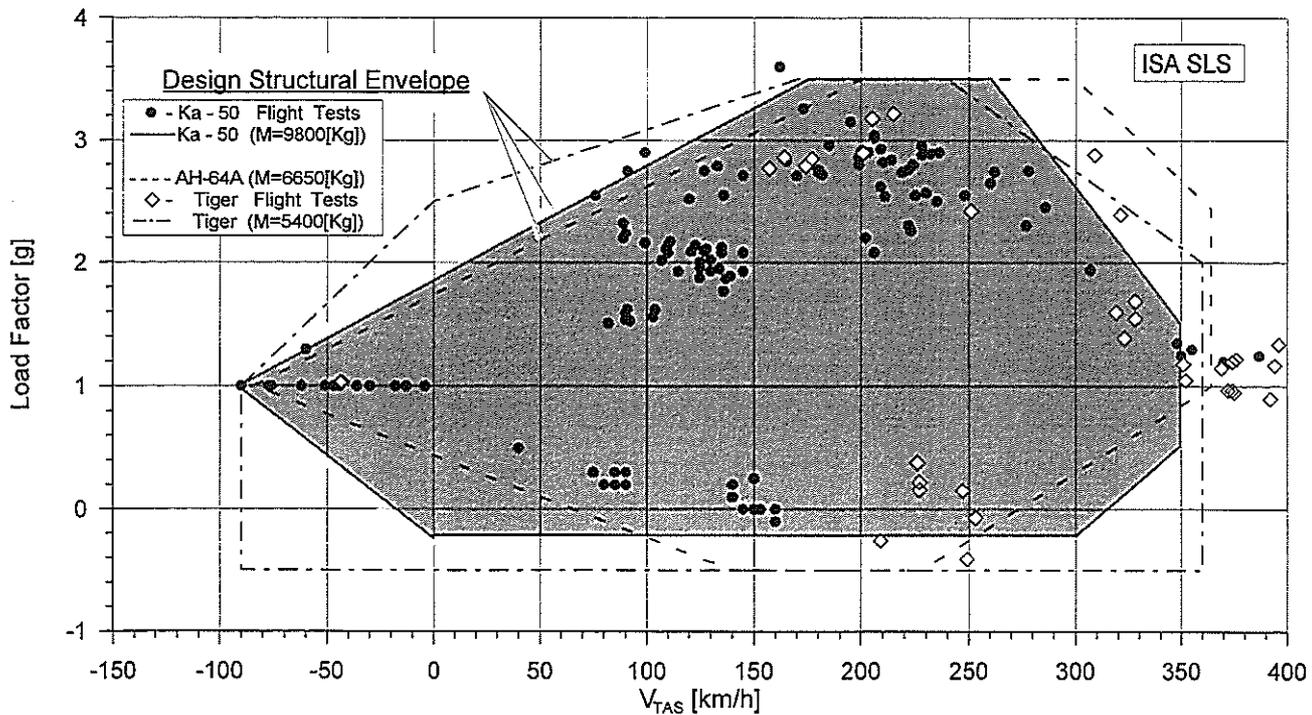
Fig.13

Ka-50 Aerobatic Maneuvries

MANEUVER	The Measured Parameter Values (min/max)				DESCRIPTION
	Airspeed V$_{TAS}$ [km/h]	Load Factor [g]	Pitch attitude [deg]	Roll attitude [deg]	
Hard Turn (Right/Left)	280 ÷ 60	1.0 → 2.9 → 1.0	20 ÷ 50	0 ÷ -70	Unsteady Turn with Pitch & Roll
Flat Turn (Right/Left)	220 ÷ 0	1.0 → 1.5 → 1.0	±5	±20	Jaw Attitude ±80 ÷ ±90 [deg]
Hammerhead (Right/Left)	280 ÷ 0	1.0→ 2.9 → 1.0 → 2.9 → 1.0	0 ÷ ±90	±90	
Dive	0 ÷ 390	1.0 → 0.25 → 2.9 → 1.0	0 ÷ -90	±30	Push-Down, Dive & Pull-Out
Skewed Loop (Right/Left)	280 ÷ 70	1.0 → 2.9 → 1.2 → 3.5 → 1.0	0 ÷ 360	±150	
Quick Stop (Right/Left)	150 ÷ 40	1.0 → 2.0 → 1.2 → 1.0	0 ÷ 40	±55	Pitch / Roll Decceleration
Pull-Up with the Tail Forward	-90 ÷ 0	1.0 → 1.5 → 1.0	0 ÷ -70	±10	Backward Acceleration & Pull-Up with the Tail Forward/Up

Flight Path While Performing Skewed Loop

Fig.14