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PREDICTION OF CRITICAL CONFIGURATIONS OF HELICOPTER - EXTERNAL SLUNG LOAD SYSTEM USING BIFURCATION THEORY AND CONTINUATION METHODS

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Abstract

Application of helicopters to transport of heavy and bulky loads creates the stability problems specially of hovering with hanging loads. The presence of an external load modifies the flight dynamics and handling qualities characteristics of a helicopter because the load behaves like a pendulum, and it can change natural frequencies and mode shapes of the low frequency modes of the helicopter. Additionally, the aerodynamic characteristics of the load may make it unstable in certain flight conditions, with obvious repercussions on the stability and the safety of the entire helicopter/load system. The paper presents a study of the fight dynamics of an articulated rotor helicopter carrying a suspended load. The rotorcraft model includes rigid body dynamics, individual flap and lag blade dynamics, and inflow dynamics. The load is modelled as pendulum with a single suspension point. Helicopter with suspended load is the inherently non-linear and time varying system. Manoeuvrability of this system in wide flight regimes involves non-linear aerodynamics and inertial coupling. Dynamical systems theory provides a methodology for studying non-linear systems of ordinary differential equations. Bifurcation theory is a part of that theory which is considering changes in the stability lead to qualitatively different responses of the system. Results from dynamical systems theory were used to predict the nature of the instabilities caused by bifurcations and the response of the rotorcraft and suspended after a bifurcation was studied.

1 Introduction

Carrying external suspended loads has always been one of the traditional missions of the helicopter. Both military and commercial operators have exploited the capability of the helicopter to rapidly move heavy loads to locations where the use of ground based equipment would be impractical or impossible. The presence of an external load can modify the fight dynamic and handling qualities characteristics of a helicopter because the load behaves like a pendulum, and it can change natural frequencies and mode shapes of the low frequency modes of the helicopter. Additionally, the aerodynamic characteristics of the load may make it unstable in certain fight conditions, with obvious repercussions on the stability and the safety of the entire helicopter/load system. The dynamics of a helicopter with external suspended loads received considerable attention in the late 1960’s and early 1970’s. Two reasons for this interest were the extensive external load operations in the Vietnam war, and the Heavy-Lift Helicopter program (HLH). This interest has been renewed recently, prompted by the re-evaluation and extension of the ADS-33 [1] Helicopter Handling Qualities Specifications to transport helicopters, and in the expectation of new cargo helicopter procurements.

One of the first theoretical studies of the dynamics of a helicopter with a slung load is due to Lucassen and Sterk [2]. A simple 3-degree of freedom modelled the hover longitudinal dynamics of the helicopter and the angular displacement of the load. A single suspension point was assumed and the aerodynamic forces and moments on the load were neglected. In general, the pole associated with the load pendulum mode was stable; the phugoid remained unstable, but its frequency decreased with increasing cable length. For some combination of parameters the helicopter mode became unstable while the load mode was stabilised.

Szustak and Jenney [3] pointed out that a conventional stability augmentation system was not adequate for precision hover and load release, and could result in pilot-induced oscillations (PIO). A more effective solution consisted of an inner loop in which the relative motion of aircraft and load was fed back to cyclic, and an outer loop in which the aircraft position above ground was fed back, again to cyclic.

Dukes studied the basic stability characteristics of a helicopter with a slung load, and possible feedback stabilisation schemes [4], and appropriate piloting strategies for various manoeuvres [5]. A 3-degree of freedom longitudinal helicopter/load model was used. Positive pitch damping, whether provided by the rotor alone or also by a fight control system, did not necessarily increase the stability of the pendulum mode of the load. This mode, essentially undamped, could become unstable for certain configurations (i.e., cable lengths, load weight, and relative position of attachment point and aircraft centre of mass). Pitch damping provided at best a modest increase in the damping of the mode. A feedback control scheme in which the attachment point was actively moved longitudinally proved very effective on paper, but its practical feasibility was not explored. The previous studies were limited to hover or low speed fight, and therefore the aerodynamics of the suspended load did not play a significant role. Slung loads are rarely aerodynamically shaped bodies. The typical loads are bluff bodies that may be subject to dynamic instabilities triggered by unsteady aerodynamics.

Poli and Cromack [6] studied the stability in forward fight of a helicopter carrying a container and a circular cylinder. The results indicated that long cables, high speeds, and low weights increased the stability of the loads.

A stability study in forward fight by Cliff and Bailey [7] partially confirmed the results of Ref. [6] because decreasing the weight improved stability, but longer cables were found to be destabilising. The differences may be due to the different aerodynamics of the load, which was a much more idealised representation in Ref. [7]. Lowering drag increased stability. Lateral-directional and longitudinal stability was governed by the same parameters, but the conditions for lateral stability proved more stringent. Łucjanek and Sibilski [8] confirmed result from Ref. [7].

Few years later Sibilski and Łucjanek addressed the stability of a single-point suspension load configuration [9]. The analysis model was much more sophisticated than in any of the studies previously mentioned, and included full nonlinear equations for helicopter motion and 3-degress of freedom suspended load dynamics. The equations were then linearized for stability analysis, and the effect of several configuration parameters was investigated. Cable length, fore/aft and vertical position of the suspension point, and load weight were all found to affect stability. Depending on the combination of parameters some modes could be stabilised and others destabilised, but overall all the instabilities were quite weak.

Concurrently Nagabhushan in his analysis [10] addressed the low-speed stability of a single-point suspension load configuration. The analysis model included full nonlinear equations for rigid body aircraft motion and rotor flap dynamics. Results from Ref. [10] confirmed, that cable length, fore/aft and vertical position of the suspension point, and load weight affect stability of helicopter-suspended load system.

More recently, Cicolani *et al.* have reported the results of fight tests of a UH-60 helicopter [11], including frequency responses obtained using system identification techniques. While the study focused primarily on system identification and simulation validation, several conclusions were presented on the effect of the loads on fight dynamics and handling qualities. Increasing load weight reduced lateral bandwidth; further increases could reduce the bandwidth to a value below that of the pendulum frequency. Longitudinal stability margins were not very sensitive to the load, but lateral stability margins were degraded. The effect on bandwidth and phase delay was highly variable depending on the load configuration.

At last, Fusato, Guglieri, and Celi explore some fundamental aspects of the dynamics of an articulated rotor helicopter with an external load suspended from a single attachment point [12]. The results indicated that external load affects the trim state primarily through the overall increase in the weight of the aircraft, both in straight and in turning flight. The influence of the length of the cable is negligible. Substantial dynamic coupling occur between the Dutch roll mode. A load mode consists primarily of the lateral motion of the load. The effect of the load on the phugoid is very small. A suspended load modifies the on-axis roll frequency response by adding a notch to the gain curves and a roughly 180-degree jump in the phase curves. The modifications of the frequency response introduced by the load occur primarily at frequencies lower than those used to determine bandwidth and phase delay according to the ADS-33 specifications. The phase shifts cause additional crossings of the 135-degree delay line that, at least formally, can reduce the phase bandwidth considerably. If these additional crossings are ignored, the changes in bandwidth and phase delay are generally small.

A helicopter carrying a suspended load is the inherently non-linear and time varying system. Therefore linear model is adequate for basic studies of the fight dynamics of helicopter with suspended loads, but it cannot describe some important practical problems. For example, it is not possible to model using linear approach, the sling load "vertical bounce" phenomenon. Another type of problem that the linear model cannot capture is the aerodynamic instabilities due to unsteady inflow, and/or the non-streamlined shape of many suspended loads. These instabilities, described for example by Gabel and Wilson [13], Poli and Cromack [6], Sheldon [14], Cicolani *et al*. [15], and Simpson and Flower [16], often limit the maximum speed of the helicopter.

Recent development in the field of numerical analysis of non-linear equations created a class of computer algorithms known as continuation methods. Those methods use predictor-corrector techniques to follow solution curves of systems of non-linear equations of motion represented by functions of a number of variables and parameters, respectively. This approach was successfully demonstrated for a helicopter with a suspended load flight dynamics analysis.

Continuation methods are class of predictor corrector techniques for the solution of systems of non-linear algebraic equations, which are functions of a number of parameters, over a specified range of the parameters. The general technique is to fix all parameters but one and trace the steady states of the system as a function of this parameter. The stability of each steady state can be determined by calculating the eigenvalues of the linearized system. Any changes in stability from one steady state to the next will signify a bifurcation.

Dynamical system theory has provided a powerful tool for analysis of non-linear phenomena of aircraft behaviour. In the application of this theory, numerical continuation methods and bifurcation theory have been used to study roll-coupling instabilities, stall/spin phenomena, and high angles of attack dynamics of a number of aircraft models. Results of great interest have been reported in several papers (it can be mentioned papers by Jahnke and Culick [17], Carroll and Mehra [18], Guicheteau [19], Avanzini and de Matteis [20], Sibilski [21], [22], [23], Marusak, Pietrucha and Sibilski. [24]). Continuation methods are numerical techniques for calculating the steady states of systems of ordinary differential equations and can be used to study roll coupling instabilities and high-angle of attack instabilities

In the present paper, after a brief description of the methodology and associated procedures, some fundamental aspects of the nonlinear dynamics of an articulated rotor helicopter with an external suspended load are studied using continuation and bifurcation methods, by means of checking the stability characteristics related to unstable equilibria. Numerical simulations are used to verify the predictions. The mathematical model of the helicopter used in this study is a nonlinear blade element type model that includes fuselage, rotor, main rotor inflow, and propulsion system dynamics. The 6-degree of freedom rigid body motion of the aircraft, 3-degree of freedom rigid body motion of slung load, and articulated, four-bladed main rotor with rigid blades is assumed. The aerodynamic loads are unsteady forces and moments in the direction determined by the local airflow (defined by the suspended load angle of attack and slip angle). Unsteady aerodynamic effects are modelled using the ONERA dynamic inflow model [25]. The state vector has a total of 31 elements: flap and lag displacements and rates for each of the 4 blades (16 states); 9 rigid body velocities, rates, and attitudes; and 3 external load angles with their respective rates. The formalism of theoretical mechanics allows to present dynamic equations of motion of the coupled load/helicopter dynamic system in quasi-co-ordinates, giving incredibly interesting and comfortable tool for construction of equation of motion. An example can be Boltzmann-Hamel equations, which are generalisation of LaGrange equations of the second kind for quasi-co-ordinates. These equations are written in the form allowing to create procedures meant for their automatic formulation, (e.g., by means of such well known commercial software as Mathematica® or Maple V®).

The objectives of this paper are:

1. To present formulations and solutions of a mathematical model of an articulated rotor helicopter carrying an external load,
2. To study, the effects of cable length and load weight on helicopter/load system dynamic characteristics, especially in hover flight.

2 Theoretical background

2.1 Dynamical systems theory

Dynamical systems theory (DST) provides a methodology for studying systems of ordinary differential equations. The most important ideas of DST used in the paper will be introduced in the following sections. More information on DST can be found in the book of Wiggins [26].

The first step in the DST approach is to calculate the steady states of the system and their stability. Steady states can be found by setting all time derivatives equal to zero and solving the resulting set of algebraic equations. The Hartman-Grobman theorem (p. 234 in reference [26]) proves that the local stability of a steady state can be determined by linearizing the equations of motion about the steady state and calculating the eigenvalues.

The implicit function theorem (Ioos and Joseph [27], in Chap.2) proves that the steady states of a system are continuous function of the parameters of the system at all steady states where the linearized system is non-singular. A singular linearized system is characterised by a zero eigenvalue. Thus, the steady states of the equations of motion for a helicopter-suspended load system are continuous functions of the cable length and/or suspended load mass for example. Stability changes can occur as the parameters of the system are varied in such a way that the real parts of one or more eigenvalues of the linearized system change sign. Changes in the stability of a steady state lead to qualitatively different responses for the system and are called bifurcations. Stability boundaries can be determined by searching for steady states, which have one or more eigenvalues with zero real parts. There are many types of bifurcations and each has different effects on the aircraft response. Qualitative changes in the response of the helicopter - suspended load system can be predicted by determining how many and what types of eigenvalues have zero real parts at the bifurcations point. Bifurcations for which one real eigenvalue is zero lead to the creation or destruction of two or more steady states. Bifurcations for which one pair of complex eigenvalues has zero real parts can lead to the creation or destruction of periodic motion. Bifurcations for which more than one real eigenvalue or more than one pair of complex eigenvalues has zero real parts lead to very complicated behaviour.

2.2. Bifurcation Theory

For steady states of aircraft motion, very interesting phenomena appear when even if one negative real eigenvalue crosses the imaginary axis when control vector varies. Two cases can be considered.

- The steady state is regular, i.e. when the implicit function theorem works and the equilibrium curve goes through a limit point. It should be noted that a limit point is structurally stable under uncertainties of the differential system studied.
- The steady state is singular. Several equilibrium curves cross a pitchfork bifurcation point, and bifurcation point is structurally unstable.

If a pair of complex eigenvalues cross the imaginary axis, when control vector varies, Hopf bifurcation appears [28], [29]. Hopf bifurcation is another interesting bifurcation point. After crossing this point, a periodic orbit appears. Depending of the nature of nonlinearities, this bifurcation may be sub-critical or supercritical. In the first case, the stable periodic orbit appears (even for large changes of the control vector). In the second case the amplitude of the orbit grows in portion to the changes of the control vector.

2.4. Continuation technique

Continuation methods are a direct result of the implicit function theorem, which proves that the steady states of a system are continuous functions of the parameters of the system al all steady states except for steady states at which the linearized system is singular. The general technique is to fix all parameters except one and trace the steady states of system as a function of this parameter. If one steady state of the system is known, a new steady state can be approximated by linear extrapolation from the known steady state [30], [31]. The slope of the curve at the steady state can be determined by taking the derivative of the equation given by setting all time derivatives equal to zero. If two steady states are known, a new steady state can be approximated by linear extrapolation through the two known steady states. The stability of each steady state can be determined by calculating the eigenvalues of the linearized system. Any changes in stability from one steady state to the next will signify a bifurcation.

2.4 Methodology scheme

Taking into account experience of many researches, one can formulate the following tree-step methodology scheme (being based on bifurcation analysis and continuation technique) for the investigation of nonlinear aircraft behaviour:

- During the first step it is supposed that all parameters are fixed. The main aim is to search for all possible equilibria and closed orbits, and to analyze their local stability. This study should be as thorough as possible. The global structure of the state space (or *phase portrait*) can be revealed after determining the asymptotic stability regions for all discovered attractors (stable equilibria and closed orbits). An approximate graphic representation plays an important role in the treating of the calculated results.

- During the second step the system behaviour is predicted using the information about the evolution of the portrait with the parameters variations. The knowledge about the type of encountered bifurcation and current position with respect to the stability regions of other steady motions are helpful for the prediction of further motion of the aircraft. The rates of parameters variations are also important for such a forecast. The faster the parameter change, the more the difference between steady state solution and transient motion can be observed.

Last, the numerical simulation is used for checking the obtained predictions and obtaining transient characteristics of system dynamics for large amplitude state variable disturbances and parameter variations**.**

3 Mathematical model of helicopter with slung load motion

3.1. Boltzmann-Hamel equations of motion

The formalism of theoretical mechanics allows to present dynamic equations of motion of mechanical systems in quasi-co-ordinates, giving incredibly interesting and comfortable tool for construction of equation of motion of aircraft. An example can be Boltzmann-Hamel equations, which are generalisation of LaGrange equations of the second kind for quasi-co-ordinates. Boltzmann-Hamel equations have the following form [32], [33]:

$$\frac{d}{dt}\left(\frac{\partial T^*}{\partial \omega_\sigma}\right) - \frac{\partial T^*}{\partial \pi_\sigma} + \sum_{\mu=1}^{k}\sum_{\lambda=1}^{k} \gamma^{\mu}_{\sigma\lambda} \frac{\partial T^*}{\partial \omega_\mu}\omega_\lambda = Q^*_\sigma \qquad (1)$$

where: T^* – kinetic energy (function of quasi co-ordinates and quasi-co-ordinates), ω_σ – quasi-velocity, π_σ – quasi co-ordinate, q_λ, q_σ – generalized co-ordinates, $Q^*_\sigma = \sum_{\sigma=1}^{k} Q_\sigma b_{\sigma\mu}$ – a co-ordinate of generalized force vector, k – number of degree of freedom of mechanical system, $\gamma^{r}_{\mu\alpha}$ – Boltzmann symbols [32], [33]:

$$\gamma^{r}_{\mu\alpha} = \sum_{r=1}^{k}\sum_{\alpha=1}^{k}\left(\frac{\partial a_{r\sigma}}{\partial q_\lambda} - \frac{\partial a_{r\lambda}}{\partial q_\sigma}\right) b_{\sigma\mu} b_{\lambda\alpha} \qquad (2)$$

where $a_{r\sigma}, b_{r\sigma}$ - elements of transformation matrix.

Relations between quasi-velocities and generalized velocities are shown in equations:

$$\begin{aligned} \omega_\sigma &= \sum_{\alpha=1}^{k} a_{\sigma\alpha}(q_1, q_2, .., q_k)\cdot \dot{q}_\alpha, \\ \dot{q}_\sigma &= \sum_{m=1}^{k} b_{\sigma\mu}(q_1, q_2, .., q_k)\cdot \omega_\mu, \quad \sigma = 1, ..., k \end{aligned} \qquad (3)$$

Eqs. (3) can be written in the matrix form:

$$\mathbf{\Omega} = \mathbf{A}_T \dot{\mathbf{q}}, \quad \dot{\mathbf{q}} = \mathbf{A}_T^{-1}\mathbf{\Omega} = \mathbf{B}_T \mathbf{\Omega} \qquad (4)$$

where $\mathbf{\Omega}$ - vector of quasi-velocities, $\mathbf{q}$ – vector of generalized co-ordinates

$$\begin{aligned} \mathbf{\Omega} &= [\omega_1, \omega_2, ..., \omega_k]^T, \\ \mathbf{q} &= [q_1, q_2, ..., q_k]^T \end{aligned} \qquad (5)$$

The construction of matrix $\mathbf{A}_T$ depends on explored issue.

3.2. Assumed systems of co-ordinates

Non-linear equations of motion of rotorcraft-slung load system and the kinematics relations are expressed using moving co-ordinate systems. It is applied the following systems of co-ordinates:

Systems of co-ordinates attached to the aircraft, the common origin of which is located at the arbitrary accept point inside aircraft body (Fig. 1):

– A vertical moving system of co-ordinates $Ox_gy_gz_g$, the Oz_g axis of that is vertical and directed downwards;

– A system of co-ordinates *Oxyz* attached to the aircraft. The *Oxz* plane is coinciding with the symmetry plane of the aircraft and *Oz* axis is directed downwards.

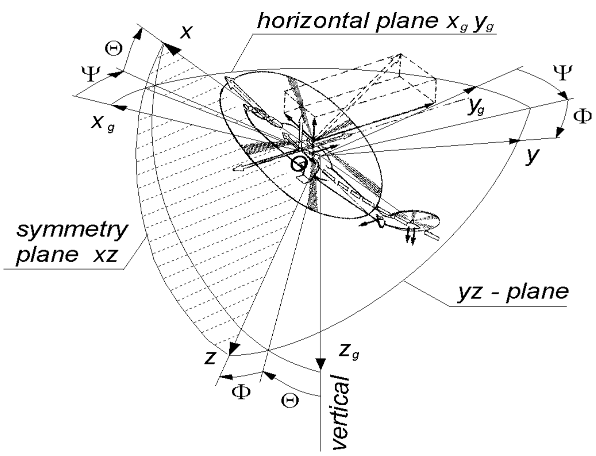

Fig. 1 System of co-ordinates attached to helicopter

Systems of co-ordinates attached to the external slung load (Fig. 2).

– A system of co-ordinates $O_4x_4y_4z_4$, the origin of system O_4 overlap a suspension point, all axis are parallel to slung load axis of inertia, Ox_4 axis of that system is directed forwards, and Oz_4 axis is directed downwards;

– A system of co-ordinates $O_5x_5y_5z_5$ attached to the suspended load. Origin of this system overlap the suspended load centre of mass The axis of those system are parallel to the axis of system of co-ordinates *Oxyz*.

– A system of co-ordinates $O_6x_6y_6z_6$ attached to the suspended load. Origin of this system overlap the suspended load centre of mass The axis of those system are overlap the slung load axis of inertia.

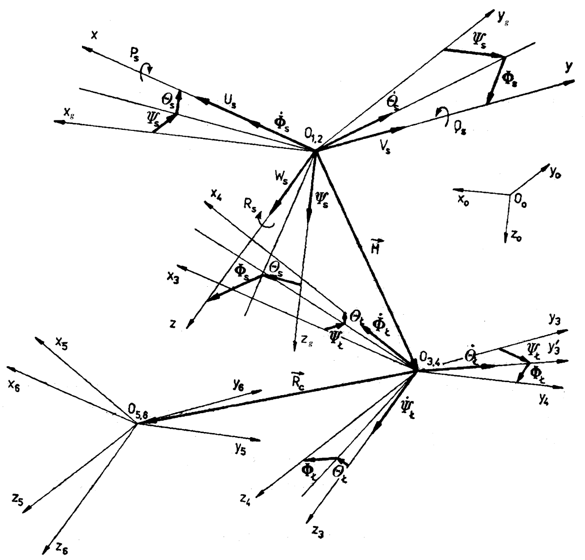

Fig. 2 Systems of co-ordinates attached to slung load

Systems of co-ordinates attached to the main rotor blades (Fig. 3).

– A system of co-ordinates $Ox_{wn}y_{wn}z_{wn}$, the origin of those system overlap a centre of main rotor hub, all axis are parallel to the system of co-ordinates *Oxyz* attached to aircraft.

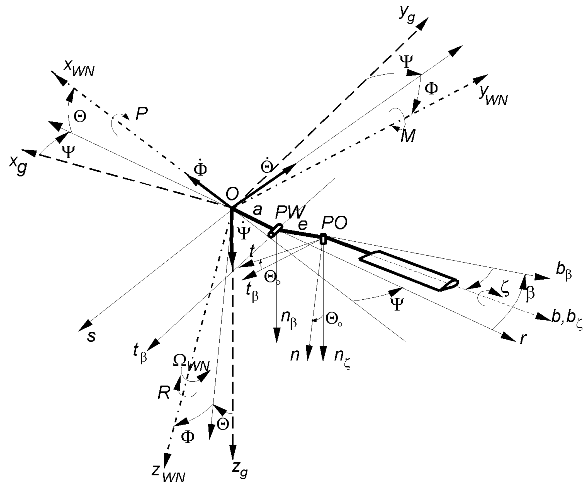

Fig. 3 Systems of co-ordinates attached to main rotor and main rotor blades

– Systems of co-ordinates of main rotor hub $Or_is_iz_{WN}$ (*i=1,2,..n; n* – number of main rotor blades) attached to the main rotor hub. Those systems turn with main rotor rate *Ω*.

– Systems of co-ordinates attached to the main rotor blades. The rotor blade is mounted to the hub on a universal join – free to flab (flapping hinge *PW*, system of co-ordinates $PWt_\beta b_\beta n_\beta$), lead or lag (lag hinge *PO*, system of co-ordinates $POt_\zeta b_\zeta n_\zeta$,), but fixed in pitch (feathering hinge, system of co-ordinates *POtbn*).

3.3. Equation of motion

Relations between quasi-velocities and generalized velocities are shown in equations:

$$\omega_\sigma = \sum_{\alpha=1}^{k} a_{\sigma\alpha}(q_1, q_2, .., q_k)\cdot \dot{q}_\alpha, \qquad \dot{q}_\sigma = \sum_{m=1}^{k} b_{\sigma\mu}(q_1, q_2, .., q_k)\cdot \omega_\mu, \quad \sigma=1,...,k \tag{6}$$

Eqs. (7) can be written in the matrix form:

$$\mathbf{\Omega} = \mathbf{A}_T\dot{\mathbf{q}}, \qquad \dot{\mathbf{q}} = \mathbf{A}_T^{-1}\mathbf{\Omega} = \mathbf{B}_T\mathbf{\Omega} \tag{7}$$

where $\mathbf{\Omega}$ - vector of quasi-velocities, $\mathbf{q}$ – vector of generalized co-ordinates

$$\mathbf{\Omega} = [\omega_1, \omega_2, ..., \omega_k]^T, \qquad \mathbf{q} = [q_1, q_2, ..., q_k]^T \tag{8}$$

The construction of matrix $\mathbf{A}_T$ depends on explored issue. In case when we consider model of a helicopter carrying suspended load treated as systems containing rigid fuselage and *n* rigid blades of the main rotor fixed to hub by means of three hinges, and three degrees of freedom hinging load, quasi-velocities and generalised co-ordinates have following forms:

$$\mathbf{\Omega} = [u, v, w, p, q, r, \Omega, \dot\beta_1, ...\dot\beta_n, \dot\zeta_1, ...\dot\zeta_n, \dot\theta_1, ...\dot\theta_n, \dot\Psi_L, \dot\Theta_L, \dot\Phi_L]^T \tag{9}$$

$$\mathbf{q} = [x_s, y_s, z_s, \Phi, \Theta, \Psi, \psi, \beta_1, ...\beta_n, \zeta_1, ...\zeta_n, \theta_1, ...\theta_n, \Psi_L, \Theta_L, \Phi_L]^T \tag{10}$$

The matrix $\mathbf{A}_T$ has a construction:

$$\mathbf{A}_T = \begin{bmatrix} \mathbf{A}_G & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_T & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix} \tag{11}$$

The matrices $\mathbf{A}_G$ and $\mathbf{C}_T$ are classical matrices of transformations of kinematics relations and can be found in Ref. [34], the unit matrix $\mathbf{I}$ has dimension (3*n*+1) x (3*n*+1), *n* – number of the main rotor blades.

Matrices $\mathbf{D}_i$ can be determine as follows

$$\mathbf{D}_i = \frac{d\mathbf{a}_i}{d\mathbf{q}} = \begin{bmatrix} \frac{\partial a_{11}}{\partial q_1} & \dots & \frac{\partial a_{1k}}{\partial q_k} \\ \dots & \dots & \dots \\ \frac{\partial a_{k1}}{\partial q_1} & \dots & \frac{\partial a_{kk}}{\partial q_k} \end{bmatrix} \quad (12)$$

where the vector $\mathbf{a}_i$ means i-th row of the matrix $\mathbf{A}_T$.

In the matrix notation the Boltzmann symbols can be presented in the form of elements of block matrix $\mathbf{\Gamma}(k \text{ x } (k \text{ x } k))$, k – number degrees of freedom desired dynamic system:

$$\mathbf{\Gamma} = \begin{bmatrix} \mathbf{\Gamma}^1 \\ \mathbf{\Gamma}^2 \\ \dots \\ \mathbf{\Gamma}^k \end{bmatrix} = \begin{bmatrix} \mathbf{B}_T{}^T \left(\mathbf{D}_1 - \mathbf{D}_1{}^T\right)\mathbf{B}_T \\ \mathbf{B}_T{}^T \left(\mathbf{D}_2 - \mathbf{D}_2{}^T\right)\mathbf{B}_T \\ \dots\dots\dots\dots \\ \mathbf{B}_T{}^T \left(\mathbf{D}_k - \mathbf{D}_k{}^T\right)\mathbf{B}_T \end{bmatrix} \quad (13)$$

The matrix $\mathbf{\Gamma}$ can be presented in the short form:

$$\mathbf{\Gamma} = \mathbf{B}_T^T \left(\mathbf{D} - \mathbf{D}^T\right)\mathbf{B}_T \quad (14)$$

Finally, Boltzmann-Hamel equations written in the matrix form can be presented as follows:

$$\frac{d}{dt}\left(\frac{\partial T^*}{\partial \mathbf{\Omega}}\right) + \left(\mathbf{\Gamma}^T \mathbf{\Omega}\right)\frac{\partial T^*}{\partial \mathbf{\Omega}} - \mathbf{B}_T^T \frac{\partial T^*}{\partial \mathbf{q}} = \mathbf{Q} - \mathbf{U} \quad (15)$$

The vector $\mathbf{Q}$ is the sum of aerodynamic loads and another non-potential forces acting on the helicopter slung load system, $\mathbf{U}$ is the vector of potential forces acting on the helicopter and external hanging load. Eq. (15) is very comfortable to use in procedures of automatic formulation of equation of motion.

In our case the subject of consideration are problems of dynamics of helicopter with sling heavy load. The quasi-velocities vector is given by Eq. (9). Total kinetic energy of the system is the sum of the kinetic energy of the rigid fuselage of helicopter, rotor blades, and slung load:

$$T^* = T_H^* + \sum_{i=1}^{n} T_{Bi}^* + T_L^* \quad (16)$$

According to the general theorem, the kinetic energy of airframe is [34]:

$$T_H^* = \frac{1}{2} m \mathbf{V}^2 + \frac{1}{2} \mathbf{\Omega}_k^T \mathbf{J}_A \mathbf{\Omega}_k \quad (17)$$

The kinetic energy i-th rotor blade is [34]:

$$T_{Bi}^* = \frac{1}{2} m_{Bi} \left[\mathbf{V} + \mathbf{\Omega}_k \times \mathbf{x}_{Bi} + \left(\mathbf{\Omega}_k + \boldsymbol{\omega}_{Bi}\right) \times \mathbf{R}_{Bi}\right]^2 + \\ + \frac{1}{2}\left(\mathbf{\Omega}_k + \boldsymbol{\omega}_{Bi}\right)^T \mathbf{J}_{Bi} \left(\mathbf{\Omega}_k + \boldsymbol{\omega}_{Bi}\right) \quad (18)$$

where: $\mathbf{x}_{Bi}$ – vector connecting centre of gravity of aircraft with centre of main rotor hub; $\mathbf{R}_{Bi}$ – vector connecting centre of main rotor hub with blade centre of gravity , $\mathbf{J}_{Bi}$ – moment of inertia of i-th rotor blade respect to flapping axis, $\boldsymbol{\omega}_{Bi}$ – vector relative angular velocity of rotor blade.

$$\boldsymbol{\omega}_{Bi} = \Omega \cdot z_{wn} + \frac{d\beta_i}{dt} \cdot \bar{r}_{\beta_i} + \frac{d\zeta_i}{dt} \cdot n_{\zeta_i} + \frac{d\theta_i}{dt} \cdot \bar{b} \quad (19$$

The kinetic energy of slung load is :

$$T_{Li}^* = \frac{1}{2} m_{Bi} \left[\mathbf{V} + \mathbf{\Omega}_k \times \mathbf{H} + \left(\mathbf{\Omega}_k + \boldsymbol{\omega}_{Li}\right) \times \mathbf{R}_{Ci}\right]^2 + \\ + \frac{1}{2}\left(\mathbf{\Omega}_k + \boldsymbol{\omega}_{\dot{S}i}\right)^T \mathbf{I}_L \left(\mathbf{\Omega}_k + \boldsymbol{\omega}_L\right) \quad (20)$$

where: $\mathbf{H}$ – vector connecting helicopter centre of gravity with point of suspension of external load; $\mathbf{R}_C$ – vector connecting point of suspension with centre of gravity of external slung load, $\mathbf{I}_L$ tensor of inertia of slung load.

After making conversions, relation for kinetic energy can be presented in the form:

$$T^* = \frac{1}{2} \mathbf{\Omega}^T \mathbf{E} \mathbf{\Omega} \quad (21)$$

The matrix $\mathbf{E}$ depends on the mass distribution of airframe and control surfaces, and has the form:

$$\mathbf{E} = \begin{bmatrix} \mathbf{M} & -\mathbf{S}_1 & \mathbf{S}_2^{(E)} \\ \mathbf{S} & \mathbf{J}_A & \mathbf{N}^{(E)} \\ \left(\mathbf{S}_2^{(E)}\right)^T & \left(\mathbf{N}^{(E)}\right)^T & \mathbf{I}_S^{(E)} \end{bmatrix} \quad (22)$$

Elements of matrixes $\mathbf{M}$, $\mathbf{N}^{(E)}$, $\mathbf{J}_A$, $\mathbf{S}_1$, $\mathbf{S}_2^{(E)}$, $\mathbf{I}_L^{(E)}$, can be found in Refs [34, 35].

After making differentiation and conversions we obtain a set of equations describing motion of an articulated rotor helicopter carrying external suspended load:

$$\mathbf{E}\dot{\mathbf{\Omega}} + \left[\left(\mathbf{\Gamma}^T \mathbf{\Omega}\right)\mathbf{E} - \mathbf{B}^T \mathbf{\Omega}^T \frac{d\mathbf{E}}{d\mathbf{q}}\right] \cdot \mathbf{\Omega} = \mathbf{Q} - \mathbf{U} \quad (22)$$

Eq (23) together with kinematic relations make non-linear set of ordinary differential equations of first kind describing the motion of a helicopter- suspended load system. Kinematic relations can be found in Ref. [9], [34], [35], [36]. These equations are written in the form allowing to create procedures meant for their automatic formulation, (e.g., by means of such well known commercial software as Mathematica® or Mathcad®).

3.4. Modelling of aerodynamic loads

Precise describing of aerodynamic forces and moments found in equations of motion is fundamental source of difficulties. In each phase of flight dynamics and aerodynamics influence each other, which disturbs the precise mathematical description of those processes. The requirements for method on aerodynamic load calculations stem both from flow environment and from algorithms used in analysis of helicopter flight. The bifurcation approach is very fruitful when the sources and nature of aerodynamic phenomena are considered. It is assumed that the airframe model consists of the fuselage, horizontal tail, vertical tail, and landing gear. The fuselage model is based on wind tunnel test data (as function of angle of attack α and slip angle β). The horizontal tail and vertical tail are treated as aerodynamic lifting surfaces with lift and drag coefficients computed from data tables as functions of angle of attack α and slip angle β. The tail rotor is linear model using strip-momentum theory with an uniformly distributed inflow. The influence of rotor dynamic inflow on airframe and suspended load aerodynamic forces and moments are included in the model. The technique used provides the essential effects of increased interference velocity with increased rotor load and decreased interference as the rotor wake deflects reward with increased forward speed [37]. On the basis of results presented in ref. [38], [39], [40] the effects of changing induced velocity due helicopter angular rates are included. Special techniques are proposed to calculate the aerodynamic forces and moments acting on external load taking into account the dynamics inflow and interference of rotor wake on slung load local inflow velocity and angles of attack and slip [35]. Aerodynamic data for a NACA 23012 airfoil in the angle of attack range +/-23^o and the compressibility effects have been included. Those data have been blended with suitable low speed data for the remainder of the 360^o range to model the reversed flow region and fully stalled retreating blades. Semi-empirical methods, that use differential equations, have been used to predict the unsteady aerodynamic loads and dynamic stall effects. The model developed by ONERA [25] has been used. The ONERA model is a semi-empirical, unsteady, non-linear model which uses experimental data to predict aerodynamic forces on an oscillating airfoil which experiences dynamic stall. State variable formulations of aerodynamic loads to allow use existing codes for rotorcraft flight simulation.

3.4. Solution technique

Now, it must be pointed up that dynamic systems theory provides a methodology for studying of complicated nonlinear ordinary differential equations (ODE). In addition, recent development in the area of numerical analysis of non-linear equations created a class of computer algorithms known as continuation method [30]. The set of ordinary differential equations described helicopter-slung load system can was solved using the continuation and bifurcation software AUTO97 (available at Internet address: ftp://ftp.cs.concordia.ca/pub/doedel/auto).

This very useful freeware gives all desired bifurcation points for different values of dynamical system parameters (in our case cable length or suspended load mass).

4. Results

The helicopter configuration selected for this study is representative of a PZL W-3 "Sokół". The weight without the external load is 5000 kg, corresponding to a C_T/σ = 0:065. The altitude for all the results is 100 m. Most results refer to combinations of cable lengths from range 0<l<55 m, and load masses from range 0<m<1500 kg. The external load is assumed as container 2m x 2m x 5m. The present results are for a bare airframe configuration.

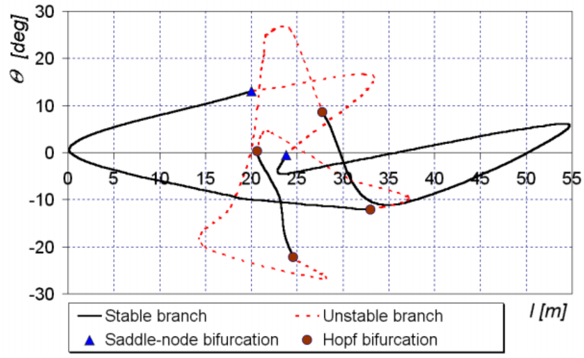

Fig. 4 Steady states for the hovering helicopter carrying the suspended load

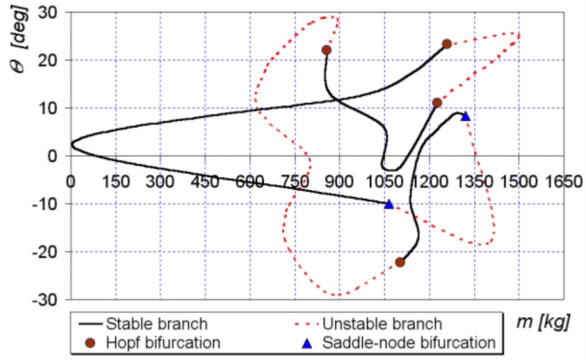

Fig. 5 Steady states for the hovering helicopter carrying the suspended load

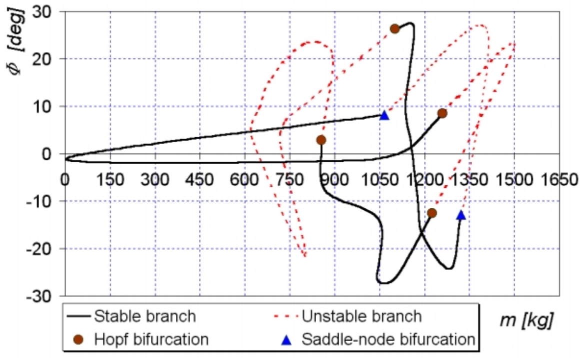

Fig. 6 Steady states for the hovering helicopter carrying the suspended load

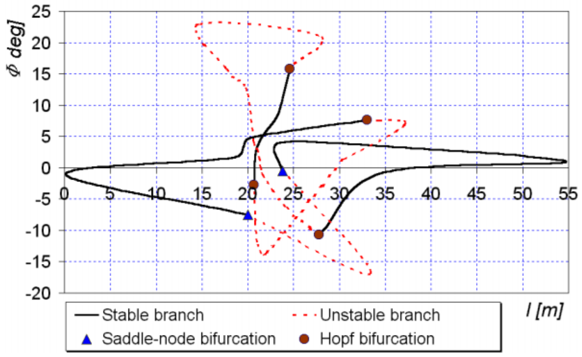

Fig. 7 Steady states for the hovering helicopter carrying the suspended load

Figures 4-7 show the steady states of the PZL "Sokół" helicopter - suspended load system as a function of a cable length and a suspended load mass. Those figures show, that multiple steady states exists for most cable lengths and suspended load masses. For example, a vertical line representing 15 m of cable length intersects four steady states. Two of them are stable the others are unstable, so the helicopter could exhibit any of these four steady states. The segment of unstable steady states containing the trim conditions from range of cable length 20 m $<l<$ 32 m, and slung load mass 850 kg $<m_L<$ 1500 kg, because of six saddle-node or Hopf bifurcations, that occur at 20 m, 20.5, 24, 25, 28, and 38 m of cable lengths. The cable lengths and slung load masses from those ranges can be assumed as unsafe and dangerous slung load configurations.

Figs. 8-17 show the time simulation of the helicopter-suspended load system motion in which the cable length is assumed l=20 m, and suspended load mass m_L=1050kg. The values of parameters assumed in that simulation put the helicopter-suspended load system in region of unstable steady states. These figures show rapidly developing aircraft oscillations. The lung load oscillations grow slower

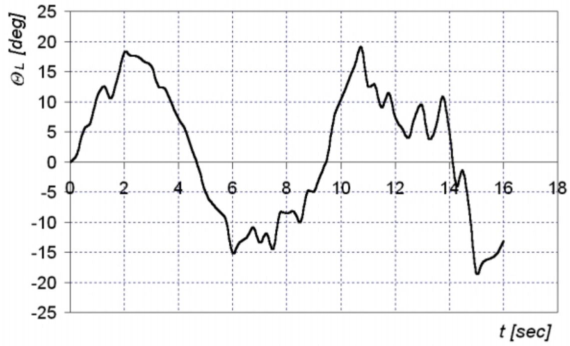

Fig. 8 Course of slung load longitudinal deflection

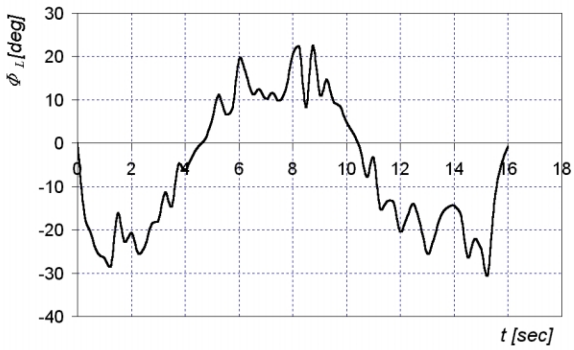

Fig. 9 Course of slung load lateral deflection

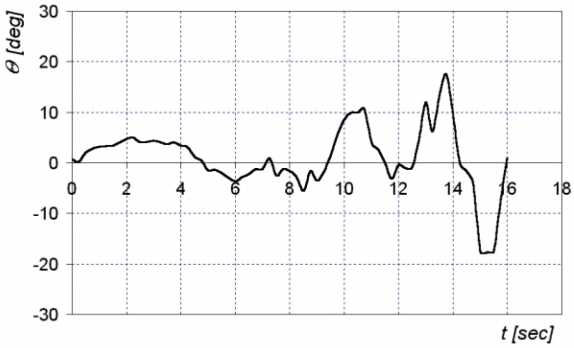

Fig. 10 Course of airframe pitch angle

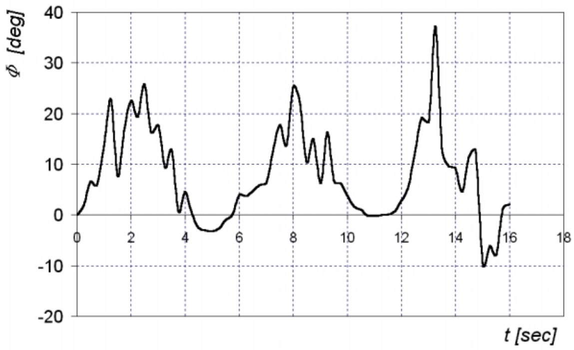

Fig. 11 Course of airframe roll angle

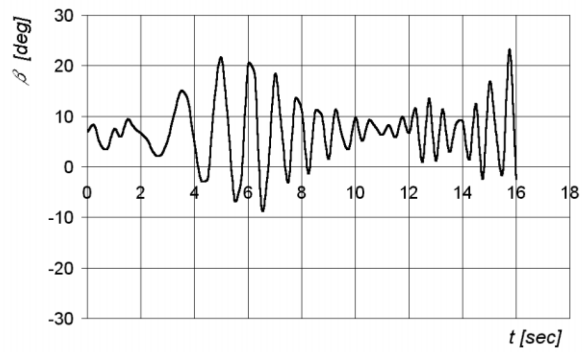

Fig. 12 Course of main rotor blade flapping angle

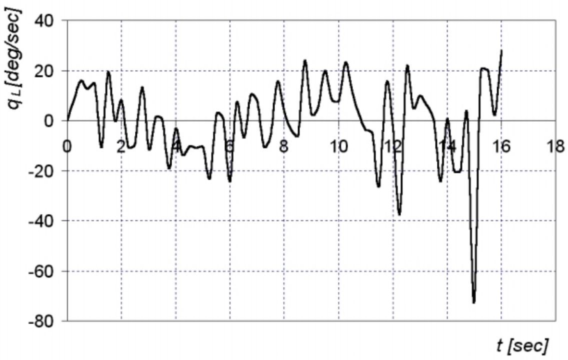

Fig. 13 Course of slung load longitudinal deflection rate

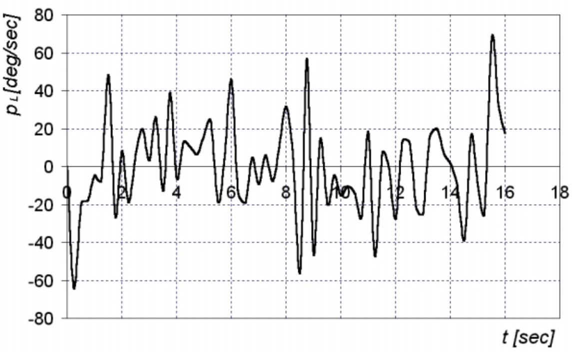

Fig. 14 Course of slung load lateral deflection rate

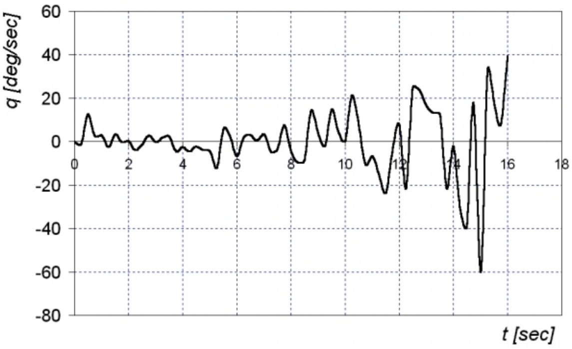

Fig. 15 Course of airframe pitch rate

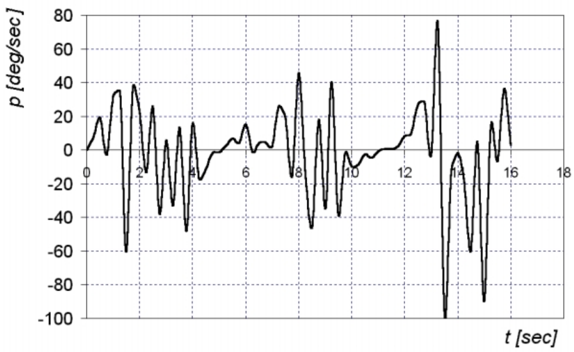

Fig. 16 Course of airframe roll rate

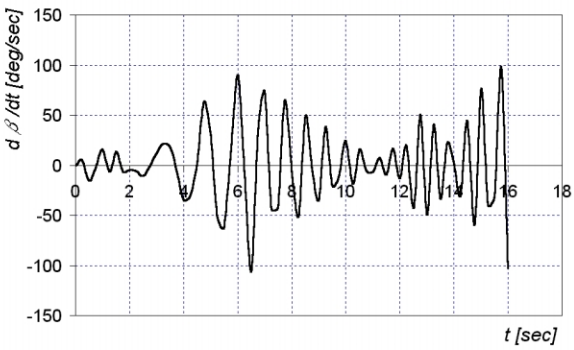

Fig. 17 Course of main rotor blade flapping rate

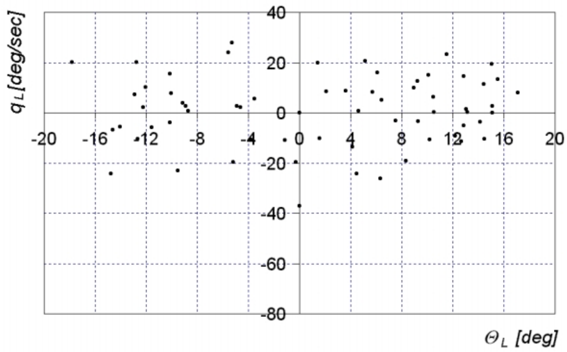

Fig. 18 Slung load longitudinal motion – Poincare map

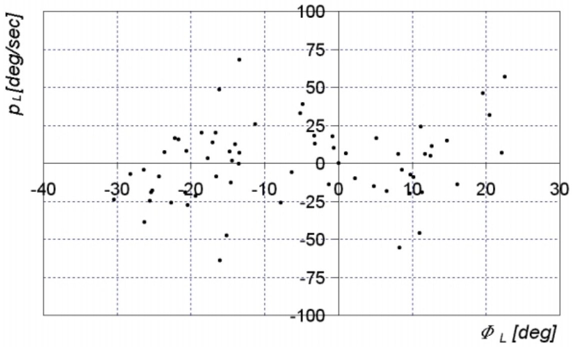

Fig. 19 Slung load lateral motion – Poincare map

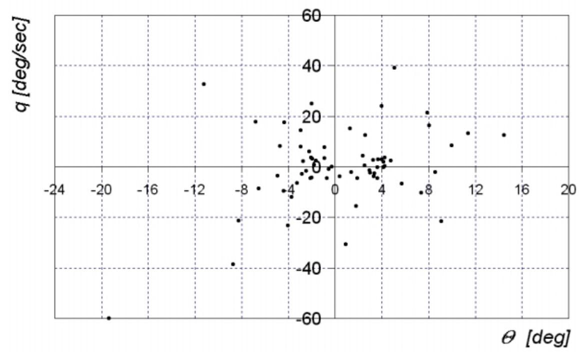

Fig. 20 Airframe phugoid motion – Poincare map

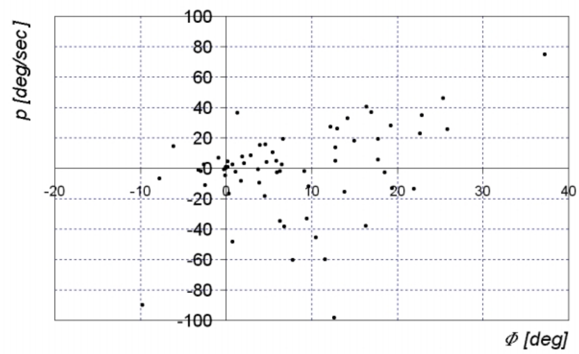

Fig. 21 Airframe Dutch roll – Poincare map

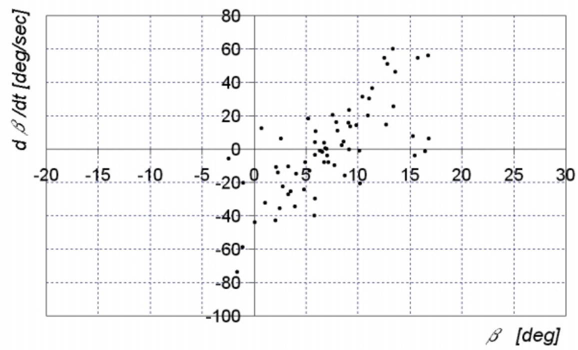

Fig. 22 Main rotor blade flapping motion – Poincare map

The results indicated that external load affects the helicopter motion. Substantial dynamic coupling occur between the Dutch roll mode. A load mode consists of the lateral and longitudinal motion of the load. The effect of the load on the phugoid is rather small. Magnitude and frequency of those oscillations are irregular and have chaotic character.

Figs. 18-22 show the Poincare maps of selected state parameters. It can be stated that taking into consideration unsteady rotor-blade aerodynamic model and hysteresis of aerodynamic coefficients, one counters significant irregularities in solution of equations of motion, that are characteristic for chaotic motion. When the condition for the onset of chaotic motion is satisfied, flapping, pitching and rolling motions appear to have chaotic oscillations. Se results obtained by Tang and Dowell, confirmed, that unsteady aerodynamics, including deep stall phenomena together with a strong non-linear rotorcraft model can lead to a chaotic response of the system [41].

5. Summary and Conclusions

The paper presented a study of the fight dynamics of an articulated rotor helicopter carrying a suspended load. The aircraft model included rigid body dynamics, individual flap and lag dynamics of each blade, and inflow dynamics. The external load was modelled, as a 3-degrees of freedom pendulum suspended from a single point. The aerodynamic load was an unsteady force in the direction determined by the local airflow (defined by angle of attack and slip angle of slung load). The main aim of the study was to apply modern methods of investigation ODE for prediction of critical configurations of helicopter - external slung load system. Based on the investigation described above, the following conclusions can be drawn:

1. The present results show the value of using continuation and bifurcation methods for analyzing the equations of rotorcraft carrying an external slung load motion;
2. The efficiency of the methods makes it possible to analyse complicated aerodynamic models using the complete equations of motions for the whole range of system parameters;
3. Knowledge of such configurations of helicopter-slung load system, which cause bifurcation, allows us to select the unsafe configurations of mass of hanging load and cable length.
4. The need for a precise description of aerodynamic loads is a fundamental cause of difficulties.
5. Substantial dynamic coupling can occur between the Dutch roll mode and a load motion that consists primarily of the lateral motion of the load. Because of this coupling, the Dutch roll damping can decrease with a consequent deterioration of handling qualities.

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